The dynamics of capital and labor misallocation: a critical assessment

Maurice J.G. Bun* and Jasper M. de Winter†

this version: October 31, 2018

Abstract

We analyze the impact of misallocation of labor and capital on productivity levels in the Netherlands. We use the marginal revenue products of labor and capital to measure the extent of misallocation. We find that there is substantial heterogeneity in the marginal revenue products at the level of the firm. Compared to a counterfactual efficient allocation we find that misallocation has doubled in the period 2001–2016. We analyze the evolution of marginal revenue products over time with a dynamic panel data specification allowing for heterogeneous coefficients. We find that the observed increase in misallocation is caused by a small fraction of firms, with a unit root in marginal revenue products. Finally, we show that in our sample the measurement of misallocation is largely insensitive to observed heterogeneity in the production function and to the presence of capital adjustment costs.

Keywords: misallocation; panel data; productivity.
JEL-Codes: C23, D24, O47.

1 Introduction

In this paper we estimate the negative impact of misallocation of labor and capital on productivity levels in the Dutch business sector. In our empirical analysis we use the model of Hsieh and Klenow (2009). This set up is attractive because it estimates distortions or

*Economics and Research Division, De Nederlandsche Bank and University of Amsterdam, m.j.g.bun@dnb.nl.
†Economics and Research Division, De Nederlandsche Bank, j.m.de.winter@dnb.nl.

wedges in output and capital at the level of the firm for each year. These estimates are then used to measure misallocation of labor and capital within industries. Furthermore, we construct counter-factual productivity levels assuming an efficient allocation of production factors. Comparing this counter-factual with realized productivity, we conclude that misallocation in the period 2001–2016 has doubled.

The recent literature on misallocation has questioned the assumptions of the Hsieh-Klenow model. In this paper we therefore analyze the impact of these assumptions on the measurement of misallocation. Crucial question to answer is whether we should attribute measured distortions to misallocation, or alternative explanations for the observed distortions. In other words, are the estimated distortions due to misallocation or misspecification? We address a number of alternative explanations for observed distortions. First, what is the impact of an incorrect specified production function? We analyze the impact of heterogeneous production functions at the level of the firm. We also explain the implications for the model outcomes when a Constant Elasticity of Substitution (CES) production function is assumed instead of the Cobb-Douglas (CD) functional form adopted by Hsieh and Klenow (2009). Second, capital adjustment costs are important and relevant, but are not modeled by Hsieh and Klenow (2009). We address how much capital adjustment costs attribute to the misallocation measure.

We conclude that the misallocation measure of Hsieh and Klenow (2009) is largely robust to these extensions. We next analyze the empirical properties of the misallocation measure, i.e. the Marginal Revenue Product of Capital (MRPK). For this part of the analysis we use a balanced panel of approximately 8,000 firms for which we have complete data on MRPK for the period 2001–2016. We use a variety of panel data model specifications to estimate the nature of the MRPK process over time and across firms. Among other things, we assess the degree of persistence in MRPK allowing for heterogeneity in intercept, slope and variance parameters. Our empirical model of MRPK is able to reproduce the main empirical fact that misallocation, i.e. the cross-sectional variance of MRPK, increases over time.

The remainder of this paper is structured as follows. Section 2 describes the data set, Section 3 describes our baseline model, Section 4 introduces persistence and dynamics in the model. Section 5 analyses the robustness of our baseline model to recent criticism in the literature. Section 6 concludes.

2 Data set

Our data set contains the complete population of Dutch firms that had to declare corporate income tax during the period 2001–2016 (i.e., all firms with a legal person). These confidential micro data are provided by Statistics Netherlands (CBS) and are based on a merger of the Dutch general business register (ABR) and corporate tax declarations (NFO). The matched data set includes annual balance sheets as well as profit and loss statements. We restrict our sample to non-agricultural private firms in the non-financial sectors. For multi-establishment firms we take the two-digit industry code (NACE Rev. 2 two-digit) of the firm with the largest number of employees. The raw data set contains 2,642,811 firm-year observations, and 507,891 unique firms for the period we analyzed (2001-2016). We categorize all firms as either micro, small, medium-sized, or large.[1]

The original data come as a repeated cross-section, with unique identifiers for the firm, and can be viewed as highly unbalanced panel data. We measure the firm's capital stock as the value of fixed tangible assets on the balance sheet, and labor as the firm's number of employees. We clean our data set for outliers and minimum number of observation per industry following previous research of e.g. Gamberoni et al. (2016) and Gopinath et al. (2017) by taking the following steps. First, we drop firm-year observations for which no size-class or fixed tangible assets are available. These steps reduce our sample to 2,189,256 firm-year observations and 402,574 unique firms. Second, we drop observation when the ratio of tangible assets to the balance sheet total is greater than one. This step reduces the sample by 955 observations. Next, we drop firm-year observations where the ratio of the wage bill to value added is in the top/bottom 1% of the distribution. This step reduces the sample by 40,705 observations. In addition, following Gopinath et al. (2017), we drop firm-year observations if the wage bill to value added ratio is larger than 1.1 or smaller than 0.1. This step reduces the sample by 255,784 and 52,626 observations, respectively. Finally, we drop industries where the average number of yearly observations is less than 100. This step reduces our sample by 4,716 observations. After cleaning, our final data set contains 1,834,470 firm-year observations and 367,869 unique firms.

[1] Micro (small) enterprises have less than 10 (50) employees and an annual balance sheet total of below EUR 2 (10) million. Medium-sized (large) enterprises have less (more) than 250 employees and an annual balance sheet total of no more (less) than EUR 43 million. A firm's number of employees includes the firm owners. This means that sole proprietors are classified as firms with one employee.

3 Measuring misallocation

To quantitatively assess the effect of within-industry resource misallocation on aggregate total factor productivity we build on the framework developed by Hsieh and Klenow (2009), recently applied by i.e. Gopinath et al. (2017), Gamberoni et al. (2016), Calligaris (2015), Calligaris et al. (2017) and many others. The Hsieh-Klenow model formally shows that frictions distorting the MRPK or the marginal revenue product of labor (MRPL) lower aggregate factor productivity.

The model has two main assumptions. First, firms are heterogeneous in their productivity level and in the extent of factor-market and size distortions they face. Second, every firm supplies a heterogeneous good which is priced individually in the market. Aggregate economy wide output is defined by a CD production technology:

$$Y_t = \prod_{S=1}^{S} Y_{s,t}^{\theta_s} \text{ where } \sum_{s=1}^{S} \theta_s = 1, \tag{1}$$

where $Y_{s,t}$ denotes industry-specific output in industry s in year t and Y_t is the product of all industry-specific outputs $Y_{s,t}$ raised to their individual industry-output share θ_s. At the industry level, s, the output is a CES-aggregate of M_s differentiated products:

$$Y_{s,t} = \left(\sum_{i=1}^{M_s} Y_{is,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $Y_{is,t}$ denotes output of firm i –i.e. the heterogeneous good produced by firm i– in industry s at time t. The parameter σ is the time-invariant elasticity of substitution between firm value added or the substitutability of competing manufacturers. The higher σ the more substitutable the goods become and the less the firm can control the market price via it's mark-up. For comparability with Hsieh and Klenow (2009) and Gopinath et al. (2017) we set σ at 3.[2]

Each individual firm produces its unique good according to a standard CD production function:

$$Y_{is,t} = A_{is,t} K_{is,t}^{\alpha_s} L_{is,t}^{(1-\alpha_s)}, \tag{3}$$

Where $A_{is,t}$, $K_{is,t}$ and $L_{is,t}$ are total factor productivity, real capital and labor input of firm i, in industry s at time t, respectively. We measure the firm's nominal value added $P_{is,t}Y_{is,t}$ as the difference between gross turnover minus materials used in production. Real firm-level output $Y_{is,t}$ equals nominal value added deflated with a form-specific output price

[2] Note that as σ goes to ∞ the economy leaves monopolistic competition and approaches perfect competition.

deflator $P_{is,t}$. Since we do not observe prices at the firm level in our data set, we revert to using two-digit industry deflators, following e.g. Dias et al. (2016), Gorodnichenko et al. (2018) and Gopinath et al. (2017). This implies we calculate $Y_{is,t}$ as $P^Y_{s,t}Y_{is,t}/P^Y_{s,t}$, where $P^Y_{s,t}$ is the industry deflator. We measure labor input $L_{is,t}$ as the firm's wage bill deflated by the same two-digit industry deflators used to deflate nominal firm-level output, i.e. $P^Y_{s,t}L_{is,t}/P^Y_{s,t}$. We use the nominal wage bill instead of employment to control for differences in the quality of the workforce across firms, as argued in e.g. Hsieh and Klenow (2009) and Gopinath et al. (2017). We measure the firm-specific real capital stock $K_{is,t}$ with the book value of fixed tangible assets deflated with the total deflator of non-financial corporations gross fixed capital formation, P^K_t. This implies we calculate $K_{is,t}$ as $P^K_t K_{is,t}/P^K_t$.

The idea of the Hsieh-Klenow model is that each firm is a monopolist in its differentiated product and faces two types of distortion: a capital wedge, $\tau^K_{is,t}$, that changes the relative marginal revenue product of capital with respect to labor, and an output wedge $\tau^Y_{is,t}$, that changes the marginal product of capital and labor by the same proportion. Market distortions appear in the firm's profit equation:

$$\pi_{is,t} = (1 - \tau^Y_{is,t})P_{is,t}Y_{is,t} - w_t L_{is,t} - (1 + \tau^K_{is,t})R_s K_{is,t}, \tag{4}$$

where w_t is the wage faced by all firms and R_s is the time-invariant industry specific real rental price of capital, i.e. the sum of the nominal interest rate per industry r_s and the industry-specific depreciation rate δ_s minus the average headline inflation, $\Delta HICP_t$, where $HICP_t$ is the natural logarithm. We assume an average inflation rate of 2% in our sample, in line with the average HICP-inflation rate in the Netherlands during the period 2001–2016. We estimate r_s and δ_s from the firm-level data in two steps. First, we calculate the firm-specific time-varying implicit interest rate $r_{is,t}$ and depreciation rate $\delta_{is,t}$, by dividing the firm's interest payment by total debt and depreciation divided by total fixed tangible assets. Second, we aggregate the firm-specific time-varying implicit interest rate and depreciation rate to industrial averages by weighting with the average industrial real value added shares over the period 2001–2016.[3] Before aggregation we clean $\delta_{is,t}$ and $r_{is,t}$ by setting $\delta_{is,t}$ or $r_{is,t}$ equal to missing when it is in the top/bottom 5% of the respective distributions.

