fem-1d

August 18, 2019

0.1 Exercícios 2 e 3

```
In [1]: import pandas as pd
        import numpy as np
        import matplotlib.pyplot as plt
```

```
In [12]: def thomas(data):
             N = data.A.size
             C_prime = np.zeros(N)
             D_prime = np.zeros(N)
             for i in xrange(1,N+1):
                 if (i==1):
                     C_prime[i-1] = data.C[i-1] / data.B[i-1]
                     D_prime[i-1] = data.D[i-1] / data.B[i-1]
                 else:
                     C_prime[i-1] = data.C[i-1] / (data.B[i-1]-(data.A[i-1]*C_prime[i-2]))
                     D_prime[i-1] = (data.D[i-1]-(data.A[i-1]*D_prime[i-2])) / \
                                    (data.B[i-1]-(data.A[i-1]*C_prime[i-2]))

             x = np.zeros(N)
             for i in xrange(N-1,-1,-1):
                 if (i==N-1):
                     x[i] = D_prime[i]
                 else:
                     x[i] = D_prime[i] - (C_prime[i]*x[i+1])
             return C_prime,D_prime,np.round(x,4)
```

```
In [13]: path = './src/data.txt'
         data = pd.read_csv(path,sep='\t',dtype=float)
         data['C_prime'],data['D_prime'],data['c_k'] = thomas(data)
```

```
In [14]: data.head(20)
```

```
Out[14]:         A       B       C         D    C_prime    D_prime     c_k
         0  0.0000  4.4258  7.2007   37.4490   1.626983   8.461521  3.2210
         1  1.7557  4.4258  7.2007   62.8916   4.588460  30.609513  3.2210
         2  9.8515  5.3964  7.2893   78.2298  -0.183117   5.610090  5.9690
         3  5.0398  6.2351  3.2890   74.7451   0.459488   6.492253  1.9600
         4  0.7593  3.5378  9.6730  120.2838   3.033324  36.173550  9.8637
```

```
5    5.7762  9.0621  0.9250  139.4377   -0.109351     8.217056  8.6736
6    8.8807  5.3009  4.3282  116.3878    0.690081     6.921952  4.1751
7    9.4295  2.3142  1.7413   53.8097   -0.415295     2.733380  3.9805
8    3.3948  5.2373  2.3549   30.1569    0.354272     3.140841  3.0029
9    3.0899  5.7860  9.3674   37.8386    1.996746     5.996955  0.3893
10   2.0695  4.1548  8.8003   77.5434  390.519914  2890.312488  2.8084
11   7.0594  8.8422  4.1095  106.7115   -0.001495     7.386173  7.3940
12   9.2154  5.6766  5.9753  101.0697    1.050070     5.799815  5.2334
13   2.2369  9.3450  9.6948   79.0538    1.385744     9.445293  0.5394
14   6.7204  8.3313  8.2718   92.1310   -8.428113   -29.196348  6.4268
15   9.2321  6.1959  2.0718  101.0969    0.024663     4.412120  4.2267
16   1.0288  1.8913  8.6296   53.5542    4.624833    26.268452  7.5180
17   3.0574  4.7805  8.1953   70.4819   -0.875616     1.050409  4.0543
18   5.6834  6.1846  1.7236   60.3024    0.154430     4.868034  3.4306
19   6.3482  6.7943  0.6228   89.4178    0.107122    10.064507  9.3080
```

