Lab 9

Topics:

In this lab we try to cover the implementation of the following:

- Alpha-Beta Pruning
- Minimax Algorithm

Questions:

1. Implement Minimax Algo in python.(pseudocode is given in chapter slides as well as in this manual below)

2. Implement alpha beta pruning in python. .(pseudocode is given in chapter slides as well as in this manual below)

Alpha Beta pruning of Minimax Algorithm:

Alpha-Beta pruning is not actually a new algorithm, rather an optimization technique for minimax algorithm. It reduces the computation time by a huge factor. This allows us to search much faster and even go into deeper levels in the game tree. It cuts off branches in the game tree which need not be searched because there already exists a better move available. It is called Alpha-Beta pruning because it passes 2 extra parameters in the minimax function, namely alpha and beta.

Let’s define the parameters alpha and beta.
Alpha is the best value that the **maximizer** currently can guarantee at that level or above.
Beta is the best value that the **minimizer** currently can guarantee at that level or above.

Pseudocode :

```
function minimax(node, depth, isMaximizingPlayer, alpha, beta):

    if node is a leaf node :
        return value of the node

    if isMaximizingPlayer :
        bestVal =-INFINITY
```

```
        for each child node :
            value = minimax(node, depth+1, false, alpha, beta)
            bestVal = max( bestVal, value)
            alpha = max( alpha, bestVal)
            if beta <= alpha:
                break
        return bestVal

    else :
        bestVal = +INFINITY
        for each child node :
            value = minimax(node, depth+1, true, alpha, beta)
            bestVal = min( bestVal, value)
            beta = min( beta, bestVal)
            if beta <= alpha:
                break
        return bestVal
// Calling the function for the first time.
minimax(0, 0, true, -INFINITY, +INFINITY)
```

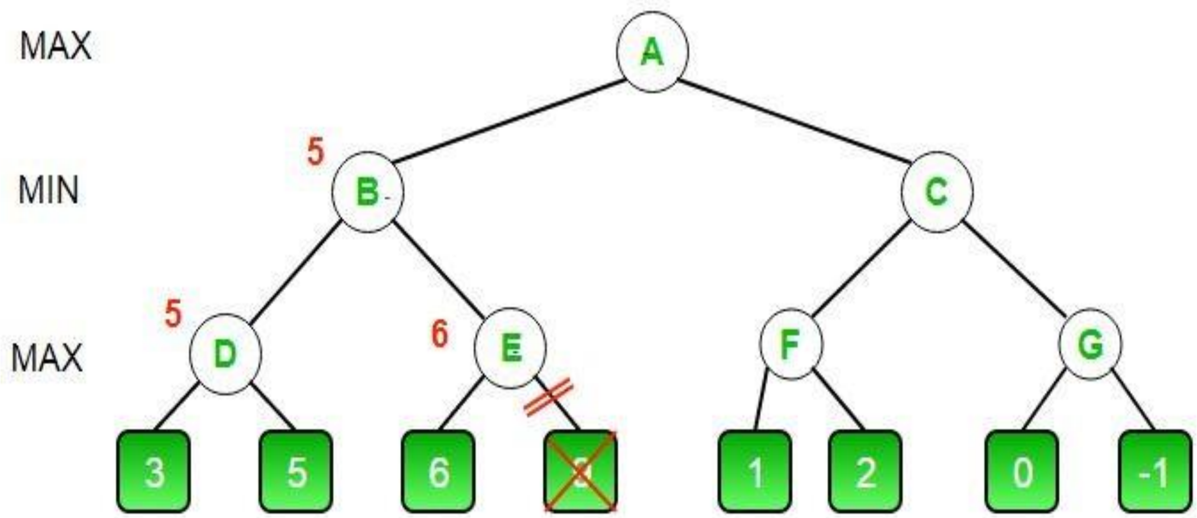
Let's make above algorithm clear with an example.