As a result of the wedges $\tau^Y_{is,t}$ and $\tau^K_{is,t}$, there will be differences in the marginal products of labor and capital across firms. The first order conditions for profit maximization with

[3] All deflators, the HICP index and the real value added shares are publicly available via the website of Statistics Netherlands here

respect to labor and capital, are given by:

$$P_{is,t}\frac{\partial Y}{\partial L} = MRPL_{is,t} = (1-\alpha_s)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{P_{is,t}Y_{is,t}}{L_{is,t}}\right) = \left(\frac{1}{1-\tau^Y_{is,t}}\right)w_t, \tag{5}$$

$$P_{is,t}\frac{\partial Y}{\partial K} = MRPK_{is,t} = \alpha_s\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{P_{is,t}Y_{is,t}}{K_{is,t}}\right) = \left(\frac{1+\tau^K_{is,t}}{1-\tau^Y_{is,t}}\right)R_{s,t}, \tag{6}$$

Following Foster et al. (2008) we make a distinction between revenue productivity (TFPR) and physical productivity (TFPQ), i.e. the classical Solow-residual. This is not only convenient, but also necessary because firm survival ultimately depends on TFPR instead of TFPQ. Using this terminology we can define TFPQ by rewriting equation (3):

$$TFPQ_{is,t} = A_{is,t} = \frac{Y_{is,t}}{K^{\alpha_s}_{is,t}L^{(1-\alpha_s)}_{is,t}}, \tag{7}$$

and TFPR as:

$$TFPR_{is,t} = P_{is,t}A_{is,t} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{MRPK_{is,t}}{\alpha_s}\right)^{\alpha_s}\left(\frac{MRPL_{is,t}}{1-\alpha_s}\right)^{1-\alpha_s}, \tag{8}$$

As is evident, in this model TFPR does not vary across firms within an industry unless firms face output and/or capital distortions. The idea is that in a frictionless economy more capital and labor should be allocated to firms with higher physical productivity, $A_{is,t}$, to the point where higher output results in lower price and the same TFPR as that of firms with lower physical productivity. However, when there are frictions, economy wide output can be (much) lower. Imagine and economy with two firms that have identical technology but in which one firm benefits from subsidized credit (say from a state-owned bank), and the other firm can only borrow at high rates from informal capital markets. Assuming that both firms equate the MRPK with the interest rate, the MRPK of the firm with access to subsidized credit will be lower than the MRPK of the firm that only has access to informal financial markets. This is a clear case of capital misallocation: aggregate output would be higher if capital was reallocated from the firm with a low MRPK to the firm with a high MRPK. The misallocation of capital results in lower aggregate output per worker and TFPQ (see e.g. Gopinath et al., 2017).

We follow Hsieh and Klenow (2009), and estimate the impact of misallocation thus defined on the TFPQ level by defining the "efficient" level of TFPQ, as the TFPQ-level we would observe in the first-best allocation in absence of dispersion in MRPK, MRPL and TFPR such that $TFPR_{is,t} = \overline{TFPR}_{s,t}$, where:

$$\overline{TFPR}_{s,t} = \frac{P_{s,t}Y_{s,t}}{K^{\alpha_s}_{s,t}L^{(1-\alpha_s)}_{s,t}}, \tag{9}$$

It can then be shown that the difference in log(TFPQ) arising from misallocation – the misallocation-gain $\Lambda_{s,t}$– can be written as a combination of the variables introduced in equations (3)–(9). See equations (1)–(11) in Gopinath et al. (2017) for a formal derivation of our misallocation measure. In Section 3.1 we will express our misallocation-measure $\Lambda_{s,t}$ in percentage point differences with respect to the first-best allocation in absence of dispersion in MRPK, MRPK and TFPR in 2001.

Our misallocation measure represents an upper limit to intra-industry misallocation, given that *all* variations in the marginal revenue product are attributed to misallocation. This is assuming that firms are at their long-term static equilibrium at any given moment in time, irrespective of the scope, type and frequency of firm-specific shocks. Each diversion from the equilibrium is regarded as misallocation, which constitutes a substantial assumption. The model assumptions are relatively strict: all firms have a productivity level with the same mark-up and the same capital intensity, adjustment costs are absent, and the substitution elasticity with respect to capital and labor is equal to a CD production function with constant revenues of scale. Despite these limitations, the model is nevertheless a suitable initial indication of misallocation in the economy. In Section 5 we show that the Hsieh-Klenow measure of misallocation is quite robust to recent proposed extensions, including functional form of the production function, firm-specific heterogeneity in the production function and adjustment costs in our sample of firms.

3.1 Estimation results misallocation

Figure 1 shows the allocative efficiency for both factors of production, labor and capital, in the 2001-2016 period according to the Hsieh-Klenow model outlined above, reflecting the standard deviation of the log of the marginal revenue product of capital (MRPK, black line) and the log of the marginal revenue product of labor (MRPL, grey line).[4] The larger the standard deviation, the higher the degree of misallocation. Capital misallocation shows a discernible upward trend, interrupted briefly between 2004 and 2007. On balance, capital misallocation in the Netherlands increased by almost 20% in the 2001–2016 period. Labor misallocation did not develop in line with the same trend, but remained more or less constant over the entire period. In the 2002–2006 period labor misallocation even dropped below the level recorded in 2001. Labor misallocation fluctuated between 1% and 4% during and after the crisis years, in 2016 on balance exceeding the 2001 level by a "mere" 4%. This outcome resembles other recent studies of misallocation elsewhere in Europe

[4] Economy-wide measures are calculated by the weighted mean, using real industrial value added weight calculated as the average share in real value added over the period 2001–2016.

(e.g. Gamberoni et al., 2016 and Gopinath et al., 2017). For example, Gopinath et al. (2017) report that capital misallocation increased sharply in Spain, Italy and Portugal, but more or less stabilized in France, Germany and Norway.

Under the Hsieh and Klenow (2009) framework that we adopt, increasing dispersion of log(MRPK) together with stable dispersion of the log(MRPL) implies that the covariance between log(TFPR) and $\log(\frac{K}{L})$ across firms is decreasing over time, as explained in more detail in Gopinath et al. (2017). In the Netherlands the variance of $\log(\frac{K}{L})$ has increased in the period 2001–2016, which is equal to the result found for Spain in Gopinath et al. (2017). Moreover, the combination of a steeply rising dispersion of MRPK and more or less steady dispersion of MRPL is not consistent with heterogeneous markups as an alternative explanation. Increasing dispersion of markups would induce increasing dispersion in both MRPK and MRPL, as documented in Gopinath et al. (2017).

[INSERT FIGURE 1 ABOUT HERE]

Figure 2 shows the possible gain from reducing misallocation to zero on the measured TFPQ level in the total economy, our misallocation-gain measure $\Lambda_{s,t}$ as introduced earlier in this section. It turns out that, removing all distortions would boost aggregate TFPQ in the Netherlands from 7% in 2001 to 14% in 2016, suggesting that since 2001 there has been a decline in allocative efficiency by about 7 percentage points. Gains from the absence of misallocation remain quite constant in the period from 2002 up until the Great Recession (2008), and from there onwards start to rise. Therefore, distortions increase significantly in the year following the Great Recession, and the European Debt Crisis (2011) when the Netherlands entered a prolonged recession period. The results could indicate the lack of a large "cleansing" during these crisis. Comparing our results to recent findings in the literature seems to indicate that the increase in misallocation since 2001 is more comparable to the Southern EU member states than the Northern memberstates. Similar to the Netherlands, Gopinath et al. (2017) observe significant increased in misallocation in Spain and Italy, but quite stable levels of misallocation in Germany, France and Norway. Calligaris (2015) and Calligaris et al. (2017) also find steadily increasing misallocation in Italy.

[INSERT FIGURE 2 ABOUT HERE]

In the observed period, the increase in misallocation was not evenly spread across industries. First, misallocation in the services sector exceeds that in the manufacturing sector.[5]

[5] The manufacturing sector is comprised of (NACE Rev. 2 two-digit codes in brackets: manufacturing

This is clear from Figure 3, which plots the measured capital and labor misallocation in the total economy, alongside the misallocation of capital and labor in manufacturing (striped line) and in the services sector (dotted line). Panel A shows the misallocation measures for capital (MRPK) and Panel B shows our misallocation measure for labor (MRPL). This results matches the outcome of earlier studies. For Portugal, Dias et al. (2016) find that misallocation in the services sector exceeds that in the manufacturing sector by a large amount, as do Busso et al. (2013) for a number of Latin American countries.

[INSERT FIGURE 3 ABOUT HERE]

Second, the development of labor misallocation in manufacturing is markedly different from that in the services sector (Panel B of Figure 3). In manufacturing it has witnessed a strong increase since 2005, whereas growth in the services sector is considerably smaller. Dias et al. (2016) indicate that the higher misallocation measures in the services sectors seem to be driven by lower competition coupled with limited international tradeability, regulatory barriers, and the fact that services are highly location-specific. Unlike the diverging development of labor misallocation, capital misallocation rose during the crisis years in both manufacturing and the services sector.[6]

Besides, it could be relevant to policy makers to analyze the impact of firms' technological level on misallocation, as pointed out recently by Calligaris (2015). To this purpose, we divide firms in the industry sector according to their technological intensity into high, medium-high, medium-low and low-technological intensity and in services according to knowledge intensive services and less knowledge intensive services based on the Eurostat classification.[7]

Figure 4 reports the increase in the dispersion of MRPK according to technological and knowledge intensity in the manufacturing and services sector, respectively. MRPK dispersion is expressed against MRPK dispersion in the total sample in 2001. We do not find

industry (10–33) and construction (41–43). The services sector is comprised of wholesale and retail trade; repair of motor vehicles and motorcycles (45–47), transportation and storage (49–53), accommodation and food services (55–56), information and communication (58–63), professional, scientific and technical activities (69–75), administrative and support service (77–82).