0.2 Exercício 4

Resolver o problema de contorno usando o método de elementos finitos

$$-\frac{d^2y}{dx^2} + 20\frac{dy}{dx} + 10y(x) - 1 = 0$$

com as condições de contorno essenciais

$$y(0) = 0 \;\; y(1) = 0$$

```
In [15]: def baseUp(x):
             x0, x1 = x[0],x[-1]
             w = (x-x0) / (x1-x0)
             dw = 1.0 / (x1-x0)
             return w,dw

         def baseDw(x):
             x0, x1 = x[0],x[-1]
             w = 1 - (x-x0) / (x1-x0)
             dw = - 1.0 / (x1-x0)
             return w,dw

         def calcIntegral(y,dx):
             aux = y[1:-1] * dx
             r = np.sum(aux)
             r = r + (y[0]*dx/2.) + (y[-1]*dx/2.)
             return r

         def calcK(wA,dwA,wB,dwB):
             auxK = (dwA*dwB) + (20.*wA*dwB) + (10.*wA*wB)
             return auxK
```

```
def rigidity(N,x):
    K = np.zeros(shape=(N,2,2))
    w = np.zeros(shape=(2,np.size(x[0])))
    dw = np.zeros(shape=(2,np.size(x[0])))
    for e in xrange(N):
        for n in xrange(2):
            for m in xrange(2):
                w[0],dw[0] = baseDw(x[e])
                w[1],dw[1] = baseUp(x[e])
                if (e%2==0):
                    auxK = calcK(w[n],dw[n],w[m],dw[m])
                    K[e,n,m] = calcIntegral(auxK,x[e,1]-x[e,0])
                else:
                    auxK = calcK(w[n],dw[n],w[m],dw[m])
                    K[e,n,m] = calcIntegral(auxK,x[e,1]-x[e,0])
    return K

def force(N,x):
    F = np.zeros(N)
    wAUX,dwAUX = baseDw(x[0])
    F[0] = calcIntegral(wAUX,x[0,1]-x[0,0])
    wAUX,dwAUX = baseUp(x[-1])
    F[-1] = calcIntegral(wAUX,x[-1,1]-x[-1,0])
    for e in xrange(1,N-1):
        wUP,dwUP = baseUp(x[e-1])
        wDW,dwDW = baseDw(x[e])
        F[e] = calcIntegral(wUP,x[e-1,1]-x[e-1,0]) + calcIntegral(wDW,x[e,1]-x[e,0])
    return F
```

```
In [16]: N = 100
         a = 0.0
         b = 1.0

         nn = 1001

         x = np.zeros(shape=(N,nn))
         for e in xrange(N):
             x[e] = np.linspace(e*b/N,(e+1)*b/N,nn)

         K = rigidity(N,x)
         F = force(N+1,x)

         # print K

         #condicoes de contorno
         K[0,0,0] = 1.
         K[0,0,1] = 0.
         K[-1,-1,1] = 1.
         K[-1,-1,0] = 0.
         F[0] = 0.
         F[-1] = 0.
```

```
In [17]: path2 = './src/exercicio4.txt'
         f = open(path2,'w')
         f.write("A\tB\tC\tD\n")
         f.write("0\t"+str(K[0,0,0])+"\t"+str(K[0,0,1])+"\t"+str(F[0])+"\n")
         for e in xrange(N-1):
             aux1 = K[e,1,0]
             aux2 = K[e,1,1] + K[e+1,0,0]
             aux3 = K[e+1,0,1]
             aux4 = F[e+1]
             f.write(str(aux1)+"\t"+str(aux2)+"\t"+str(aux3)+"\t"+str(aux4)+"\n")
         f.write(str(K[-1,-1,0])+"\t"+str(K[-1,-1,1])+"\t0.0\t"+str(F[-1])+"\n")
         f.close()
```

```
In [18]: e = 2.71828182846
         x = np.linspace(0,1,N+1)

         aux1 = -e**10. + e**np.sqrt(110.)
         aux2 = e**((10.+np.sqrt(110.))*x)
         aux3 = 10.*(e**10.)*(1.-e**(2.*np.sqrt(110.)))

         aux4 = e**(-10.+np.sqrt(110.))
         aux5 = e**(10.+np.sqrt(110.)) -1.
         aux6 = e**((10.-np.sqrt(110.))*x)
         aux7 = 10.*(1.-e**(2.*np.sqrt(110.)))

         y = (aux1*aux2/aux3) + (aux4*aux5*aux6/aux7) + 1./10.

         path = './src/exercicio4.txt'
         data = pd.read_csv(path,sep='\t',dtype=float)
         data['C_prime'],data['D_prime'],data['c_k'] = thomas(data)

         data.head()
```

```
Out[18]:             A           B          C     D   C_prime   D_prime     c_k
         0    0.000000    1.000000   0.000000  0.00  0.000000  0.000000  0.0000
         1 -109.983333  200.066667 -89.983333  0.01 -0.449767  0.000050  0.0005
         2 -109.983333  200.066667 -89.983333  0.01 -0.597500  0.000103  0.0010
         3 -109.983333  200.066667 -89.983333  0.01 -0.669760  0.000159  0.0015
         4 -109.983333  200.066667 -89.983333  0.01 -0.711869  0.000217  0.0019
```

