Fourier Series Expansion

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Description:

The purpose of this report is to understand the Fourier Series Expansion and acquire a graphical intuition for this representation. We will be deriving the Fourier Series Expansion for a simple function, graphing the phasors corresponding to the fourier expansion and plotting the projection of the real axis with respect to time. We will understand how the phasors move with respect to the unit circle by analyzing how each fourier coefficient corresponds to the magnitude of a complex exponential that determines the vector's rotational speed.

Method:

Problem 1:

Formulate the Fourier Series expansion of the signal $f(t) = -1 \quad -4 \leq t < 0$, $f(t) = 1 \quad 0 \leq t < 4$ in the complex form.

To determine the fourier series expansion, I will find a general form for the fourier coefficients F_n.

$$F_n = \frac{1}{T} \int_{<T>} f(t) e^{-jnw_0 t} \partial t$$

$$F_n = \frac{1}{8} \int_{-4}^{0} -1 * e^{-jnw_0 t} \partial t + \frac{1}{8} \int_{0}^{4} 1 * e^{-jnw_0 t} \partial t$$

$$F_n = \frac{1}{8}\left(\frac{-1}{-jnw_0} * (1 - e^{4jnw_0}) + \frac{1}{-jnw_0} * (e^{-4jnw_0} - 1)\right)$$

$$w_o = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$F_n = \frac{1}{8}\left(\frac{-4}{-jn\pi} * (1 - e^{jn\pi}) + \frac{1}{-jn\pi} * (e^{-jn\pi} - 1)\right)$$

$$F_n = \frac{1}{8}\left(\frac{8}{jn\pi} - \frac{4}{jn\pi} e^{jn\pi} - \frac{4}{jn\pi} e^{-jn\pi}\right)$$

$$e^{jn\pi} = e^{-jn\pi} = 1 \; for \; n \; even, \; -1 \; for \; n \; odd$$

$$F_n = \frac{1}{8}\left(\frac{8}{jn\pi} - \frac{8}{jn\pi}\right) = 0; \; n \; even$$

$$F_n = \frac{1}{8}\left(\frac{8}{jn\pi} + \frac{8}{jn\pi}\right) = \frac{2}{jn\pi}; \; n \; odd$$

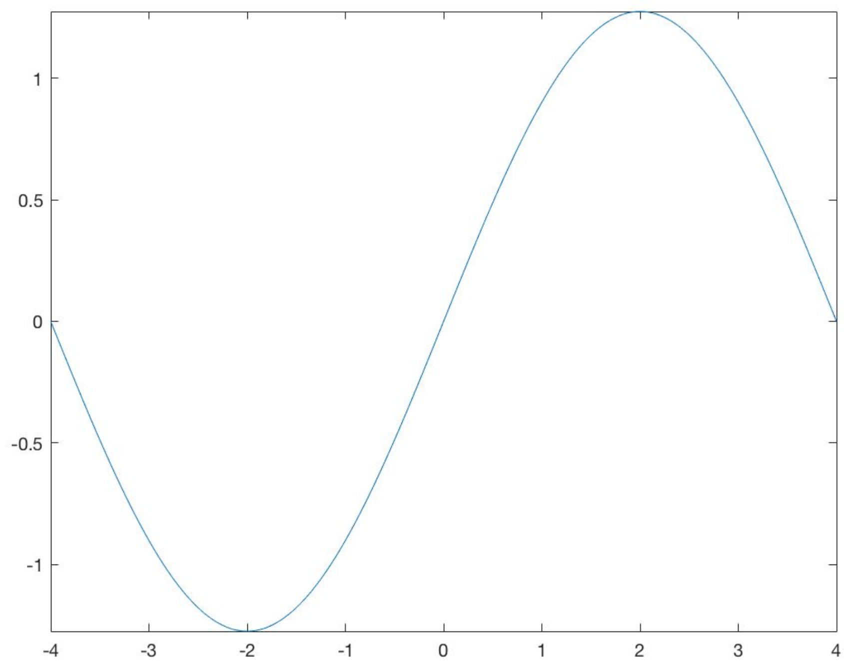
Given our derivation, all coefficients for which n is odd are non-zero with value of $\frac{2}{jn\pi}$.

Problem 2:

For this periodic signal, add the harmonics sequentially, for n = ± 1, ± 2, ± 3…, and observe the convergence.

