Page Replacement

Demand paging is the operating system's attempts to improve the computer system's utilization and throughput.

Page replacement take the following approach. If no frame is free to store the page of particular process, then we find one that is not currently being used and free it. We can free frame by writing its contents to swap space and changing * page table to indicate that the page is no longer in memory. We can now use the freed frame to hold the page for which the process faulted. For this we are using page-replacement algorithm. There are many different page-replacement algorithms. Every operating system probably has its own replacement scheme.

We evaluate an algorithm by running it on a particular string of memory references and computing the number of page faults. The string of memory references is called a reference string.

1) First in First out (FIFO) page Replacement algorithm ⇒

(1) The simplest page-replacement algorithm is a first-in, first-out (FIFO) algorithm. A FIFO replacement algorithm associates with each page the time when that page was brought into memory. When a page must be replaced, the oldest page is chosen.

(1) The given reference string is

7 0 1 2 0 3 0 4 2 3 0 3 2 1 2 0 1 7 0 1

Soln ⇒ No. of Hits = 5 | Hit Ratio = $\frac{\text{No. of Hits}}{\text{No. of References}} = \frac{5}{20} \times 100 = 25\%$

Page fault / page miss = 15

	7	0	1	2	0	3	0	4	2	3	0	3	2	1	2	0	1	7	0	1
f1	7	7	7	2	2	2	4	4	4	4	0			0	0					
f2		0	0	0	3	3	3	2	2	2	2			1	1					
f3			1	1	1	0	0	0	0	3	3			3	2					
	*	*	*	*	Hit	*	*	*	*	*	*	Hit	Hit	*	*	Hit	Hit			

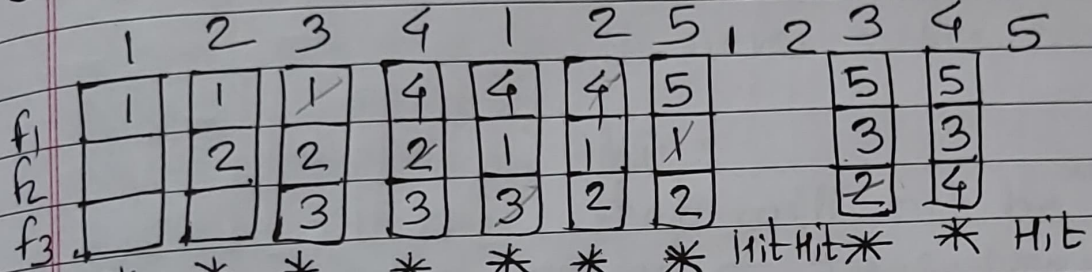
0	1
7	7
0	0
2	1
*	*
(miss)	(miss)

$\text{Miss Ratio} = \frac{\text{No. of miss}}{\text{No. of References}} = \frac{15}{20} \times 100 = 75\%$