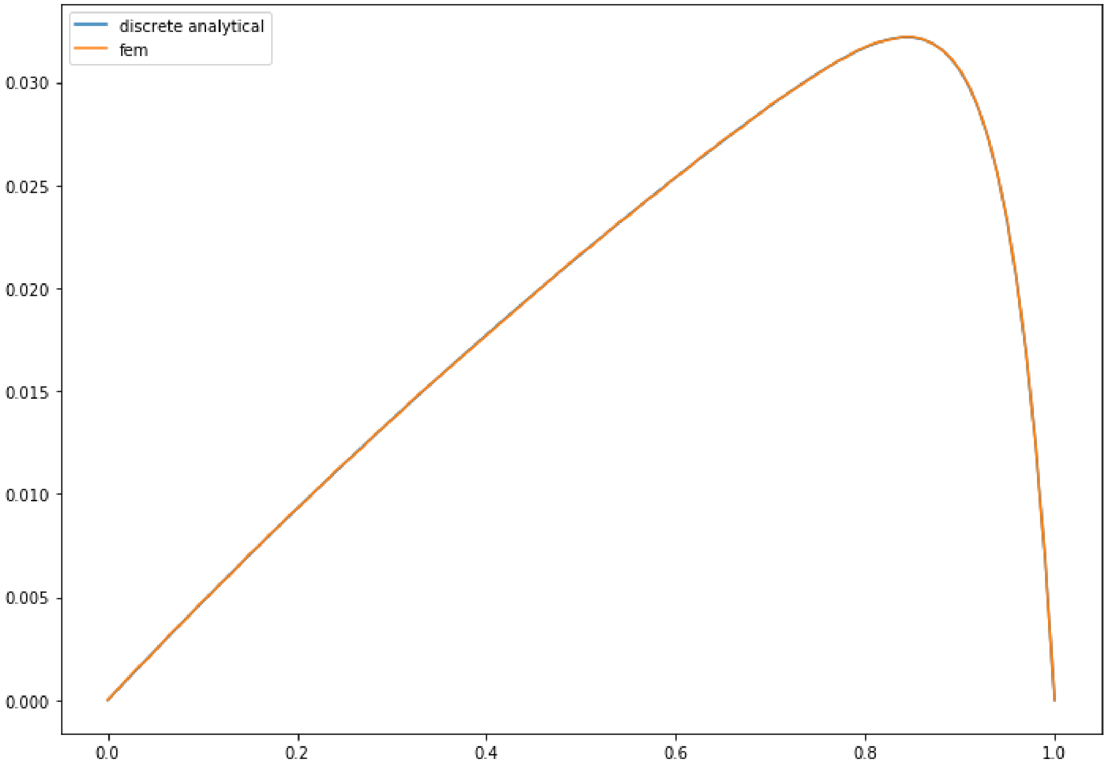
```
In [19]: data.tail()
```

```
Out[19]:              A           B          C     D   C_prime   D_prime     c_k
         96   -109.983333  200.066667 -89.983333  0.01 -0.814171  0.006704  0.0204
         97   -109.983333  200.066667 -89.983333  0.01 -0.814171  0.006762  0.0169
```