[6] Figure A.3 in Appendix A shows a more detailed picture by comparing our capital misallocation measure at the two-digit industry level in 2001 and 2016, respectively. Panel A shows that within manufacturing, capital misallocation in the 2001–2016 period increased most sharply in construction, from 10 percentage points below to 40% above the average in 2016. In the services sector, the highest degree of misallocation in both 2001 and 2016 was recorded in the storage industry, while the degree of misallocation is lowest in the courier industry.

[7] Outlays on research and development in percent of value added served as a criterion of classification of industries in manufacturing into high-technology, medium high-technology, medium low-technology and low-technology industries. Services are aggregated into knowledge-intensive services and less knowledge-intensive services based on the share of tertiary educated persons in the workforce. We refer to Eurostat's website, here,for a breakdown of industry into technological/knowledge-intensity.

large differences in the increase in misallocation according to the technological or knowledge intensity. We find that capital distortions increased over time for all groups. For firms at low-technological/knowledge intensity the MRPK dispersion is higher, suggesting that misallocation is more important in industries at high levels of technology/knowledge. Overall, the trends of the series follow a pretty similar path and there is not an unequivocal relation between technological intensity and misallocation. This conclusion is in line with previous evidence (see e.g. Calligaris, 2015 and Calligaris et al., 2017).

[INSERT FIGURE 4 ABOUT HERE]

Misallocation is also related to firm size for various reasons. Larger firms tend to be older than smaller firms, and may be able to self-finance themselves more easily via build up reserves than the smaller ones and, hence, being less exposed to financial constraints, with a consequent effect on misallocation. Moreover, the dimension of firms might be a signal of good management practices and more efficient allocation of resources (see e.g. Calligaris, 2015). Recently, Gopinath et al. (2017) found that credit restrictions for small enterprises explain the sharp increase in capital misallocation, providing model-based and empirical evidence that credit restrictions in a low-interest environment prompt large enterprises to invest too much and small enterprises to invest too little.

[INSERT FIGURE 5 ABOUT HERE]

Figure 5 reports the increase in the dispersion of MRPK according to firm size, relative to the total misallocation in 2001. During the period 2001–2012 misallocation increased for all business sizes. Evidently, there is a strong negative relation between capital misallocation and the size of the firm: averaged over the period 2001–2016 the MRPK dispersion of micro-firms is around 13% higher than the MRPK dispersion of large firms. Remarkably, this difference was much larger in the years leading up to the financial crisis of 2008–2009. In the period 2001–2007 the difference was 21%, in the years 2008–2011 the standard deviation of the MRPK of large firms increased much faster than for small firm firms decreasing the gap in MRPK dispersion between micro and large firms to 8% averaged over the period 2008–2011. In 2011 the gap was "just" 1%. After 2001 the difference in MRPK dispersion increased again, mainly due to a decrease in the MRPK dispersion amongst large firms. Our model outcomes suggest that the increase in misallocation in large enterprises is related to a "subsidy" for capital costs, while in smaller enterprises it usually is "tax".[8]

[8] The median $\tau^K_{is,t}$ for large firms is smaller than the median $\tau^K_{is,t}$ for small firms. Results available upon request with the authors)

4 Dynamics of misallocation

Section 3.1 indicates that capital misallocation is trending upward over time and that there are substantial differences in the dispersion of MRPK depending on firm-size and industry. A related question is for what period of time the observed differences in MRPK and MRPL persist. To model the evolution of MRPK and MRPL over time we exploit an empirical specification from the literature on individual earnings dynamics based on Guvenen (2007) and Guvenen (2009), see Browning and Ejrnæs (2013) for a recent survey of the earning dynamics literature. Key property of this model is that it allows for heterogeneity in the parameters at the level of te firm.

4.1 Persistence of misallocation

To estimate the firm dynamics of misallocation we restrict our analysis to a balanced subsample of 8,186 firms for which we have complete data over the period 2001–2016[9]. We estimate the following panel data model:

$$y_{i,t} = \lambda_i + z_{i,t}, \tag{10}$$

$$z_{i,t} = \rho_i z_{i,t-1} + \varepsilon_{i,t}, \tag{11}$$

where $y_{i,t}$ is either log(MRPK) or log(MRPL) of firm i in year t. equation (10) contains firm specific effects λ_i in marginal revenue products. Equation (11) postulates an AR(1) model for the idiosyncratic term $z_{i,t}$ in marginal revenue products, with firm-specific autoregressive coefficients ρ_i. Combining (10) and (11) the estimated model is:

$$\begin{aligned} y_{it} &= (1-\rho_i)\lambda_i + \rho_i y_{i,t-1} + \varepsilon_{i,t}, \\ &= \alpha_i + \rho_i y_{i,t-1} + \varepsilon_{i,t}, \end{aligned} \tag{12}$$

where we assume that $\varepsilon_{i,t} \sim i.i.d.(0, \sigma_i^2)$. We estimate the dynamic regression model (12) by Ordinary Least Squares (OLS) for each firm individually, and calculate the cross-sectional distribution and moments (mean, standard deviation, etc.) of the estimates $\hat{\alpha}_i$, $\hat{\rho}_i$ and $\hat{\sigma}_i^2$.

[9] Balancing our sample does not alter the main findings in Section 3 as can be seen in Figure A.1 and Figure A.2. The misallocation gain in the balanced panel is 11% (unbalanced: gain of 14%). The smaller gain can not be attributed to a lesser increase in the dispersion of MRPK, but because the dispersion in MRPL decreased by 5% over the sample (balanced: *in*crease of 5%).

4.2 Fraction of persistent firms

We observed that MRPK dispersion increases over time, while the cross-sectional variance for MRPL does not reveal a trend. The linear trend in variance for MRPK indicates that the distribution of MRPK over time is nonstationary. One possibility is that some of the firms have a unit root in MRPK. We use the estimator of Ng (2008) to quantify the fraction of the firms that has an autoregressive unit root ($\rho_i = 1$) and the fraction of firms with a stationary process ($\rho_i < 1$). The basic idea of this estimator is that, if some firm specific marginal revenue products are non-stationary, then the cross-sectional variance embodies a linear trend. Ng (2008) shows that:

$$V_{t,\infty} = \Lambda_\infty + \theta \cdot t + c, \tag{13}$$

where $V_{t,\infty}$ is the cross-sectional variance of $y_{i,t}$ in period t. Furthermore, Λ_∞ is the asymptotic cross-sectional average of λ_i, θ is the fraction of firms for which $\rho_i = 1$ and c is a positive constant. Note that $\Delta V_{t,\infty} = \theta$, hence the fraction θ is estimated by:

$$\hat{\theta} = \frac{1}{T} \sum_{t=1}^{T} \Delta V_{t,N}, \tag{14}$$

where $V_{t,N}$ is the sample cross-sectional variance. Model (12) can be generalized allowing for heterogeneous deterministic linear trends. In Section C.4 we show that there is no significant evidence for the existence of these deterministic trends in our sample, however.

4.3 Estimation results dynamics of misallocation

The estimation results of the regression model in equation (12) for MRPK and MRPL are summarized in Panel A and B of Table I, respectively. The average autoregressive parameter of MRPK is 0.59, suggesting moderate persistence. The cross-sectional standard deviation of 0.27 indicates considerable variation, however.

[INSERT TABLE I ABOUT HERE]

The percentiles of the cross-sectional distribution of ρ_i give a further indication of the wide range of possible values for the autoregressive parameter. The 1^{st} and 99^{nd} percentiles show that estimates smaller than zero or larger than one are possible too. The results for MRPL show much less persistence than MPRK, as can be seen from the average autoregressive parameter which is 0.45. The skewness is close to zero, indicating that the MRPL distribution is close to symmetric. The cross-sectional standard deviation of 0.30

indicates a large degree of heterogeneity, similar to MRPK. The percentiles of the cross-sectional distribution of ρ_i give a further indication of the wide range of possible values for the autoregressive parameter. Regarding large values the prevalence of an unit root or explosive values is less than for MRPK. Table I indicates considerable heterogeneity in the other parameters of model (12), but compared to MRPK there is much less heterogeneity in MRPL.

We observe that MRPK dispersion increases over time, but the cross-sectional variance of MRPL does not reveal a trend. Given the observed linear trend in variance, we expect to estimate a positive fraction of unit roots θ for MRPK, but $\theta \approx 0$ for MRPL. This is confirmed by the empirical estimates. Using the estimator (14) we find $\hat{\theta} = 0.042$ for MRPK with an estimated standard error equal to 0.012. The resulting t-test suggest statistical evidence (at all conventional significance levels) of a unit root in MRPK for 4.2% of the firms. For MRPL results are different, as expected. We estimate $\hat{\theta} = -2.9 \times 10^{-4}$, which is not significantly different from zero.

The relatively large cross-sectional variances of the model parameters in equation (12) (α_i, ρ_i and σ_i^2) indicate that there are important, yet not modeled, explanatory factors for the dispersion in MRPK and MRPL. To determine the link between MRPK and MRPL dispersion and observable firm-characteristics we conducted cross-section regressions to explain the cross-sectional pattern of the model parameters. In more detail, we separately regress the estimated parameters $\hat{\alpha}_i, \hat{\rho}_i$ and $\hat{\sigma}_i^2$ on a constant, firm-size dummies (micro, small, medium-sized and large firms) and a sectoral dummy (manufacturing sector or services sector). The regression results are reported in Table II.

[INSERT TABLE II ABOUT HERE]

For MRPK we find that micro firms have a higher equilibrium level of MRPK, but the differences with other firms are small and not always significant. Manufacturing firms have lower level of misallocation than services firms, which result is significant. Furthermore, we find that the persistence over time (ρ_i) increase with firm size, while the variance of idiosyncratic shocks to MRPK (σ_i^2) decreases with firm size.

Besides the results shown in Table II, we also calculate θ depending on firm size and sector. Regarding MRPK we find that the fraction of unit roots decreases with firm size. Around 5.0% (standard error is 1.4%) of the micro firms have a unit root in MRPK, while for large firms this is only 2.0% (standard error is 3.2%). Furthermore, firms from the services sector have a lower fraction of unit roots than manufacturing firms (2.9% versus 4.9%).