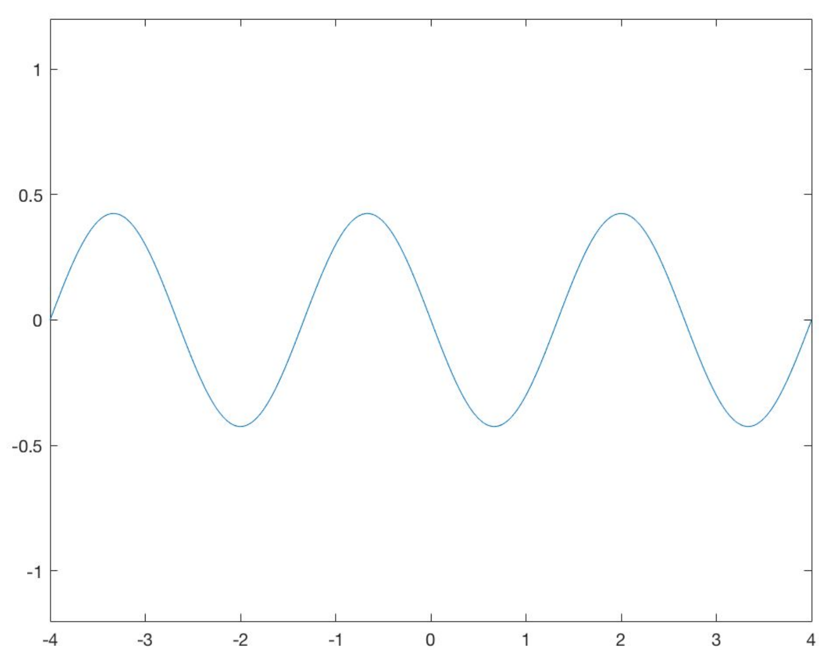
For n = 1:

$F_1 = \frac{2}{j\pi}$



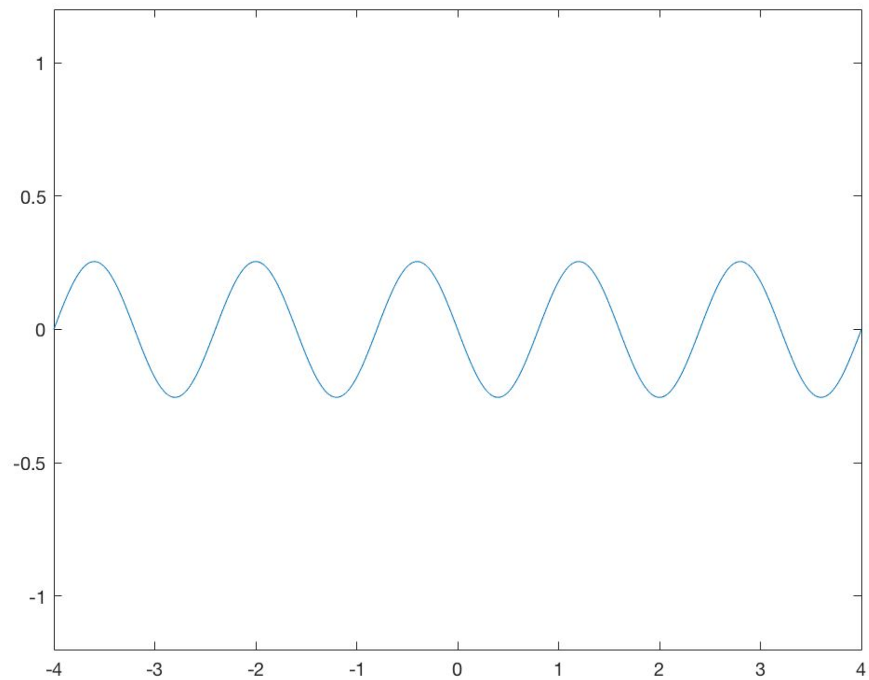
For n = 3:

