

Computational Techniques I

Course zero

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(WITH MATERIAL FROM PREVIOUS COURSES
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Outline

- 1. Introduction
- 2. Data types
- 3. Declaration statements
- 4. Operators
- 5. Arrays
- 6. Program control: IF statements and DO loops
- 7. Formatted variable
- 8. I/O statements
- 9. Indexed variables
- 10. Procedures
- 11. Libraries
- 12. Other statements
- 13. Makefiles

Introduction

(Design and execution of a computer code)

High-level programming languages

- Computers work in binary system (machine language)
- Initially, numerical codes were used to represent operations such as “add the number in register A to the number in register B”
- By the beginning of 50’s it was possible to use sets of keys to replace the numerical code for expressions like “RCL A; SUM B” (assembler)
- Clearly some kind of programming language which allowed a series of calculations to be expressed in something resembling mathematical notation was required
- In 1956 the first manual of the IBM Mathematical FORmula TRANslating System appears by John Backus *et al.* (high-level language)
- With it, also a translation program that converted the “text” into the numerical codes which the computer understood (compiler)

FORTRAN standards

FORTRAN (I-V, 66)



FORTRAN 77 (A lot of extensions: allocate, case-insensitive...)



FORTRAN 90/95



Extensions standardization
Pointers, modules
Matrix features

FORTRAN 2003 (Object-oriented, C-interoperability...)



FORTRAN 2018 (More strict rules)

Format of the fortran statements

FREE SOURCE FORMAT (f90 style, file.f90)

no limitation on placing in the line.

& at the end of the line for continuation.

Blanks are significant

! commented the line

Format of the fortran statements

FIXED SOURCE FORMAT (f77 style, file.f)

c: commented the line

Each column in each line counts!:

Column 1-5 are used for statement label number (usually blank)

Column 6: used ONLY to indicate line continuation (if non-blank character)

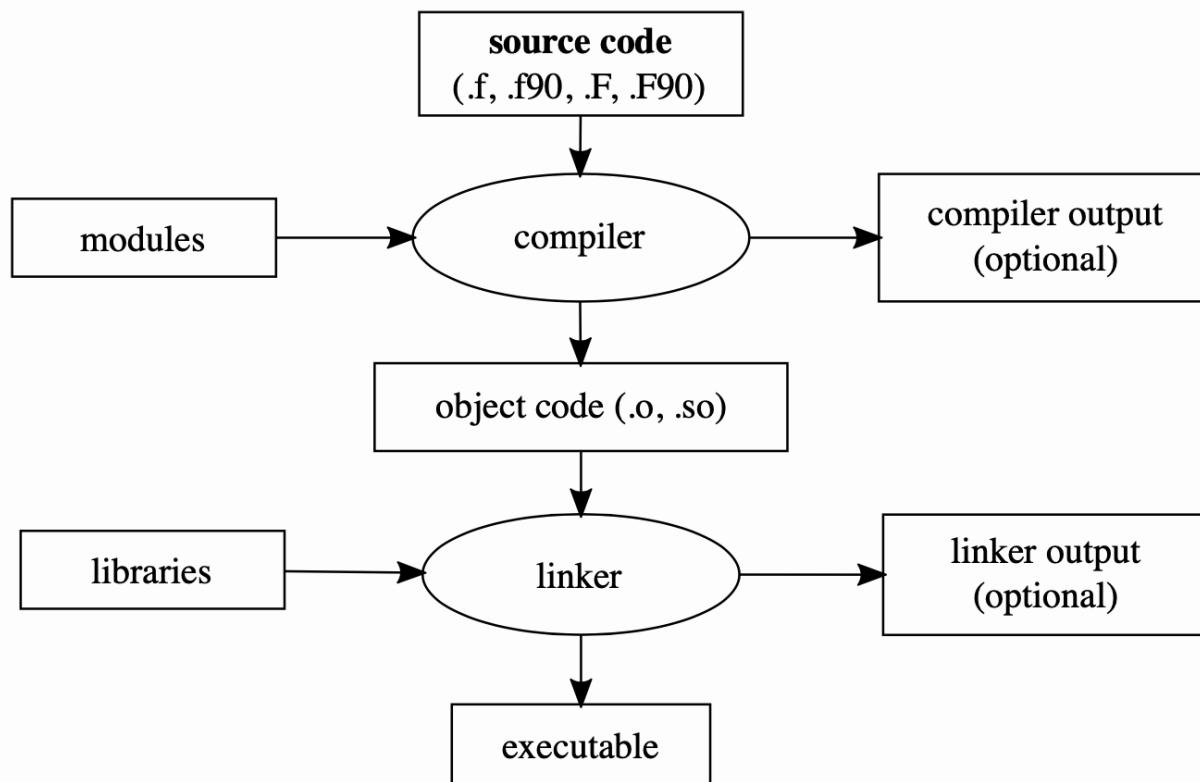
Column 7-72: fortran statements.; Column 73-80: ignored

For continuation*: & (or number) at column 6 in the next line.

Possible to indicate some lines of optional compilation (a D in the 1st column)

Several instructions in the same line (separated by ;)*

Running FORTRAN



Structured Programs

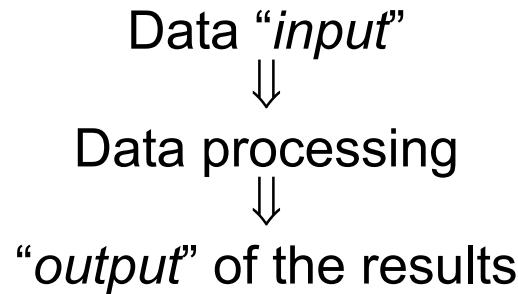
The design of a program has various phases:

- Definition of the problem → Input data and results
- “*top-down*” design of the program → Divide the program in modules and files.
- Design of each module → Selection of the best algorithm and connection with the rest.

Basic goal: divide the problem into as many basic tasks as possible.

Algorithms and program flow

- Selection of algorithm:
 - Mathematical or numeric reasons
 - Computational
- Structure of an algorithm:



- Type of logical structures:
 - Execution control
 - Assignment (=)
 - Decision (IF)
 - Repetition (DO)

The structure of a fortran program

!----- Declaration area -----

PROGRAM Example

! Declaration statements

!

IMPLICIT NONE

!

INTEGER :: s ! integer variables

REAL :: a, b, sum, product, power ! real variables

!

!----- Execution area -----

s = 2

a = 4.

b = 6.

sum = a + b

product = a*b

power = a**s

!

WRITE(*,*) sum, product, power

!----- End area -----

!

STOP

END PROGRAM Example

!-----

Data types

Declaration of variables

! ----- Declaration area -----

PROGRAM Example

! Declaration statements

!

IMPLICIT NONE

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INTEGER :: s ! integer variables

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! ----- Execution area -----

s = 2

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sum = a + b

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!

WRITE(*,*) sum, product, power

! ----- End area -----

!

STOP

END PROGRAM Example

! -----

Data types

- A data type can be constant or variable.
- A *data type* implies:
 - A *finite* set of values
 - A way to represent them
 - A set of allowed operations among them
- There are *five basic intrinsic data types* □ defined in fortran by default
- There can be *defined derived data types* □ defined by the programmer
- Intrinsic Data Types:
 - Numeric: Integer, Real and Complex
 - Non-numeric: Character and logical

Memory storage. Units

- **bit** : Minimum unit to represent information
(binary + digit : bit)
- **byte**: a group of 8 bits, that constitutes the minimum unit of storage in memory
- **kilobyte (kB)** : 1024 bytes
- **megabyte (MB)** : set of 1024x1024 bytes
- **gigabyte (GB)** : set of 1024x1024x1024 bytes
- **terabyte (TB)**: 1024 GB

Memory storage. Variables

- **Integer Numbers**: they imply a translation from decimal -> binary. Exact representation, but in practice, there are limits to the magnitude of the integer number we can represent.
- **Real numbers**: represented by a mantissa + exponent. For instance, $110.5 = +0.1105 \times 10^3$. Both are represented as integer numbers. In general, representation not exact, as we can not represent a mantissa of infinite digits.
- **Complex numbers**: represented as a pair of real numbers (Re, Im).
- **Alphanumeric characters**: They are stored character by character as real numbers following the ASCII code.
- **Logical variables**: They are stored in one bit; Its value (.TRUE. or .FALSE.) is identified by 0 or 1.

Memory usage. The KIND attribute

The amount of memory stored for a variable can be specified with KIND:

KIND=n (n=1,2,4,8)

where n is the number of bytes (#bits = 8 n) stored.

Integer variables

- Declaration of an integer variable in the “Declaration area”:

INTEGER :: s

INTEGER([KIND=]n) :: s

INTEGER*n :: s

- Frequent subtypes $n=\{1,2,4,8\}$.

- O.S. of 32 bits → maximum $n=4$
 - O.S. of 64 bits → maximum $n=8$

- The range of integer numbers are: -2^{m-1} to $2^{m-1}-1$ ($m= 8 n$)

$n=1$ -128 to 127 (using “two’s complement”)

$n=2$ -32768 to 32767

-  $s=1$; $s=0$; $s=-33$; $s=+33$; $s=-67_2$ (equivalent to KIND=2)
-  $s=9999999999999999$; $s=3.14$; $s=32,767$; $s=33_3$

Real variables

- Declaration of real variables in the “Declaration area”:

REAL :: variables

REAL([KIND=]n) :: variables (with n= 4,8 or 16)

REAL*n :: variables

DOUBLE PRECISION

REAL(KIND=4)

3.14159 ; 3.14159_4; 62. ; -.00127 ; +5.0E3 ; 2E-3_4

REAL(KIND=8) (or DOUBLE PRECISION):

123456789D+5 ; 123456789E+5_8 ; +2.7843D00 ; 2E200_8 ; 2.3_8

REAL(KIND=16) :

123456789Q4000 ; -1.23Q-400 ; +2.72Q0 ; 1.88_16

- Valid ranges: 10^{37} - 10^{-37} (_4), 10^{307} - 10^{-307} (_8) and 10^{4931} - 10^{-4931} (_16)

Complex variables

- Declaration of complex variables in the “Declaration area”

COMPLEX :: c

COMPLEX([KIND=]n) :: c

COMPLEX*n :: c

DOUBLE COMPLEX

- Valid examples of **COMPLEX(KIND=4)** :

(1.7039,-1.70391) ; (44.36_4,-12.2E16_4) ; (+12739E3,0.) ;
(1.,2.)

