Homework 10

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1

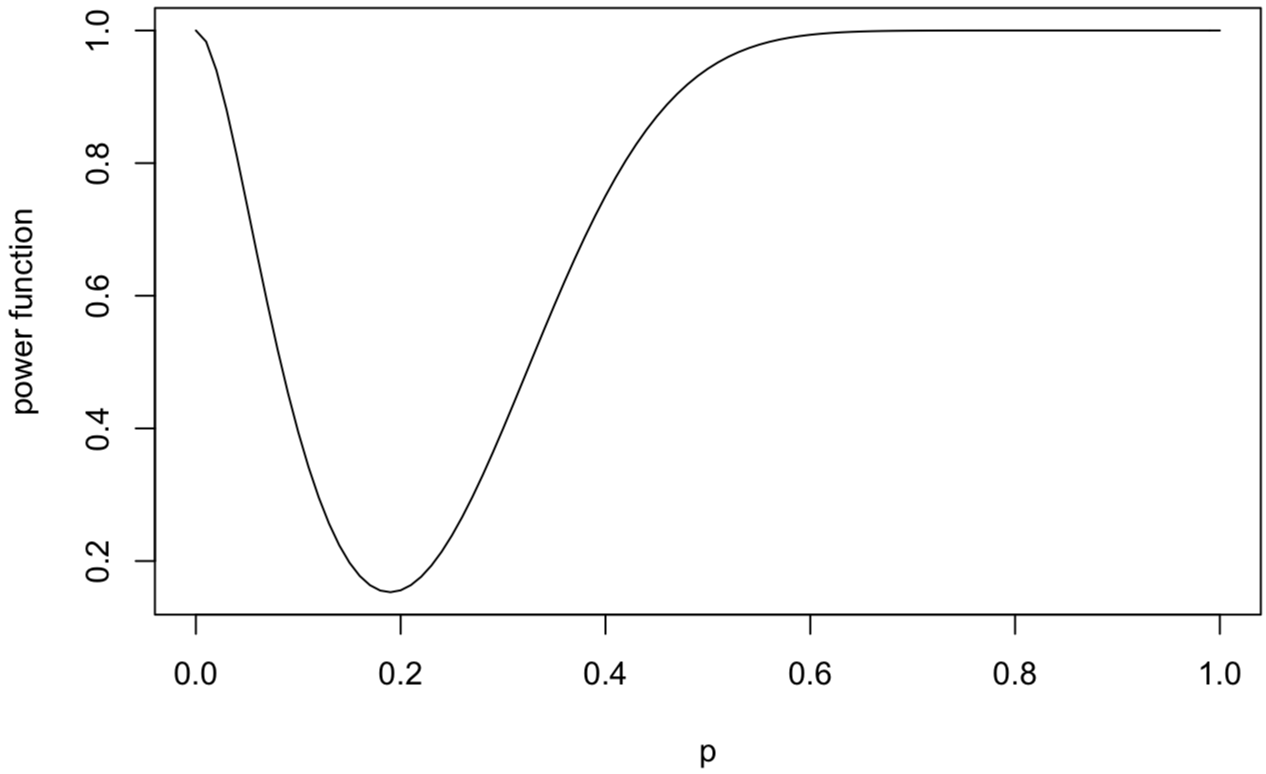
i.

```
n<-11
x<-array(1:11)

for(i in 1:n)
{
p<-0.1*(i-1)
p
power1<-pbinom(1,size=20,prob=p)+1-pbinom(6,size=20,prob=p)
x[i]<-power1
}
x
```

```
##  [1] 1.0000000 0.3941331 0.1558678 0.3996274 0.7505134 0.9423609 0.9935345
##  [8] 0.9997390 0.9999982 1.0000000 1.0000000
```

```
curve(expr=pbinom(1,size=20,prob=x)+1-pbinom(6,size=20,prob=x),from=0,to=1,xlab="p",y
lab="power function")
```



ii.

$$\text{type two error is: } 1 - \beta^*_\phi(0.2) = 0.84413$$
$$\alpha = \beta^*_\phi(0.2) = 0.15587$$

2

i.