$F_3 = \frac{2}{j3\pi}$



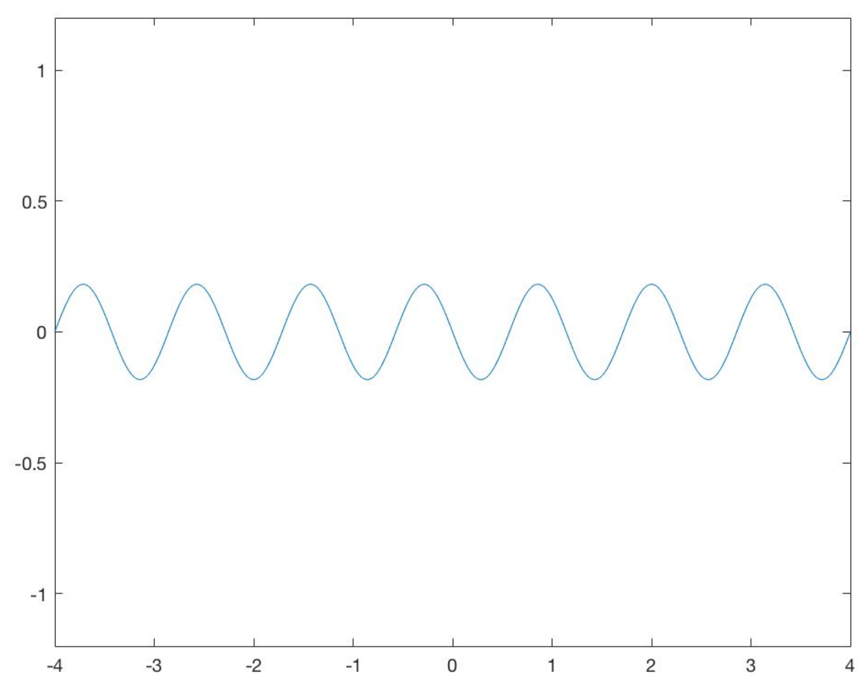
For n = 5:

$F_5 = \frac{2}{j5\pi}$



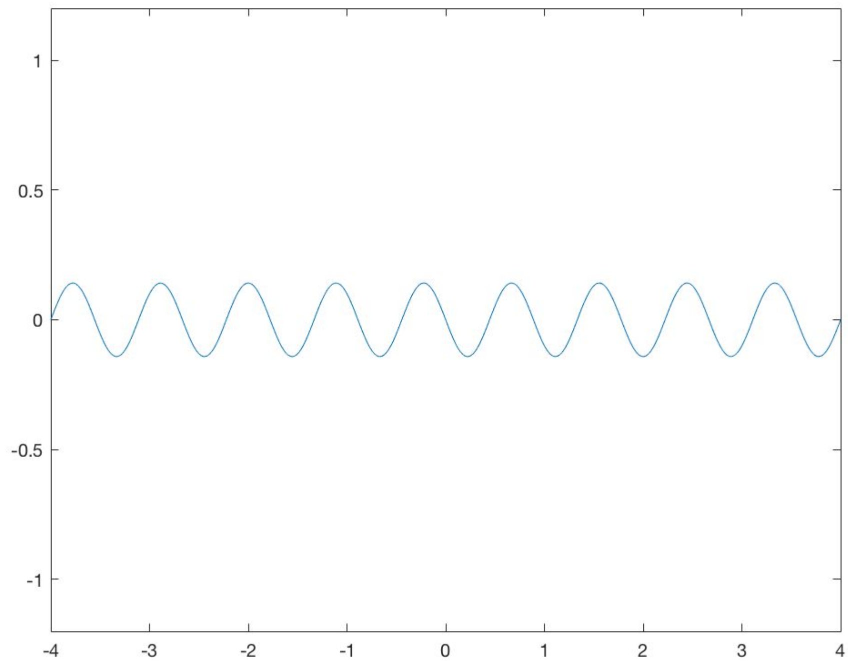
For n = 7:

$F_7 = \frac{2}{j7\pi}$



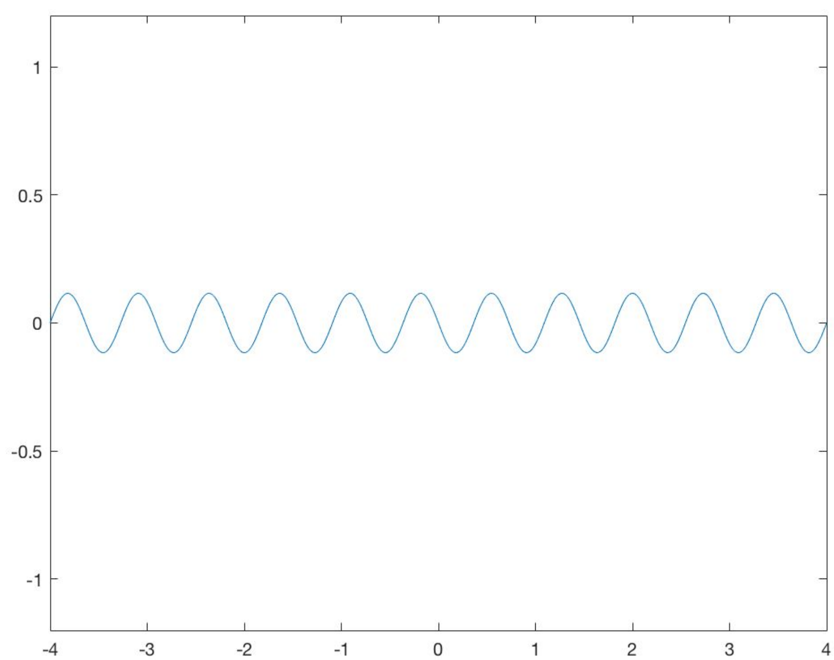
For n = 9:

$F_9 = \frac{2}{j9\pi}$

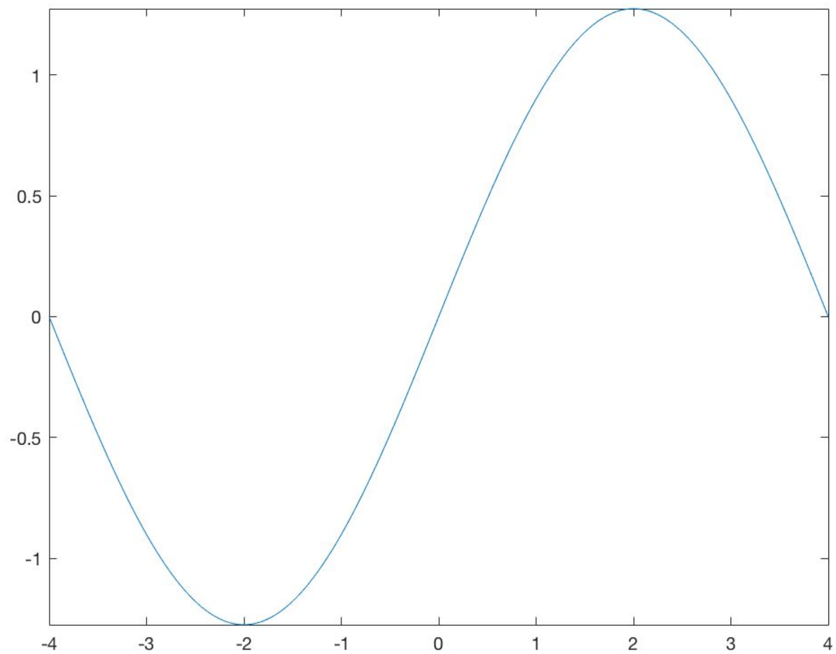


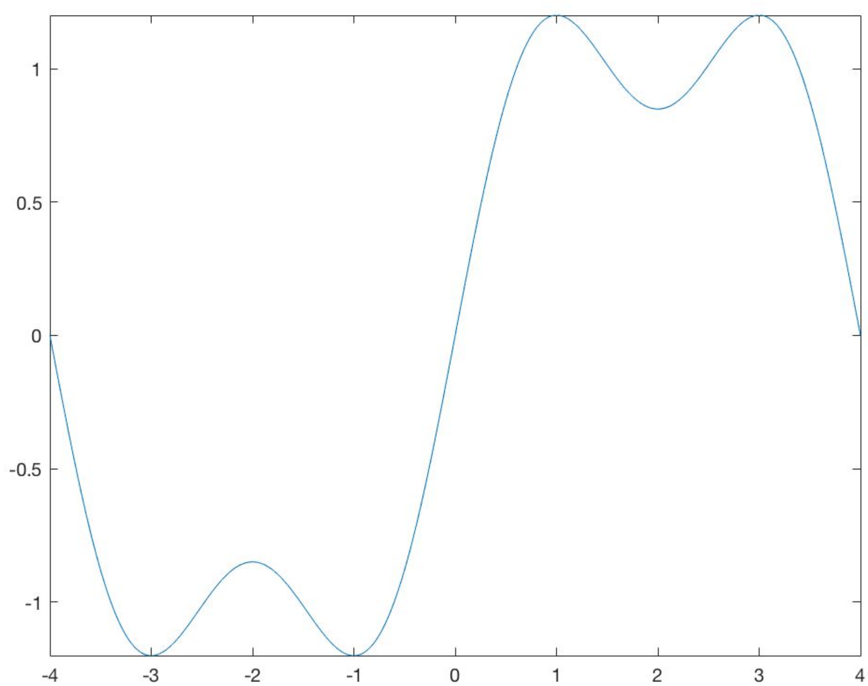
For n = 11:

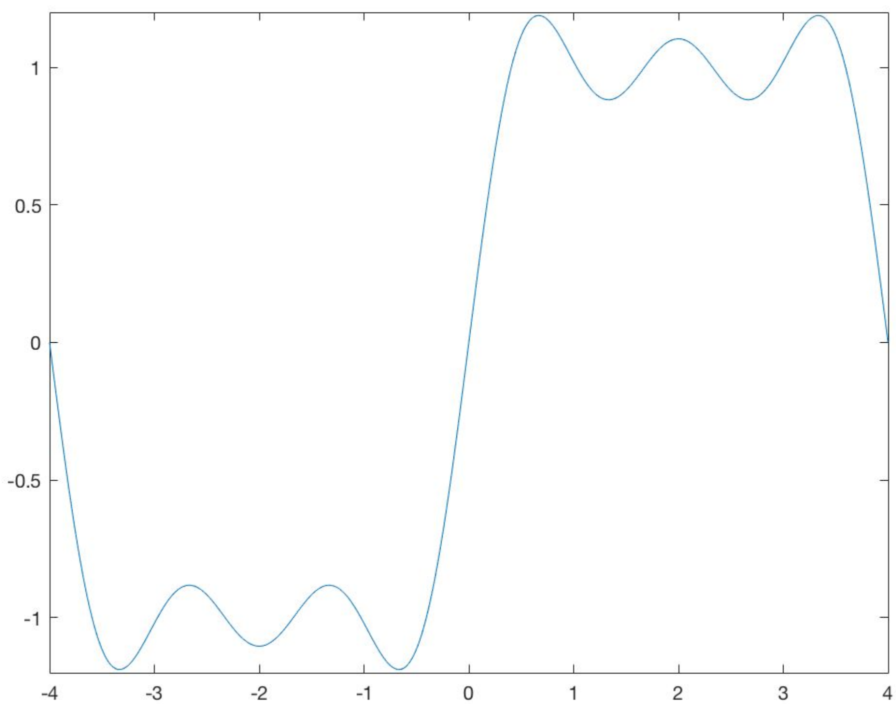
$F_{11} = \frac{2}{j11\pi}$



Next, I have plotted the added harmonics for n = ± 1, ± 2, ± 3… ± 11. Furthermore, notice that as long our function has hermitian symmetry we can find the single-sided band and double the magnitude of each coefficient projection onto the real axis.



