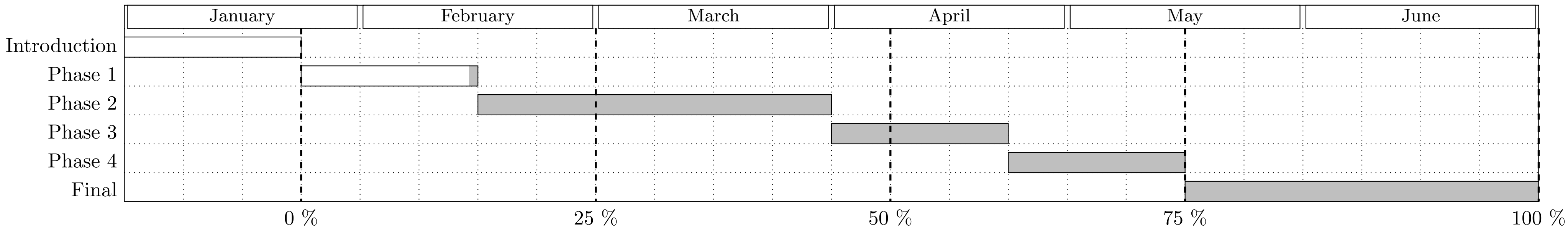
Master Plan - Isak Hammer

Solving Cahn-Hilliard Equation using CutCIP

Version: February 14, 2023

	Phase 1	**Phase 2**	**Phase 3**	**Phase 4**	**Report**
Estimated time	2-3 Weeks	4-5 Weeks	2 Weeks	3 Weeks	
Problem	CutDG for $-\Delta u = f$	CutCIP for $\Delta^2 u = f$	CutCIP for $\partial_t u + \Delta^2 u = g$	CutCIP for $\partial_t u + \Delta^2 u + f(u) = g$	Progress in report
Goals	• Analysis ☑ Coercivity ☑ Boundedness ☑ A priori estimates ☑ Condition number ☑ Constructing face based g_h • Implementation ☑ Poisson CutDG ☑ L^2 convergence ☑ H^1 convergence ☑ $a_h, *$ convergence ☑ Implement $\mathcal{P}^2$ elements	• Analysis ☐ Initial problem setup ☐ Coercivity ☐ Boundedness ☐ A priori estimates ☐ Condition number † • Implementation ☐ First plot ☐ L^2 convergence ☐ H^1 convergence	• Analysis ☐ BDF analysis • Implementation ☐ First plot ☐ L^2L^2 convergence ☐ L^2H^1 convergence	• Implementation ☐ Fixed point method ☐ L^2L^2 convergence ☐ L^2H^1 convergence	☐ Introduction ☐ Mathematical background ☐ CutDG $-\Delta u = f$ ☑ Bounded and coercive ☐ A priori estimates ☐ Condition number ☐ Constructing g_h ☐ Numerical experiments ☐ CutCIP for $\Delta^2 u = f$ ☐ Weak form in H^4 ☐ Construction of CutCIP ☐ Well-posedness ☐ A priori estimates ☐ Numerical experiments ☐ CutCIP for $\partial_t u + \Delta^2 u = g$ ☐ Time discretization ☐ Numerical experiments ☐ CutCIP for $\partial_t u + \Delta^2 u + f(u) = g$ ☐ Fixed point methods ☐ Numerical experiments ☐ Conclusion
Comments	Mostly based on (Gürkan and Massing, 2019)	† Not prioritized			• Marked done **only** if it is 95% done. • Page counter: 21
Digression		Aggregated FEM (Badia, Verdugo, and Martín, 2018) for $\mathcal{P}^k$, $k = 1, 2, 3$		Solve $\partial_t u + \kappa(u)\Delta^2 u = g$	



What have I done this week?

- Very productive since last time. Mostly been busy verifying every single sentence in Gürkan and Massing, 2019. There is some minor details here and there, but I agree very much with the proofs.
- Here is some numerical results.

What am I planning to do next week?

- Seems like the next 5 weeks will be the most intense part of my master thesis.
- Start easy with coercivity and boundedness for Biharmonic Equation.

Other

1) Maybe exam in mid of April?
2) Easter 5.-10. April

h/L	$\|e\|_{L^2}$	**EOC**	$\|e\|_{H^1}$	**EOC**	$\|e\|_{a_h,*}$	**EOC**
$\frac{1}{4}$	1.073E-01		8.184E-01		1.395E+00	
$\frac{1}{8}$	2.909E-02	1.883E+00	4.184E-01	9.680E-01	6.186E-01	1.173E+00
$\frac{1}{16}$	6.795E-03	2.098E+00	2.162E-01	9.524E-01	2.959E-01	1.064E+00
$\frac{1}{32}$	1.620E-03	2.069E+00	1.089E-01	9.896E-01	1.414E-01	1.066E+00
$\frac{1}{64}$	3.928E-04	2.044E+00	5.453E-02	9.978E-01	6.871E-02	1.041E+00

Table 1: Order 1

h/L	$\|e\|_{L^2}$	**EOC**	$\|e\|_{H^1}$	**EOC**	$\|e\|_{a_h,*}$	**EOC**
$\frac{1}{4}$	5.910E-03		7.007E-02		1.697E-01	
$\frac{1}{8}$	8.971E-04	2.720E+00	1.876E-02	1.901E+00	4.505E-02	1.914E+00
$\frac{1}{16}$	9.344E-05	3.263E+00	4.314E-03	2.121E+00	9.251E-03	2.284E+00
$\frac{1}{32}$	9.717E-06	3.265E+00	1.013E-03	2.091E+00	2.088E-03	2.147E+00
$\frac{1}{64}$	1.113E-06	3.126E+00	2.453E-04	2.046E+00	4.989E-04	2.065E+00

Table 2: Order 2