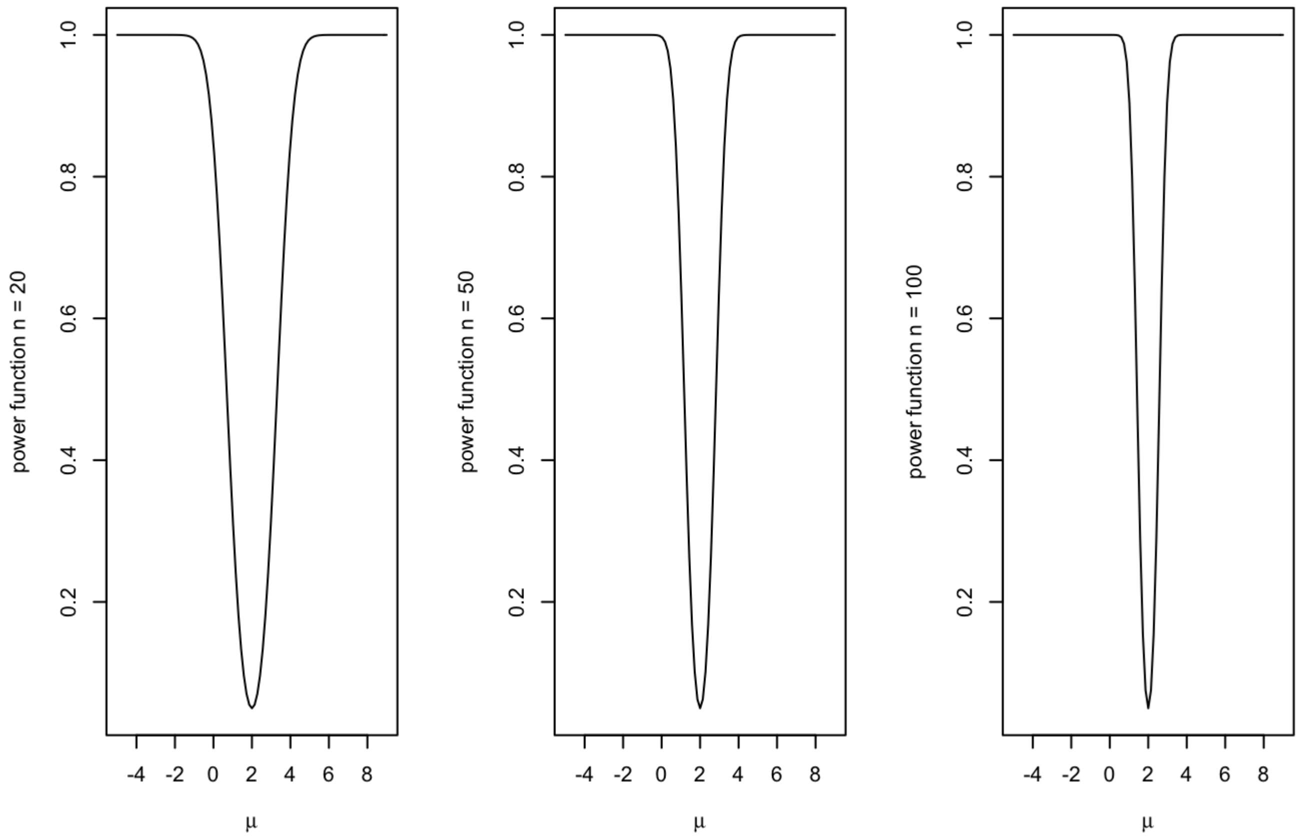
$$\text{we can have: } c = \frac{\sigma u_{\alpha\, 2}}{\sqrt{n}} = \frac{5.88}{n}$$

ii.