- Valid examples of **COMPLEX(KIND=16)** :

(1.7039,-1.7039Q2) ; (547.3E0_16,-1.44) ; (+12739Q3,0.Q0)

Logical variables

- Declaration of logical variables in the “Declaration area”

LOGICAL :: I

LOGICAL([KIND=]*n*) :: I (with *n*=1,2,4,8)

LOGICALn* :: I**

- The logical constants are written as

.TRUE.[_*n*]

.FALSE.[_*n*]

Character variables

- Declaration of character variables in the “Declaration area”

CHARACTER :: a

CHARACTER([KIND=]*n*) :: a

(with $0 \leq \text{len} \leq 2000$, len being #characters)

CHARACTER([LEN=]*len*) :: a

CHARACTER([LEN=*len*],[*KIND=n*]) :: a

CHARACTER(KIND=*n* [,LEN=*len*]) :: a

CHARACTER*len[,] :: a

- ## ■ Valid examples:

"Orthogonal Vectors "

'YESTERDAY'S RESULTS'

“This isn’t wrong”

1

Derived data types

```
type atom
    character(len=2) :: name
    Integer :: Z
    real :: mass
    real :: q
    real, dimension(1:3) :: coord
end type atom
```

- Therefore, we can now define variables of *atom* type in the following way:
`type(atom) :: carbon`
- Assigning values to these variables:
`carbon = atom("C ", 6, 12.011, 0.35, (/0.0, 1.0, 2.0/))`
- We can select an element, for instance the atomic number: `carbon%Z`
- These structures can be combined into new *derived data types*

```
type molecule
    integer :: natom
    type(atom), dimension(1:1000) :: at
end type molecule
```

Declaration statements

Default declaration

If variables are not declared as belonging to a specific type, they are implicitly declared by default as follows:

Variables starting with : I, J, K, L, M, N:

Integer (_4) type by default

Variables starting with : A to H or O to Z

Real (_4) type by default

IMPLICIT statement

However, it is recommended to declare all variables to be used in a program. Thus, to prevent defaults, the following statement is used

IMPLICIT NONE

An error will be produced if an invoked variable has not been declared

IMPLICIT statement

Sacrificing safeness in the sake of simplicity

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

or even

IMPLICIT DOUBLE PRECISION (A-H,O-Y)

IMPLICIT LOGICAL Z

**No error will be generated but no checking
will be done**

Parameter statement

It allows to give a name to a particular constant

! define and assign the speed of light and its square

```
REAL, PARAMETER :: C = 2.9979251E8,C2= C**2
```

! define π number

```
real*8, parameter :: pi = acos(-1.0d0)
```

! define “useful” real numbers

```
real*8, parameter :: one = 1.0d0, two = 2.0d0
```

! not restricted to numbers

```
logical, parameter :: dbgprt = .true.
```

```
character(15), parameter:: separation = '-----'
```

```

program prog_1
!
implicit none
!
integer :: i
character (len=12) :: hi
character (len=20) :: hi2
character (len=10) :: nam
!
!
hi = 'Hello, I am'
hi2 = hi
nam = "Alfredo"
!
! 1) Free format
!
print*, hi,nam
write(6,*) hi,nam
print*, 
!
print*, hi2,nam
write(6,*) hi2,nam
!
write(6,*) (' ',i=1,4)
!
! 2) Fixed format
!
print '(2a)', hi,nam
write(6,'(a,3x,a)') hi, nam
print*, 
!
print '(2a)', hi2,nam
write(6,'(a12,a)') hi2, nam
!
!
stop
end program prog_1

```

Hello, I am	Alfredo
Hello, I am	Alfredo
Hello, I am	Alfredo
Hello, I am	Alfredo
Hello, I am	Alfredo
Hello, I am	Alfredo
Hello, I am	Alfredo
Hello, I am	Alfredo

EXAMPLE 1

```

program prog_2
!
implicit none
!
integer*4 :: i, num, fact, isum
integer (kind = 8) :: bigfac
real*8 :: xlim4, xlim8
real*8, parameter :: one = 1.0d0, two = 2.0d0
!
write(6,*) 'Number?'
read(5,*) num
write(6,*)
write(6,*)
!
isum = 0
fact = 1
bigfac = 1_8
!
do i = 1,num
    isum = isum + i
    fact = fact * i
    bigfac = bigfac * i
    write(6,'(i3,2i25)') i,fact,bigfac
end do
!
write(6,*)
write(6,100) 'isum :', isum
write(6,100) 'fact :', fact
write(6,100) 'bigfac :', bigfac
100 format(a,i25)
!
xlim4 = two**31 - one
xlim8 = two**63 - one
write(6,'(//,a,d15.4)') 'Maximum integer*4 number', xlim4
write(6,'(a,d15.4)') 'Maximum integer*8 number', xlim8
!
stop
end program prog_2

```

Number?
22

1	1	1
2	2	2
3	6	6
4	24	24
5	120	120
6	720	720
7	5040	5040
8	40320	40320
9	362880	362880
10	3628800	3628800
11	39916800	39916800
12	479001600	479001600
13	1932053504	6227020800
14	1278945280	87178291200
15	2004310016	1307674368000
16	2004189184	20922789888000
17	-288522240	355687428096000
18	-898433024	6402373705728000
19	109641728	121645100408832000
20	-2102132736	2432902008176640000
21	-1195114496	-4249290049419214848
22	-522715136	-1250660718674968576

isum : 253
fact : -522715136
bigfac : -1250660718674968576

Maximum integer*4 number 0.2147D+10
Maximum integer*8 number 0.9223D+19

EXAMPLE 2

Exercises

1. Write a “Hello world” program and modify it to experiment with different data types and arithmetic operations
2. Fibonacci numbers are a sequence of integers defined by the recurrence relation $F_n = F_{n-1} + F_{n-2}$ with the initial values $F_0=0$ and $F_1=1$. Print out the first 20 Fibonacci numbers. Using trial and error, determine the last $F_n < 1000000$. Print out the first 100 Fibonacci numbers.
3. Use the Fibonacci numbers above to estimate the golden ratio

$$\varphi = \frac{1 + \sqrt{5}}{2} = 1.61803 \dots = \lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n}$$

4. Numerically verify that $\varphi^2 - \varphi = \varphi - 1/\varphi = 1$

Arrays

Defining arrays

There are several forms of defining arrays:

1. Indicating between parenthesis the corresponding dimensions (The maximum number of dimensions is 7)

integer :: idx(23)

real :: amat(4,6)

real(kind=8) :: dpmatrix(3,4,5,6)

character (len = 80) :: screen(24)

2. Using the dimension attribute

3. Using the dimension statement (In this case, the type is determined by default naming or previous implicit statement)

Defining arrays

There are several forms of defining arrays:

1. Indicating between parenthesis the corresponding dimensions (The maximum number of dimensions is 7)
2. Using the dimension attribute

```
integer , dimension(23) :: idx  
real , dimension (4,6) :: amat  
real(kind=8) , dimension (3,4,5,6) :: dpmatrix  
character (len = 80), dimension (24) :: screen
```

3. Using the dimension statement (In this case, the type is determined by default naming or previous implicit statement)

Defining arrays

There are several forms of defining arrays:

1. Indicating between parenthesis the corresponding dimensions (The maximum number of dimensions is 7)
2. Using the dimension attribute
3. Using the dimension statement (In this case, the type is determined by default naming or previous implicit statement)

dimension b(13), c(0:5)

dimension mat(5,8)

Indexing

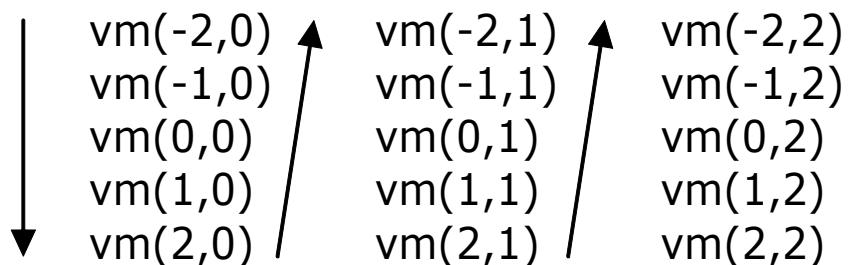
- By default, indexing from 1 to n:

real*8 vec(3) ! Defines vec(1), vec(2), vec(3)

real*8 vec(-1:1) ! Defines vec(-1), vec(0), vec(1)

- Matrices are stored by columns (unlike C or Mathematica)

real, dimension(-2:2,0:2) :: vm



Assigning arrays elements

■ Elements:

`vm(1,0) =1.`

`vm(j,k) =0.` ! $-2 \leq j \leq 2 \text{ & } 0 \leq k \leq 2$

`B(5:7) = (/1.D0, 2.D0, 3.D0/)`

`vector(0:10) = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]` ! New standard

■ Sections:

`B(5:7)=1.D0` ! assigning 1.D0 to B(5),B(6) and B(7) elements

`vm(-2:2,1)=0.` ! assigning zero to the second column

`vm=0.` ! assigning zero to the whole matrix

`vector(1:3) = (/ (2*i, i=1,5,2) /)` ! implicit cycle/loop with step = 2

`lhs(1:3, 0:9) = rhs(-2:0, 20:29)` ! this is ok

`lhs(1:2, 0:9) = rhs(-2:0, 20:29)` ! but this isn't

`a(1:49:2, 2:50:2) = 4.0` ! elements in odd rows and even columns (*start:end:step*)

Array syntax

Less loops required:

```
integer :: m = 100, n = 200
```

```
real :: a(m,n), x(n), y(m)
```

```
integer :: i, j
```

```
!
```

```
y = 0.0
```

```
!
```

```
do j = 1, n
```

```
    y(:) = y(:) + a(:,j) * x(j)
```

```
end do
```

```
do j = 1, n
    do i = 1, m
        y(i) = y(i) + a(i,j) * x(j)
    end do
end do
```

