# Bearing Angle Estimation

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# Description:

The purpose of this report is to understand and implement a bearing angle estimation method with matlab. We will be deriving the bearing angle estimation method through analysis of correlation in continuous time signals and implemented in a sampled signal in the discrete domain. The idea behind this application is to estimate the direction of a sound source based on the time delay between two sensors spaced by a certain distance d. This application could be used to determine the direction at which gunshots are being fired in a battlefield or even a camera tracker based on voice.

# Method:

This method of deriving a bearing angle estimation relies on finding the angle $\theta$ at which the source of a sound lies in reference to the sensors.

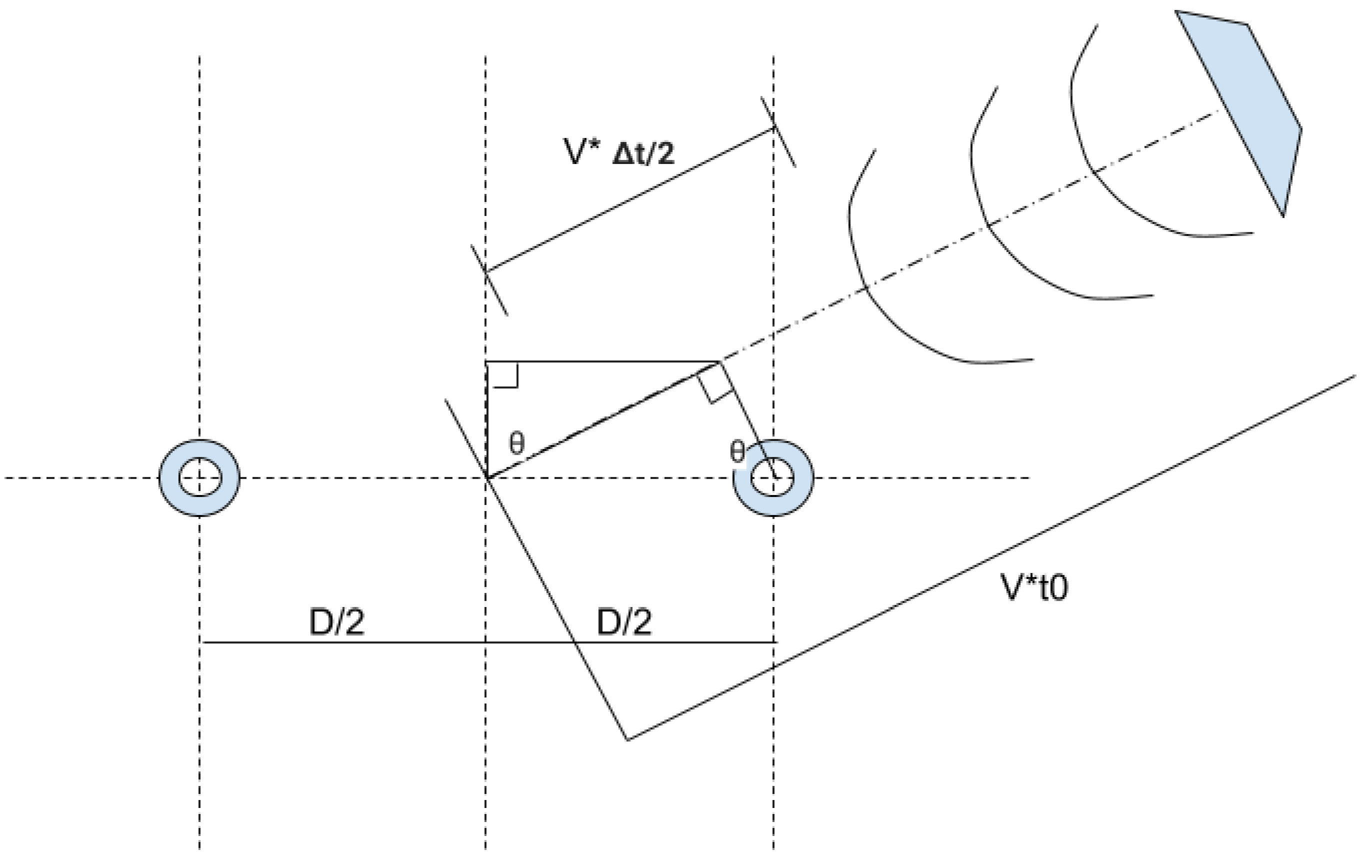


**Figure 1**

Figure 1 illustrates how we derived our formula for determining the bearing angle. A sound source at an angle $\theta$ produces a sound picked up by both sensors, and because one is farther away, the signal recorded by this sensor has a time delay in comparison to the first sensor signal.

