French for Homological Algebra via Tohoku

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26 mai 2015

Résumé

This exposition focuses on the foundational paper 'Sur quelques points d'algèbre homologique' by Grothendieck (otherwise known as 'Tohoku'). Related papers are used initially as a source of examples. The goal is to quickly develop the confidence required to read [Tok] (and other related material such as [BBD]) in French with the occasional use of a dictionary and without the blind use of technology.

1 Introduction

For acquiring a broad reading ability of the sciences in French, the books [6] and [7] are recommended. The strength of these books is in their method of presentation which skillfully leverage the reader's knowledge of English and teaches the most commonly encountered constructs first. Unfortunately for us, even with the focus of these books on the sciences, their vocabulary coverage is still too general for a time-constrained mathematician to memorize. This article, while heavily influenced by their presentation, will shift focus to the language used within [Tok]. Essentially, these notes are the record of the author's own attempt to learn mathematical French but organised in a way that may also assist others. **This is a work in progress.**

2 French vocabulary

We will obtain our vocabulary and examples from 'Faisceaux Pervers', with [Tok], [BBG], [EGA] and [Ill] being used for further examples. Note that while many French words carry across to English with little or no change in meaning we will use a more formal and mathematical meaning. For example 'beaucoup' in conversation may mean 'a lot' but here we will translate it as 'many'.

2.1 Cognates

The most well behaved vocabulary class is that of the cognates. History has provided an overlap between French and English but care must be taken with the so called false-friends - words that look the same but have a very different meaning.

2.1.1 Exact Cognates

These words have the same spelling as the corresponding English word but may have broader meaning.

assertion	base	cause
section	notation	définition
détail	référence	introduction
condition	construction	notes
application	international	observation
construction	contravariant	conséquence
consideration	interpretation	général
situation	image	notion
excellent	complète	coincide
multiplicative	noethérien	dimension
catégories	complexes	affine
extension	général	index
dual	existence	équivalent
quasi-	bi-	hyper-
image	intersection	précise
limite	membre	morphisme
module	preprint	naturel
notation	évidence	paire
passage	position	problème
rotation	stable	structure
spectral	families	

translation	variable	stable
solution	note	long
fraction	analogue	triangle
injection	projection	quotient
inversion	couple	structure
inclusion	flasque	inductive
unique	definition	simple
base	restriction	semi-simple
group	additive	axiom
global	cone	type
rotation	t-structure	t-exact
pull-back	proposition	précise

Furthermore, some words are easily recognizable since their spellings only differ slightly from their English counterparts.

axiome	complexe	diagonale
résulte(*)	exacte	exemple
morphisme	isomorphisme	épimorphisme
monomorphisme	intervalle	objet
essentiel	groupe	transforme
terme	théorème	triviale
résulte	orthogonale	problème
formalisme	universelle	propriété
usuel	resulte	générateur
résolvant	généralités	critères (criteria)

2.1.2 Cognates ending in -ie or -é

By replacing an **-ie** or **-é** ending within a French noun with *-y* the corresponding English word may often be recognized.

catégorie	dualité	théorie
géométrie	homologie	cohomologie
variété	homotopie	identité
ambiguité	quantité	topologie

2.1.3 Cognates ending in -iqué

By replacing a **-ique** ending within a French adjective with *-ic* or *-ical* the corresponding English word may often be recognized.

mathematique	identique	symetrique
geometrique	homologique	cohomologique
acyclique	specifique	homotopique
réciproque	topologique	réciproque

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2.1.4 False friends

These words appear to be exact cognates but have different meanings. Some important examples have been added from [Stack]

actuel, actuelle	current,present-day
application	function
but	target(of a morphism),purpose, goal
car	for ; becasue
comment	? ? ?
commodité	convenience
conference	lecture
court, -e	short
encore	? ? ?
inconvénient	disadvantage
lecture	reading
or	now ;but ;gold
phrase	sentence
pour	for, in order to
démonstrations	demonstration, show
limitation	restriction
large	wide, large (but also) generous
on	one
résumé	résumé (but also) summary
moment	moment (but also) occasion
départ	depart (but also) start
plus	more
a	has
à	to, at, in
schema	scheme (but also) diagram
propos	purpose, talk
suites	consequence, series, sequence, continuation
encore	still, again, yet
former	to form
prolongement	extension ; continuation
rapport	link, connection ; report ; return,yield

premier first, principal, distinguished

2.1.5 Definite article ("the") - *le,la,l',les*

The words *le,la,l',les* are translated as *the*. The first three are used with singular nouns and **les** is used for plural nouns.