Operators and intrinsic functions

Arithmetic operators

Operations are carried up in the same order than in standard Math

real :: x, y

integer :: i

!

i = 3

!

x = 2.0**(-i) ! power and negation: maximum priority

x = x*real(i) ! multiplication and type change : “medium” priority

y = x/2.0 ! division: “medium” priority

x = 2.0+y ! Addition and subtraction: minimum priority

Use parenthesis when needed

Be careful with integer division (e.g. $4/5 = 0$)

Relational operators

Operator	Alternative (f77)	Description
<code>==</code>	<code>.eq.</code>	Tests for equality of two operands
<code>/=</code>	<code>.ne.</code>	Test for inequality of two operands
<code>></code>	<code>.gt.</code>	Tests if left operand is strictly greater than right operand
<code><</code>	<code>.lt.</code>	Tests if left operand is strictly less than right operand
<code>>=</code>	<code>.ge.</code>	Tests if left operand is greater than or equal to right operand
<code><=</code>	<code>.le.</code>	Tests if left operand is less than or equal to right operand

Logical operators

Operator	Description
.and.	TRUE if both left and right operands are TRUE
.or.	TRUE if either left or right or both operands are TRUE
.not.	TRUE if right operand is FALSE
.eqv.	TRUE if left operand has same logical value as right operand
.neqv.	TRUE if left operand has the opposite logical value as right operand
.xor.	TRUE if only left or right but not both operands are TRUE

Numerical functions

ABS (A)	Absolute value	FLOOR (A) [*]	Greatest integer .ie. A
AIMAG (Z)	Imaginary part of Z	INT (A [, KIND])	Conversion to integer
AINT (A) [*]	Truncation to whole n°	MAX (X, Y [,Z,...])	Maximum value
ANINT (A) [*]	Nearest whole number	MIN (A1, A2 [..])	Minimum value
CEILING (A) [*]	Least integer .ge. A	MOD (A, P)	Remainder function
CMPLX (X [, Y]) [*]	Conversion to complex	MODULO (A, P)	Modulo function
CONJG (Z)	Conjugate of Z	NINT (A [, KIND])	Nearest integer
DBLE (A)	Conversion to real *8	REAL (A [, KIND])	Conversion to real
DIM (X, Y)	Positive difference	SIGN (A, B)	Transfer of sign

(*) Optional KIND argument admitted (normally standard from 2003)

Mathematical functions

Function	Description		Function	Description
ACOS (X)	Arccosine		LOG (X)	Natural logarithm
ASIN (X)	Arcsine		LOG10 (X)	Base 10 logarithm
ATAN (X)	Arctangent		SIN (X)	Sine
ATAN2 (Y, X)	Arctangent		SINH (X)	Hyperbolic sine
COS (X)	Cosine		SQRT (X)	Square root
COSH (X)	Hyperbolic cosine		TAN (X)	Tangent
EXP (X)	Exponential		TANH (X)	Hyperbolic tangent

Archaic forms: DCOS(X), DASIN(X)...

Extensions: ERF(X), BESSEL_J1(X) (BESJ1(X))...

Character functions

Function	Description	Function	Description
ACHAR (I)	Character in ASCII	//	Concatenation operator
ADJUSTL (ST)	Adjust left	LGE (ST1,ST2)	Lexically .ge.
ADJUSTR (ST)	Adjust right	LGT (ST1,ST2)	Lexically .gt.
CHAR (I [, KIND])	Character in processor	LLE (ST1,ST2)	Lexically .le.
IACHAR (C)	ASCII code of C	LLT (ST1,ST2)	Lexically .lt.
ICHAR (C)	Processor code of C	REPEAT (ST, NC)	Repeated concatenation
INDEX (ST, SUBST [, BACK])	Starting position of a substring	SCAN (STRING, SET [, BACK])	Position of first match of STRING in SET
LEN (STRING)	Length of string	TRIM (STRING)	Remove trailing blanks
LEN_TRIM (ST)	Length without trailing blank characters	VERIFY (STRING, SET [, BACK])	Position of first non-match of STRING in SET

Control structures

Conditional structures (IF)

Do something or another depending on a condition, which may be from a simple comparison to a complex combination using logical operators. Two forms:

```
if (condition) ... ! do something
```

Example:

```
if (a .ge. 0) b = sqrt(a)
```

```
if (condition) then
    ....           ! do something
else if (condition2) then
    .....         ! or maybe this
else
    ...
end if
```

Conditional structure (SELECT)

- Allows a variable to be tested for equality against a list of values
- The first match prevents further checking
- Normally with integer or character variables
- Each case block can contain multiple statements

```
program selex
!
implicit none
character :: a ; integer :: icode
!
write(6,*) 'Type in a character'
read(5,*) a
!
! Get code (code of '0' is 48; of '1', 49..)
!
icode = ichar(a) - 48
select case (icode)
  case (0,1,2,3,4)
    write(6,*) 'Number .lt. 5'
  case (5:9)
    write(6,*) 'Number .ge. 5'
  case default
    write(6,*) 'Not a number'
end select
!
stop      ! Actually, not needed'
end
```

Repetitive structures (DO)

Three loop formats available in Fortran:

1. Integer counter (fixed number of iterations)
2. Condition controlled (do until condition is false)
3. Explicit exit statement

do {control clause}

..... ! execute something again and again until stopped

end do

Repetitive structures (DO)

Three loop formats available in Fortran:

1. Integer counter (fixed number of iterations)
2. Condition controlled (do until condition is false)
3. Explicit exit statement

```
a = 0.0_8  
do i = 0,20,2  
    a = a + real(i,8)  
end do
```

```
a = 0.0d0  
do 100 i = 0,20,2  
    a = a + real(i,8)  
100 continue
```

Repetitive structures (DO)

Three loop formats available in Fortran:

1. Integer counter (fixed number of iterations)
2. Condition controlled (do until condition is false)
3. Explicit exit statement

```
i = 0
do while (i <= 100)
    if (i == 13) cycle      ! Goes to the end of the loop (13 will not be included)
    i = i + 1
end do
```

Repetitive structures (DO)

Three loop formats available in Fortran:

1. Integer counter (fixed number of iterations)
2. Condition controlled (do until condition is false)
3. Explicit exit statement

```
i = 0
do
    i = i + 1
    if (i > 100) exit      ! Leaves the loop
end do
```

Construct names

- We can put a name to control structures (IF, DO, SELECT)
- It helps in long structures
- cycle or exit can refer to name (for e.g. exit the outer loop from the inner one)
- Alternatively (not recommended!) use label style

```
program loop
!
implicit none
integer :: i,j,ij
real, dimension(10,10) :: amat
!
!
amat = 0.0
!
do i =1,10
    do j = 1,10
        ij = 10*i + j
        amat(i,j) = real(ij)
    end do
end do
!
write(6,'(/a/)') 'The first matrix'
do i = 1,10
    write(6,'(10f8.1)') (amat(i,j), j=1,10)
end do
!
amat = 0.0
!
```

```

!
loop1 : do i = 1,10
    loop2 : do j = 1,10
        ij = 10*i + j
        test : if (ij == 13) then
            cycle
        else if (ij == 25) then
            cycle loop1
        else if (ij == 33) then
            exit
        else if (ij == 68) then
            exit loop1
        else
            amat(i,j) = real(ij)
        end if test
    end do loop2
end do loop1
!
write(6,'(/a/)') 'The second matrix'
do i = 1,10
    write(6,'(10f8.1)') (amat(i,j), j=1,10)
end do
!
end program loop

```

The first matrix

11.0	12.0	13.0	14.0	15.0	16.0	17.0	18.0	19.0	20.0
21.0	22.0	23.0	24.0	25.0	26.0	27.0	28.0	29.0	30.0
31.0	32.0	33.0	34.0	35.0	36.0	37.0	38.0	39.0	40.0
41.0	42.0	43.0	44.0	45.0	46.0	47.0	48.0	49.0	50.0
51.0	52.0	53.0	54.0	55.0	56.0	57.0	58.0	59.0	60.0
61.0	62.0	63.0	64.0	65.0	66.0	67.0	68.0	69.0	70.0
71.0	72.0	73.0	74.0	75.0	76.0	77.0	78.0	79.0	80.0
81.0	82.0	83.0	84.0	85.0	86.0	87.0	88.0	89.0	90.0
91.0	92.0	93.0	94.0	95.0	96.0	97.0	98.0	99.0	100.0
101.0	102.0	103.0	104.0	105.0	106.0	107.0	108.0	109.0	110.0

DO EXAMPLE (3/4)

The second matrix

11.0	12.0	0.0	14.0	15.0	16.0	17.0	18.0	19.0	20.0
21.0	22.0	23.0	24.0	0.0	0.0	0.0	0.0	0.0	0.0
31.0	32.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
41.0	42.0	43.0	44.0	45.0	46.0	47.0	48.0	49.0	50.0
51.0	52.0	53.0	54.0	55.0	56.0	57.0	58.0	59.0	60.0
61.0	62.0	63.0	64.0	65.0	66.0	67.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

DO EXAMPLE (4/4)

Exercises

5. Write a simple program to explore INT, NINT, FLOOR,...
6. Repeat exercise 2 using arrays and proper control structures
7. Interpret the outputs of the do example.
8. Cholesky decomposition allows to write a positive definite matrix as

$$\mathbb{M} = \mathbf{L} \mathbf{L}^T \longrightarrow M_{pq} = \sum_{J=1}^n L_p^J L_q^J \quad \text{with} \quad L_i^J = \frac{M_{i,J}^{J-1}}{\sqrt{M_{J,J}^{J-1}}}$$

Construct a positive definite 5×5 matrix and Cholesky-decompose it

Input and output

Formatted and unformatted files

Fortran files are identified by numbers

By default, almost all fortran files are not human-readable (they are unformatted), the only exceptions being units 5 (input) and 6 (output).

As already seen, the two basic commands for I/O are READ and WRITE. Their syntax is the same:

READ(file[,format]) var1 [,var2,var3...]

WRITE(file [,format]) [var1,var2,var3...]

