Homework 6

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(i)

$$E(X) = \int_\theta^\infty xe^{-(x-\theta)}dx = -xe^{-(x-\theta)}\Big|_\theta^\infty - e^{-(x-\theta)}\Big|_\theta^\infty = \theta - 1$$

so we have:

$$\theta - 1 = \bar{X} = \frac{1}{n}\sum_{i=1}^n x_i \to \hat{\theta} = \frac{1}{n}\sum_{i=1}^n x_i + 1$$

(ii)

$$\text{because } X_i \text{ i.i.d } EX_i^k = EX^k = \mu_k E(\hat{\theta}) = \frac{1}{n}\cdot n\mu + 1 = \mu + 1 = \bar{X} + 1 = \theta$$

actually, $EM_k = \frac{1}{n}\sum EX_i^k = \mu_k$ sample moment estimation is always the unbiased estimation of population.

(iii)

$$E(X^2) = \int_\theta^\infty x^2e^{-(x-\theta)}dx = -x^2e^{-(x-\theta)}\Big|_\theta^\infty + 2(\theta-1) = \theta^2 + 2\theta - 2 D(\hat{\theta}) = E(\hat{\theta})^2 - (E\hat{\theta})^2 = \theta^+ 2\theta - 2 - (\theta-1)^2 = 4\theta - 3$$

2

we will first calculate the k moment of gamma distribution:

$$E(X^k) = \int_0^\infty \frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha+k-1}e^{-\frac{x}{\beta}}dx \text{let } x = \mu\beta E(X^k) = \frac{\beta^k}{\Gamma(\alpha)}\int_0^\infty \mu^{\alpha+k-1}e^{-\mu}d\mu = \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)}\cdot\beta^k \text{when k} = 1 \to EX = \alpha\beta =$$

solve the equation we have:

$$\beta = \frac{\frac{1}{n}\sum_{i=1}^n x_i^2 - \bar{x}^2}{\bar{x}}\alpha = \frac{\bar{x}^2}{\frac{1}{n}\sum_{i=1}^n x_i^2 - \bar{x}^2}$$