Summarizing, our results deliver a couple of interesting insights into the dynamics of misallocation. First, there is a great deal of heterogeneity in the dynamic process underlying MRPK and MRPL. As the variance of the firm specific effect is large, MRPK levels converge to widely different equilibrium levels over time. Secondly, shocks to MRPK die out very quickly for some firms ($\rho_i \approx 0$), while having permanent effects on other firms ($\rho_i \approx 1$). Thirdly, increasing MRPK dispersion is caused by a small fraction of firms having a unit root in MRPK.

4.4 Validation by Monte Carlo simulation

To determine if model (12) is capable of reproducing the pattern of the estimation results in Figure 1 we run a Monte Carlo experiment. We generate artificial data according to equation (12). Regarding the error components we assume $\eta_i \sim$ i.i.n.$(0, \sigma_\eta^2)$ independent from $\varepsilon_{it} \sim$ i.i.n.$(0, \sigma_\varepsilon^2)$. Regarding the firm specific autoregressive coefficients we assume $\lambda_i \sim$ i.i.n.$(\mu_\lambda, \sigma_\rho^2)$ but we truncate draws $\rho_i > 1$ to the unit root $\rho_i = 1$. We furthermore choose $\mu_\rho = 0.5$ and for the variance parameters $\sigma_\rho^2 = 0.10$, $\sigma_\varepsilon^2 = 0.25$ and $\sigma_\lambda^2 = 0.75$, which implies $\sigma_\alpha^2 \approx 0.25$ in the simulations. We choose these values, because they are close to the empirical estimates in Table I. Finally, we assume that the process initiated S periods before we start observing the data at period 1 and we assume:

$$y_{i,1-S} = \alpha_i + \varepsilon_{i,1-S}, \tag{15}$$

with $\varepsilon_{i,t-S} \sim$ i.i.n.$(0, \sigma_\varepsilon^2)$. Note that this implies that the process is mean stationary, but due to the fact that some $\rho_i = 1$ it is not covariance stationary. Finally we choose $T = 16$ and $N = 8,000$, which is close to the sample size in the empirical analysis ($T = 16$ and $N = 8,189$). We create 1,000 data sets and calculate the time series of cross-sectional variances for each replication. The mean of these 1,000 replications is then compared to the empirical cross-sectional variances. Panel A of Figure 6 shows the empirical variances (normalized at 1 in base year 2001), show the standard deviation of the MRPK in our sample, and the simulated average mean and 95%-confidence interval standard deviation of the Monte Carlo replications when the number of start-up observations is $S = 5$. This means that the age of all firms is equal to 5 years, which is roughly in line with the average firm age in our sample.

[INSERT FIGURE 6 ABOUT HERE]

Although the empirical estimates do show some irregular development, especially in the crisis years 2007–2009, the simulated cross-sectional variances are capable of reproducing the upward trend in the data. The initial conditions do matter for the pattern of the simulation results. However, when $S = 50$ –when the average age of the firms is very high–, a different picture emerges, as shown in Panel B. of Figure 6. When when $S = 50$, the upward trend in the mean MRPK-dispersion is much less pronounced than for $S = 5$. This could indicate that older firms have a smaller fraction of firms with an autoregressive unit root and/or are less persistent in MRPK.

5 Robustness tests

Recently, several studies have raised concern about the Hsieh-Klenow model. A first source of concern is that capital might be subject to adjustment costs in investment ("time-to-build"), which can lead to a higher dispersion simply due to technology-driven adjustment processes, which in itself are not inefficient (see e.g. Asker et al., 2014, Cooper and Haltiwanger, 2006 and David and Venkateswaran, 2018). The Hsieh and Klenow (2009) model neglects this distinction between technology-driven adjustment costs, such as the natural time needed to build a new plant, and wasteful frictions, such as the bureaucratic procedures of authorization that may delay the construction and activation of a new plant. We estimate the importance of these non-wasteful adjustment costs for our measurement in Section 5.1. Another concern is the functional form. Recently Haltiwanger et al. (2018) showed that the model is only valid under quite strict assumptions which generally do not hold in reality. In order to investigate the impact of the functional form we estimate the effect of allowing for firm-specific labor and capital shares, and a different functional form of the production function (i.e. CES instead of a CD functional form) in Section 5.2.[10]

5.1 Adjustment costs of capital

In order to explore whether adjustment costs are a significant driver of our measurement of misallocation we use the methodology of David and Venkateswaran (2018). The model is an extension of the Hsieh and Klenow (2009) framework to include dynamic considerations in firm's investment decisions. In the model a number of forces can contribute to the observed dispersion in MRPK and MRPL, i.e.: (i) capital adjustment costs, (ii) informational frictions, in the form of imperfect knowledge about firm-level fundamentals

[10] Another concern with our quantification relates to measurement error in firms' revenues and inputs. As Bils et al. (2018) point out, mismeasurement distorts our analysis as a firm's TFPR is higher when revenues are overstated and/or inpts are understated, the dispersion of measured TFPR is unequivocally biased upward. In our sample this form of mismeasurement is likely not a main concern, since we use tax-data. Though firm-reported, these data are thoroughly checked by the tax authorities.

and (iii) firm-specific factors, meant to capture all other forces influencing investment decisions, such as unobserved heterogeneity in markups and/or production technologies, financial frictions or institutional distortions. Section C.3 in the Appendix presents the main model equations. David and Venkateswaran (2018) model capital adjustment costs as a quadratic function:

$$\Phi\left(K_{is,t+1}, K_{is,t}\right) = \frac{\hat{\xi}}{2}\left(\frac{K_{is,t+1}}{K_{is,t}} - (1-\delta)\right)^2 K_{is,t}, \tag{16}$$

where δ is the depreciation rate. [11] The coefficient $\hat{\xi}$ determines the magnitude of adjustment costs. For example, if $\delta = 0.10$ and capital is doubled form time t tot $t+1$ ($K_{is,t+1} = 2K_{is,t}$), $\hat{\xi}$ implies that the adjustment costs are 60.5% of the investment ($\Delta K_{is,t+1}$). There is a rather large variation in estimates of $\hat{\xi}$ in the literature. Investment-regressions, derived from the Q-theory of investment usually find values for $\hat{\xi}$ (> 10), see for instance Hayashi and Inoue (1991). Estimates based on the method of moments are usually much lower, e.g. between 0.8 and 1.6 in Eberly et al. (2008)) or close to zero in Cooper and Haltiwanger (2006) and Bloom (2009). This could be partly driven by different country samples. Asker et al. (2014) show that there is quite a large difference in the estimated $\hat{\xi}$ for the US (4.4) on the one hand and France (0.1) on the other.

The relevant model parameters to determine $\hat{\xi}$ can be estimated from the following dynamic model of the capital stock:

$$k_{is,t} = \pi_1 k_{is,t-1} + \pi_2 a_{is,t-1} + \eta_i + u_{is,t}, \tag{17}$$

where, $k_{is,t}$ en $a_{is,t}$ are the log of capital $k_{is,t}$ and $TFPQ_{is,t}$, respectively. Moreover, π_1 measures capital adjustment costs, η_i measures firm-specific factors influencing investment decisions and the disturbance term $u_{is,t}$ captures all other possible frictions. In the hypothetical case of no adjustment costs of capital, $\pi_1 = 0$ and when there are adjustment cost $\pi_1 > 0$. In the limit, when $\hat{\xi} \to \infty$ it holds that $\pi_1 \to 1$. Identification of adjustment costs is straightforward, where the null-hypothesis is: $H_0 : \pi_1 = 0$ and the alternative hypothesis is: $H_1 : \pi_1 > 0$. We estimate the dynamic panel data model (17) directly using OLS.[12]

Our estimate for π_1 is 0.62 and the standard error is 0.002, which implies a value of $\hat{\xi} \approx 0.4$. Our estimate indicates adjustment costs are a relevant source of misallocation, although the coefficient of 0.4 is on the low-end of the estimations found in the

[11] More general specification that also allow non-convex elements can be found in e.g. Cooper and Haltiwanger (2006), Bloom (2009) and Asker et al. (2014).

[12] Our estimation method is a simplification of the method of moments estimator of David and Venkateswaran (2018), who use a set of non-linear moment conditions to estimate the parameters. We are interested only in the adjustment costs parameter, hence the linear model in equation (17).

literature. Based on the estimated coefficients David and Venkateswaran (2018) derive a measurement of the empirical relevance of adjustment costs. First, they assume that adjustment costs are the sole source of between-firm variation in MRPK. In that special case, model (17) reduces to:

$$k_{is,t} = \pi_1 k_{is,t-1} + \pi_2 a_{is,t-1}, \tag{18}$$

where $a_{is,t}$ is assumed to follow a stationary AR(1) process is. We can use equation (18) to determine the variance of MRPK (σ^2_{MRPK}) when adjustment costs are the only source of misallocation. Section C.3 in the Appendix presents the formal derivation of σ^2_{MRPK} in this special case. Using our estimate of π_1 we find $\sigma^2_{MRPK} = 0.16$. This estimate is less than 10% of the measured total MRPK variance of 2.97 in our sample. We therefore conclude adjustment costs are not a sizable source of misallocation in our sample.

5.2 Alternative functional form

Another concern of the Hsieh and Klenow (2009) approach is the strict assumptions on the functional form of the production function, i.e. a CD production function with firm invariant capital elasticities and a substitution elasticity of 1 between labor and capital. We relax these assumptions one-by-one in Section 5.2.1 and Section 5.2.2. Our main finding is that only a small fraction of the total measures misallocation can be attributed to one of these factors.

5.2.1 Firm level heterogeneity in the production function

In the standard Hsieh and Klenow (2009) model the CD production function is specified has industry varying labor and capital shares, but these shares are constant within an industry, as can be seen from equation (3). In other words: the capital and labor elasticities are assumed to be equal for all firms. It follows from the model equations in equations (1)–(8) that *all* firm-specific variation in labor or capital-elasticities automatically lead to an increase in misallocation. We are able to relax this strong assumption, because we are able to calculate firm-specific capital shares based on the datawe have. In Section C.1 in the Appendix we derive an expression for the dispersion in (the log of) MRPK where misallocation is solely caused by differences in firm-specific capital elasticities. By comparing this measure with the observed dispersion in (the log of) MRPK in the data we can get a sense of the impact on our results of the assumption of equal capital shares within an industry. We find that the overall impact is rather small. The total variance of MRPK in the data is 2.97, whilst our alternative firm-specific variation is only 0.03. This implies that only approximately 1% of the observed variance of the (log of) MRPK can be attributed to differences in firm-specific capital-elasticities.