(an empty line is written if no variable is given)

Format and edit descriptors

The format consists in a set of edit descriptors

The format can be given either as a label either as a character variable or constant:

```
WRITE(6,100) ivar1,rvar1,rvar2,rvar3,rvar4  
100 FORMAT(I3,4F15.5)
```

```
FMT = '(I3,4F15.5)'  
WRITE(6,FMT) ivar1,rvar1,rvar2,rvar3,rvar4
```

```
WRITE(6, '(I3,4F15.5)') ivar1,rvar1,rvar2,rvar3,rvar4
```

Format and edit descriptors

Some possible edit descriptors are:

- Iw w positions for an integer number (variable or constant)
- Fw.d w positions for a real number with d decimal digits
- Ew.d w positions with d decimal digits in scientific notation
- Dw.d w positions with d decimal digits in scientific notation
- Gw.d Fw.d or Ew.d as convenient
- Aw w positions for characters
- Xw w empty positions

Other I/O statements

- Files others than 5 or 6 should be explicitly opened:

open([unit=]n [,file=..] [,status=..],[,...])

- file = 'filename'
- status = {'old', 'new', 'replace', 'scratch', 'unknown'}
- access = {'sequential', 'direct'}
- form = {'formatted', 'unformatted'}
- recl = 'record length'

- Once a file is not going to be used, it should be closed:

close ([unit=]n [,status={'keep','delete'})

- If a file is to be read/written from its beginning:

rewind ([unit=]n)

- To go back a record

backspace ([unit=]n)

Arrays intrinsic functions

Arrays: functions and manipulations

Since Fortran90 there are a lot of functions to work with matrices

They allow for

- Determining dimension, number of elements...
- Finding specific numerical values
- Manipulating matrices
- Carrying up mathematical operations

Arrays: functions and manipulations

We will review them with an example

```
program arrays
!
! implicit none
!
real(kind=8) :: xmat(-1:1,5) , vec(-7:7), vmat(3,5),
&                  bmat(5,5), tmat(5,3), zmat(3,3),
&                  pad_dat(7)
real(kind=8), parameter :: two = 2.0d0, one = 1.0d0, half = 0.5d0
integer(kind=4) :: i, j
character(len=15), parameter :: fmti ='(a20,5i5)',
&                               fmtr ='(a20,5f12.5)'
!
vec = (/ (real(i,8), i=-7,7) /)      ! Fortran 2003 [...] == (/.../)
pad_dat = (/ (real(i,8)+half, i=1,7) /)
call random_number(xmat)
xmat = xmat - half
!
```

```

        write(6,'(/,a,7f10.3,,/8f10.3,/)' ) 'The vector',
&           (vec(j), j=-7,-1), (vec(j), j=0,7)
!
write(6,fmti) 'size vec :',size(vec),size(vec,1)
write(6,fmti) 'shape vec :',shape(vec)
write(6,fmti) 'lbound vec :', lbound(vec)
write(6,fmti) 'ubound vec :', ubound(vec)
write(6,*)
write(6,fmtr) 'minval vec :',minval(vec),minval(vec,1),
&                           minval(vec,mask=abs(vec)<3.5)
write(6,fmti) 'minloc vec :',minloc(vec),minloc(vec,1),
&                           minloc(vec,mask=abs(vec)<3.5)
write(6,*)

```

The vector	-7.000	-6.000	-5.000	-4.000	-3.000	-2.000	-1.000
	0.000	1.000	2.000	3.000	4.000	5.000	6.000

size vec :	15	15	
shape vec :	15		
lbound vec :	-7		
ubound vec :	7		
minval vec :	-7.00000	-7.00000	-3.00000
minloc vec :	1	1	5

integer :: a (-1:1,4) ! Equivalent to a(-1:1,1:4)

$$A = \begin{pmatrix} 8 & 0 & -3 & -4 \\ -1 & 2 & 6 & 5 \\ 9 & 5 & 3 & -7 \end{pmatrix}$$

shape(a)	→	3 4	! 3 rows and 4 columns
size(a)	→	12	! 12 elements in total
size(a,1)	→	3	! 3 rows
size(a,2)	→	4	! 4 columns
lbound(a)	→	-1 1	! indexing of rows starts at -1; columns at 1
lbound(a,1)	→	-1	! indexing of rows starts at -1
lbound(a,2)	→	1	! indexing of columns starts at 1
ubound(a)	→	1 4	! indexing of rows ends at 1; columns at 4
ubound(a,1)	→	1	! indexing of rows ends at 1
ubound(a,2)	→	4	! indexing of columns ends at 4

In the case of matrices, each dimension can be analyzed

```
write(6,*) 'The matrix'
do i = -1,1
    write(6,'(5f12.6)') (xmat(i,j), j=1,5)
end do
write(6,*)
!
write(6,fmti) 'shape xmat :',shape(xmat)
write(6,fmti) 'size xmat :',size(xmat),size(xmat,1),size(xmat,2)
write(6,fmti) 'lbound xmat :', lbound(xmat)
write(6,fmti) 'lbound xmat(dim1) :', lbound(xmat,1)
write(6,fmti) 'lbound xmat(dim2) :', lbound(xmat,2)
write(6,fmti) 'ubound xmat :', ubound(xmat),ubound(xmat,1),
&                               ubound(xmat,2)

shape xmat :      3      5
size xmat :     15      3      5
lbound xmat :    -1      1
lbound xmat(dim1) :   -1
lbound xmat(dim2) :    1
ubound xmat :     1      5      1      5
```

```
integer :: a (-1:1,4) ! Equivalent to a(-1:1,1:4)
```

$$A = \begin{pmatrix} 8 & 0 & -3 & -4 \\ -1 & 2 & 6 & 5 \\ 9 & 5 & 3 & -7 \end{pmatrix}$$

```
maxval(a) → 9 ! Maximum value of matrix elements
```

```
maxval(a,mask=mod(xmat,2.0)==0.0)  
→ 8 ! Maximum value of even matrix elements
```

```
maxval(a,1) → 9 5 6 5 ! Maximum value of each column
```

```
maxval(a,2 ) → 8 6 9 ! Maximum value of each row
```

```
maxloc(a) → 3 1 ! The indices of maxval(a)
```

```
maxloc(a,1) → 3 3 2 2 ! In which rows are maxval of each column
```

```
maxloc(a,2) → 1 3 1 ! In which columns are maxval of each row
```

```

!
write(6,fmtr) 'minval xmat :',minval(xmat),
&                                     minval(xmat,mask=xmat>0.0)
write(6,fmtr) 'minval xmat_col :,minval(xmat,1)
write(6,fmtr) 'maxval xmat_row :,maxval(xmat,2)
write(6,fmti) 'minloc xmat :,minloc(xmat)
write(6,fmti) 'minloc xmat_col :,minloc(xmat,1)
write(6,fmti) 'maxloc xmat_row :,maxloc(xmat,2)
write(6,*)
!
```

The matrix

-0.027667	0.240682	0.010644	-0.112907	-0.336583
-0.040477	0.406349	-0.104848	-0.416482	-0.053095
-0.067364	0.372761	-0.049117	0.101061	-0.055366

minval xmat :	-0.41648	0.01064			
minval xmat_col :	-0.06736	0.24068	-0.10485	-0.41648	-0.33658
maxval xmat_row :	0.24068	0.40635	0.37276		
minloc xmat :	2	4			
minloc xmat_col :	3	1	2	2	1
maxloc xmat_row :	2	2	2		

integer :: a (-1:1,4) ! Equivalent to a(-1:1,1:4)

$$A = \begin{pmatrix} 8 & 0 & \boxed{-3 & -4} \\ -1 & 2 & \boxed{6 & 5} \\ 9 & 5 & 3 & -7 \end{pmatrix}$$

sum(a)	→ 23	! Sum of all matrix elements
sum(a, a.gt.0)	→ 38	! Sum of positive matrix elements
sum(a,dim=1)	→ 16 7 6 -6	! Sum of each column
sum(a,dim=2)	→ 1 12 10	! Sum of each row
sum(a(-1:0,3:4))	→ 4	! Sum of the red submatrix elements

```

! write(6,'(a20,3f17.1)') 'sum vec elements :',sum(vec),
&                               sum(vec,vec.gt.0)
 write(6,'(a20,3f17.1)') 'product vec ele. :',product(vec),
&                               product(vec,vec.ne.0),
&                               product(vec,vec.gt.0)
 write(6,*)
 write(6,fmtr) 'sum xmat elements :',sum(xmat),
&                               sum(xmat,xmat.gt.0)
 write(6,fmtr) 'sum xmat_col ele. :',sum(xmat,dim=1)
 write(6,fmtr) 'sum xmat_row ele. :',sum(xmat,dim=2)
 write(6,fmtr) 'sum xmat section :',sum(xmat(-1:0,4:5))
 write(6,*)

```

The matrix

-0.027667	0.240682	0.010644	-0.112907	-0.336583
-0.040477	0.406349	-0.104848	-0.416482	-0.053095
-0.067364	0.372761	-0.049117	0.101061	-0.055366

sum vec elements :	0.0	28.0			
product vec ele. :	-0.0	-25401600.0	5040.0		

sum xmat elements :	-0.13241	1.13150			
sum xmat_col ele. :	-0.13551	1.01979	-0.14332	-0.42833	-0.44504
sum xmat_row ele. :	-0.22583	-0.20855	0.30198		
sum xmat section :	-0.91907				

The vector	-7.000	-6.000	-5.000	-4.000	-3.000	-2.000	-1.000
0.000	1.000	2.000	3.000	4.000	5.000	6.000	7.000

```

vmat = reshape(vec,(/3,5/))
do i = 1,3 ; write(6,'(5f12.6)') (vmat(i,j), j=1,5) ; end do
write(6,*)
!
```

```

vmat = reshape(vec,(/3,5/),order=(/1,2/))
do i = 1,3 ; write(6,'(5f12.6)') (vmat(i,j), j=1,5) ; end do
write(6,*)
!
```

```

vmat = reshape(vec,(/3,5/),order=(/2,1/))
do i = 1,3 ; write(6,'(5f12.6)') (vmat(i,j), j=1,5) ; end do