- The initial call starts from **A**. The value of alpha here is **-INFINITY** and the value of beta is **+INFINITY**. These values are passed down to subsequent nodes in the tree. At **A** the maximizer must choose max of **B** and **C**, so **A** calls **B** first
- At **B** it the minimizer must choose min of **D** and **E** and hence calls **D** first.
- At **D**, it looks at its left child which is a leaf node. This node returns a value of 3. Now the value of alpha at **D** is max(-INF, 3) which is 3.
- To decide whether its worth looking at its right node or not, it checks the condition beta<=alpha. This is false since beta = +INF and alpha = 3. So it continues the search.
- **D** now looks at its right child which returns a value of 5.At **D**, alpha = max(3, 5) which is 5. Now the value of node **D** is 5
- **D** returns a value of 5 to **B**. At **B**, beta = min(+INF, 5) which is 5. The minimizer is now guaranteed a value of 5 or lesser. **B** now calls **E** to see if he can get a lower value than 5.
- At **E** the values of alpha and beta is not -INF and +INF but instead -INF and 5 respectively, because the value of beta was changed at **B** and that is what **B** passed down to **E**
- Now **E** looks at its left child which is 6. At **E**, alpha = max(-INF, 6) which is 6. Here the condition becomes true. beta is 5 and alpha is 6. So beta<=alpha is true. Hence it breaks and **E** returns 6 to **B**
- Note how it did not matter what the value of **E**'s right child is. It could have been +INF or -INF, it still wouldn't matter, We never even had to look at it because the minimizer was guaranteed a value of 5 or lesser. So as soon as the maximizer saw the 6 he knew the minimizer would never come this way because he can get a 5

on the left side of **B**. This way we dint have to look at that 9 and hence saved computation time.

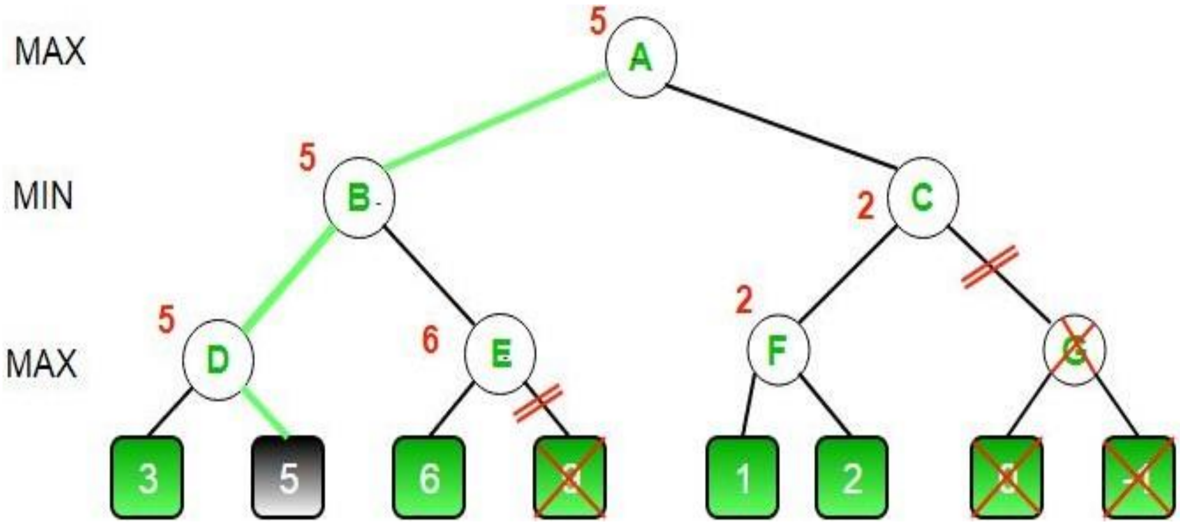
- **E** returns a value of 6 to **B**. At **B**, beta = min(5, 6) which is 5.The value of node **B** is also 5

So far this is how our game tree looks. The 9 is crossed out because it was never computed.



- **B** returns 5 to **A**. At **A**, alpha = max(-INF, 5) which is 5. Now the maximizer is guaranteed a value of 5 or greater. **A** now calls **C** to see if it can get a higher value than 5.
- At **C**, alpha = 5 and beta = +INF. **C** calls **F**
- At **F**, alpha = 5 and beta = +INF. **F** looks at its left child which is a 1. alpha = max(5, 1) which is still 5.
- **F** looks at its right child which is a 2. Hence the best value of this node is 2. Alpha still remains 5
- **F** returns a value of 2 to **C**. At **C**, beta = min(+INF, 2). The condition beta <= alpha becomes true as beta = 2 and alpha = 5. So it breaks and it does not even have to compute the entire sub-tree of **G**.
- The intuition behind this break off is that, at **C** the minimizer was guaranteed a value of 2 or lesser. But the maximizer was already guaranteed a value of 5 if he choose **B**. So why would the maximizer ever choose **C** and get a value less than 2
  ? Again you can see that it did not matter what those last 2 values were. We also saved a lot of computation by skipping a whole sub tree.
- **C** now returns a value of 2 to **A**. Therefore the best value at **A** is max(5, 2) which is a 5.
- Hence the optimal value that the maximizer can get is 5

This is how our final game tree looks like. As you can see **G** has been crossed out as it was never computed.