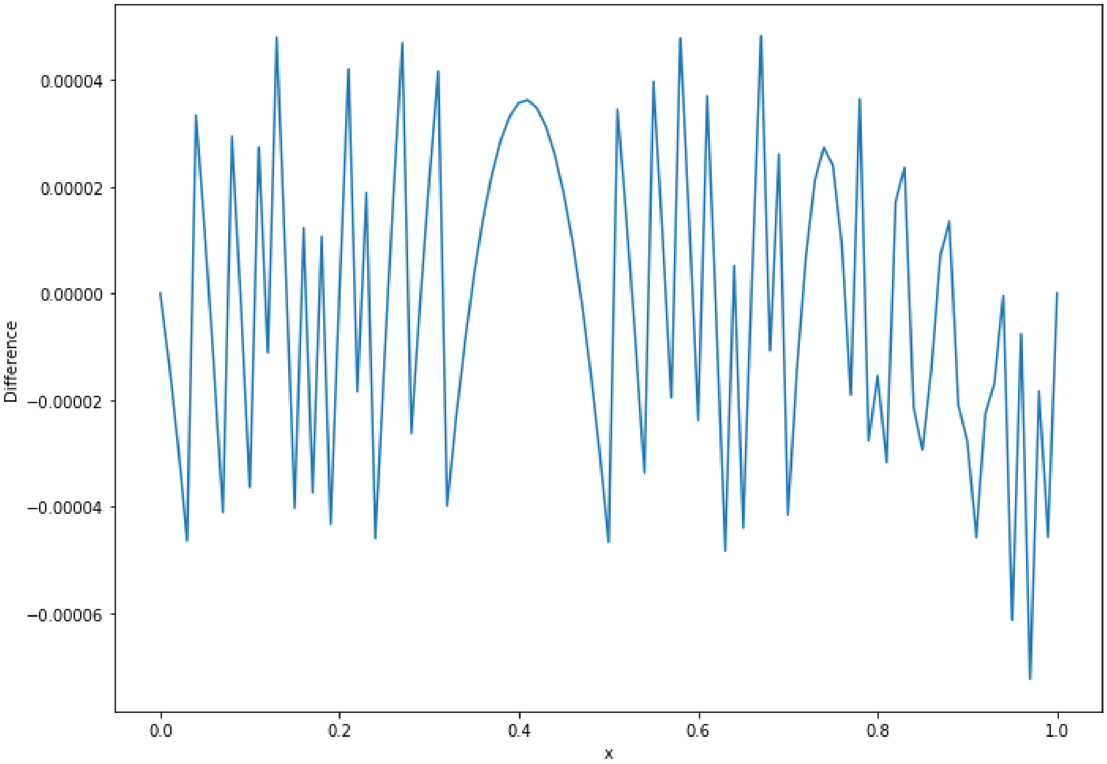
```
98   -109.983333  200.066667 -89.983333  0.01 -0.814171  0.006819  0.0124
99   -109.983333  200.066667 -89.983333  0.01 -0.814171  0.006876  0.0069
100     0.000000    1.000000   0.000000  0.00  0.000000  0.000000  0.0000
```

```
In [20]: xx = np.linspace(0,1,N+1)

         plt.figure(figsize=(10,7))
         plt.plot(x,y,"-",label="discrete analytical")
         plt.plot(x,data.c_k,"-",label="fem")

         plt.legend()
         plt.tight_layout()
         print(y.max(),data.c_k.max())
```

```
(0.03217861744836471, 0.0322)
```



```
In [21]: plt.figure(figsize=(10,7))
         plt.plot(x,(y[xx==x]-data.c_k))
         plt.xlabel("x")
         plt.ylabel("Difference")
         plt.tight_layout()
```


0.00004
0.00002
0.00000
−0.00002
−0.00004
−0.00006
Difference
0.0
0.2
0.4
0.6
0.8
1.0
x