

# Replication of ‘The Likelihood of Mixed Hitting Times’

Jaap H. Abbring\*

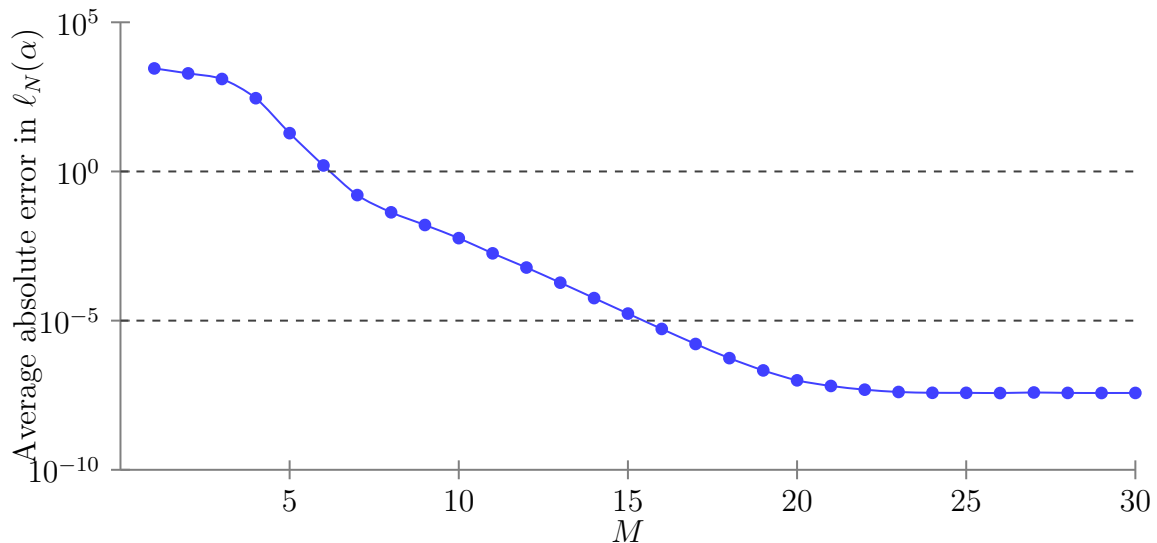
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*This document reports on the replication of Abbring and Salimans (2021a). It was generated by running `make` in the current version of the replication package (Abbring and Salimans, 2021b).*

## 1 Main Results

Figure 1: Approximation Error of the Log Likelihood for Various  $M$



Note: Mean calculation times for Figure 1 are  $4.56781e - 04$  seconds (analytical) and  $5.23704e - 03$  seconds (numerical inversion), so that mean time numerical =  $11.47 \times$  mean time analytical.

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Figure 2: Approximation Error of the Log Inverse Gaussian Density Function