(2) The given reference string is

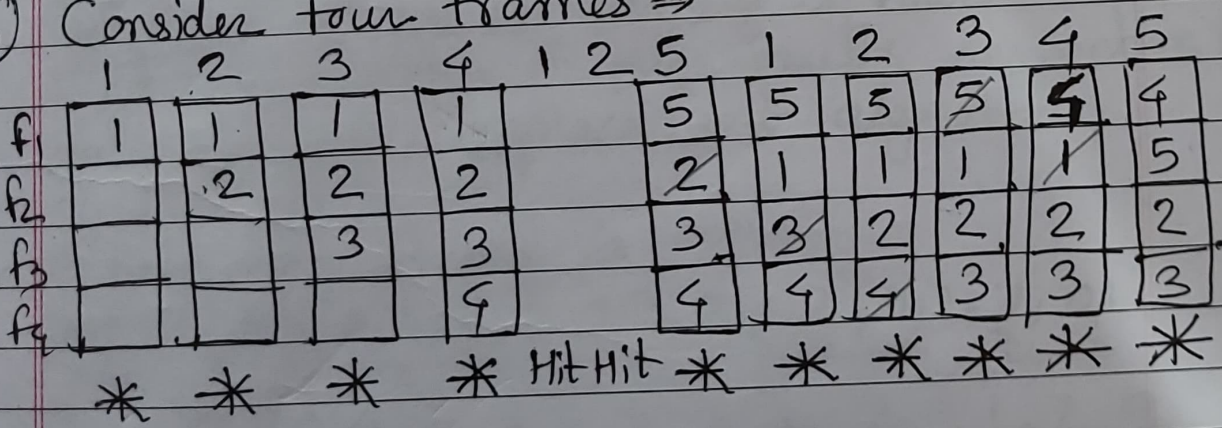
1 2 3 4 1 2 5 1 2 3 4 5

(a) Consider three frames ⇒



No. of Hits = 3 and page faults = 9

(b) Consider four frames ⇒



No. of Hits = 2 and page faults = 10

Here we have increased no. of frames from 3 to 4, means we have allocated more space. Pages of more processes can come at once and page faults should decrease. But here No. of hits are less and no. of page faults are more.

This most unexpected result is known as Belady's anomaly.

This problem will occur in FIFO algorithm, the page-fault rate may increase as the number of allocated frames increases.

II) Optimal Page Replacement Algorithm ⇒

One result of the discovery of Belady's anomaly was the search for an optimal page-replacement algorithm. This algorithm has the lowest page-fault rate of all algorithms and will never suffer from Belady's anomaly.

It is simply this:

Replace the page that will not be used for the longest period of time. Use of this page-replacement algorithm guarantees the lowest possible page-fault rate for a fixed number of frames.

1) Consider a f/s reference string:
7,0,1,2,0,3,0,4,2,3,0,3,2,1,2,0,1,7,0,1.
the no. of frames in memory is four.
Find out number of page faults respective to: optimal page replacement algorithm.

Soln ⇒ Reference string is
7,0,1,2,0,3,0,4,2,3,0,3,2,1,2,0,1,7,0,1

	7	0	1	2	0	3	0	4	2	3	0	3	2	1	2	0	1	7	0	1
f1	7	7	7	7	7	3	3	3	3	3	3	3	3	3	3	3	3	7	7	7
f2		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
f3			1	1	1	1	1	4	4	4	4	4	4	1	1	1	1	1	1	1
f4				2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	*	*	*	*	Hit	*	Hit	*	Hit	Hit	Hit	Hit	Hit	*	Hit	Hit	* Hit	*	Hit	Hit

No. of hits = 12 Hit Ratio = $\frac{12}{20} \times 100 = 60\%$

page faults = 08

Page missed (faults) Ratio = $\frac{8}{20} \times 100 = 40\%$

(2) Consider a reference string : 4, 7, 6, 1, 7, 6, 1, 2, 7, 2 the no. of frames in the memory is 3. Find out number of page faults respective to optimal page replacement algorithm.

Soln ⇒

	4	7	6	1	7	6	1	2	7	2
f1	4	4	4	1	1	1	1	2	2	2
f2		7	7	7	7	7	7	7	7	7
f3			6	6	6	6	6	6	6	6
	Miss	Miss	Miss	Miss	Hit	Hit	Hit	Miss	Hit	Hit

OR

	4	7	6	1	7	6	1	2	7	2
f1	4	4	4	1	1	1	1	1	1	1
f2		7	7	7	7	7	7	7	7	7
f3			6	6	6	6	6	2	2	2
	Miss	Miss	Miss	Miss	Hit	Hit	Hit	Miss	Hit	Hit

No. of Hits = 5 & No. of Miss = 5

Hit Ratio = $\frac{5}{10} \times 100$ = 50%

Miss Ratio = $\frac{5}{10} \times 100$ = 50%

III) Least Recently Used (LRU)

Algorithm ⇒

LRU replacement associates with each page the time of that page's last use. When a page must be replaced, LRU chooses the page that has not been used for the longest period of time.