5.2.2 Constant Elasticity of Substitution production function

Another possible explanation for the observed misallocation misallocation could be that the assumption of unity substitution in the CD production function in equation (3) is invalid. An alternative –often used– functional form is a CES production function, where the substitution-elasticity between capital and labor is freely estimated and not a priori equalized to unity, i.e.:

$$Y_{is,t} = A_{is,t} \left(\alpha_s K_{is,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha_s) \left(L_{is,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{19}$$

where σ is the subsitution elasticity between capital and labor. The CES production function reduces to the Hsieh and Klenow (2009) imposed CD production function (3) when $\sigma = 1$. MRPK is defined as $P_{is,t}\frac{\partial Y}{\partial K}$, the product of marginal revenue ($P_{is,t}$) and marginal product ($\frac{\partial Y}{\partial K}$). In Section C.2 in the Appendix we derive that in this case the MRPK can be written as:

$$MRPK_{is,t} = \alpha_s^{1-\sigma} R_{s,t}^{\sigma} \left(\alpha_s^{\sigma} R_{s,t}^{1-\sigma} + (1-\alpha_s)^{\sigma} w_t^{1-\sigma} \right), \tag{20}$$

Notice that when $\sigma = 1$ (CD production function), the expression reduces to $MRPK = R$. In the standard Hsieh-Klenow model the only source of variation in MRPK are firm-specific disturbances in marginal revenue (P), the cost of capital R, $\tau_{is,t}^{Y}$ and $\tau_{is,t}^{K}$. In the case of a CES production-function ($\sigma \neq 1$) this is not any different, from the standard Hsieh-KLenow model, since the parameters α and σ are not firm-specific, but only vary between industries. Section C.2 in the Appendix also considers a CES production function with labor-augmenting technological process. We derive that labor-augmenting technology can only have impact on our measure of misallocation if it varies between firms in *the same* industry. In policy discussions a concern is the negative impact of industrial robots on employment growth (Acemoglu and Restrepo, 2017), which is an example of a labor saving technology. Only if the use of robots has unequal effects on employment growth across firms within an industry, the misallocation estimation based on a CES production function will lead to different results compared to the base model of Hsieh and Klenow (2009). We leave it to future research to determine hoe relevant the variation in the adaption of such technologies varies between firms in the same industry. We conclude that, relaxing the (strong) assumption of a unity substitution elasticity between capital and labor does not alter our estimate of misallocation.

6 Conclusion

Misallocation of capital has been on the rise since the turn of the millennium in several European countries (see e.g. Gopinath et al., 2017, Gamberoni et al., 2016 and Calligaris et al., 2017). Our analysis focuses on this development. We use a very rich data set for the Netherlands, containing annual balance sheets as well as profit and loss statements for all Dutch firms that had to declare corporate income tax during the period 2001–2016. Our results shed new light on the development of capital and labor misallocation and underlines the importance of looking at the level and persistence of misallocation at the firm-level. Our main findings can be summarized as follows.

First, We find that there is substantial heterogeneity in the marginal revenue products at the level of the firm. We find a steeply rising dispersion of the marginal revenue product of capital, together with a stable dispersion of the revenue product of labor, indicating that capital frictions account for most of the measured misallocation in the Netherlands. Compared to a counter-factual efficient allocation we find that misallocation has doubled in the period 2001–2016.

Second, the richness of our data set allows splitting up the measured misallocation to industries and size class. We find that the level of misallocation is much larger in the manufacturing sector than in the services sector. This result might be driven by less competitive pressures in the services sector compared to the manufacturing sector. The *increase* in the misallocation of capital over the period 2001–2016 was of the same orde of magnitude in both sectors.

Third, the misallocation of capital is inversely related to firm size, i.e. the variance of the marginal revenue products is smaller amongst large firms than under small firms. Strikingly, the *increase* in the misallocation of capital was much larger for large firms than for smaller firms. The increase in misallocation of capital for large firms was concentrated in the period 2009–2012, which seems to indicate that the Great Recession and the Debt Crisis had a relatively large impact on the dispersion on the misallocation of capital amongst large firms.

Fourth, The increase in the misallocation of capital was caused by a relatively small fraction of total firms with a unit root in marginal revenue products. We verify this result with a Monte Carlo simulation, and conclude that we can largely replicate our empirical findings.

Fifth, we examine how robust our findings are against recently raised concerns with the Hsieh-Klenow model. We show that in our sample the measurement of misallocation is largely insensitive to observed heterogeneity in the production function and to the presence of capital adjustment costs.

Acknowledgements

The opinions expressed in this paper are the authors personal views and do not necessarily reflect the position of De Nederlandsche Bank. We are grateful to Job Swank and Peter van Els for their valuable comments and stimulating suggestions.

References

Acemoglu, D. and P. Restrepo (2017). Robots and jobs: Evidence from US labor markets. NBER Working Papers 23285, National Bureau of Economic Research, Inc.

Asker, J., A. Collard-Wexler, and J. De Loecker (2014). Dynamic inputs and resource (mis)allocation. *Journal of Political Economy 122*(5), 1013–1063.

Bils, M., P. J. Klenow, and C. Ruane (2018). Misallocation or mismeasurement? Working Papers 599, Stanford Center for International Development.

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica 77*(3), 623–685.

Browning, M. and M. Ejrnæs (2013). Heterogeneity in the dynamics of labor earnings. *Annual Review of Economics 5*(1), 219–245.

Busso, M., L. Madrigal, and C. Pagés (2013). Productivity and resource misallocation in Latin America. *The B.E. Journal of Macroeconomics 13*(1), 903–932.

Calligaris, S. (2015). Misallocation and total factor productivity in Italy: Evidence from firm-level data. *Labour 29*(4), 367–393.

Calligaris, S., M. Del Gatto, F. Hassan, G. I. Ottaviano, and F. Schivardi (2017). The productivity puzzle and misallocation: An Italian perspective. Working Papers 1520, Centre for Economic Performance.

Cooper, R. W. and J. C. Haltiwanger (2006). On the nature of capital adjustment costs. *Review of Economic Studies 73*(3), 611–633.

David, J. M. and V. Venkateswaran (2018). The sources of capital misallocation. Technical report, NYU Stern.

Dias, D. A., C. R. Marques, and C. Richmond (2016). Misallocation and productivity in the lead up to the eurozone crisis. *Journal of Macroeconomics 49*, 46–70.

Eberly, J., S. Rebelo, and N. Vincent (2008). Investment and value: A neoclassical benchmark. NBER Working Papers 13866, National Bureau of Economic Research, Inc.

Foster, L., J. Haltiwanger, and C. Syverson (2008). Reallocation, firm turnover, and efficiency: selection of productivity or profitabiity? *American Economic Review 98*(1), 394–425.

Gamberoni, E., C. Giordano, and P. Lopez-Garcia (2016). Capital and labour (mis)allocation in the euro area: some stylized facts and determinants. Working Paper Series 1981, European Central Bank.

Gopinath, G., e. Kalemli-Özcan, L. Karabarbounis, and C. Villegas-Sanchez (2017). Capital allocation and productivity in South Europe. *The Quarterly Journal of Economics 132*(4), 1915–1967.

Gorodnichenko, Y., D. Revoltella, J. Svejnar, and C. T. Weiss (2018). Resource misallocation in european firms: the role of constraints, firm characteristics and managerial decisions. NBER Working Papers 24444, National Bureau of Economic Research, Inc.

Guvenen, F. (2007). Learning your earning: Are labor income shocks really very persistent? *The American Economic Review 97*(3), 687–712.

Guvenen, F. (2009). An empirical investigation of labor income processes. *Review of Economic Dynamics 12*(1), 5879.

Haltiwanger, J., R. Kulick, and C. Syverson (2018). Misallocation measures: The distortion that ate the residual. NBER Working Papers 24199, National Bureau of Economic Research, Inc.

Hayashi, F. and T. Inoue (1991). The relation between firm growth and Q with multiple capital goods: Theory and evidence from panel data on Japanese firms. *Econometrica 59*(3), 731–753.

Hsieh, C.-T. and P. J. Klenow (2009). Misallocation and manufacturing TFP in China and India. *The Quarterly Journal of Economics 124*(4), 1403–1448.

Ng, S. (2008). A simple test for nonstationarity in mixed panels. *Journal of Business & Economic Statistics 26*(1), 113–127.

Raval, D. (2018). The micro elasticity of substitution and non-neutral technology. Technical report, Federal Trade Commission.

A Figures and tables main text

Figure 1: Development of MRPK and MRPL, 2001–2016, the Netherlands.

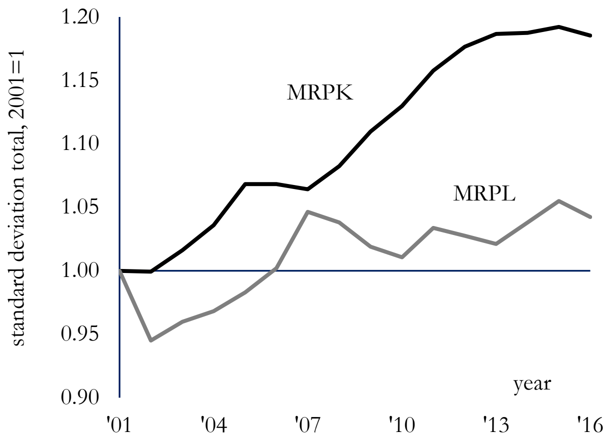


Figure 2: Misallocation in the Netherlands, 2001–2016.

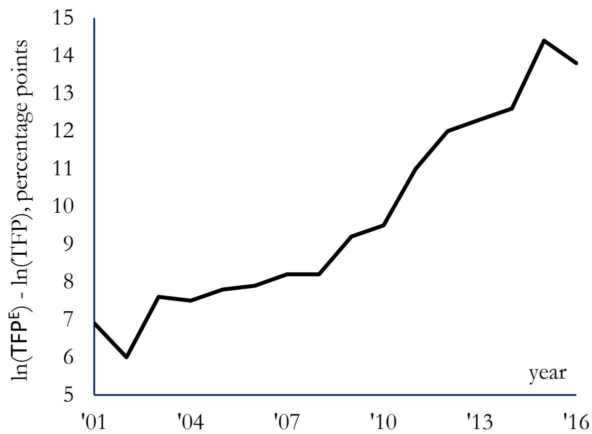


Figure 3: Development of MRPK and MRPL in manufacturing and services sector, 2001–2016, the Netherlands.

