

Homework 7

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(i)

the interval is: $z_{\alpha/2} \frac{\sigma}{\sqrt{n}} * 2$ so if we want to halve it: $m(n) = 4n$

(ii)

$$z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = \frac{l}{2} \text{ so } n = \frac{4z_{\alpha/2}^2 \sigma^2}{l^2}$$

(iii)

$$n = \frac{4 * 1.645^2}{0.1^2} \approx 1083$$

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(i)

the CDF of X: $F(x) = 1 - e^{-(x-\theta)}$ ($x > \theta$) so the p.d.f of Y1 is $f_Y(x) = n(1-F(x))^{(n-1)} f(x) = ne^{-n(x-\theta)}$ ($x > \theta$)

(ii)

T is the linear transformation of Y1, the p.d.f of T is: $f_T(y) = \frac{1}{2} e^{-\frac{1}{2}y} I_{y \geq 0}$ according to the definition of chi square distribution

(iii)

T has the function related to θ but the distribution is irrelevant so T is the pivot variable $P(\chi_2^2(1-\alpha/2) \leq T \leq \chi_2^2(\alpha/2))$

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$T = (n-1)S^2/\sigma^2$ is the pivot variable which is $\sim \chi_{(n-1)}^2$ so the confidence interval of σ is: $[\sqrt{\frac{(n-1)S^2}{\chi_{(n-1)}^2(\alpha/2)}}, \sqrt{\frac{(n-1)S^2}{\chi_{(n-1)}^2(1-\alpha/2)}}]$

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(i) [0.57202651, 0.16659375, 0.11972054, -0.33681715, 0.04054679, -0.05460625, 0.37101069, -0.00665182]

(ii) shown in the folder

(iii) shown in the folder

(iv) shown in the folder

code: written in python, also in the folder. can be opened by jupyter notebook