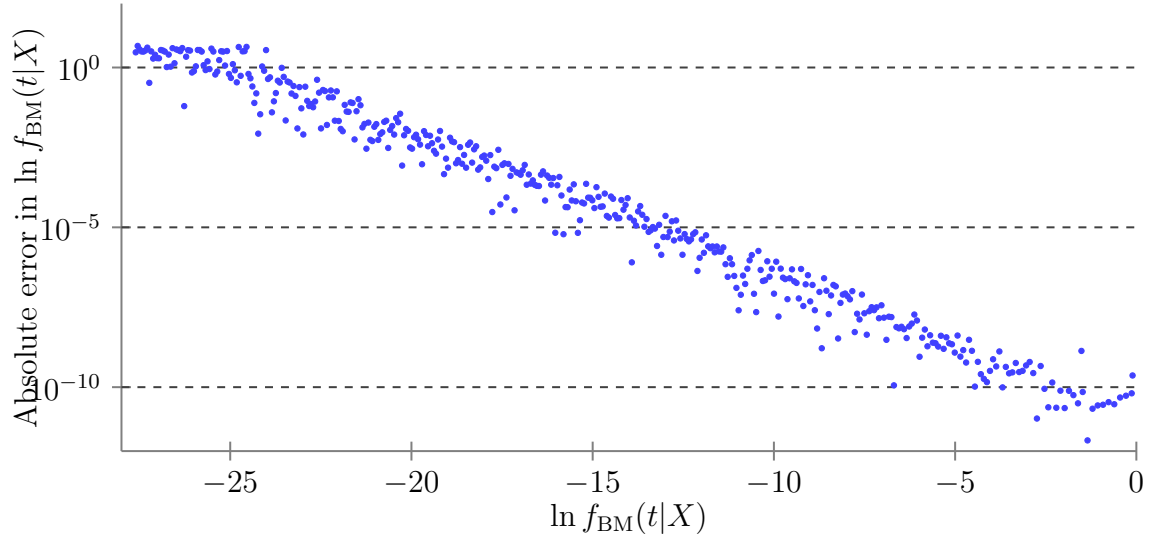
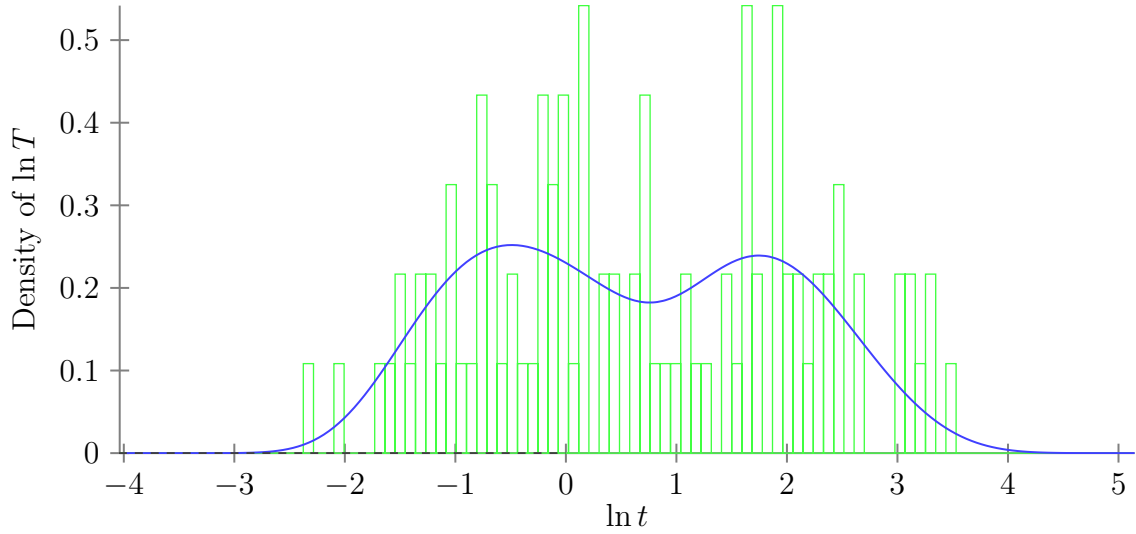


Figure 3: Approximate Probability Density and Histogram of Simulated Values of  $\ln T$  for a Specification With Shocks and Heterogeneity



Note: Calculating the pdf of  $\ln(t)$  in Figure 3 100,000 times takes 1.141 seconds.

Table 1: Maximum Likelihood Estimates for Kennan's (1985) Strike Duration Data

	I	II	III	IV	V	VI
$\mu$	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
$\sigma^2$	19.659 (3.157)	6.218 (0.863)	2.067 (0.403)	1.227 (0.217)	1.197 (0.218)	0.542 (0.315)
$\lambda$						0.019 (0.021)
$\nu$						-5.133 (2.546)
$\beta$	-0.931 (0.601)	-1.772 (0.687)	-1.085 (0.643)	-0.867 (0.628)	-0.862 (0.629)	-0.579 (0.611)
$v_1$	6.260 (0.467)	2.543 (0.199)	1.537 (0.142)	1.105 (0.113)	1.031 (0.175)	0.755 (0.177)
$v_2$		8.751 (0.520)	5.888 (0.390)	3.209 (0.452)	1.756 (1.032)	2.083 (0.510)
$v_3$			18.161 (1.011)	7.165 (0.560)	3.518 (0.763)	4.138 (0.842)
$v_4$				18.557 (0.698)	7.303 (0.645)	7.412 (0.552)
$v_5$					18.575 (0.693)	17.004 (1.220)
$\pi_1$	1 (0)	0.399 (0.044)	0.353 (0.034)	0.252 (0.038)	0.199 (0.117)	0.198 (0.040)
$\pi_2$		0.601 (0.044)	0.492 (0.034)	0.283 (0.050)	0.098 (0.133)	0.201 (0.073)
$\pi_3$			0.154 (0.023)	0.315 (0.053)	0.256 (0.083)	0.223 (0.062)
$\pi_4$				0.151 (0.019)	0.297 (0.064)	0.238 (0.064)
$\pi_5$					0.150 (0.019)	0.140 (0.020)
$\ell_N$	-1658.9	-1588.7	-1583.0	-1576.3	-1576.1	-1575.4

Note: The drift is normalized to 1 per week. All specifications include a single covariate, Kennan's (1985) deseasonalized and detrended log industrial production. Asymptotic standard errors are in parentheses.