$$Given\ Left\ signal : x(t - t_0 - \Delta t)\ and\ right\ signal : x(t - t_0)$$
$$x(t - t_0 - \Delta t)\ \leftrightarrow X(jw)e^{-jwt_0}\ and\ x(t - t_0) \leftrightarrow X(jw)e^{-jwt_0}e^{-jw\Delta t}$$
$$x(t - t_0 - \Delta t)\ *(correlated)\ x(t - t_0)\ =\ R_x(t - \Delta t)$$
$$Eq.\ 1 : R_x(t - \Delta t) \leftrightarrow |X(jw)|^2 e^{-jw\Delta t}$$

Equation 1 shows that correlating the two signal corresponds to a new signal with it's peak at $\Delta t$ . This method allows us to measure the time delay between both signals and allows us to continue with our bearing angle estimation method. Next, we will be using Eq. 2 to estimate the angle given the time delay $\Delta t$. Following figure 1, we can determine that:

$$V\,\frac{\Delta t}{2} = \frac{D}{2}sin(\theta)$$
$$Dsin\theta/v\ =\ \Delta t$$
$$Eq.\ 2 : \theta\ = sin^{-1}(\frac{v*\Delta t}{D})$$

In equation 2, D is the distance between the sensors, which for our given situation is 2.5 meters. V is the velocity of sound through air at 343.6 m/s and $\Delta t$ is the time delay picked up by the sensors. This equation gives us a direct way to compute the angle of the source given the time delay.

# Outcome:

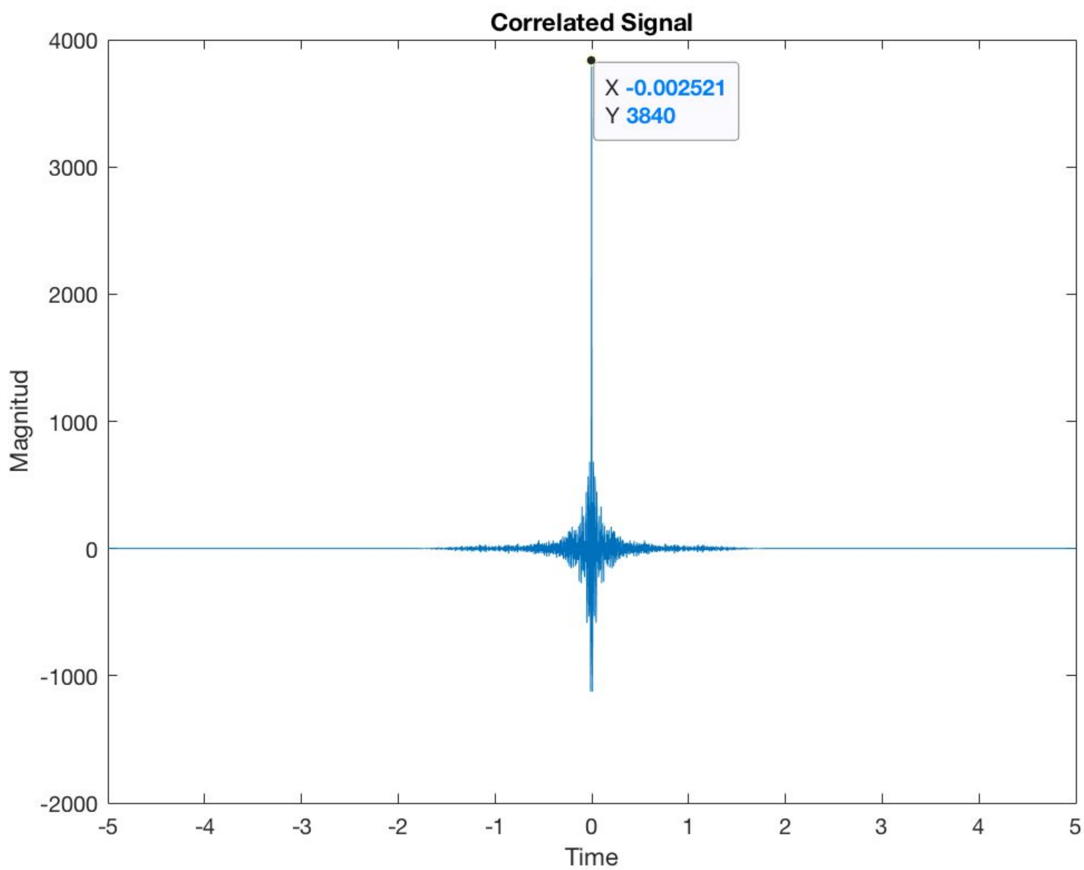


**Figure 2**

Figure 2 shows the time delay of the correlated signal as -2.521 ms from which we can determine that our angle will be negative and our source located closer to the sensor on the left in Figure 1.