le is used for masculine nouns - **le morphisme**.

la is used before feminine nouns - **la catégorie**

l' is used before any singular noun beginning with a vowel sound - **l'axiome**

les is used before any plural noun - **les morphismes, les catégories, les axiomes**

The gender of nouns is indicated in the vocabulary via the definite article as follows
masculine noun e.g., **le complexe**
feminine nouns e.g., **la définition**
Note that a small number of nouns have the same form as the plural.
Also, for adjectives ending in **-au** or **-al** the plural ending has **-aux**. For example

singular	plural
local	locaux
global	globaux
principal	principaux

2.2 Exercises 1

Translate the following into English.

la limite	le théorème
la situation	l'analogue
le but	le triangle
la construction	la suite
l'equivalence	la t-structure
la dimension	la proposition
l'image	l'inclusion
l'automorphisme	la dualitié
algébrique	geometrique
canonique (?)	homologique
historique	systematique
identique	axiomatique
les images	les t-structures
les foncteurs	les assertions
les suites	les axiomes

2.2.1 Indefinite article - *un,une,des*

The words **un** and **une** can be translated as *a* or *an*. Masculine singular nouns use **Un** while feminine use **une**. The word **Des** means *some* and is used before plural nouns - often it can be left untranslated. Some examples from [Ill] and [BGG] are

un triangle	a triangle	des triangles	(some) triangles
une condition	a condition	des conditions	(some) conditions
une catégorie	a categorie	des catégories	(some) categories
une t-structure	a t-structure	des t-structures	(some) t-structures

2.2.2 Plurals of nouns

Usually nouns become plural by simply adding '-s' but some nouns in their singular form have the endings '-s','-x' or '-z'. The plural form of these singular nouns does not

change. In this case, the article 'les' or 'le' can be used to check whether the noun is singular or plural. Some examples are

le faisceaux	the sheaf	les faisceaux	the sheaves
le cas	the case	les cas	the cases
le choix	the choice	les choix	the choices
l'anneaux	the ring	les anneaux (shortened form ?)	the rings

2.2.3 -ment words

uniformément	uniformly
partiellement	partially
naturellement	naturally
essentiellement	essentially
respectivement	respectively
nécessairement	necessarily
suffisamment	sufficiently (enough)
dualement	dually
correctment	correctly
automatiquement	automatically
homotopiquement	homotopically
recouvrement	collection ?,covering ?ensemble ?
localement	locally
isomorphiquement	isomorphically
linéairment	linearly
évidemment	obviously
fonctoriellement	functorially
également	equally, likewise, also
canoniquement	canonically
uniquement	uniquely
directement	directly
respectivement	respectively
inversement	conversely
réciproquement	reciprocally
immédiatement	immediately
successivement	successively
simplement	simply
facilement	easily
trivialement	trivially
précisément	precisely
fidèlement	faithfully
purement	purely
généralement	generally, usually
forment	form
seulement	only
pleinement(*)	fully

Some phrases are given to cosolidate the above material. Note that adjectives usually follow the noun they modify as seen from these examples :

les catégories additives	the additive categories
un espace topologique	a topological space
la dimension cohomologique	the cohomological dimension
une catégorie abélienne	an abelian category
une résolution injective	an injective resolution
un triangle exact	an exact triangle
un faisceau d'anneaux	a sheaf of rings
foncteur identique	identity functor (?)
les deux suites exactes	the two exact sequences
les systèmes locaux	the local systems
invariants cohomologiques globaux	global cohomological invariants
les espaces localement compacts	locally compact spaces
évidemment un ensemble constructible	... obviously a constructable sequence ...
résulte trivialement	trivial result