```

```

-7.000000   -4.000000   -1.000000    2.000000    5.000000
-6.000000   -3.000000    0.000000    3.000000    6.000000
-5.000000   -2.000000    1.000000    4.000000    7.000000

```

```

-7.000000   -4.000000   -1.000000    2.000000    5.000000
-6.000000   -3.000000    0.000000    3.000000    6.000000
-5.000000   -2.000000    1.000000    4.000000    7.000000

```

```

-7.000000   -6.000000   -5.000000   -4.000000   -3.000000
-2.000000   -1.000000    0.000000    1.000000    2.000000
 3.000000    4.000000    5.000000    6.000000    7.000000

```

The vector	-7.000	-6.000	-5.000	-4.000	-3.000	-2.000	-1.000
	0.000	1.000	2.000	3.000	4.000	5.000	6.000
							7.000

```

! bmat = reshape(vec,(/5,5/),pad=pad_dat)
! write(6,fmtr) 'pad_dat(1:5) :',(pad_dat(i),i=1,5)
! write(6,fmtr) 'pad_dat(6:7) :',(pad_dat(i),i=6,7)
! write(6,*)
! do i = 1,5 ; write(6,'(5f12.6)') (bmat(i,j), j=1,5) ; end do
! write(6,*)

! pad_dat(1:5) :      1.50000      2.50000      3.50000      4.50000      5.50000
! pad_dat(6:7) :      6.50000      7.50000

-7.000000   -2.000000    3.000000    1.500000    6.500000
-6.000000   -1.000000    4.000000    2.500000    7.500000
-5.000000     0.000000    5.000000    3.500000    1.500000
-4.000000     1.000000    6.000000    4.500000    2.500000
-3.000000     2.000000    7.000000    5.500000    3.500000

```

!

```
forall(i=1:3, j=1:5) vmat(i,j) = vec(3*(j-1)+i-8) / two  
do i = 1,3 ; write(6,'(5f12.6)') (vmat(i,j), j=1,5) ; end do  
write(6,*)
```

!

```
where(vmat > 0) vmat = one / vmat  
do i = 1,3 ; write(6,'(5f12.6)') (vmat(i,j), j=1,5) ; end do  
write(6,*)
```

-3.500000	-2.000000	-0.500000	1.000000	2.500000
-3.000000	-1.500000	0.000000	1.500000	3.000000
-2.500000	-1.000000	0.500000	2.000000	3.500000
-3.500000	-2.000000	-0.500000	1.000000	0.400000
-3.000000	-1.500000	0.000000	0.666667	0.333333
-2.500000	-1.000000	2.000000	0.500000	0.285714

The previous forall using explicits loops

```
!  
! write(6,'(3a3,2(3x,a4,3x))') ' i ',' j ',' k ',  
&                                'vec','vmat'  
do j = 1,5  
  do i = 1,3  
    k = 3*(j-1)+i-8 ! Just to be clear  
    vmat(i,j) = vec(k) / two  
    write(6,'(3i3,2f7.2)') i,j,k,vec(k),vmat(i,j)  
  end do  
end do
```

i	j	k	vec	vmat
1	1	-7	-7.00	-3.50
2	1	-6	-6.00	-3.00
3	1	-5	-5.00	-2.50
1	2	-4	-4.00	-2.00
2	2	-3	-3.00	-1.50
3	2	-2	-2.00	-1.00
1	3	-1	-1.00	-0.50
2	3	0	0.00	0.00
3	3	1	1.00	0.50
1	4	2	2.00	1.00
2	4	3	3.00	1.50
3	4	4	4.00	2.00
1	5	5	5.00	2.50
2	5	6	6.00	3.00
3	5	7	7.00	3.50

The subsequent where statement takes only the strictly positive elements of vmat (those inside the red rectangle) and invert them

```

        write(6,fmt) 'dot product vec·vec:',dot_product(vec,vec)
        write(6,*)

!
vmat = reshape(vec,(/3,5/))
xmat = one + vmat**two
do i = -1,1
    write(6,'(5f12.6)') (xmat(i,j), j=1,5)
end do
write(6,*)

!
tmat = transpose(vmat)
do i = 1,5 ; write(6,'(5f12.6)') (tmat(i,j), j=1,3) ; end do
write(6,*)

dot product vec·vec   280.00000 
$$\left( \sum_{n=-7}^7 n^2 \right)$$

50.00000   17.00000   2.00000   5.00000   26.00000
37.00000   10.00000   1.00000   10.00000  37.00000
26.00000   5.00000   2.00000   17.00000  50.00000

-7.00000   -6.00000   -5.00000
-4.00000   -3.00000   -2.00000
-1.00000   0.00000   1.00000
 2.00000   3.00000   4.00000
 5.00000   6.00000   7.00000

```

```
xmat = vmat + vmat  
do i = -1,1 ; write(6,'(5f12.6)') (xmat(i,j), j=1,5) ; end do  
write(6,*)
```

```
xmat = vmat * vmat  
do i = -1,1 ; write(6,'(5f12.6)') (xmat(i,j), j=1,5) ; end do  
write(6,*)
```

-14.000000	-8.000000	-2.000000	4.000000	10.000000
-12.000000	-6.000000	0.000000	6.000000	12.000000
-10.000000	-4.000000	2.000000	8.000000	14.000000
49.000000	16.000000	1.000000	4.000000	25.000000
36.000000	9.000000	0.000000	9.000000	36.000000
25.000000	4.000000	1.000000	16.000000	49.000000

```

! zmat = matmul(vmat,transpose(vmat))
do i = 1,3 ; write(6,'(5f12.6)') (zmat(i,j), j=1,3) ; end do
write(6,*)

! bmat = matmul(transpose(vmat),vmat)
do i = 1,5 ; write(6,'(5f12.6)') (bmat(i,j), j=1,5) ; end do
write(6,*)

! end program arrays

```

95.000000	90.000000	85.000000		
90.000000	90.000000	90.000000		
85.000000	90.000000	95.000000		
110.000000	56.000000	2.000000	-52.000000	-106.000000
56.000000	29.000000	2.000000	-25.000000	-52.000000
2.000000	2.000000	2.000000	2.000000	2.000000
-52.000000	-25.000000	2.000000	29.000000	56.000000
-106.000000	-52.000000	2.000000	56.000000	110.000000

Dynamic allocation of arrays

Until now, arrays have been declared at compile time:

Dimensions are fixed!
(static allocation)

They can also be declared at run time by means of the statement ALLOCATE if previously defined with the attribute ALLOCATABLE:

Dimensions are not fixed!
(dynamic allocation)

```
!...
real*8, allocatable :: vec1(:), mat(:, :)
integer :: dimvec,nrow,ncol, ierr1, ierr2
!...
read(5,*) dimvec,nrow,ncol
!...
allocate(vec1(dimvec),stat=ierr1)
if (ierr1 /= 0) stop ! Error
!...
allocate(mat(nrow,ncol),stat=ierr2)
if (ierr2 /= 0) stop ! Error
!...
deallocate(vec1,stat=ierr1)
deallocate(mat,stat=ierr2)
if ((ierr1 /= 0) .or. (ierr2 /= 0)) stop
!...
```

Derived data types

```
type atom
    character(len=2) :: name
    Integer :: Z
    real :: mass
    real :: q
    real, dimension(1:3) :: coord
end type atom
```

- Therefore, we can now define variables of *atom* type in the following way:
`type(atom) :: carbon`
- Assigning values to these variables:
`carbon = atom("C ", 6, 12.011, 0.35, (/0.0, 1.0, 2.0/))`
- We can select an element, for instance the atomic number: `carbon%Z`
- These structures can be combined into new *derived data types*

```
type molecule
    integer :: natom
    type(atom), dimension(1:1000) :: at
end type molecule
```

Exercises

9. Write a simple program to explore the different possibilities for formatting
10. Write a program that can write in a nice style a real matrix of 30 rows and 30 columns
11. Write simple programs to explore the different matrix functions.

Procedures

Modular programming

A program may (and frequently must) be split into smaller units (procedures).

This allows for:

- Faster compilation, as only new/modified units need to be (re)compiled
- Easier debugging, as errors are isolated
- Immediate reusability, as the same procedure can be used in different programs

In fortran there exist two types of procedures:

- Functions: Return a single value
- Subroutines: May return several values

Functions

```
[type] function &  
fname(a1,a2..) [result(val)]  
  
[declarations]  
  
[statements]  
  
[return] ! F77  
  
end function fname
```

In the main program:
res = fname(v1,v2...)

```
real function side3_wres(a,b,gamm) result(c)  
implicit none ; real :: a, b, gamm  
c = sqrt(a**2 + b**2 - 2*a*b*cos(gamm))  
end function side3_wres  
  
! Or simply  
  
real function side3(a,b,gamm)  
implicit none ; real :: a, b, gamm  
side3 = sqrt(a**2 + b**2 - 2*a*b*cos(gamm))  
end function side3  
!  
!  
program xxx  
!...  
x3 = side3(x1,x2,angle)  
!...  
end program xxx
```

Recursive functions

A function can be declared as recursive

Then the function calls itself

Of course, that implies that some initial value must be given

```
recursive function sumsqinv(n) result(res)
implicit none
real(kind=8) :: res, nsq ; integer :: n
if (n <= 0) then
    stop 'n must be .gt. 0'
else if (n == 1) then
    res = 1_8
else
    nsq = real(n*n,8)
    res = 1_8/nsq + sumsqinv(n-1)
end if
end function
```

Subroutines

subroutine &
subname(a1,a2,...)
[declarations]
[statements]
[return] ! F77
end subroutine fname

In the main program:
call subname(v1,v2...)

```
subroutine side3_sub(a,b,c,gamm,beta)
implicit none
real :: a, b, c, gamm, beta
!
c = sqrt(a**2 + b**2 - 2*a*b*cos(gamm))
beta=acos((a**2 + c**2 - b**2) / (2*a*c))
!
end subroutine side3_wres
!
!
!
program xxx
!...
call side3_sub(x1,x2,x3,ang_c,ang_b)
!...
end program xxx
```

