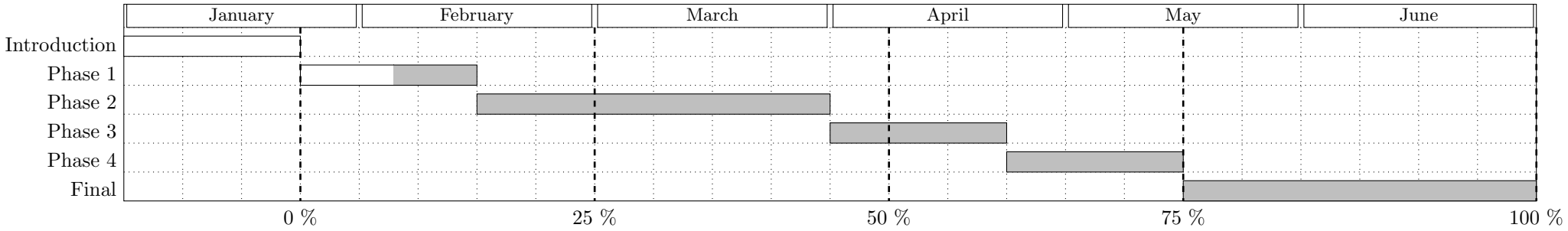
Master Plan - Isak Hammer

Solving Cahn-Hilliard Equation using CutCIP

Version: February 3, 2023

	Phase 1	**Phase 2**	**Phase 3**	**Phase 4**	**Report**
Estimated time	2-3 Weeks	4-5 Weeks	2 Weeks	3 Weeks	
Problem	CutDG for $-\Delta u = f$	CutCIP for $\Delta^2 u = f$	CutCIP for $\partial_t u + \Delta^2 u = g$	CutCIP for $\partial_t u + \Delta^2 u + f(u) = g$	
Goals	• Analysis ☐ Coercivity ☐ Boundedness ☐ Constructing g_h based on assumptions. • Implementation ☐ Poisson CutDG ☐ L^2 convergence ☐ H^1 convergence	• Analysis ☐ Coercivity ☐ Boundedness ☐ A priori estimates ☐ Condition number † • Implementation ☐ First plot ☐ L^2 convergence ☐ H^1 convergence	• Analysis ☐ BDF analysis • Implementation ☐ First plot ☐ L^2L^2 convergence ☐ L^2H^1 convergence	• Implementation ☐ Fixed point method ☐ L^2L^2 convergence ☐ L^2H^1 convergence	☐ Introduction ☐ CutDG $-\Delta u = f$ ☐ Construction of CutDG ☐ Well-posedness ☐ Numerical experiments ☐ CutCIP for $\Delta^2 u = f$ ☐ Weak form in H^4 ☐ Construction of CutCIP ☐ Well-posedness ☐ A priori estimates ☐ Numerical experiments ☐ CutCIP for $\partial_t u + \Delta^2 u = g$ ☐ Time discretization ☐ Numerical experiments ☐ CutCIP for $\partial_t u + \Delta^2 u + f(u) = g$ ☐ Fixed point methods ☐ Numerical experiments ☐ Conclusion
Comments	Mostly based on (Gürkan and Massing, 2019)	† Not prioritized			
Digression		2nd order mixed formulation	2nd order mixed formulation	Solve $\partial_t u + \kappa(u)\Delta^2 u = g$	



What have I done this week?

- Kickstarted on the report. Got quite alot of mathematical background done.
- Getting quite close to check out coercivity, boundedness
- Implementation of Poisson for DG and CutDG.
  - **Only convergence for $k = 1$ for CutPoisson?**

What am I planning to do next week?

According to plan Should I be finished with CutDG Poissoin withing the end of next week.

- Implementation of Poisson CutDG.
- Check out all boxes in analysis.
- Finish well-posedness proof.
- Seems like I underestimated the work to prove ghost penalties and how to apply the underlying assumptions. Should I focus on face-based ghost penalties, projection based penalties or both?

Other

1) Easter 5.-10. April

References

Gürkan, Ceren and André Massing (2019). "A stabilized cut discontinuous Galerkin framework for elliptic boundary value and interface problems". In: *Computer Methods in Applied Mechanics and Engineering* 348, pp. 466–499.