2.2.4   Prepositions

à	to
apres	after
avant de, before	
avec	with
d'apres	according to
dans	in
de	of ; from
par	by, with, through
pour	in order to, to-
sans	without
selon	according to
sous	under , sub-
suivant	according to, following, next
sur	in, on (a topic)

Note that **de** becomes **d'** before a vowel
Preposition examples

Dans le cas ...	In the case ...
dans la catégorie C	in the category C
un faisceau sur X	a sheaf on X
action sur les complexes	action on the complexes
un sous-complexe	a sub-complex
la sous-catégorie	the sub-category
d'après la définition	according to the definition
coincide avec le complexe	coincide with the complex
avec $\beta \circ \alpha = 0$	with $\beta \circ \alpha = 0$
sans les restrictions	without the restrictions
sans condition sur U	without the condition on U
par le symbole $\equiv$	by the symbol $\equiv$
par définition	by definition
Suivant la terminologle introduite dans ...	According to the terminology introduced in ...
Les conditions suivantes sont équivalentes	The following conditions are equivalent
selon le schéma [1] suivant	according to the following diagram

2.2.5 Infinitives - Verbs ending in '-er','-ir','-re'

Verbs with these endings are said to be in the infinitive form. As a common example we have **prouver** = *to prove*. The following examples are from [BBD]. Note 'notre' is not a verb - it means 'our'. Simillarly, 'degré' is just 'degree'.

ajouter	to add, to supplement
appliquer	to apply, to put to use
baaatir	to construct, to build
changer	to change, to modify
compléter	to supplement, to complete
déduire	to deduct, to take from, to deduce
définir	to define, to specify
disposer	to arrange, to set out
donner	to give, to show
échanger	exchange, interchange
expliquer	to explain, to show
filtré	to filter ?
généeraliser	to generalize
marquer	to label, to signify, to mark
montrer	to show, to demonstrate
partir	to go, to leave, to start
préciser	to specify
prendre	to take
renforcer	to strengthen, to reinforce
répresenter	to represent
simplifier	to simplify
supérieure	? ?
tourner	to rotate, to turn
trouver	to find, to identify, to locate
vérifier	to prove, to check

Note : The preposition **de** has the meaning 'to-' before an infinitive (see list above). Also note that **de** becomes **d'** before a vowel. Some examples of infinitive phrases are

d'appliqu**er**	to apply
Il suffit (enough,sufficient) **d'**appliqu**er**	It is sufficient/enough to apply
de déduir**e**	to deduce
d'expliquer les axiomes	to explain the axioms
Donn**er** l'exemple où (where) A est l'anneau local	Give an example where A is a local ring
Prend**re** par exemple	Take for example
Pour vérifi**er** qu'il est surjectif	To check it is surjective
Pour montr**er** ...	In order to show...
Ceci (this) permet **d'**appliqu**er** la formule de Kunneth	This allows us to apply the Kunneth formula

2.2.6   Present tense of -er verbs

On trouve   We find

Examples from EGA

on trouve un homomorphisme canonique fonctoriel
on trouve un diagramme commutatif
on trouve une suite exacte

2.2.7 The propsitions à and de

The preposition **à** is usually translated as 'at' or 'to'. It is often contracted so that that **au** is the contracted form of 'à la' and similarly 'aux' is the contracted form of 'à les'. They are both usually translated as 'to the' or 'at the'.

isomorphe au foncteur identité	isomorphic to the identiy functor
au point z	at the point z
canoniquement isomorphes aux foncteurs	canonically isomorphic to the functors
compatibles aux actions	compatible with the actions

Similarly, the preposition **de** is usually translated as 'of' or 'from'. The contracted form of 'de le' is **du** and the contracted form of 'de les' if **des** - they both have the meaning 'of the' or 'from the'

La theorie des faisceaux	The theory of sheaves
La catégorie des foncteurs contravariants exacts	The category of exact contravariant functors

We now turn to the specific mathematical vocabulary required for reading [Tok] (with some extra terms from [BBD]). The following list does not contain the obvious cognates (e.g. adjoint, projection) that have been listed earlier.