MAX
MIN
MAX
5
A
5
B
2
C
5
D
6
E
2
F
G
3
5
6
1
2

Mini-Max Algorithm in Artificial Intelligence

- Mini-max algorithm is a recursive or backtracking algorithm which is used in decision-making and game theory. It provides an optimal move for the player assuming that opponent is also playing optimally.
- Mini-Max algorithm uses recursion to search through the game-tree.
- Min-Max algorithm is mostly used for game playing in AI. Such as Chess, Checkers, tic-tac-toe, go, and various tow-players game. This Algorithm computes the minimax decision for the current state.
- In this algorithm two players play the game, one is called MAX and other is called MIN.
- Both the players fight it as the opponent player gets the minimum benefit while they get the maximum benefit.
- Both Players of the game are opponent of each other, where MAX will select the maximized value and MIN will select the minimized value.
- The minimax algorithm performs a depth-first search algorithm for the exploration of the complete game tree.
- The minimax algorithm proceeds all the way down to the terminal node of the tree, then backtrack the tree as the recursion.

Pseudo-code for MinMax Algorithm:

```
function minimax(node, depth, maximizingPlayer) is
if depth ==0 or node is a terminal node then
return static evaluation of node 4.
if MaximizingPlayer then                // for Maximizer Player
maxEva= -infinity
 for each child of node do
 eva= minimax(child, depth-1, false)
maxEva= max(maxEva,eva)                 //gives Maximum of the values
return maxEva 11.
else                                    // for Minimizer player
 minEva= +infinity
 for each child of node do
 eva= minimax(child, depth-1, true)
 minEva= min(minEva, eva)               //gives minimum of the values
```

17. **return** minEva

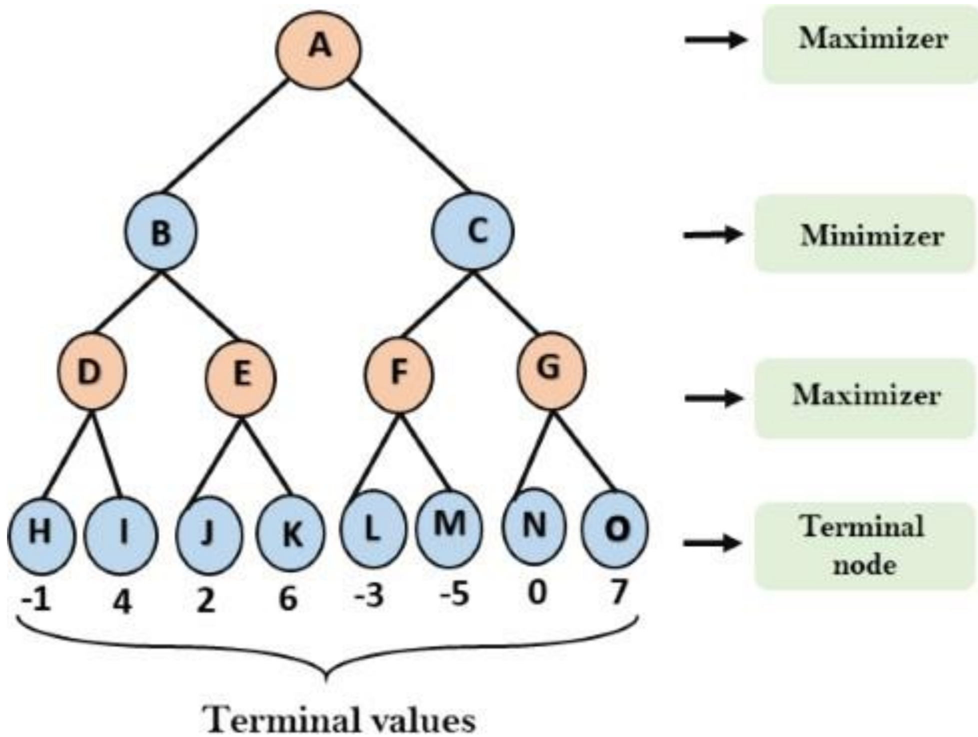
Initial call:

Minimax(node, 3, true)

Working of Min-Max Algorithm:

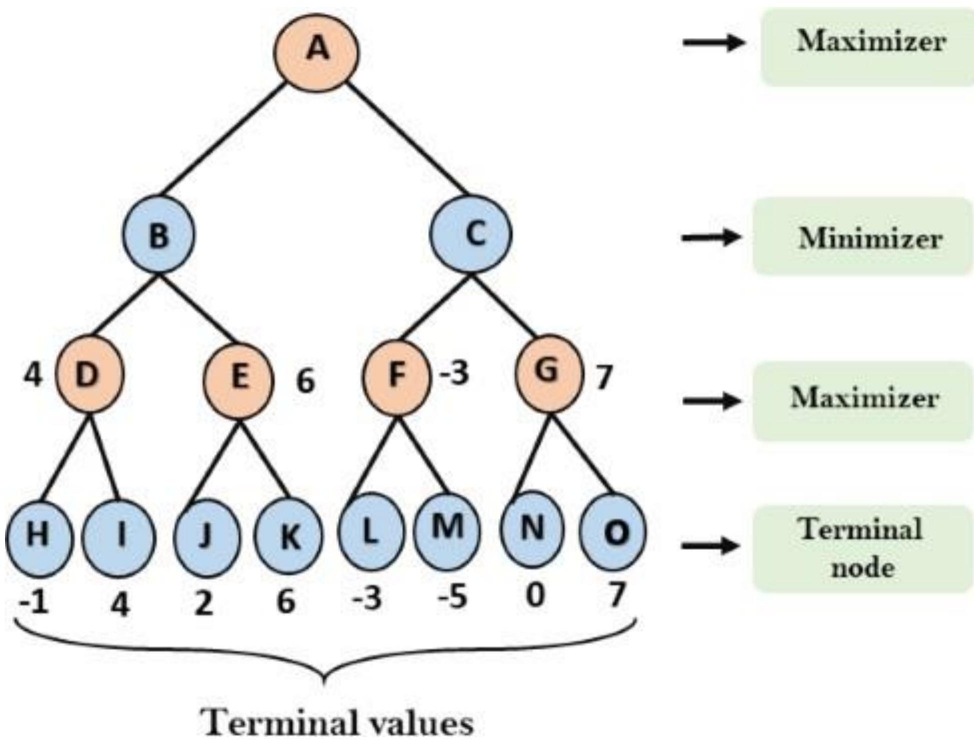
- The working of the minimax algorithm can be easily described using an example. Below we have taken an example of game-tree which is representing the two- player game.
- In this example, there are two players one is called Maximizer and other is called Minimizer.
- Maximizer will try to get the Maximum possible score, and Minimizer will try to get the minimum possible score.
- This algorithm applies DFS, so in this game-tree, we have to go all the way through the leaves to reach the terminal nodes.
- At the terminal node, the terminal values are given so we will compare those value and backtrack the tree until the initial state occurs. Following are the main steps involved in solving the two-player game tree:

Step-1: In the first step, the algorithm generates the entire game-tree and apply the utility function to get the utility values for the terminal states. In the below tree diagram, let's take A is the initial state of the tree. Suppose maximizer takes first turn which has worst-case initial value =- infinity, and minimizer will take next turn which has worst- case initial value = +infinity.



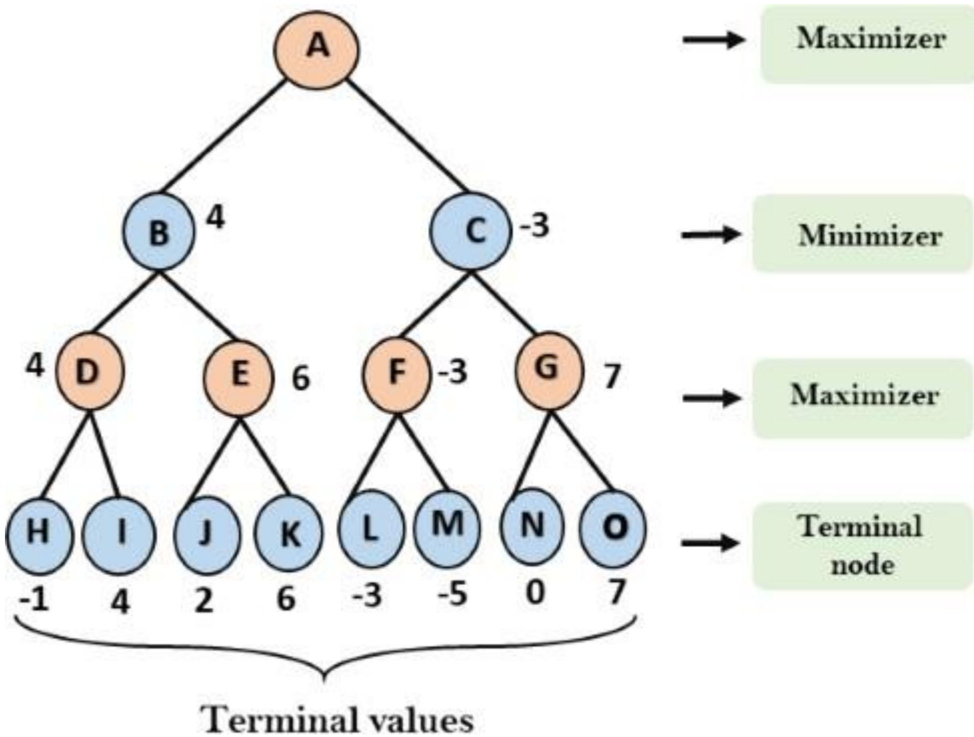
Step 2: Now, first we find the utilities value for the Maximizer, its initial value is -∞, so we will compare each value in terminal state with initial value of Maximizer and determines the higher nodes values. It will find the maximum among the all.