$$\pi(\mu) = 1 + \Phi(-u_{\alpha/2} + \frac{\sqrt{n}(\mu_0 - \mu)}{3}) - \Phi(u_{\alpha/2} + \frac{\sqrt{n}(\mu_0 - \mu)}{3})$$

iii.

```
par <- par(mfrow=c(1, 3))
curve(expr=1-pnorm(1.959964+(2-x)*(20^0.5)/3)+pnorm(-1.959964+(2-x)*(20^0.5)/3),from=
-5,to=9,xlab=expression(mu),ylab='power function n = 20')
curve(expr=1-pnorm(1.959964+(2-x)*(50^0.5)/3)+pnorm(-1.959964+(2-x)*(50^0.5)/3),from=
-5,to=9,xlab=expression(mu),ylab='power function n = 50')
curve(expr=1-pnorm(1.959964+(2-x)*(100^0.5)/3)+pnorm(-1.959964+(2-x)*(100^0.5)/3),fro
m=-5,to=9,xlab=expression(mu),ylab='power function n = 100')
```




```
par(par)
```

3

i.

$$X_{(n)} \sim \frac{1}{\theta^n} x^n$$

$$so\ \pi(\theta) = \begin{cases} 1 - \frac{c^n}{\theta^n}, & c < \theta \\ 0, & else \end{cases}$$

so it is increasing as θ increase

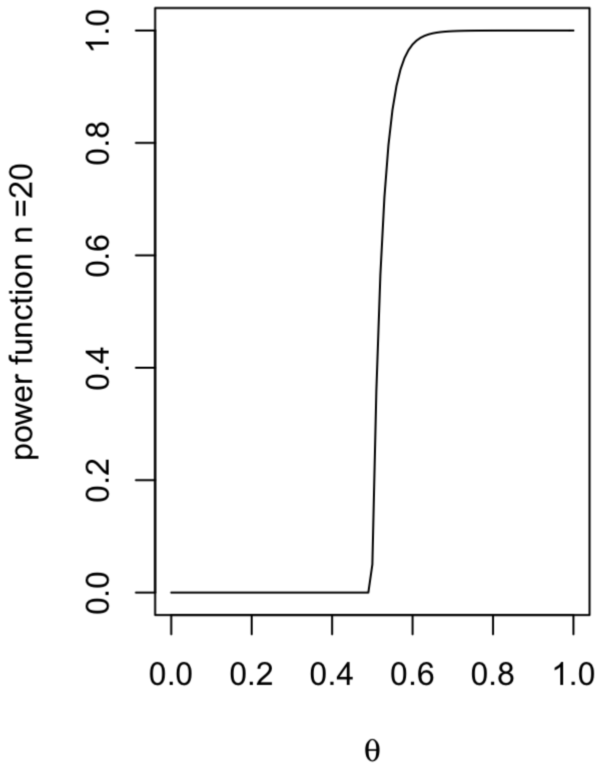
ii.