Table 2: Replicating Table 1 with  $M = 15$ 

	I	II	III	IV	V	VI
$\mu$	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
$\sigma^2$	19.659 (3.160)	6.218 (0.863)	2.067 (0.403)	1.227 (0.217)	1.197 (0.218)	0.541 (0.323)
$\lambda$						0.019 (0.022)
$\nu$						-5.113 (2.610)
$\beta$	-0.931 (0.601)	-1.772 (0.687)	-1.085 (0.643)	-0.867 (0.628)	-0.862 (0.630)	-0.579 (0.612)
$v_1$	6.260 (0.467)	2.543 (0.199)	1.537 (0.141)	1.104 (0.113)	1.031 (0.174)	0.754 (0.181)
$v_2$		8.751 (0.520)	5.888 (0.390)	3.209 (0.452)	1.756 (1.032)	2.083 (0.510)
$v_3$			18.161 (1.010)	7.165 (0.560)	3.518 (0.763)	4.138 (0.839)
$v_4$				18.557 (0.698)	7.303 (0.645)	7.410 (0.554)
$v_5$					18.575 (0.693)	16.997 (1.254)
$\pi_1$	1 (0)	0.399 (0.044)	0.353 (0.034)	0.252 (0.038)	0.199 (0.117)	0.198 (0.041)
$\pi_2$		0.601 (0.044)	0.492 (0.034)	0.283 (0.050)	0.098 (0.132)	0.201 (0.073)
$\pi_3$			0.154 (0.023)	0.315 (0.053)	0.256 (0.083)	0.223 (0.062)
$\pi_4$				0.151 (0.019)	0.297 (0.064)	0.238 (0.064)
$\pi_5$					0.150 (0.019)	0.140 (0.020)
$\ell_N$	-1658.9	-1588.7	-1583.0	-1576.3	-1576.1	-1575.4

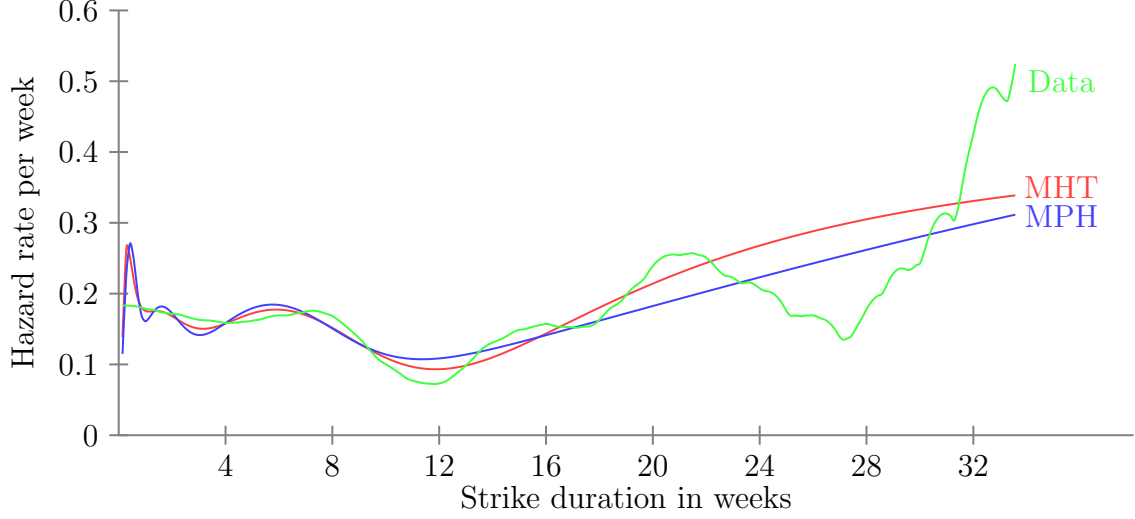
Note: The drift is normalized to 1 per week. All specifications include a single covariate, Kennan's (1985) deseasonalized and detrended log industrial production. Asymptotic standard errors are in parentheses.

