Numerical Integration Homework
2022-2023

Practical exercises to do at home **before 10th of December**. These exercises are part of the continuous evaluation of the course. They are not gradable. For any question, contact with Elena Formoso.

1. **Write a program with FORTRAN to solve the following integral,**

$$\boldsymbol{I = \int_{-1}^{1} sin(x+1)}$$

use the next Newton-Cotes methods introduced in class:

(a) Rectangle method: Simple and Composite rule.

At each iteration step the program must show: the iteration step, subinterval number, integral value and the difference between a result and the preceding one.

Stop when convergence is reached, when the difference between a result and the preceding one is smaller than 10^{-8}.

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2. This exercise does not involve an integration calculation. It is an exercises to practice with random numbers, since they are used in the Monte Carlo method: **write a FORTRAN program to calculate numerically the π number.** Some remarks:

   - The program must generate random points, that is, generate randomly the x and y components of the points. There are different alternatives to generate pseudo random numbers; one simple method is by using the "random_numbers" intrinsic subroutine:

     call random_number(x)

     call random_number(y)

   so that the x and y variables will contain a random number between 0 and 1.

   - The idea is to generate many random points that will fall inside a rectangle of h*h size. Then, check how many of these points fall inside the red circle in the figure, whose radius is h/2. To do that, calculate the distance between the circle's (or rectangle's) center and the random points. If (dist $<$ h/2), the point is inside the circle.
     (a) Note that the system's center is at the (h/2, h/2) point, but random points goes from (0,0) to (1,1). Change the values appropriately.
     (b) If the number of random points are sufficiently large, the number of points inside the circle (N_{circle}) will be equal to the circle's area, and the total number of random points (N_{Total}) equal to the rectangle's area. Therefore:

$$\frac{N_{circle}}{N_{Total}} = \frac{A_{circle}}{A_{rectangle}} = \frac{\pi(\frac{h}{2})^2}{h^2} = \frac{\pi}{4} \rightarrow \pi = 4\frac{N_{circle}}{N_{Total}}$$

   - The program should include a cycle in which initially 10 random points are generated, and the number of points is increased by 10 until a maximum of 10^6 random points. For each cycle, print the total number of points, number of points inside the circle, the computed π number, and the error in comparison with the real π number.