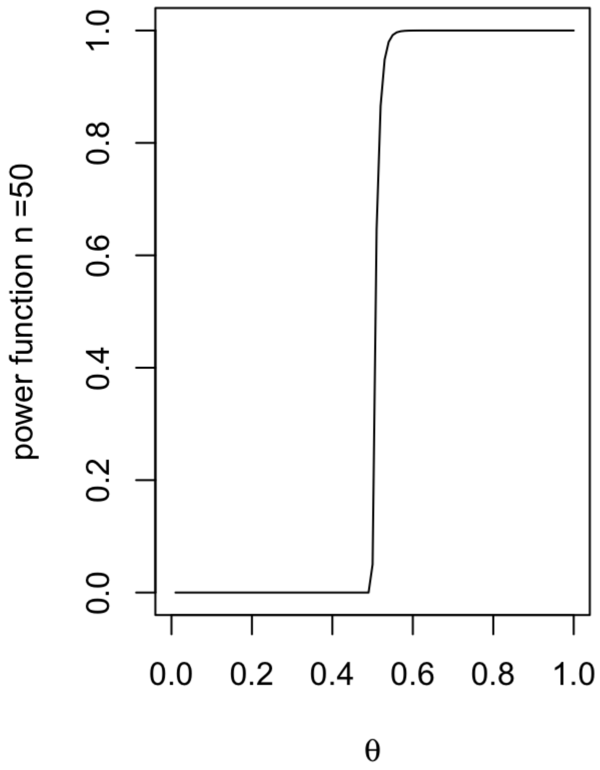
$$1 - \frac{c^n}{\theta^n} \leq 0.05$$

so we can solve that: $c = 0.95^{\frac{1}{n}}/2$

iii.

```
par <- par(mfrow=c(1, 2))
curve(expr=(1-0.95/(2*x)^20)*((1-0.95/(2*x)^20)>0),from=0.00000001,to=1,xlab=expressi
on(theta),ylab = 'power function n =20')
curve(expr=(1-0.95/(2*x)^50)*((1-0.95/(2*x)^50)>0),from=0.00000001,to=1,xlab=expressi
on(theta),ylab = 'power function n =50')
```






```
par(par)
```

iv.

$$\text{from } 1 - \frac{0.95}{1.5^n} \text{ we have: } n = 10$$

4

i.

$$Y = \sum_{i=1}^{n} X_i$$

$$\text{we reject the null hypothesis when: } Y \geq c$$

$$\text{the probablity we make type one error:}$$

$$\alpha(c) = \sum_{k=c}^{n} C_n^k p_0^k (1-p_0)^{n-k} \leq 0.05, p_0 = 0.01$$

ii.

```
x<-qnorm(0.95)*(0.01*0.99/0.08/0.92)^0.5
n<-((x-qnorm(0.01))*((0.08*0.92)^0.5)/0.07)^2
n
```

```
## [1] 128.9144
```

5

i.

$$\pi(1) = 1 - \Phi(u_\alpha + \frac{-\sqrt{n}}{\sigma})$$
$$\text{So n can bedetermined if :}\pi(1) \text{ and } \alpha \text{ are known}$$

ii. we calculate that n = 8.6. So n should be at least 9.

6

Assuming that the test statistic is $T = \sqrt{n}(\bar{X} - \mu_0)/\sigma$

We reject the null hypothesis when T < c, so c = $-u_\alpha$

Now prove that D' = {X : T < $-u_\alpha$} is rejection region.

The power funtion $\pi(\mu) = P_\mu(\sqrt{n}(\bar{X} - \mu_0)/\sigma < -u_{\alpha/2})$

=$P_\mu(\sqrt{n}(\bar{X} - \mu)/\sigma < -u_{\alpha/2} + \sqrt{n}(\mu_0 - \mu)/\sigma$ = $\Phi(-u_{\alpha/2} + \sqrt{n}(\mu_0 - \mu)/\sigma)$

it is increasing as μ increases.

So $\mu > \mu_0, \pi(\mu) < \pi(\mu_0) = \alpha$. which means that D' is the rejection region.

7

$\bar{X} = 17.52, S^2 = (31157 - 17.52^2 \times 100)/99 = 4.666$

Assuming the weight of the packages are distributed as the normal distribution:

we have: $H_0 : \mu = 18$ $H_1 : \mu \neq 18$

the rejection region is $|T| = |\frac{\sqrt{n}(\bar{X}-\mu_0)}{S}| > t_{n-1}(\alpha/2)$

T = 2.083, while $t_{99}(0.025)$ = 1.984

So we reject the null hypothesis

8

$H_0 : \mu_1 - \mu_2 = 0$ $H_1 : \mu_1 - \mu_2 \neq 0$

The test statistics $Tw = \frac{\bar{Y}-\bar{X}-\mu_0}{S_w}\sqrt{\frac{mn}{m+n}}$

$Sw = (\frac{1}{n+m-2}[(m-1)S_1^2 + (n-1)S_2^2])^0.5 = \sqrt{\frac{1}{48}(24 * 6^2 + 24 * 7^2)} = 6.519$

Tw = 2.712 t_{48}(0.025) = 2.011

So we reject the null hypothesis.