Category Theory for Programmers

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1 Category: The Essence of Composition

A category consists of objects and arrows that go between them

But the essence of a category is composition.

The composition of arrows between the objects.

> Arrows compose, so if you have an arrow from object A to object B, and another arrow from object B to object C, then there must be an arrow — their composition — that goes from A to C.

1.1 Arrows as Functions

Think of arrows, which are also called morphisms, as functions.

Arrows can be seen as:

- as **functions**
- a **relation** between objects
- *more examples...*

1.2 Properties of Composition

1. Composition is associative. If you have three morphisms, f, g, and h, that can be composed (that is, their objects match end-to-end), you don't need parentheses to compose them. In math notation this is expressed as:

$$h \circ (g \circ f) = (h \circ g) \circ f = h \circ g \circ f$$

1. For every object A there is an arrow which is a unit of composition. This arrow loops from the object to itself. Being a unit of composition means that, when composed with any arrow that either starts at A or ends at A, respectively, it gives back the same arrow. The unit arrow for object A is called id_A (identity on A). In math notation, if f goes from A to B then and $f \circ id_A =$ and $id_B \circ f = f$.

To summarize: A category consists of objects and arrows (morphisms). Arrows can be composed, and the composition is associative. Every object has an identity arrow that serves as a unit under composition.

1.3 Composition is the Essence of Programming

And how do we solve problems? We decompose bigger problems into smaller problems. If the smaller problems are still too big, we decompose them further, and so on. Finally, we write code that solves all the small problems. And then comes the essence of programming: we compose those pieces of code to create solutions to larger problems. Decomposition wouldn't make sense if we weren't able to put the pieces back together.

The Magical Number Seven, Plus or Minus Two, postulated that we can only keep 7 ± 2 "chunks" of information in our minds.

1.4 Challenges

Implement, as best as you can, the identity function in your favorite language

```
1 def id[A](a: A): A = a
```

> Implement the composition function in your favorite language. It takes two functions as arguments and returns a function that is their composition.

```
def compose[A, B, C](f: A => B)(g: B => C): A => C = a => g(f(a))
```

> Write a program that tries to test that your composition function respects identity.

```
compose[Int, Int, Int](id)(id)(4) == 4
```

```
res2: Boolean = true
```

> Is the world-wide web a category in any sense? Are links morphisms?

Yes

> Is Facebook a category, with people as objects and friendships as morphisms?

Yes - the morphism is 'relationship'.

> When is a directed graph a category?

If there is an identity arrow on each node.