- For node D max(-1,- -∞) => max(-1,4)= 4
- For Node E max(2, -∞) => max(2, 6)= 6
- For Node F max(-3, -∞) => max(-3,-5) = -3
- For node G max(0, -∞) = max(0, 7) = 7



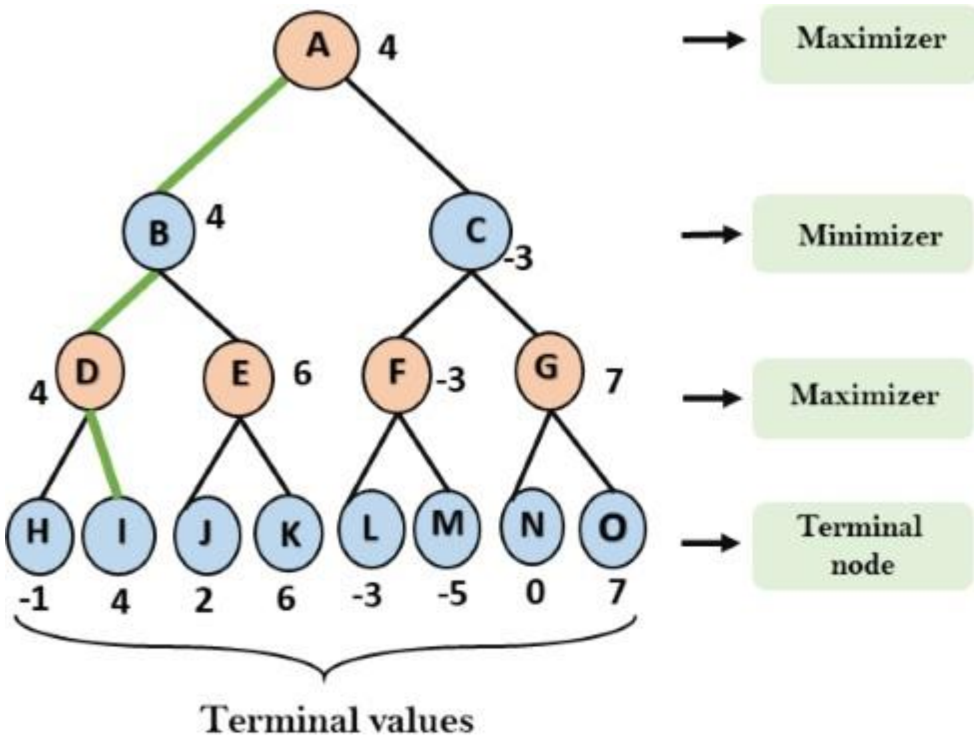
Step 3: In the next step, it's a turn for minimizer, so it will compare all nodes value with +∞, and will find the 3rd layer node values.

- For node B= min(4,6) = 4
- For node C= min (-3, 7) = -3



Step 3: Now it's a turn for Maximizer, and it will again choose the maximum of all nodes value and find the maximum value for the root node. In this game tree, there are only 4 layers, hence we reach immediately to the root node, but in real games, there will be more than 4 layers.

- For node A max(4, -3)= 4



That was the complete workflow of the minimax two player game.