Subroutine and function arguments

- Arguments are identified by their position in the arguments list, their names being irrelevant
- Actually, memory addresses and ranges are passed
- Unless if included in a module (or common in F77), variables not in the argument list are local to the procedure
- If a variable in the argument list is changed in the procedure, the main program inherits this change

The INTENT attribute

To avoid unintentional changes of variables in the argument list, the INTENT attribute can be used. There are three possibilities:

- INTENT(IN)

Value can't be changed

- INTENT(OUT)

Value must be given

- INTENT(INOUT)

Can, but not must, change

```
subroutine sub1(xinp,xout,x1,x2)
!
real, intent(in) :: xinp
real, intent(out) :: xout
! No need to modify x1 or x2
real, intent(inout) :: x1
real :: x2
intent(inout) :: x2
!...
xinp = 5.0 ! Error
!...
x1 = x2
!
! Some assignment for xout
!
xout = 3.1 ! Required
!...
end subroutine sub1
```

Types of procedures

Procedures can be classified as

- Intrinsic (fortran functions, e.g. LOG)
- Internal (declared in the same program unit by means of CONTAINS statement)
- Module (an external unit containing procedures, data definitions and/or variable declarations; defined by statement MODULE and called by statement USE)
- External (declared in a different program unit, e.g. a routine from a library)

Internal procedures: CONTAINS

Internal procedures (in the example the previous function sumsqinv) are inside the same program unit than the calling (sub)program after the statement CONTAINS

```
gfortran -std=f95 -o rec_exam.x rec_exam.f90
```

```
program rec_exam
implicit none
integer :: i ; real(kind=8) :: xsum
!
print*, 'Type in an integer' ; read(*,*) i
xsum = sumsqinv(i)
write(6,'(a,i4,a,g15.5)') 'Sum from 1 to', &
    i,' of 1/n^2 =',xsum
!
contains
recursive function sumsqinv(n) result(res)
implicit none
real(kind=8) :: res, nsq ; integer :: n
if (n <= 0) then
    stop 'n must be .gt. 0'
else if (n == 1) then
    res = 1_8
else
    nsq = real(n*n,8)
    res = 1_8/nsq + sumsqinv(n-1)
end if
end function
end program
```

Modules

- Modules are external files that may contain procedures, data declarations and/or derived data definitions
- They are defined in modules blocks MODULE ... END MODULE
- They can be called by used by other program units by means of the statement USE, maybe with the attribute ONLY
- Variables defined in the module can be PUBLIC (the default) or PRIVATE
- Modules must be compiled before linking main program

```
gfortran -c util.f    # Using F2003 iso_fortran_env
gfortran -c -std=f95 array_in_sub.f
gfortran -std=f95 -o array_in_sub.x util.o array_in_sub.o
```

```
module util
!
use, intrinsic :: iso_fortran_env, only: real64 ! F2003
integer, private, parameter :: dp = real64
!
integer :: i,j,k,l,m,n,p,q,r,s
real(kind=dp), parameter :: zero = 0._8, one = 1._8, two = 2._8
real(kind=dp), parameter :: pi = 3.14159265358979323846_8,
& hbar = 1.05457168e-34_8, toang = 0.5291772108_8,
& todeg = 180.0_8 / pi
!
contains
subroutine wrimat(xmat,nrow,ncol)
!
! Subroutine to write a matrix in blocks of 5 columns
```

```
!
! Subroutine to write a matrix in blocks of 5 columns
!
implicit none
integer :: nrow,ncol,istr, iend
real(kind=8) :: xmat(nrow,ncol)
integer, parameter :: maxcol = 5
!
iend = 0
do
    istr = iend + 1
    iend = iend + maxcol ; if (iend > ncol) iend = ncol
    do i = 1, nrow
        write(6,'(5g14.3)') (xmat(i,j),j=istr,iend)
    end do
    if (iend == ncol) exit
end do
return
end subroutine
end module
```

Arrays in procedures

The fact that only memory addresses and lengths are passed to procedures, allows for treating

- A vector as a matrix or vice versa
- Only a part of the array
- ...

Note that this can be dangerous and, therefore, requires some care!!

```
program fakedot
!
use util !to declare n and wrimat
implicit none
real(kind=8), dimension (2,2) :: a,b
real(kind=8) :: res
!
n = 2
a(:,1) = (/ 1., 3. /); a(:,2) = (/ 2., 4. /)
write(6,'(a,/)' ) 'Matrix A'
call wrimat(a,n,n)
!
b(:,1) = (/ 6., 8. /); b(:,2) = (/ 7., 9. /)
write(6,'(//,a,/)' ) 'Matrix B'
call wrimat(b,n,n)
!
```

```
.c   Calling dot_product with matrices A and B produces an error
.c   at compilation:
.c     'vector_a' argument of 'dot_product' intrinsic at (1) must be of rank 1
!
!   res = dot_product(a,b)
!
c   Instead, this works nicely
!
call cheat_dot(n,n,a,b,res)
write(6,'(///,a,f6.1,/)') 'Sum_ij (A_ij * B_ij) =', res
c
stop
!
contains
  subroutine cheat_dot(nrow,ncol,a,b,c)
    implicit none
    real(kind=8), dimension(nrow*ncol) :: a,b
    real(kind=8) :: c
    integer :: nrow,ncol
    c = dot_product(a,b)
    return
  end subroutine
end program fakedot
```

```
program test_x
!
use util
implicit none
real(kind=8), dimension(:, :) , allocatable :: a,b,c
integer :: ierr
!
allocate(a(3,3),b(5,3),c(5,3),stat=ierr)
if (ierr /= 0) stop 'Error in allocation '
!
do j = 1,3
    a(:,j) = (/ (3*(j-1)+i, i=1,3) /)
    b(:,j) = (/ (one / real(i+j,8), i=1,5) /)
end do
!
write(6,'(a,/)' ) 'In main before sub:'
write(6,1) 'a: shape, lbound, ubound', shape(a), lbound(a), ubound(a)
write(6,1) 'b: shape, lbound, ubound', shape(b), lbound(b), ubound(b)
write(6,1) 'c: shape, lbound, ubound', shape(c), lbound(c), ubound(c)
write(6,*)
1 format(a,3(2i5,4x))
!
```

```

!
m = 5 ; n = 3
call sub(m,n,a,b,c)
!

write(6,'(/,a,/)' ) 'In main after sub:'
write(6,1) 'a: shape,lbound,ubound', shape(a),lbound(a),ubound(a)
write(6,1) 'b: shape,lbound,ubound', shape(b),lbound(b),ubound(b)
write(6,1) 'c: shape,lbound,ubound', shape(c),lbound(c),ubound(c)
write(6,'(//,a,/)' ) 'Matrix c in main'
call wrimat(c,m,n)
!

contains
subroutine sub(nrow,ncol,a,b,c)
!

implicit none
integer, intent(in) :: nrow,ncol
integer :: i,j
real(kind=8), dimension(:, :), intent(in) :: a,b
real(kind=8), intent(out) :: c(-((nrow-1)/2):(nrow-1)/2,ncol)
real(kind=8) :: x(nrow,ncol)

```

Assumed-shapes arrays

Fitted-indexed array

Automatic array (local)

```

!
write(6,'(//,a,/)')
! In sub:
write(6,1) 'a: shape, lbound, ubound', shape(a),lbound(a),ubound(a)
write(6,1) 'b: shape, lbound, ubound', shape(b),lbound(b),ubound(b)
write(6,1) 'c: shape, lbound, ubound', shape(c),lbound(c),ubound(c)
write(6,1) 'x: shape, lbound, ubound', shape(x),lbound(x),ubound(x)
1 format(a,3(2i5,4x))
!
```

```

x = 2.0_8 / b
c = matmul(x,a)
```

```

return
end subroutine
end program
```

In main before sub:

a: shape, lbound, ubound	3	3	1	1	3	3
b: shape, lbound, ubound	5	3	1	1	5	3
c: shape, lbound, ubound	5	3	1	1	5	3

In sub:

a: shape, lbound, ubound	3	3	1	1	3	3
b: shape, lbound, ubound	5	3	1	1	5	3
c: shape, lbound, ubound	5	3	-2	1	2	3
x: shape, lbound, ubound	5	3	1	1	5	3

In main after sub:

a: shape, lbound, ubound	3	3	1	1	3	3
b: shape, lbound, ubound	5	3	1	1	5	3
c: shape, lbound, ubound	5	3	1	1	5	3

Matrix c in main

40.0000	94.0000	148.000
52.0000	124.000	196.000
64.0000	154.000	244.000
76.0000	184.000	292.000
88.0000	214.000	340.000

Only the indexing of c varies:

- In main: c(5,3)
- In sub: c(-2:2,3)

But it refers always to same memory position

External procedures: INTERFACE

As memory addresses are passed, kind and ranges must match

The compiler "knows" the data types used by intrinsic, internal or module procedures. But not for external procedures!

