## Numerical Integration Homework 2022-2023

Practical exercises to do at home **before 10th of December**. These exercises are part of the continuous evaluation of the course. They are not gradabable. For any question, contact with Elena Formoso.

1. Write a program with FORTRAN to solve the following integral,

$$I=\int_{-1}^1 sin(x+1)$$

## use the next Newton-Cotes methods introduced in class:

(a) Rectangle method: Simple and Composite rule.

At each iteration step the program must show: the iteration step, subinterval number, integral value and the difference between a result and the preceding one.

Stop when convergence is reached, when the difference between a result and the preceding one is smaller than  $10^{-8}$ .

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- 2. This exercise does not involve an integration calculation. It is an exercises to practice with random numbers, since they are used in the Monte Carlo method: write a FORTRAN program to calculate numerically the  $\pi$  number. Some remarks:
  - The program must generate random points, that is, generate randomly the x and y components of the points. There are different alternatives to generate pseudo random numbers; one simple method is by using the "random\_numbers" intrinsic subroutine:

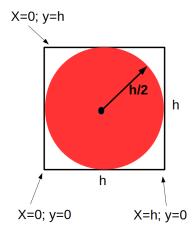
call random\_number(x) call random\_number(y)

so that the x and y variables will contain a random number between 0 and 1.

- The idea is to generate many random points that will fall inside a rectangle
  of h\*h size. Then, check how many of these points fall inside the red circle
  in the figure, whose radius is h/2. To do that, calculate the distance between
  the circle's (or rectangle's) center and the random points. If (dist < h/2), the
  point is inside the circle.</li>
  - (a) Note that the system's center is at the (h/2, h/2) point, but random points goes from (0,0) to (1,1). Change the values appropriately.
  - (b) If the number of random points are sufficiently large, the number of points inside the circle ( $N_{circle}$ ) will be equal to the circle's area, and the total number of random points ( $N_{Total}$ ) equal to the rectangle's area. Therefore:

$$\frac{N_{circle}}{N_{Total}} = \frac{A_{circle}}{A_{rectangle}} = \frac{\pi(\frac{h}{2})^2}{h^2} = \frac{\pi}{4} \rightarrow \pi = 4 \frac{N_{circle}}{N_{Total}}$$

• The program should include a cycle in which initially 10 random points are generated, and the number of points is increased by 10 until a maximum of  $10^6$  random points. For each cycle, print the total number of points, number of points inside the circle, the computed  $\pi$  number, and the error in comparison with the real  $\pi$  number.



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