① Consider a reference string:
7, 0, 1, 2, 0, 3, 0, 4, 2, 3, 0, 3, 2, 1, 2, 0, 1, 7, 0, 1.
the no. of frames in memory is four.
Find out no. of page faults respective to least recently used [LRU] algorithm.

f1	7	7	7	7	7	3	3	3	3	3	3	3	3	3	3
f2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
f3			1	1	1	1	1	4	4	4	4	4	4	4	1
f4				2	2	2	2	2	2	2	2	2	2	2	2
	*	*	*	*	Hit	*	Hit	*	Hit	Hit	Hit	Hit	Hit	*	

3	3	3	7	7	7	
0	0	0	0	0	0	
1	1	1	1	1	1	
2	2	2	2	2	2	
Hit	Hit	Hit	*	Hit	Hit	

No. of Hits = 12

No. of page faults = 8

Hit Ratio = $\frac{12}{20} \times 100 = 60\%$

7, 0, 1, 2, 0, 3, 0, 4, 2, 3, 0, 3, 2, 1, 2, 0, 1, 7, 0, 1

page missed Ratio = $\frac{8}{20} \times 100 = 40\%$

Extra Numericals on Page Replacement Algorithm

① Consider a reference string 1,2,3,4,5,1,3,1,6,3,2,3, the no. of frames in memory is 4. Find number of page faults respective to First in first out page replacement Algorithm. Also calculate Hit Ratio and miss Ratio or Page fault Ratio.

	1	2	3	4	5	1	3	1	6	3	2	3
f1	1	1	1	1	5	5	5	5	5	5	2	2
f2		2	2	2	2	1	1	1	1	1	1	1
f3			3	3	3	3	3	3	6	6	6	6
f4				4	4	4	4	4	4	3	3	3
	Miss	Miss	Miss	Miss	Miss	Miss	Hit	Hit	Miss	Miss	Miss	Hit

No. of Hits = 03 and No. of Miss = 09

ie. Total Page Fault = 9

Hit Ratio = $\frac{3}{12} \times 100 = 25\%$

Page fault Ratio = $\frac{9}{12} \times 100 = 75\%$

(2) Consider a reference string 1,2,3,4,5,1,3,1,6,3,2,3, the no. of frames in memory is 4. Find number of page faults respective optimal page replacement Algorithm. Also calculate hit ratio and page fault ratio.

	1	2	3	4	5	1	3	1	6	3	2	3
f1	1	1	1	1	1	1	1	✗	6	6	6	6
f2		2	2	2	2	2	2	2	2	2	2	2
f3			3	3	3	3	3	3	3	3	3	3
f4				4	5	5	5	5	5	5	5	5
	Miss	Miss	Miss	Miss	Miss	Hit	Hit	Hit	Miss	Hit	Hit	Hit

No. of hits = 6 and No. of page faults = 6

Hit Ratio = $\frac{6}{12} \times 100 = 50\%$

Page fault Ratio = $\frac{6}{12} \times 100 = 50\%$

OR

	1	2	3	4	5	1	3	1	6	3	2	3
f1	1	1	1	1	1	1	1	1	1	1	1	1
f2		2	2	2	2	2	2	2	2	2	2	2
f3			3	3	3	3	3	3	3	3	3	3
f4				4	5	5	5	8	6	6	6	6
	Miss	Miss	Miss	Miss	Miss	Hit	Hit	Hit	Miss	Hit	Hit	Hit

No. of hits = 6 & No. of page faults = 6

③ Consider a reference string 1,2,3,4,5,1,3,1,6,3,2,3, the no. of frames in memory is 4. Find number of page faults respective to LRU page replacement Algorithm. Also calculate hit ratio and page fault Ratio.

	1	2	3	4	5	1	3	1	6	3	2	3
f1	1	1	1	1	5	5	5	5	5	5	2	2
f2		2	2	2	2	1	1	1	1	1	1	1
f3			3	3	3	3	3	3	3	3	3	3
f4				4	4	4	4	4	6	6	6	6
	Miss	Miss	Miss	Miss	Miss	Miss	Hit	Hit	Miss	Hit	Miss	Hit

Total Page faults = 8

page fault Ratio = $\frac{8}{12} \times 100$

= 66.66 %

No. of Hits = 4

Hit Ratio = $\frac{4}{12} \times 100$

= 33.33 %