Table 3: Replicating Table 1 Using Inverse Gaussian Pdf

	I	II	III	IV	V	VI
$\mu$	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	
$\sigma^2$	19.659 (3.160)	6.218 (0.863)	2.067 (0.403)	1.227 (0.217)	1.197 (0.218)	
$\lambda$						
$\nu$						
$\beta$	-0.931 (0.601)	-1.772 (0.687)	-1.085 (0.643)	-0.867 (0.628)	-0.862 (0.630)	
$v_1$	6.260 (0.468)	2.543 (0.199)	1.537 (0.142)	1.105 (0.113)	1.031 (0.174)	
$v_2$		8.751 (0.520)	5.888 (0.390)	3.209 (0.452)	1.756 (1.033)	
$v_3$			18.161 (1.010)	7.165 (0.560)	3.518 (0.763)	
$v_4$				18.557 (0.698)	7.303 (0.645)	
$v_5$					18.575 (0.693)	
$\pi_1$	1 (0)	0.399 (0.044)	0.353 (0.034)	0.252 (0.038)	0.199 (0.117)	
$\pi_2$		0.601 (0.044)	0.492 (0.034)	0.283 (0.050)	0.098 (0.132)	
$\pi_3$			0.154 (0.023)	0.315 (0.053)	0.256 (0.083)	
$\pi_4$				0.151 (0.019)	0.297 (0.064)	
$\pi_5$					0.150 (0.019)	
$\ell_N$	-1658.9	-1588.7	-1583.0	-1576.3	-1576.1	

Note: The drift is normalized to 1 per week. All specifications include a single covariate, Kennan's (1985) deseasonalized and detrended log industrial production. Asymptotic standard errors are in parentheses.

Figure 4: Aggregate Strike End Hazard Rates



The numbers in Column IV imply that there are four unobserved types of labor conflict, on average commanding respectively 1.1, 3.2, 7.2, and 18.6 strike weeks.

Computation times (in seconds):	I	II	III	IV	V	VI
	1.1	1.5	3.4	3.9	7.7	24.6

The estimation procedure for a version of the model in Column VI with a single gamma shock stops at a similar estimate of  $\lambda$  ( $1.861e - 02$ ) and huge estimates of  $\tau$  ( $2.448e + 07$ ) and  $\omega$  ( $4.770e + 06$ ). At these estimated values of  $\tau$  and  $\omega$ , the gamma shock distribution is effectively degenerate at  $-\tau/\omega = -5.133$ . As this is very close to the estimated size of the discrete shock in Column VI, the log likelihood is virtual identical:  $-1575.4$ .

Both the MHT and the MPH models fit the empirical hazard well, but the MPH model's log likelihood, at  $-1577.9$ , is 1.6 points lower.

## 2 Other Checks

Table R1: Analytical and Numerical Gradients MHT

63.109743	63.129687
−24.600649	−24.603988
−10.666863	−10.668048
31.395502	31.391735
−16.443511	−16.447449
−63.825155	−63.827520
3.959228	3.956141
44.661001	44.661754
−62.913234	−62.910567
33.435364	33.439751
26.771924	26.769051
27.405938	27.403232
1.711623	1.714570

Table R2: Analytical and Numerical Gradients MPH

0.000095	0.000095	−521.944454	−521.944454
−0.000002	−0.000002	4.234742	4.234743
0.000014	0.000014	−71.661453	−71.661453
0.000007	0.000007	−8.757202	−8.757202
0.000001	0.000001	10.686492	10.686492
0.000002	0.000002	5.471933	5.471933
−0.000010	−0.000010	50.583811	50.583811
−0.000002	−0.000002	−42.823964	−42.823964
0.000002	0.000002	−1.252138	−1.252138

## References

- Abbring, J. H. and T. Salimans (2021a). The likelihood of mixed hitting times. *Journal of Econometrics XX*, forthcoming. [arXiv:1905.03463](#) [econ.EM].
- Abbring, J. H. and T. Salimans (2021b). The likelihood of mixed hitting times: Replication package. [doi:10.5281/zenodo.4663359](#).
- Kennan, J. (1985). The duration of contract strikes in U.S. manufacturing. *Journal of Econometrics* 28, 5–28.