Classroom Exercises
Santander 2022-2023

Students must do these exercises using FORTRAN program during the intensive course.

Part I: Integration of Functions

1. **Write a program to solve the following integral applying the composite trapezoidal method:**

$$I = \int_2^3 \frac{dx}{1+x}$$

   The program should:

   - Start with 2 subintervals, N=2
   - Automatically double the number of subintervals.
   - Show the iteration step, the integral value and the difference between a result and the preceding one at each iteration step.
   - Stop when the convergence is reached. When the difference between a result and the preceding one is smaller than a threshold value of 10^{-8}.
   - Show the number of iterations needed, subinterval number, number of abscissa points, final value of the quadrature.

2. **Write an integration program to solve the following integral using the Romberg method:**

$$I = \int_1^2 \frac{dx}{x}$$

   The program must show two different results:

   (a) the entire triangular table up to $R_{10,10}$

   (b) The element value and its position in the table, in which the method has converged. Convergence criteria: the difference between $R(k,j)$ and $R(k,j-1)$ must be lower than 10^{-8}.

Part II: Root-finding and Function Optimization

1. **Write a program that finds the root of the following function using one-dimensional Newton-Raphson method:**

$$f(x) = 3e^x - 4cos(x)$$

The program should:

- Take an initial point within the $[0, 1]$ interval.
- Impose a convergence of 10^{-8}.
- Calculate the gradient numerically (use subroutines) applying the central finite difference approximation.
- At each iteration step, print the value of x, f(x) and gradient.

2. **Write a program that minimize the following two-dimensional function using the Steepest Descent method:**

$$f(x, y) = 25x^2 + y^2$$

To make the program simple, use a fix step size value of 0.5.

- Note 1: At the minimum, the gradient vector should be zero, but due to the method's limitation (and fix step size) the program will not converge. Set a maximum iteration to finish the program ($maxiter$=40).
- Note 2: save coordinates in the $coord$($maxiter$,2) matrix. Initial coordinates: $x = 1$, $y = 3$.
- Note 3: Calculate the gradient numerically (central finite difference approximation), which must be a vector with two elements. Normalized the gradient vector.
- For each iteration step, the program must show the values of x, y, function, gradient(x), gradient(y) and normalized gradients.

Optionally, represent the x, y coordinates to observe the zig-zag pathway characteristic of the Steepest Descent method.