Homework 7

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2017/11/28

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i.

$$\bar{X} = \frac{1}{n}\sum X_i \sim N(0,1)$$
$$U = \frac{\sqrt{n}(\bar{X}-\mu)}{\sigma} \sim N(0,1)$$
$$so\ P_\mu(|U| \le u_{\alpha/2}) = 1-\alpha$$
$$\text{so the interval is: } [\bar{X} - \frac{\sigma}{\sqrt{n}}u_{\alpha/2}, \bar{X} + \frac{\sigma}{\sqrt{n}}u_{\alpha/2}]$$

ii.

$$u_{0.025} = 1.96$$
$$[\bar{X} - 0.196, \bar{X} + 0.196]$$

iii.

$$\frac{2 \times 1.96}{\sqrt{n}} \le 0.1$$
$$n \ge 39.2^2 \approx 1537$$

iv.

$$l = \frac{2\sigma u_{\frac{\alpha}{2}}}{\sqrt{n}}$$

all the variables except n are constant, so it is obvious that when n $\rightarrow \infty$ length converges in probality to 0

2

i.

$$\text{according to (4.3.21), the CI of } \mu :$$
$$[\bar{X} - \frac{S}{\sqrt{n}}u_{\alpha/2}, \bar{X} + \frac{S}{\sqrt{n}}u_{\alpha/2}]$$

ii.

$$u_{(0.025)} = 1.96$$
$$\text{so the interval is: } [\bar{X} - 0.196S, \bar{X} + 0.196S]$$

iii.

$$l = 2\frac{S}{\sqrt{n}}u_{(\alpha/2)}$$

$S \overset{p}{\sim} \sigma$ when $n \rightarrow \infty$ so it is obvious that when n $\rightarrow \infty$ length converges in probality to 0

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i. codes in python

$$interval : [\bar{X} - \frac{\sigma}{\sqrt{n}}u_{\alpha/2}, \bar{X} + \frac{\sigma}{\sqrt{n}}u_{\alpha/2}]$$
$$u_{0.025} = 1.96$$
$$\text{from the calculation we have the interval :}[2.684, 2.916]$$

ii.

$$S = 0.05,\ t_{14}(0.025) = 2.1448$$
$$\text{so the interval is:}[2.774, 2.825]$$

iii.

$$\text{the interval is: } [\frac{nS_\mu^2}{\chi^2_{(\alpha/2)}}, \frac{nS_\mu^2}{\chi^2_{(1-\alpha/2)}}]$$
$$S_\mu^2 = \sum (X_i - \mu)^2/n$$
$$\chi^2_{15}(0.025) = 27.488 \quad \chi^2_{15}(0.975) = 6.262$$
$$[0.0268, 0.1178]$$

iv.

$$S^2 = \sum \frac{(X_i - \bar{X})^2}{n-1}$$
$$\text{the interval is: } [\frac{(n-1)S^2}{\chi^2_{n-1}(\alpha/2)}, \frac{(n-1)S^2}{\chi^2_{n-1}(1-\alpha/2)}]$$
$$\chi^2_{14}(0.025) = 26.119 \quad \chi^2_{14}(0.975) = 5.629$$
$$\text{the interval is: } [0.02547, 0.11179]$$

v.

$$\text{The moment estimate is: } \bar{X}^2 + S^2, \text{ and the MLE is: } \bar{X}^2$$

codes in R code

vi. codes in R code

4

i.

$$\text{the interval is: } [\bar{Y} - \bar{X} - u_{\alpha/2}\sqrt{\sigma_1^2/m + \sigma_2^2/n}, \bar{Y} - \bar{X} + u_{\alpha/2}\sqrt{\sigma_1^2/m + \sigma_2^2/n}]$$
$$u_{\alpha/2} = 1.96\ m = 1475\ n = 123\ \sigma_1 = \sigma_2 = \sigma = 79400$$
$$\text{from the computation we have: } [70095.251359575152, 99306.2490951582]$$

ii.

$$\text{the interval is: } [\bar{Y} - \bar{X} - S_\omega t_{m+n-2}(\alpha/2)\sqrt{1/m + 1/n}, \bar{Y} - \bar{X} + S_\omega t_{m+n-2}(\alpha/2)\sqrt{1/m + 1/n}]$$
$$\text{from the computation we have: } t_{1597}(0.025) = 1.646$$
$$\text{the interval is: } [70095.251359575152, 99306.2490951582]$$

iii.

$$\sigma_1^2 \neq \sigma_2^2 \text{ both are unknown}$$
$$\frac{\sigma_1^2}{\sigma_2^2}\text{'s CI is : } [4.571, 7.720]$$
$$\text{so the CI of } \frac{\sigma_1}{\sigma_2} is : [2.138, 2.779]$$

iv.

$$\text{we can get the CI is approximately:}[\bar{X} - \bar{Y} - z_{\alpha/2}\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}, \bar{X} - \bar{Y} + z_{\alpha/2}\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}]$$

$$\text{by using bootstrap we have the interval: } [78723.7, 92352.5]$$

codes are in R code