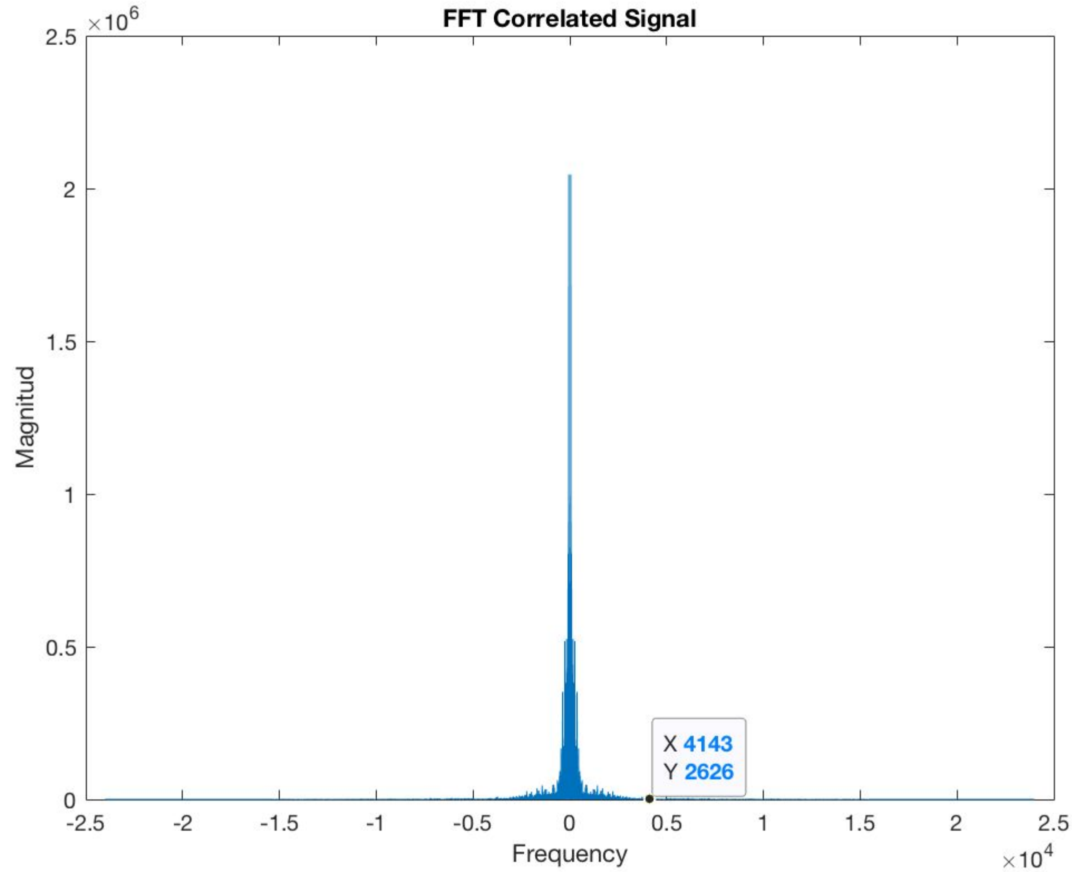


**Figure 3**

Figure 3 shows the spectrum of the correlation function which enable us to estimate the bandwidth of the signal to be approximately from -4100 to 4100.

$$4100 * 48000(sampling\, frequency)\ /\ 480001\ (number\ of\ FFT\ points)\ =\ frequency\ [Hz]$$

This translates roughly to -500 to 500 Hz.  Moreover, the time delay of the correlated signal is approximately -2.521 ms from which we can determine that our angle will be negative and our source located closer to the sensor on the left in Figure 1. To determine the accuracy level of our time delay estimate, we will look at the bandwidth of the signal. Since we know that a broad spectrum signal will give us a narrow peak in the correlated signal, we can assume that a broad spectrum signal will have a more accurate time delay measurement than a narrow band signal. Given that our bandwidth is approximately 1 KHz, we can conclude that $1/1KHz\ =\ 1ms$. This can give us an indication that our error margin is about 1ms and our measure time delay is approximately -2.521 $\pm\ 1ms$.