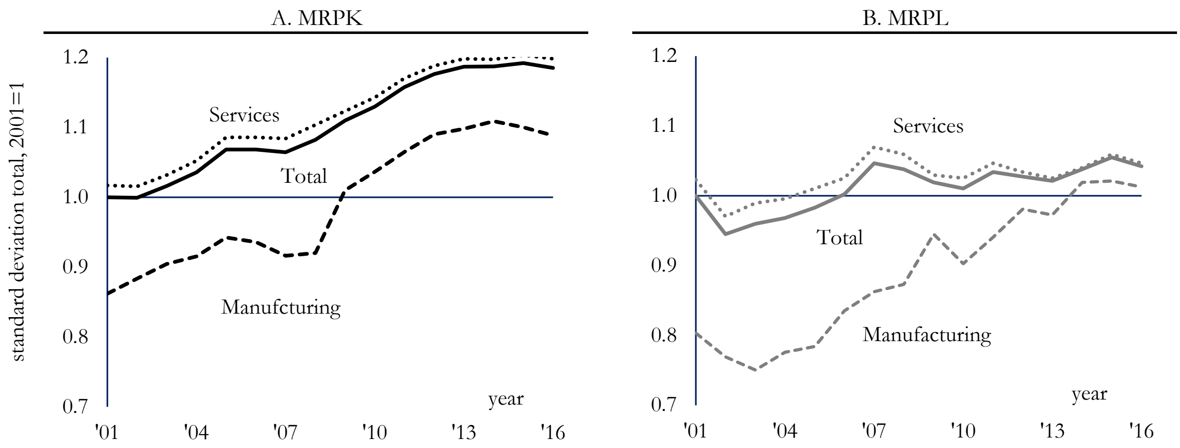


Figure 4: Development of MRPK in manufacturing and services sector according to technological intensity, 2001–2016, the Netherlands.

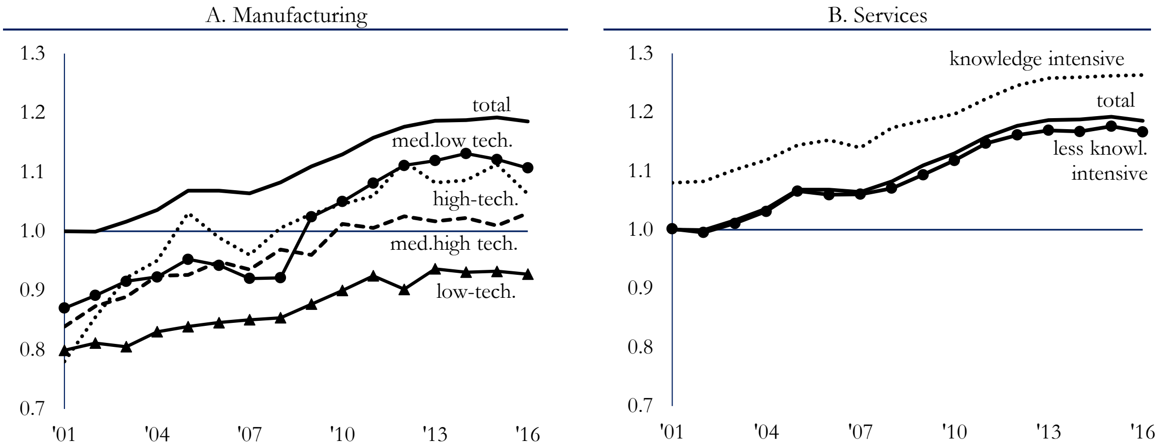


Table I: Estimates of model (12) for MRPK and MRPL, 2001–2016, the Netherlands

	mean	sd	skewness	kurtosis	1%	5%	25%	50%	75%	95%	99%
MRPK											
α_i	-0.29	0.66	-0.66	6.40	-2.31	-1.47	-0.60	-0.23	0.09	0.67	1.21
ρ_i	0.59	0.27	-0.93	8.87	-0.16	0.08	0.43	0.63	0.78	0.94	1.04
σ_i^2	0.24	0.34	4.31	32.95	0.01	0.02	0.05	0.12	0.28	0.85	1.59
MRPL											
α_i	-0.22	0.22	0.66	14.74	-0.80	-0.61	-0.35	-0.20	-0.08	0.08	0.33
ρ_i	0.45	0.30	-0.06	6.67	-0.31	-0.09	0.25	0.48	0.67	0.89	1.02
σ_i^2	0.04	0.07	8.86	124.38	0.00	0.00	0.01	0.02	0.04	0.11	0.33

Note: based on 8,186 observations.

Figure 5: Business size and development of MRPK, 2001–2016, the Netherlands.

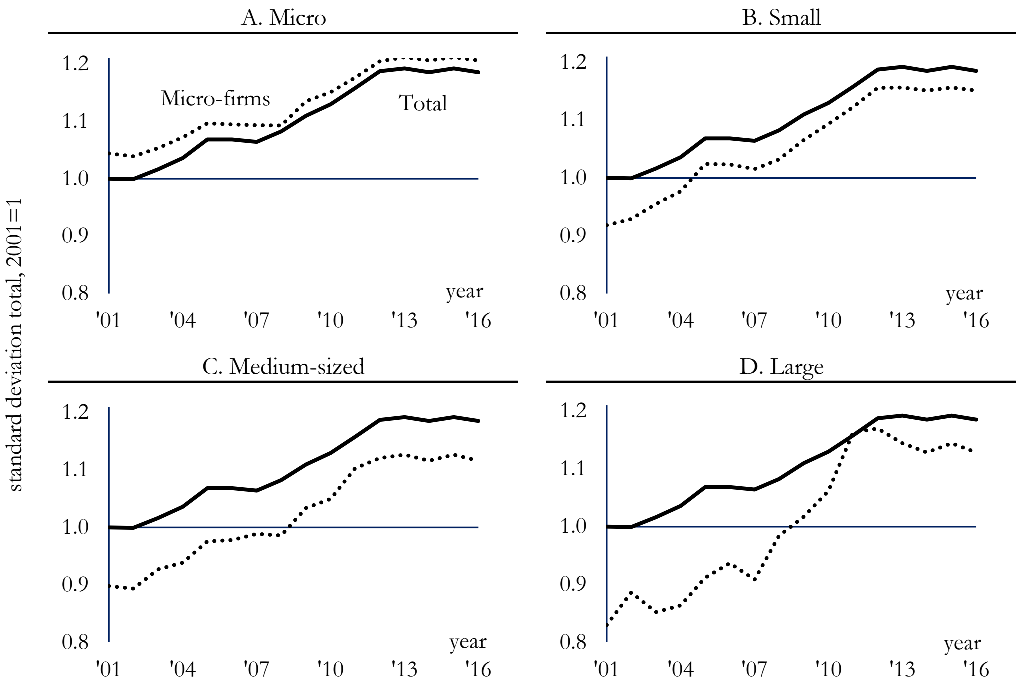
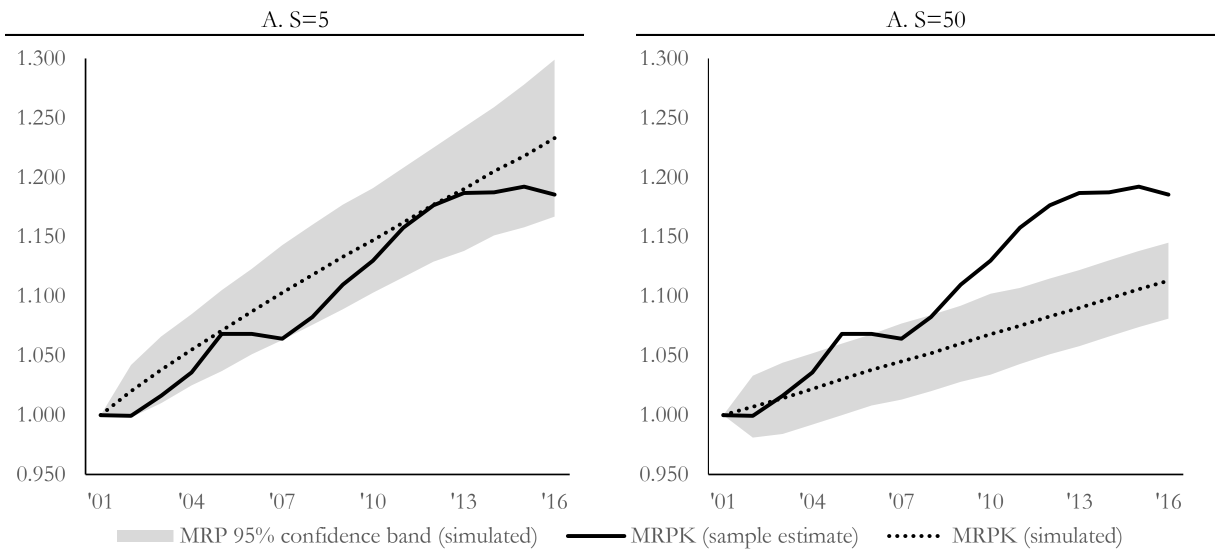


Figure 6: Empirical and simulated MRPK dispersion, 2001–2016, the Netherlands.



Note: simulated data based on 8,000 Monte Carlo replications.

Table II: Explaining cross-sectional variation in model parameters, 2001–2016, the Netherlands

	MRPK			MRPL		
	α_i	ρ_i	σ_i^2	α_i	ρ_i	σ_i^2
constant	-0.230	0.563	0.303	-0.216	0.421	0.046
	(0.012)	(0.005)	(0.006)	(0.004)	(0.005)	(0.001)
small	-0.047	0.047	-0.092	-0.001	0.055	-0.015
	(0.016)	(0.006)	(0.008)	(0.005)	(0.007)	(0.002)
intermediate	-0.007	0.039	-0.120	0.006	0.105	-0.023
	(0.026)	(0.012)	(0.012)	(0.008)	(0.012)	(0.003)
large	-0.063	0.073	-0.170	0.106	0.118	-0.017
	(0.047)	(0.020)	(0.020	(0.026)	(0.022)	(0.003)
manufacturing	-0.136	-0.001	-0.044	-0.025	-0.012	-0.004
	(0.015)	(0.007)	(0.007)	(0.005)	(0.007)	(0.002)
R^2	0.011	0.008	0.029	0.007	0.014	0.015

Note: numbers in parentheses are robust standard errors. N=8,186 firms.

