Evaluation of the triangle surface sampling formulae (Figure 6) for vertex weights $w^a = 1$, $w^b = 1$, $w^c = 1$ (uniform surface sampling) and input random variables $\xi_1 = 1$ and $\xi_2 = 1$.

Function *f*

$$\begin{aligned} X &= \left(w^b - w^a\right)/3 + \left(w^c - w^b\right)/6 = (1-1)/3 + (1-1)/6 = 0 \\ Y &= w^a/2 = 1/2 = 0.5 \\ \alpha &= X/(X+Y) = 0/0.5 = 0 \\ \beta &= Y/(X+Y) = 0.5/0.5 = 1 \end{aligned}$$

The cubic equation:

$$\begin{aligned} \alpha x^3 + \beta x^2 - \xi_1 &= 0 \\ 0x^3 + 1x^2 - 1 &= 0 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

Therefore $s = 1$.

Function *g*

$$\begin{aligned} t &= s\left(w^c - w^b\right) + 2(1-s)w^a + sw^b = 1(1-1) + 2(1-1)1 + 1 \times 1 = 0 + 0 + 1 = 1 \\ \gamma &= s\left(w^c - w^b\right)/t = 1(1-1)/1 = 0 \\ \rho &= 2\left((1-s)w^a + sw^b\right)/t = 2((1-1)1 + 1 \times 1)/1 = 2 \end{aligned}$$

The result of the function is then

$$2\xi_2/\left(\rho + \sqrt{\rho^2 + 4\gamma\xi_2}\right) = 2 \times 1/\left(2 + \sqrt{2^2 + 4 \times 0 \times 1}\right) = 2/(2+2) = 0.5$$

Function *SampleTriangle*

The function f returned $s = 1$, the function g returned $t = 0.5$. This yields the point

$$\begin{aligned} p &= (1-s)A + s(1-t)B + stC \\ &= (1-1)A + 0(1-0.5)B + 1 \times 0.5C \\ &= 0.5B + 0.5C \end{aligned}$$

which means that the point lies exactly in the middle of vertices B and C, while I would expect that the input (random) variables $\xi_1 = 1$ and $\xi_2 = 1$ should result in point lying exactly at position of vertex C.