Properties of Mini-Max algorithm:

- **Complete-** Min-Max algorithm is Complete. It will definitely find a solution (if exist), in the finite search tree.
- **Optimal-** Min-Max algorithm is optimal if both opponents are playing optimally.
- **Time complexity-** As it performs DFS for the game-tree, so the time complexity of Min-Max algorithm is **$O(b^m)$**, where b is branching factor of the game-tree, and m is the maximum depth of the tree.
- **Space Complexity-** Space complexity of Mini-max algorithm is also similar to DFS which is **O(bm)**.

Limitation of the minimax Algorithm:

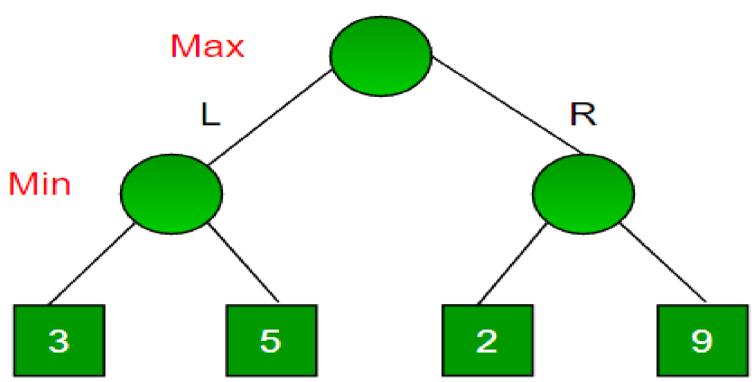
The main drawback of the minimax algorithm is that it gets really slow for complex games such as Chess, go, etc. This type of games has a huge branching factor, and the player has lots of choices to decide. This limitation of the minimax algorithm can be improved
from **alpha-beta pruning** which we have discussed in the next topic.

2nd Version

Minimax Algorithm

Minimax is a kind of backtracking algorithm that is used in decision making and game theory to find the optimal move for a player, assuming that your opponent also plays optimally. It is widely used in two player turn-based games such as Tic-Tac-Toe, Backgammon, Mancala, Chess, etc.
In Minimax the two players are called maximizer and minimizer. The **maximizer** tries to get the highest score possible while the **minimizer** tries to do the opposite and get the lowest score possible.
Every board state has a value associated with it. In a given state if the maximizer has upper hand then, the score of the board will tend to be some positive value. If the minimizer has the upper hand in that board state then it will tend to be some negative value. The values of the board are calculated by some heuristics which are unique for every type of game.

Example:
Consider a game which has 4 final states and paths to reach final state are from root to 4 leaves of a perfect binary tree as shown below. Assume you are the maximizing player and you get the first chance to move, i.e., you are at the root and your opponent at next level. **Which move you would make as a maximizing player considering that your opponent also plays optimally?**

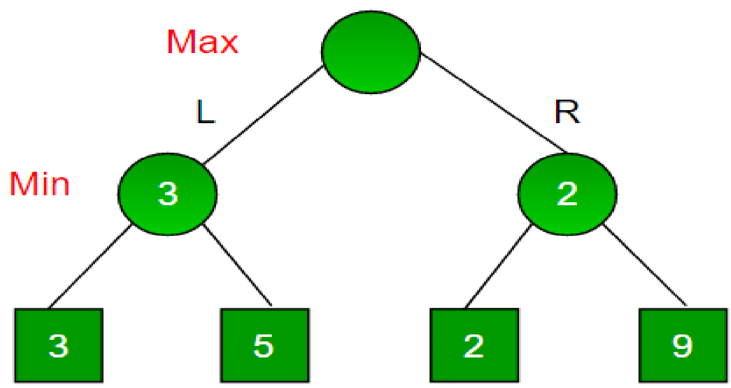


Since this is a backtracking based algorithm, it tries all possible moves, then backtracks and makes a decision.

- Maximizer goes LEFT: It is now the minimizers turn. The minimizer now has a choice between 3 and 5. Being the minimizer it will definitely choose the least among both, that is 3
- Maximizer goes RIGHT: It is now the minimizers turn. The minimizer now has a choice between 2 and 9. He will choose 2 as it is the least among the two values.

Being the maximizer you would choose the larger value that is 3. Hence the optimal move for the maximizer is to go LEFT and the optimal value is 3.

Now the game tree looks like below :



The above tree shows two possible scores when maximizer makes left and right moves.

Note: Even though there is a value of 9 on the right subtree, the minimizer will never pick that. We must always assume that our opponent plays optimally.