Categories micro-firms and services sector are included in the constant.

B Additional figures and tables

Figure A.1: Development of MRPK and MRPL, 2001–2016, the Netherlands, balanced sample vs. unbalanced sample.

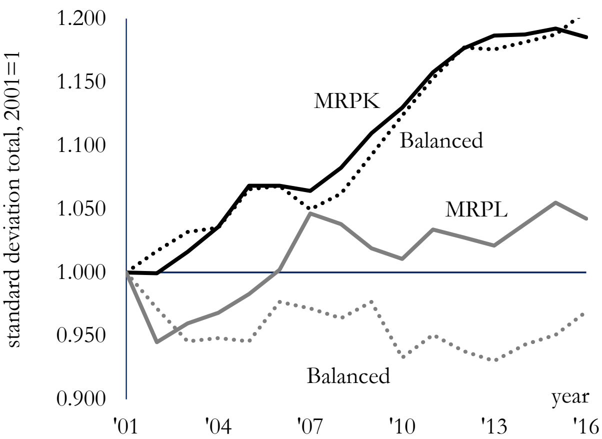


Figure A.2: Misallocation in the Netherlands, 2001–2016, balanced sample vs. unbalanced sample.

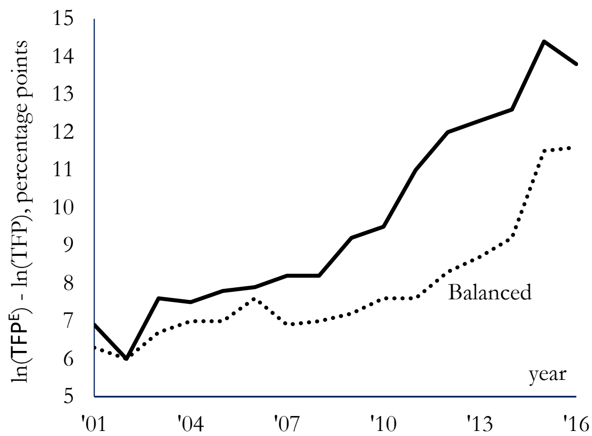


Figure A.3: Development of MRPK and MRPL in manufacturing and services sector, 2001 vs. 2016, NACE two-digit level, the Netherlands.

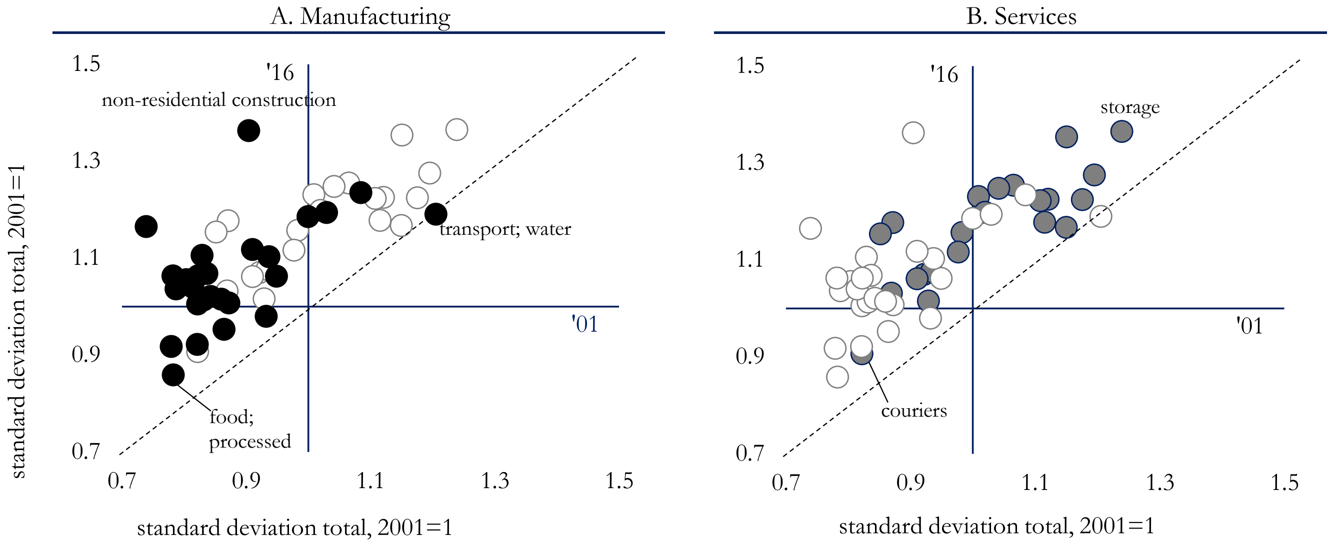


Table A.I: Estimates of model (A.36) for MRPK and MRPL, 2001–2016, the Netherlands

	mean	sd	skewness	kurtosis	1%	5%	25%	50%	75%	95%	99%
MRPK											
α_i	-0.48	0.96	-0.22	4.18	-3.09	-2.15	-1.03	-0.45	0.11	1.01	1.86
ρ_i	0.38	0.29	-0.18	10.61	-0.33	-0.11	0.21	0.42	0.58	0.78	0.93
β_i	0.00	0.06	-0.15	5.73	-0.17	-0.10	-0.03	0.01	0.04	0.10	0.17
σ_i^2	0.22	0.32	4.41	34.12	0.01	0.01	0.04	0.11	0.26	0.77	1.50
MRPL											
α_i	-0.35	0.35	0.82	10.42	-1.15	-0.90	-0.55	-0.35	-0.17	0.17	0.62
ρ_i	0.24	0.30	0.51	16.91	-0.46	-0.27	0.03	0.25	0.45	0.69	0.84
β_i	0.00	0.02	0.28	10.40	-0.06	-0.03	-0.01	0.00	0.02	0.04	0.07
σ_i^2	0.03	0.07	8.78	118.92	0.00	0.00	0.01	0.02	0.03	0.11	0.30

Note: based on 8,186 estimates.

C Technical Appendix

C.1 Derivation of σ^2_{MRPK} with firm level heterogeneity in the production function

Consider the following generalized production function:

$$Y_{is,t} = A_{is,t} K_{is,t}^{\alpha_{is,t}} L_{is,t}^{1-\alpha_{is,t}}, \tag{A.1}$$

in which capital intensities are idiosyncratic and time varying. Suppose no distortions exist. The first-order conditions for profit maximization are:

$$P_{is,t} \frac{\partial Y}{\partial L} = w_t, \tag{A.2}$$

$$P_{is,t} \frac{\partial Y}{\partial K} = R_t, \tag{A.3}$$

with

$$\frac{\partial Y}{\partial K} = A_{is,t} \alpha_{is,t} K_{is,t}^{\alpha_{is,t}-1} L_{is,t}^{1-\alpha_{is,t}} = \alpha_{is,t} \frac{Y_{is,t}}{K_{is,t}}, \tag{A.4}$$

$$\frac{\partial Y}{\partial L} = ... = (1-\alpha_{is,t}) \frac{Y_{is,t}}{L_{is,t}}. \tag{A.5}$$

Combining (A.3) and (A.4) and omitting constants we have:

$$MRPK_{is,t} = \log \frac{P_{is,t} Y_{is,t}}{K_{is,t}} \approx -\log(\alpha_{is,t}), \tag{A.6}$$

hence,

$$\mathbb{V}\text{ar}\left(MRPK_{is,t}\right) = \mathbb{V}\text{ar}\left(\log(\alpha_{is,t})\right). \tag{A.7}$$

Summarizing, in a model without distortions, but including heterogeneous technologies, all dispersion in MRPK is caused by dispersion in production technologies. Following Hsieh and Klenow (2009) we use 1 minus the labor share to estimate the elasticity of output with respect to capital. For the purpose of estimating dispersion at the level of the firm, we use firm level labor shares.

C.2 Derivation of σ^2_{MRPK} in case of a CES production function

Consider the same market structure as in Hsieh and Klenow (2009), hence the demand curve and price setting for an individual firm's product follow from the first-order condition:

$$P_s Y_s^{\frac{1}{\sigma}} Y^{-\frac{1}{\sigma}} = P. \tag{A.8}$$

However, the individual firm has the CES production function (19) with both a neutral productivity component and labor augmenting productivity (see e.g. Raval, 2018). Define $\rho = \frac{\sigma-1}{\sigma}$ with σ the substitution elasticity between capital and labor. Then we can express (19) as:

$$Y = A\left(\alpha K^{\rho} + (1-\alpha)\left(BL\right)^{\rho}\right)^{\frac{1}{\rho}}. \tag{A.9}$$

Note that CD is the special case $\sigma = 1$ or $\rho = 0$. The marginal products are:

$$\frac{\partial Y}{\partial L} = A\left(\alpha K^{\rho} + (1-\alpha)\left(BL\right)^{\rho}\right)^{\frac{1-\rho}{\rho}}(1-\alpha)\,\rho B^{\rho} L^{\rho-1}, \tag{A.10}$$

$$\frac{\partial Y}{\partial K} = A\left(\alpha K^{\rho} + (1-\alpha)\left(BL\right)^{\rho}\right)^{\frac{1-\rho}{\rho}}\alpha\rho K^{\rho-1}, \tag{A.11}$$

hence,

$$\frac{\frac{\partial Y}{\partial L}}{\frac{\partial Y}{\partial K}} = \frac{1-\alpha}{\alpha} B^{\rho}\left(\frac{K}{L}\right)^{1-\rho}. \tag{A.12}$$

In the absence of distortions profits of each producer are given by:

$$\pi = P_s Y_s^{\frac{1}{\sigma}} Y^{\frac{\sigma-1}{\sigma}} - wL - RK, \tag{A.13}$$

Each producer chooses L and K to maximize profits:

$$\max_{L,K} \pi \tag{A.14}$$

The first order conditions for profit maximization are:

$$P_s Y_s^{\frac{1}{\sigma}}\left(\frac{\sigma-1}{\sigma}\right) Y^{-\frac{1}{\sigma}}\frac{\partial Y}{\partial L} = w, \tag{A.15}$$

$$P_s Y_s^{\frac{1}{\sigma}}\left(\frac{\sigma-1}{\sigma}\right) Y^{-\frac{1}{\sigma}}\frac{\partial Y}{\partial K} = R. \tag{A.16}$$

Dividing both equations and rearranging we get for the capital-labor ratio:

$$\frac{K}{L} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\rho}} B^{\frac{-\rho}{1-\rho}}\left(\frac{w}{R}\right)^{\frac{1}{1-\rho}}. \tag{A.17}$$

Note that when $\rho = 0$, i.e. CD, we get:

$$\frac{K}{L} = \frac{\alpha}{1-\alpha}\frac{w}{R}, \tag{A.18}$$

which does not depend on B.