2.2.8 Mathematical vocabulary

anneau	ring
acyclicité	acyclicity
carré	square
conoyau	cokernel
coeur	core (heart) of a derived..?
discontinu	discontinuous
droit	right
entier	integral
espace	space
fermé	closed(set)
fidèle	faithfull, exact, true
fini	finite
flèche	arrow
foncteur	functor
gauche	left
infinis	infinite
noyau	kernel
nul	null,zero,void,no,not any
ouvert	open(set)
plein	full, whole
recouvrement	cover (topological)
si et seulement si	if and only if
sous-	sub-
plongement	embedding (from immersion, plunge)

Mathematical vocabulary examples

Pour tout entire a	For all integral a
carré commutatif	commutative square
un faisceau d'anneaux O	the sheaf of rings O
faisceau de O-modules à gauche	sheaf of left O-modules
plongements de sous-complexes $X \hookrightarrow Y$	embeddings of sub-complexes $X \hookrightarrow Y$

UPTO - STACK - Upto page 19, section 25 - present tense 'ent' type verbs. UPTO - I've done about 170 words. Need 60 more for pages 18 and 19. Perhaps do 30 useful ones and leave single appearance words for footnotes.

2.2.9 Frequent vocabulary

toujours	always
ce,cet,cette	this
ces	these, those
ceci	this
cela	that
celle	that, that one
celles	those
elle	this
et	and
en	in
nous	we
sont	are
est	is
si	if
seul	single, only

2.2.10 Commonly used words and phrases

pour	for
pour tout	for all
toujours	always
méme	same
donc	then, thus, hence, therefore
On a donc	therefore (?)
alors	therefore, hence,thus
Soit...	Suppose...
Soient...	Let there be...
Lorsque...	When...
Il existe...	There exists...
Rappelons..	Recall...

Le premier first, principal En particulier In particular dernier last, ending

2.2.11 Exercize 1

All of the following phrases are taken from [Ill],[BBD],[EGA] or [SGA]. Translate these phrases (answers may be found in the appendix).

immédiatement des définitions	immediately from the definitions
dans la catégorie des groupes abéliens	from the category of abelian groups
dans le cas général	in the general case
Lorsque F est exact à gauche	when F is left exact
Soit X une variété topologique de dimension n	Suppose X is a topological variety of dimension n.
Il existe une résolution	There exists a resolution

2.3   Sentence starters

At this point we break with the order of presentation in [blah] and focus on the common ways to begin a mathematical sentence.

2.3.1   Pour

The word 'pour' means 'for' or 'in order to'. The phrase 'pour tout' 'for all' is very common. Another common usage is with the '-er' (indicative) form of a verb. For example 'Pour prouver' 'to prove' and 'Pour vérifier' 'to check'

Pour la definition	
Pour tout/toute/tous/toutes	For all
Pour vérifier	To
Pour A une...	
Pour prouver	To prove...
Pour ètre	

2.3.2   Misc Vocabulary

etude	study
calcul	calculation

3   Appendix 1 - SGA II Chapter 1 first 3 pages

Frequency of vocabulary

ouvert(e) 16	cover
dans 11	in
pour 9	for, in order to
par 9	through
au,aux 8	to the, with the, at the
sous 7	sub-
donc 7	then, therefore
cas 6	case
partie 6	part
si 5	so, however ; if, whether
si et seulement	iff
alors 5	then, at that time
fermé 5	closed (set) (also firm, steady)
comme 5	as, like, as if
etre 4	to be, to exist, to belong
Bien 4	good, well
bien entendu	of course
bien que	although, though
bien sur	of course, certainly
tel 5	so, so much
nous 4	we
tout 4	all, any, every (? check elsewhere)
support 3	
localement 3	
fermée 3	
mĹeme 3	
notons 3	
connu 3	
définir 3	

UPTO : Next...(1). Underline and include nouns from original's first 3-4 pages. (2) May need bit more on verb endings (3) Take a look at [5] J. Nekovar, Introduction to Mathematical English. http ://people.math.jussieu.fr/ nekovar/co/en/