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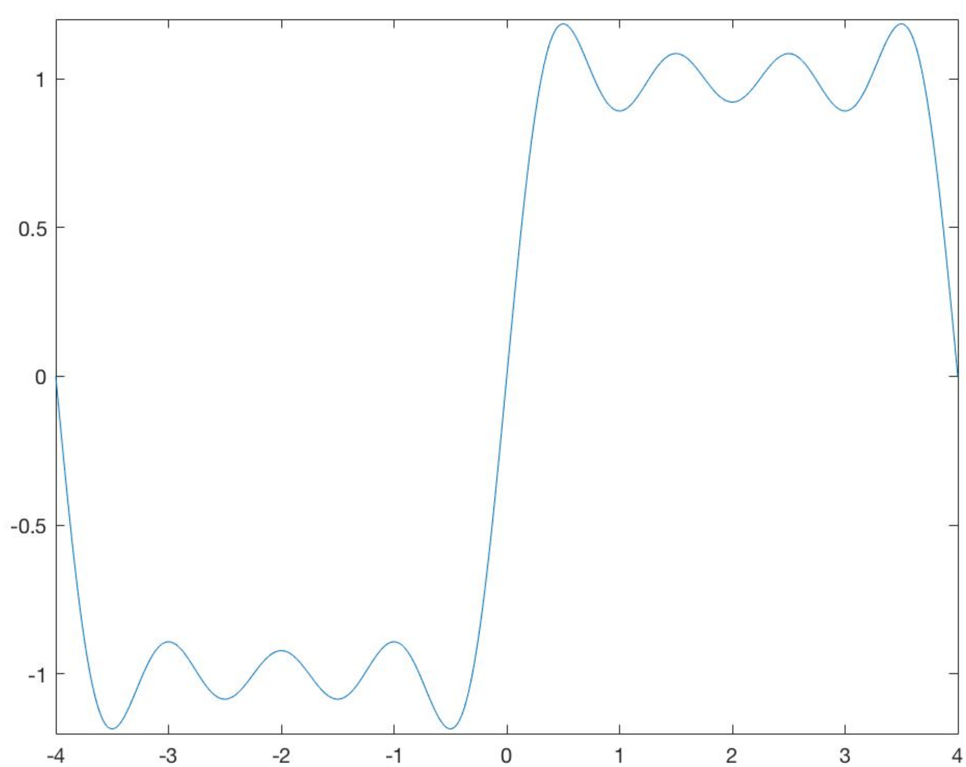
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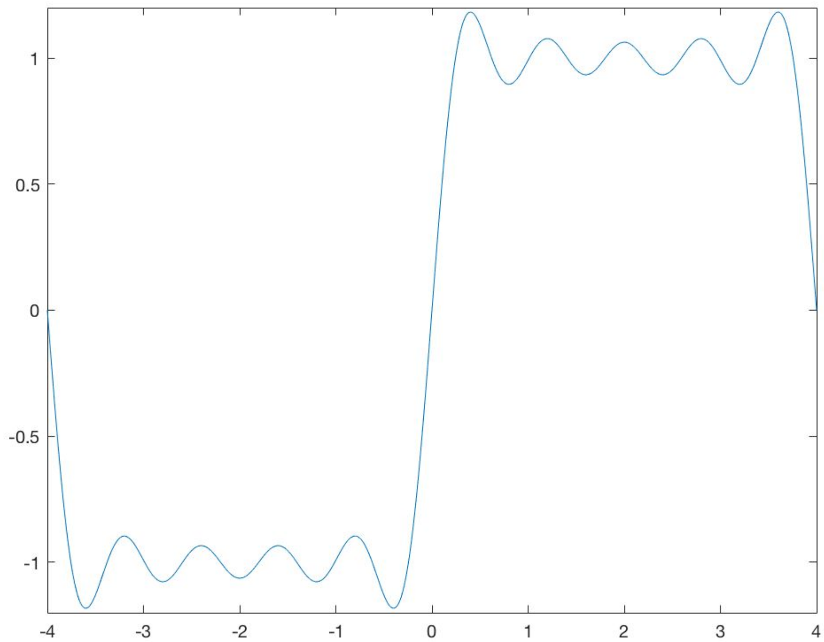
