Camera Matrix

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} f_x & & c_x \\ & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Point location in camera coords

2d location:

$\left(\frac{u}{w}, \frac{v}{w}\right)$ w is z distance from camera

Suppose we are trying to find the camera ray from a pixel $\left(\frac{u}{w}, \frac{v}{w}\right)$... Divide eqn by w, let $u_0 = \frac{u}{w}$, $v_0 = \frac{v}{w}$

$$\begin{bmatrix} u_0 \\ v_0 \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & & c_x \\ & f_y & c_y \\ & & 1 \end{bmatrix} \begin{bmatrix} x/w \\ y/w \\ z/w \end{bmatrix}$$

$z/w = 1$ since $z = w$

$$\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & & c_x \\ & f_y & c_y \end{bmatrix}}_{\text{SVD into } USV^{+}} \begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} s_1 & & 0 \\ & s_2 & 0 \end{bmatrix} \begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix}^{+} \begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix} \quad \text{let } V^{+} \begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix} =$$

$$\underbrace{U^{+} \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}}_{\text{define as } \begin{bmatrix} q \\ r \end{bmatrix}} = \begin{bmatrix} s_1 & & 0 \\ & s_2 & 0 \end{bmatrix} \begin{bmatrix} V_1^{+} \\ V_2^{+} \\ V_3^{+} \end{bmatrix} \begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix}$$

Then $q = s_1 V_1^{+} \begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix}$, $r = s_2 V_2^{+} \begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix}$

Or $V_1 \cdot \begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix} = \frac{q}{s_1}$ and $V_2 \cdot \begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix} = \frac{r}{s_2}$

Or $\begin{bmatrix} V_{11} & V_{21} & \\ V_{12} & V_{22} & . \end{bmatrix} \begin{bmatrix} x/w \\ y/w \end{bmatrix} = \begin{bmatrix} q/s_1 - V_{31} \\ r/s_2 - V_{32} \end{bmatrix}$

So end up with

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \end{bmatrix}^{-1} \begin{bmatrix} q/s_1 - v_{31} \\ r/s_2 - v_{32} \end{bmatrix}$$

where $\frac{x}{w} = c_1$, $\frac{y}{w} = c_2$ where $w = z$

Ray leaving $(x, y, z) = (0,0,0)$

vector $(c_1, c_2, 1)$

Derivative w.r.t u_0:

From U^t, evaluate $\frac{\partial q}{\partial u_0}$ and $\frac{\partial r}{\partial u_0}$

$$\frac{\partial}{\partial u_0} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{s_1} \frac{\partial q}{\partial u_0} \\ \frac{1}{s_2} \frac{\partial r}{\partial u_0} \end{bmatrix}$$

Camera - to - object transform A (4×4)

$A \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow$ transform focal point

$A \begin{bmatrix} c_1 \\ c_2 \\ 1 \\ 0 \end{bmatrix} \rightarrow$ transform vector

then ray - to - polygon intersection