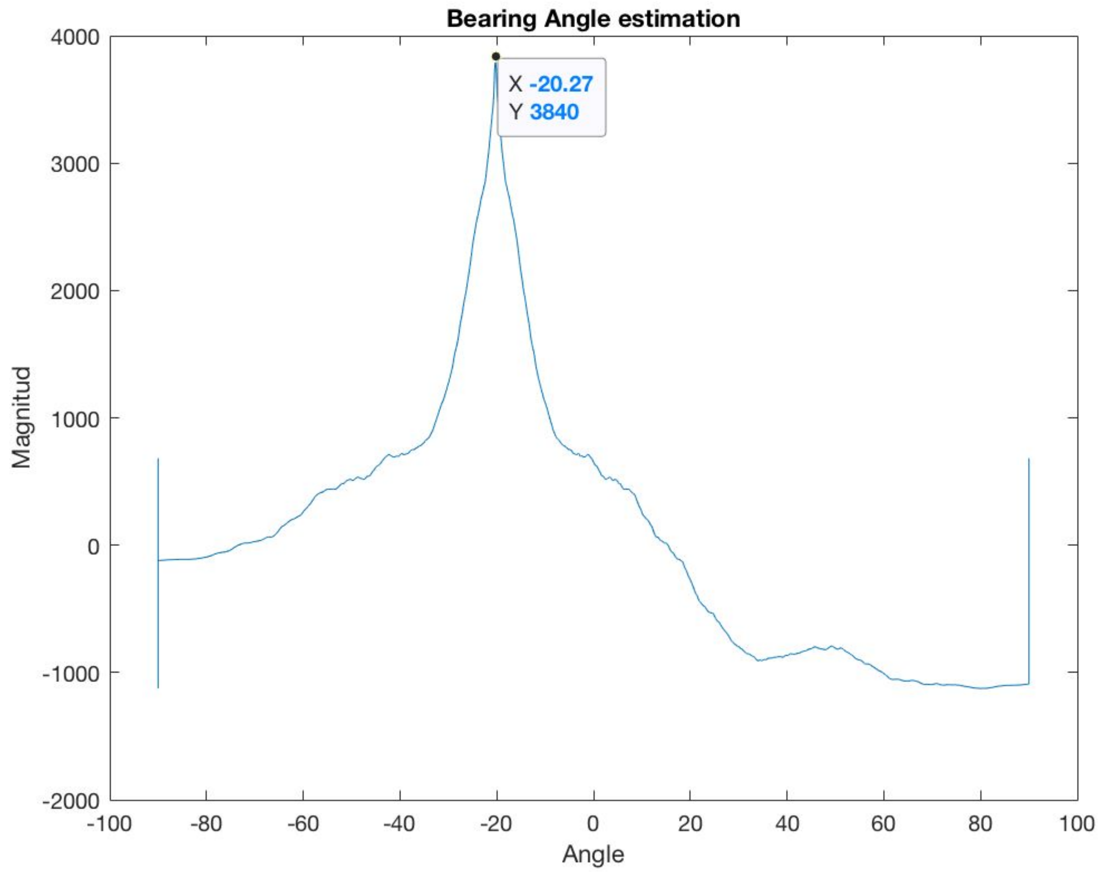


**Figure 4**

Figure 4 shows our bearing angle estimation, we can determine that $-20.27\circ$ is our best calculation based on our time delay measurements. Furthermore, we can determine the accuracy of our angle by considering the time delay range. We found our time delay to be -2.521 $\pm\ 1ms.$ This gives a range between -3.521 ms and -1.521 ms, for which we get an angle range from $-12.1\circ\ to\ -28.9\circ$ using equation 2 derived previously. We can conclude that our bearing angle has an accuracy of $\pm 8\circ$.

# Discussion:

From completing this assignment, I have acquired a better understanding of correlation and it's possible implementations. I learned the link between correlation and convolution and how correlation gives us the time difference between two signals. Furthermore, I noticed a few ways to increase our time delay and bearing angle accuracy. First of all, separating the distance between the sensors would give us a more accurate time delay measurement which is controllable, but if the signal has a narrow spectrum, which is out of our control, then this approach might not be the most robust. Another method of increasing the accuracy would include an array of sensors in different orientations and distances. If we build an array of sensors in a straight line separated by the same distance d, we could derive and calculate the bearing angle much like the derivation shown in this assignment. Furthermore, if we were to arrange the sensors in a different manner as in a circle or triangle, the complexity of the analysis would increase greatly but would give us the most accurate results. Depending on the requirements of the bearing angle estimation implementation, it might be enough to use our shown method and hardware arrangement.