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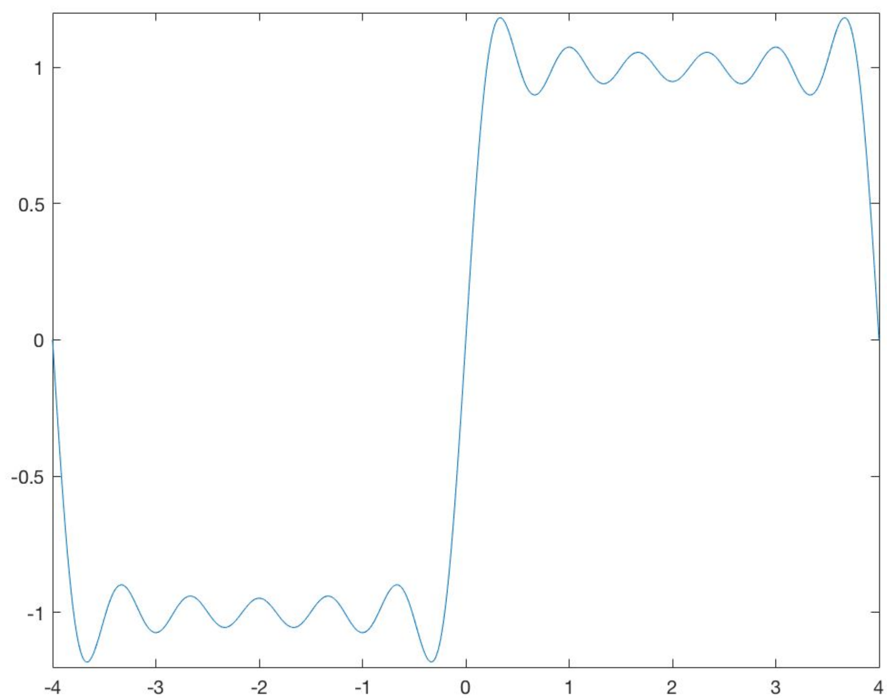
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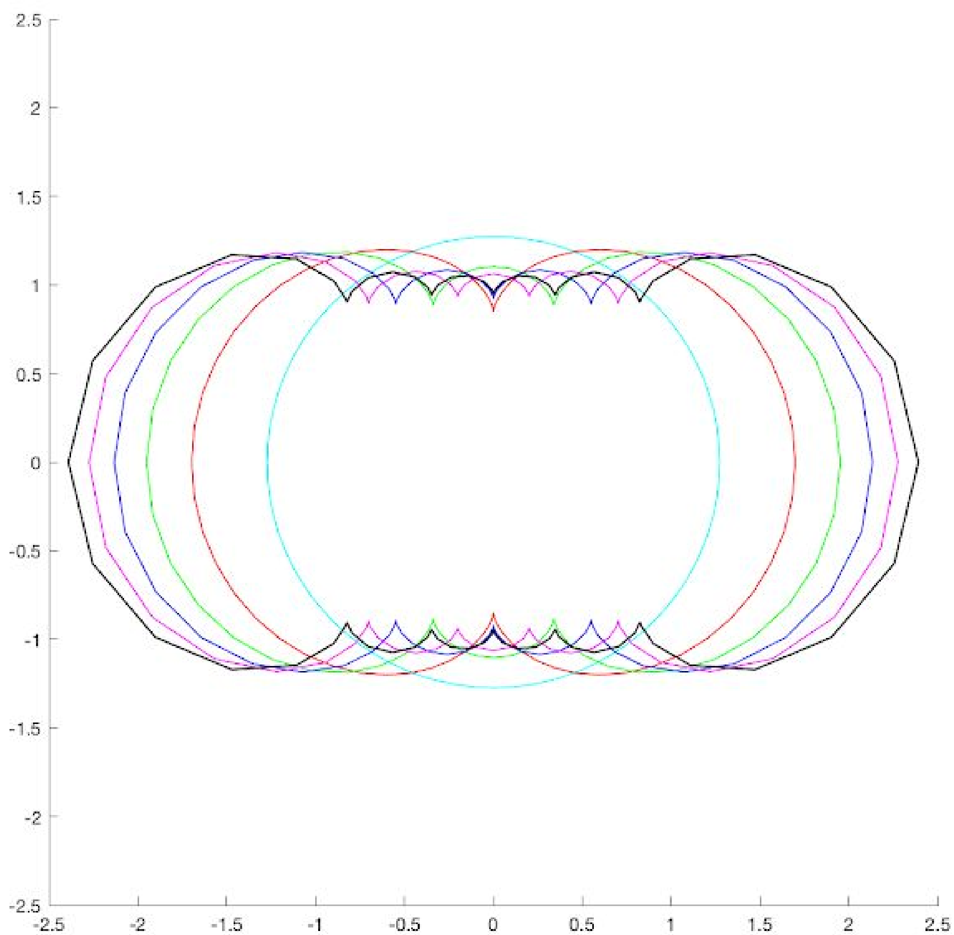
Problem3:

Develop Matlab code to implement each complex component of the Fourier series expansion in the form of a rotating point on the screen. The radius of the rotation is the amplitude of the Fourier coefficient and the angular speed of the rotation is the angular frequency of the corresponding component.

I have plotted the phasors in Matlab to graphically visualize how the vector corresponding to fourier series expansion behave in the unit circle.

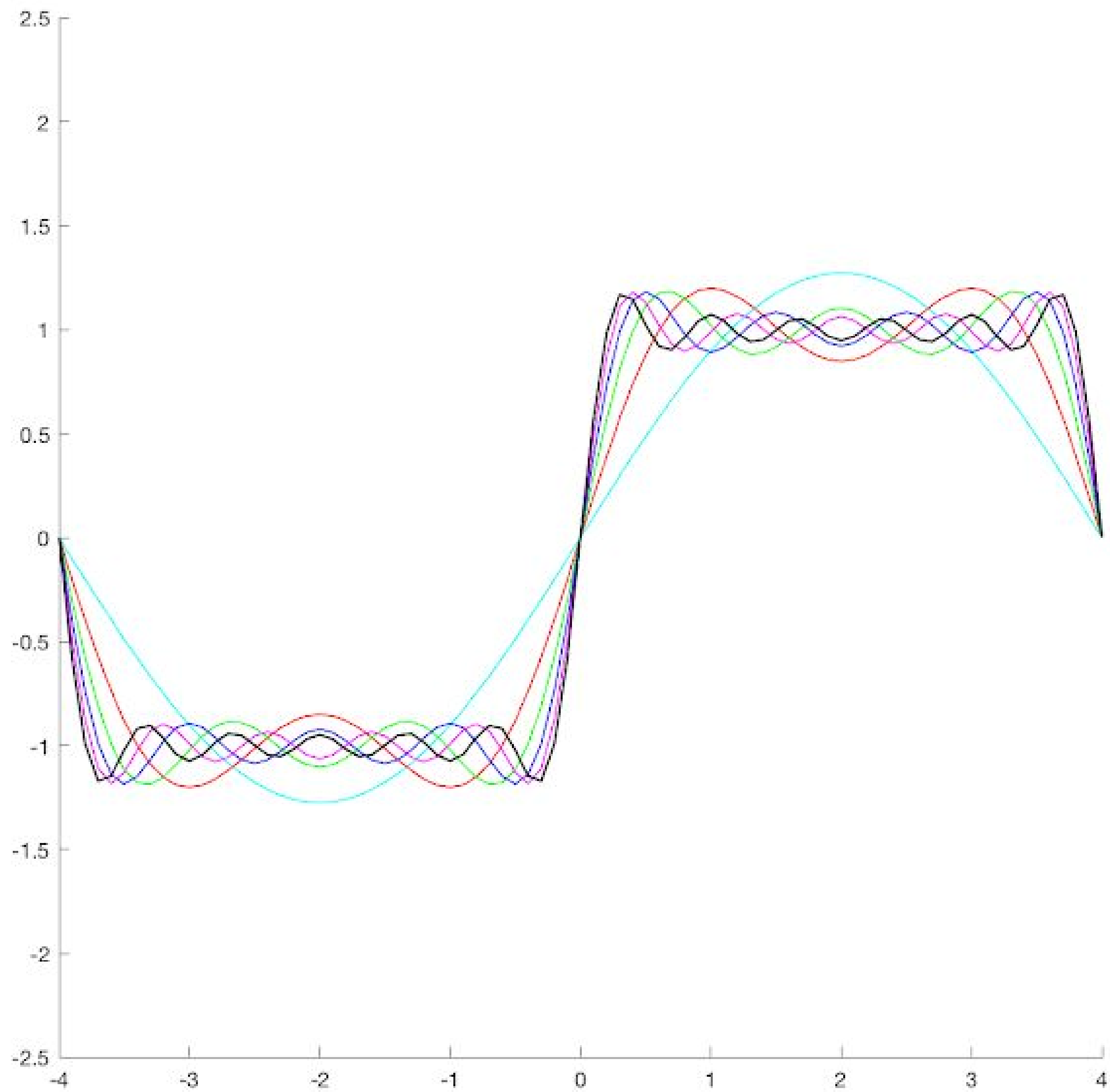


$Im(F_n * e^{jnw_0t})$

Problem 4:

Sequentially add the complex components, in the form of vector addition, for n = 1, 2, 3, …. , and observe the variation of the trace on the screen. Observe and trace the projection (shadow) of the pointer on the real axis with respect to time.

I have plotted the projection of the real axis with respect to time in Matlab to graphically visualize how the vector corresponding to fourier series expansion result in our given function.



Discussion:

From completing this assignment, I have acquired a better understanding of how signals can be decomposed into different frequency sinusoidal signals. Furthermore, I have gotten a physical intuition of the fourier series expansion and how adding these complex components sequentially, we are able to get back our signal $f(t)$. For our given function, we are able to approximate our desired signal by adding up a large number of fourier coefficients. I approximated the results by taking n from 0 to 11. Another key take-away is that as long our function has hermitian symmetry we can find single sided band and double the magnitude of each coefficient projection onto the real axis as in part 2.