INTERFACE provides such information in order the compiler to be able of checking for mismatches

```
gfortran -c -std=f95 recfunc.f90  
gfortran -c -std=f95 recfunc_ext.f90  
gfortran -o recfunc_ext.x -std=f95 recfunc.o recfunc_ext.o
```

```
program rec_exam  
!  
implicit none  
integer :: i ; real(kind=8) :: xsum  
!  
interface  
    recursive function sumsqinv(n) result(res)  
        real(kind=8) :: res, nsq ; integer :: n  
    end function  
end interface  
!  
print*, 'Type in an integer' ; read(*,*) i  
xsum = sumsqinv(i)  
write(6,'(a,i4,a,g15.5)') 'Sum from 1 to', &  
    i,' of 1/n^2 =',xsum  
!  
end program
```

INTERFACE: other use

INTERFACE can also be used to pass the name of a procedure as argument to other procedure

```
subroutine sub(namefun,x,...)
!
implicit none
!....
interface
    function namefunc(p,q)
        real*8 :: namefunc,p,q
    end function
end interface
```

```

program newraph
c
implicit none
real(kind=8) :: x0, x1, diff, fun, der
integer(kind=4) :: iter
real(kind=8), parameter :: thres = 1.0d-8
c
interface
    function f(x)
        real(kind=8) :: f,x
    end function
end interface
c
write(6,*) 'Initial guess?' ; read(5,*) x0
write(6,'(/,a4,2(6x,a6,5x),4x,2(a7,7x))') "Iter","x_0",
&                                         "f(x_0)","f'(x_0)","x_1"
c
iter = 0
do
    iter = iter + 1 ; fun  = f(x0)
    call mkder(f,der,x0)
    x1  = x0 - fun/der
    write(6,'(i3,4f16.8)') iter,x0,fun,der,x1
c

```

```

        diff = abs(x1-x0)
        if (diff .lt. thres) then
            exit
        else
            x0 = x1
        end if
    end do
    stop

c
contains

c
subroutine mkder(f,d,x)
real(kind=8) d,x
real(kind=8), parameter :: h = 1.0d-3, two = 2.0d0
c
    interface
        function f(x)
        real(kind=8) :: f,x
        end function
    end interface
c
    d = (f(x+h) - f(x-h)) / (two*h)
c
    return
end subroutine
end program

```

The function is programmed in a separate file

```
function f(x)
real(kind=8) :: f,x
f = 4.0_8*x**3 - 0.1223_8*x*x + 0.1223_8*x - 0.03058_8
return
end function
```

To compile, link and run

```
[macold_556 % gfortran -c newraph.f
[macold_557 % gfortran -c funcion_f.f
[macold_558 % gfortran -o newraph.x funcion_f.o newraph.o
[macold_559 %
[macold_559 % ./newraph.x
  Initial guess?
0.1

Iter      x_0          f(x_0)        f'(x_0)        x_1
  1    0.10000000   -0.01557300   0.21784400   0.17148694
  2    0.17148694    0.00696847   0.43325152   0.15540280
  3    0.15540280    0.00048414   0.37409285   0.15410862
  4    0.15410862    0.00000292   0.36960264   0.15410073
  5    0.15410073    0.00000000   0.36957540   0.15410073
[macold_560 %
```

Exercises

12. Convert your code to calculate the n^{th} -term of a Fibonacci series into a function
13. Convert your code to sum the first n^{th} -terms of the succession $1/n^2$ into a function to sum $1/n^k$, where k is an integer given also in the argument list
14. Convert your codes for matrix multiplication, matrix writing and Cholesky decomposition into three subroutines
15. Use previous procedures to write a program that Cholesky decompose the matrix $\sum_{i=1}^3 \vec{v}_i \vec{w}_i$, where \vec{v}_i are column vectors built with elements calculated using Fibonacci series and \vec{w}_i are row vectors built with elements calculated by summing successions of powers of inverse natural numbers. The output must look nice.

Libraries

Creating static libraries

Object files can be conveniently grouped in libraries.

Static libraries (lib.a)

- The executable contains a copy of the library.
- Then, larger size but faster
- Created and managed by means of ar utility
- Linked using options -l and -L

```
[513 % ar -r libfunc.a function_*.o
[514 % ar -t libfunc.a
___.SYMDEF
function_1.o
function_2.o
function_4.o
function_5.o
function_3.o
[515 % ar -d libfunc.a function_3.o
516 % gfortran -o prog.x main.o -lfunc
```

Creating dynamic libraries

Object files can be conveniently grouped in libraries.

Dynamic libraries (lib.so)

- Accessed at run time through environment variable LD_LIBRARY_PATH
- Executable of smaller size, but slower
- Created using the compiler
- Examined by nm

```
132 % gfortran -c -fPIC function_*.f
133 % gfortran -shared -o libfunc.so function_*.o
134 % gfortran -o newraph.x newraph.o -L. -lfunc
135 % mv libfunc.so $HOME
136 % ./newraph.x
./newraph.x: error while loading shared libraries: libfunc.so:
cannot open shared object file: No such file or directory
137 % export LD_LIBRARY_PATH=$HOME:$LD_LIBRARY_PATH
138 % ./newraph.x
Initial guess?
139 %
```

BLAS and LAPACK libraries

There exist pre-programmed libraries for different purposes. Among them, two very often used in Physics and Chemistry are

- BLAS libblas.a (libblas.so)
<https://www.netlib.org/blas/>
<https://math-atlas.sourceforge.net/> (tunable implementation)
- LAPACK liblapack.a (liblapack.so)
<https://www.netlib.org/lapack/>

gfortran file.o –L <library_directory> -lblas -llapack

BLAS level 1 routine DCOPY

- ◆ **dcopy()**

```
subroutine dcopy ( integer           N,  
                  double precision, dimension(*)  DX,  
                  integer                      INCX,  
                  double precision, dimension(*) DY,  
                  integer                      INCY  
                )
```

DCOPY

Purpose:

DCOPY copies a vector, x , to a vector, y .
uses unrolled loops for increments equal to 1.

BLAS level 1 routine DDOT

- ◆ **ddot()**

```
double precision function ddot ( integer           N,  
                               double precision, dimension(*) DX,  
                               integer             INCX,  
                               double precision, dimension(*) DY,  
                               integer             INCY  
 )
```

DDOT

Purpose:

DDOT forms the dot product of two vectors.
uses unrolled loops for increments equal to one.

BLAS level 1 routine DSCAL

- ◆ dscal()

```
subroutine dscal ( integer N,  
                  double precision DA,  
                  double precision, dimension(*) DX,  
                  integer INCX  
 )
```

DSCAL

Purpose:

DSCAL scales a vector by a constant.
uses unrolled loops for increment equal to 1.

BLAS level 1 routine DAXPY

◆ daxpy()

```
subroutine daxpy ( integer           N,  
                   double precision      DA,  
                   double precision, dimension(*) DX,  
                   integer                 INCX,  
                   double precision, dimension(*) DY,  
                   integer                 INCY  
 )
```

DAXPY

Purpose:

DAXPY constant times a vector plus a vector.
uses unrolled loops for increments equal to one.

BLAS level 2 routine DGEMV (1/2)

DGEMV

Purpose:

DGEMV performs one of the matrix-vector operations

$y := \alpha \cdot A \cdot x + \beta \cdot y$, or $y := \alpha \cdot A^T \cdot x + \beta \cdot y$,

where α and β are scalars, x and y are vectors and A is an m by n matrix.

BLAS level 2 routine DGEMV (2/2)

- **dgemv()**

```
subroutine dgemv ( character          TRANS,  
                  integer            M,  
                  integer            N,  
                  double precision     ALPHA,  
                  double precision, dimension(lda,*) A,  
                  integer            LDA,  
                  double precision, dimension(*)    X,  
                  integer            INCX,  
                  double precision     BETA,  
                  double precision, dimension(*)    Y,  
                  integer            INCY  
                )
```

BLAS level 3 routine DGEMM (1/2)

DGEMM

Purpose:

DGEMM performs one of the matrix-matrix operations

$C := \alpha \cdot \text{op}(A) \cdot \text{op}(B) + \beta \cdot C,$

where $\text{op}(X)$ is one of

$\text{op}(X) = X \quad \text{or} \quad \text{op}(X) = X^{**T},$

alpha and beta are scalars, and A, B and C are matrices, with $\text{op}(A)$ an m by k matrix, $\text{op}(B)$ a k by n matrix and C an m by n matrix.

BLAS level 3 routine DGEMM (2/2)

◆ dgemm()

```
subroutine dgemm ( character          TRANSA,  
                  character          TRANSB,  
                  integer            M,  
                  integer            N,  
                  integer            K,  
                  double precision    ALPHA,  
                  double precision, dimension(lda,*) A,  
                  integer            LDA,  
                  double precision, dimension(lbd,*) B,  
                  integer            LDB,  
                  double precision    BETA,  
                  double precision, dimension(lcd,*) C,  
                  integer            LDC  
                )
```

LAPACK routine DSYEV

Purpose:

◆ dsyev()

DSYEV computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A.

```
subroutine dsyev ( character          JOBZ,  
                  character          UPLO,  
                  integer            N,  
                  double precision, dimension( lda, * ) A,  
                  integer            LDA,  
                  double precision, dimension( * )      W,  
                  double precision, dimension( * )      WORK,  
                  integer            LWORK,  
                  integer            INFO  
                )
```

LAPACK routine DGETRI (1/2)

Purpose:

DGETRI computes the inverse of a matrix using the LU factorization computed by DGETRF.

This method inverts U and then computes $\text{inv}(A)$ by solving the system $\text{inv}(A)*L = \text{inv}(U)$ for $\text{inv}(A)$.

LAPACK routine DGETRI (2/2)

- ◆ dgetri()

```
subroutine dgetri ( integer N,  
                    double precision, dimension( lda, * ) A,  
                    integer LDA,  
                    integer, dimension( * ) IPIV,  
                    double precision, dimension( * ) WORK,  
                    integer LWORK,  
                    integer INFO  
    )
```

Some other statements

Pointers

- Pointers are variables that instead of storing values store memory addresses
- Inherited from C, where arguments passed by procedures are values instead of memory addresses as in Fortran
- Make faster codes, but they are not always easy to use
- Pointer variables are associated to target variables of the same type by the operator =>
- Pointers are dissociated from targets with the statement `NULLIFY(pointer_name)`
- The status of a pointer can be checked with the statement `ASSOCIATED(pointer_name [, target_name])`
- Pointers are very useful for iterative procedures

```
program pointer_ex1
use util
c
implicit none
integer, parameter :: nrow = 9, ncol = 5
real(kind=8), target :: matrix(nrow,ncol)
real(kind=8), pointer :: p_matrix(:, :), p_vec(:)
c
do j = 1,5
    matrix(:,j) = (/ (real(10*i + j,8), i=1,9) /)
end do
write(6,'(/,a,/}') 'Original matrix'
call wrimat(matrix,nrow,ncol)
c
p_matrix => matrix(3:5,2:4) ; p_matrix = one
write(6,'(/,a,/}') 'First modification'
call wrimat(matrix,nrow,ncol)
c
p_vec => matrix(8,1:5) ; p_vec = two
write(6,'(/,a,/}') 'Second modification'
call wrimat(matrix,nrow,ncol)
c
stop
end
```

```
do j = 1,5
    matrix(:,j) = (/ real(10*i + j,8), i=1,9) /)
end do
```

Original matrix

11.0000	12.0000	13.0000	14.0000	15.0000
21.0000	22.0000	23.0000	24.0000	25.0000
31.0000	32.0000	33.0000	34.0000	35.0000
41.0000	42.0000	43.0000	44.0000	45.0000
51.0000	52.0000	53.0000	54.0000	55.0000
61.0000	62.0000	63.0000	64.0000	65.0000
71.0000	72.0000	73.0000	74.0000	75.0000
81.0000	82.0000	83.0000	84.0000	85.0000
91.0000	92.0000	93.0000	94.0000	95.0000

```
C
p_matrix => matrix(3:5,2:4) ; p_matrix = one
C
```

First modification

11.0000	12.0000	13.0000	14.0000	15.0000
21.0000	22.0000	23.0000	24.0000	25.0000
31.0000	1.00000	1.00000	1.00000	35.0000
41.0000	1.00000	1.00000	1.00000	45.0000
51.0000	1.00000	1.00000	1.00000	55.0000
61.0000	62.0000	63.0000	64.0000	65.0000
71.0000	72.0000	73.0000	74.0000	75.0000
81.0000	82.0000	83.0000	84.0000	85.0000
91.0000	92.0000	93.0000	94.0000	95.0000

```
c  
    p_vec => matrix(8,1:5) ; p_vec = two  
c
```

Second modification

11.0000	12.0000	13.0000	14.0000	15.0000
21.0000	22.0000	23.0000	24.0000	25.0000
31.0000	1.00000	1.00000	1.00000	35.0000
41.0000	1.00000	1.00000	1.00000	45.0000
51.0000	1.00000	1.00000	1.00000	55.0000
61.0000	62.0000	63.0000	64.0000	65.0000
71.0000	72.0000	73.0000	74.0000	75.0000
2.00000	2.00000	2.00000	2.00000	2.00000
91.0000	92.0000	93.0000	94.0000	95.0000

```
macold_666 % time ./rel_pointer.x
```

```
Iter    maxdif
 1  0.38E+01
 2  0.15E+01
 3  0.34E+00
 4  0.62E-01
 5  0.31E-01
 6  0.16E-01
 7  0.78E-02
 8  0.39E-02
 9  0.19E-02
10  0.97E-03

Maximum original element (index, value, square root) :
2257892   8.5999985232   2.9325754079

Maximum difference :  0.17E-02
Average difference :  0.33E-09

real    0m1.333s
user    0m1.206s
sys     0m0.124s
```

```
macold_667 % time ./rel_nopointer.x
```

```
Iter    maxdif
 1  0.38E+01
 2  0.15E+01
 3  0.34E+00
 4  0.62E-01
 5  0.31E-01
 6  0.16E-01
 7  0.78E-02
 8  0.39E-02
 9  0.19E-02
10  0.81E-03
```

```
Maximum original element (index, value, square root) :
7361916   8.5999995236   2.9325755785
```

```
Maximum difference :  0.28E-03
Average difference :  0.31E-10
```

```
real    0m1.762s
user    0m1.326s
sys     0m0.096s
```

F77 statements

COMMON

- F77 command
- Allows to share information among routines that used it
- Superseded by MODULE

```
common /name_c/ dpvar1, spvar1,  
&           xvec(6), ivar3
```

DATA

- F77 command
- Allows to initialize variables
- Superseded by F90 variable declarations

```
data dpvar1, ivar3, xvec  
&      /0.5d0, 4, 6*0.0/
```

Makefiles

Definitions

- A makefile is a file that compiles the code.
- When changes are made to some of the code, only the updated files need to be recompiled.
- The code must be linked again to create the new executable.
- Makefile knows the pieces of a large program that need to be recompiled.
- Makefile has all the sentences to compile and link the code up.
- Such sentences must be prefixed by a TAB character

Important points

- Makefile will recompile those changed from last compilation.
- Comments: must be delimited by hash marks (#).
- Continuation character: backslash (\) the end of the line.
- By default, the first target file appearing in the makefile is the one built.
- The order of the other targets does not matter.
- Makefile is invoked from the command line typing
make [-f <name_other_than_M(m)akefile] [target]
- In the makefile variables can be defined.
- More useful when preprocessor directives are used

```
program pointer_test
c
c      Calculate iteratively square root of the elements of a big vector
c
use module_test
c
implicit none
c
#if defined (debug)
    integer, parameter :: n = 40
#else
    integer, parameter :: n = 10000000
#endif
c
real(kind=8) :: xmat(n)
c
#if defined (no_pointers)
    real(kind=8) :: old(n), new(n), tmp(n)
#else
    real(kind=8), target :: old(n), new(n)
    real(kind=8), pointer :: p_old(:), p_new(:), p_tmp(:)
#endif
c
```

```
c
real(kind=8) :: diffmx, valmx
real(kind=8), parameter :: thres = 0.001_8
integer :: indmx, iter
c
call random_number(xmat)
xmat = 8.6_8 * xmat
valmx = maxval(xmat)
indmx = maxloc(xmat,1)
c
#if !defined (no_pointers)
    p_old => old
    p_new => new
    p_old = 1.0_8
#else
    old = 1.0_8
#endif
c
write(6,'(/,a,3x,a)') 'Iter', 'maxdif'
iter = 0
do
    iter = iter + 1
c
```

```

c
#if defined (no_pointers)
    new = approx_sqrt(n,xmat,old)
    diffmx = maxval(abs(new-old))
#else
    p_new = approx_sqrt(n,xmat,p_old)
    diffmx = maxval(abs(p_new-p_old))
#endif
        write(6,'(i3,e11.2)') iter, diffmx
c
    if (diffmx .ge. thres) then
#endif defined (no_pointers)
        tmp = old
        old = new
        new = tmp
#else
        p_tmp => p_old
        p_old => p_new
        p_new => p_tmp
#endif
        else
            exit
        end if
    end do

```

```
c
    write(6,'(/,2a,/i8,2f16.10)') 'Maximum original element ',
    &                                '(index, value, square root) :',
    &                                indmx,xmat(indmx),new(indmx)
c
#if defined(debug)
    call mk_dbg(n,xmat,new)
#else
    call mk_diff(n,xmat,new)
#endif
c
stop
end program
```

```
cpp -Dno_pointers pointer_2.F -o rel_nopointer.f
```

```
# 1 "pointer_2.F"
# 1 "<interno>"
# 1 "<línea-de-orden>"
# 1 "pointer_2.F"
      program pointer_test
c
c Calculate iteratively square root of the elemen
c
c      use module_test
c
c      implicit none
c

      integer, parameter :: n = 100000000

      real(kind=8) :: xmat(n)
c

      real(kind=8) :: old(n), new(n), tmp(n)
```

```
gfortran -E pointer_2.F -o rel_pointer.f
```

gfortran -E ≡ cpp

PREPROCESSOR OUTPUTS

```

module module_test
c
implicit none
c
contains
c
function aprox_sqrt(n,x,r)            $y = \sqrt{x} \longrightarrow y_{i+1} = \frac{y_i + x/y_i}{2}$ 
integer :: n
real(kind=8), dimension(n) :: aprox_sqrt
real(kind=8), dimension(:) :: x, r
real(kind=8) :: two = 2.0_8
aprox_sqrt = (r + x/r) / two
return
end function
c
c
subroutine mk_diff(ndim,xmat,aprox)
c
c.....[red bar]
c
return
end subroutine

```

```

subroutine mk_dbg(ndim,xmat,aprox)
c
implicit none
c
integer :: ndim, i
real(kind=8), dimension(:) :: xmat, aprox
real(kind=8), dimension(ndim) :: exact
real(kind=8) :: diff, maxdif, avedif
c
exact = sqrt(xmat)
c
maxdif = maxval(abs(exact-aprox))
avedif = sum((exact-aprox)**2)
avedif = sqrt(avedif) / ndim
c
write(6,'(/,a,e11.2)') 'Maximum difference :',maxdif
write(6,'(a,e11.2,//)') 'Average difference :',avedif
c
write(6,'(a3,4a15,/)' ) ' I ', ' Original ', }
&                      ' Approx root ', ' Exact root ', }
&                      ' Error '
do i = 1, ndim
    write(6,'(i3,3f15.10,e11.2)') i,xmat(i), aprox(i),
&                                exact(i), abs(aprox(i)-exact(i))
end do
c
return
end subroutine
c
end module

```

Lines added for
debug routine

```
#  
# Makefile example  
#  
FC      = gfortran  
#  
STD_FLAG = -std=f95 -O2 -c -o  
DBG_FLAG = -std=f95 -O0 -c -o  
LNK_FLAG = -std=f95 -o  
#  
LIBS = -I/Library/Developer/CommandLineTools/SDKs/MacOSX.sdk/usr/include -L/Library/Developer/CommandLineTools/SDKs/MacOSX.sdk/usr/lib  
#  
#  
def: module_test rel_pointer  
    $(FC) $(LNK_FLAG) rel_pointer.x module_test.o rel_pointer.o $(LIBS)  
#  
rel_no: module_test rel_nopointer  
    $(FC) $(LNK_FLAG) rel_nopointer.x module_test.o rel_nopointer.o $(LIBS)  
#  
dbg: module_test dbg_pointer  
    $(FC) $(LNK_FLAG) dbg_pointer.x module_test.o dbg_pointer.o $(LIBS)  
#  
dbg_no: module_test dbg_nopointer  
    $(FC) $(LNK_FLAG) dbg_nopointer.x module_test.o dbg_nopointer.o $(LIBS)  
#  
"
```

(TAB)

```
#  
module_test:  
    $(FC) $(STD_FLAG) module_test.o module_test.f  
#  
#  
rel_pointer:  
    $(FC) $(STD_FLAG) rel_pointer.o pointer_2.F  
#  
rel_nopointer:  
    $(FC) -Dno_pointers $(STD_FLAG) rel_nopointer.o pointer_2.F  
#  
dbg_pointer:  
    $(FC) -Ddebug $(DBG_FLAG) dbg_pointer.o pointer_2.F  
#  
dbg_nopointer:  
    $(FC) -Ddebug -Dno_pointers $(DBG_FLAG) dbg_nopointer.o pointer_2.F  
#  
#  
clean:  
    rm *.o *.mod
```