The firm's output price P will be set as a fixed mark up $\frac{\sigma}{\sigma-1}$ over the firm's marginal costs. In case of the CES production function (19), marginal costs are:

$$MC(R,w) = \frac{\alpha^{\sigma} R^{1-\sigma} + (1-\alpha)^{\sigma}\left(\frac{w}{B}\right)^{1-\sigma}}{A\left(\alpha\left(\frac{R}{\alpha}\right)^{1-\sigma} + (1-\alpha)\left(\frac{w}{(1-\alpha)B}\right)^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}}, \tag{A.19}$$

hence,

$$P = \left(\frac{\sigma}{\sigma - 1}\right) MC(R, w). \tag{A.20}$$

Combining these equations after some algebra we find that:

$$\begin{aligned} MRPK &= P\frac{\partial Y}{\partial K} \\ &= \alpha^{1-\sigma} R^{\sigma} \left(\alpha^{\sigma} R^{1-\sigma} + (1-\alpha)^{\sigma} \left(\frac{w}{B}\right)^{1-\sigma}\right) \end{aligned} \tag{A.21}$$

Without component B, i.e. the standard CES production function, the result in (20) follows. For labor augmenting or labor saving productivity to cause variation in MRPK, it should vary across firms within the same industry.

C.3 Misallocation with capital adjustment costs

David and Venkateswaran (2018) develop a model to distinguish the various sources of measured capital misallocation, i.e. dispersion in MRPK. They distinguish capital adjustment costs, informational frictions and firm-specific factors. Section C.3 describes the main model equations in David and Venkateswaran (2018). Section C.3 describes the derivation of the variation of MRPK when adjustment costs are the only source of misallocation.

Main model equations David and Venkateswaran (2018)

Firms produce intermediate goods according to a CD production function. The intermediate goods are bundled to produce a single final good using a standard CES aggregator.

The main equations of the DGP, which are in logarithms, are as follows. An idiosyncratic firm-specific fundamental, which can be interpreted as demand shifter and/or level of efficiency, is generated by:

$$a_{it} = \rho a_{i,t-1} + \mu_{it}, \quad \mu_{i,t} \sim i.i.d.(0, \sigma_{\mu}^2), \tag{A.22}$$

where $0 < \rho < 1$ measures the persistence of firm fundamentals. The capital distortion is generated by

$$\tau_{i,t} = \gamma a_{i,t} + \varepsilon_{i,t} + \chi_i, \quad \varepsilon_{i,t} \sim i.i.d.(0, \sigma_{\varepsilon}^2), \quad \chi_i \sim i.i.d.(0, \sigma_{\chi}^2), \tag{A.23}$$

where γ models the correlation of the distortion with the firm fundamental. Furthermore, ε_{it} and χ_i are the uncorrelated time-varying and permanent components respectively. The equation for capital is:

$$k_{i,t+1} = \psi_1 k_{i,t} + \psi_2 (1+\gamma)\, \mathbb{E}_{i,t}\left[a_{i,t+1}\right] + \psi_3 \varepsilon_{i,t+1} + \psi_4 \chi_i, \tag{A.24}$$

where

$$
\begin{aligned}
\xi\left(\beta\psi_1^2+1\right) &= \psi_1\left((1+\beta)\,\xi+1-\alpha\right), \\
\psi_2 &= \frac{\psi_1}{\xi\left(1-\beta\rho\psi_1\right)}, \\
\psi_3 &= \frac{\psi_1}{\xi}, \\
\psi_4 &= \frac{1-\psi_1}{1-\alpha}.
\end{aligned}
$$

The parameter β is the discount rate, which is an element of the optimized dynamic profit function of the firm. The parameter $\alpha = \frac{\alpha_1}{1-\alpha_2}$ where α_1 and α_2 are proportional to the capital and labor elasticities of the firm-level CD production function. Finally, the marginal revenue product of capital (MRPK) is equal to:

$$
MRPK_{i,t} = p_{i,t} + y_{i,t} - k_{i,t}, \tag{A.25}
$$

where $p_{i,t}$ is the price of good i and $y_{i,t}$ is firm-level output.

Derivation of σ^2_{MRPK} when adjustment costs are the only source of misallocation

Using the model's equations we can rewrite equation (A.25) as:

$$
\begin{aligned}
MRPK_{i,t} &= -\frac{1}{\theta}y_{i,t} + \frac{1}{\theta}y_t + \hat{a}_{i,t} + y_{i,t} - k_{i,t} \\
&= \left(1-\frac{1}{\theta}\right)\left(\hat{\alpha}_1 k_{i,t} + \hat{\alpha}_2 l_{i,t}\right) + \left(1-\alpha_2\right)a_{i,t} - k_{i,t} + \frac{1}{\theta}y_t \\
&= \alpha_1 k_{i,t} + \alpha_2 l_{i,t} + \left(1-\alpha_2\right)a_{it} - k_{it} + \frac{1}{\theta}y_t \\
&\approx \alpha_1 k_{i,t} + \alpha_2\left(a_{it} + \frac{\alpha_1}{1-\alpha_2}k_{i,t}\right) + \left(1-\alpha_2\right)a_{i,t} - k_{i,t} \\
&= \frac{\alpha_1+\alpha_2-1}{1-\alpha_2}k_{i,t} + a_{i,t},
\end{aligned} \tag{A.26}
$$

where $\approx$ means that we left out terms without cross-sectional dimension. The reason is that these terms will not contribute to dispersion in MRPK, which is defined as:

$$
\sigma^2_{MRPK} = \left(\frac{\alpha_1+\alpha_2-1}{1-\alpha_2}\right)^2\sigma_k^2 + \sigma_a^2 + 2\left(\frac{\alpha_1+\alpha_2-1}{1-\alpha_2}\right)\sigma_{ka}, \tag{A.27}
$$

where

$$
\sigma_a^2 = \frac{\sigma_\mu^2}{1-\rho^2},
$$

due to the assumption that the firm fundamental follows a stationary AR(1) process.

The model distinguishes three main sources of dispersion in MRPK: (1) capital adjustment costs; (2) imperfect information; (3) distortions. In absence of adjustment costs we have:

$$\psi_1 = 0, \quad \psi_2 = \psi_3 = \psi_4 = \frac{1}{1-\alpha},$$

while in case of perfect foresight we have:

$$\mathbb{E}_{it}\left[a_{i,t+1}\right] = a_{i,t+1},$$

and when there are no distortions we have:

$$\gamma = 0, \quad \sigma_\varepsilon^2 = \sigma_\chi^2 = 0.$$

Each of these three origins will show up in MRPK dispersion mainly through dispersion in capital (σ_k^2).

David and Venkateswaran (2018) assume the following:

$$\mathbb{E}_{i,t}\left[a_{i,t+1}\right] = \rho a_{i,t} + s_{i,t+1}^*, \tag{A.28}$$

where the error $s_{i,t+1}^*$ depends on the information a firm has on the next innovation $\mu_{i,t+1}$. We then have:

$$k_{i,t+1} = \psi_1 k_{i,t} + \psi_2(1+\gamma)\rho a_{i,t} + u_{i,t+1} + \psi_4\chi_i, \tag{A.29}$$

where $u_{i,t+1} = s_{i,t+1}^* + \psi_3\varepsilon_{i,t+1} \sim i.i.d.(0,\sigma_u^2)$. Using the lag operator, the model can be written as

$$k_{i,t+1} = \psi_2(1+\gamma)\rho\frac{1}{(1-\psi_1 L)(1-\rho L)}\mu_{it} + \frac{1}{(1-\psi_1 L)}u_{i,t+1} + \frac{\psi_4}{1-\psi_1}\chi_i. \tag{A.30}$$

The development in firm capital is the sum of three orthogonal components (μ_{it}, $u_{i,t+1}$ and χ_i are uncorrelated), which are an AR(2), AR(1) and i.i.d. process respectively. The AR(2) process:

$$\phi_{i,t+1} = \frac{1}{(1-\psi_1 L)(1-\rho L)}\mu_{i,t+1}$$

has variance:

$$\sigma_\phi^2 = \left(1 - (\psi_1+\rho)\operatorname{corr}\left(\phi_{i,t+1},\phi_{it}\right) + \psi_1\rho\operatorname{corr}\left(\phi_{i,t+1},\phi_{i,t-1}\right)\right)^{-1},$$

with

$$\begin{aligned}
\operatorname{corr}\left(\phi_{i,t+1},\phi_{i,t}\right) &= \frac{\psi_1+\rho}{1+\psi_1\rho}, \\
\operatorname{corr}\left(\phi_{i,t+1},\phi_{i,t-1}\right) &= \frac{(\psi_1+\rho)^2}{1+\psi_1\rho} - \psi_1\rho.
\end{aligned}$$

Hence, we find that dispersion in capital is:

$$\sigma_k^2 = (\psi_2(1+\gamma)\rho)^2 \sigma_\phi^2 + \frac{1}{1-\psi_1^2}\sigma_u^2 + \left(\frac{\psi_4}{1-\psi_1}\right)^2 \sigma_\chi^2.$$

Consider now the case $\sigma_u^2 = \sigma_\chi^2 = 0$, i.e. only adjustment costs matter for dispersion in MRPK. The model reduces to

$$k_{i,t+1} = \psi_1 k_{i,t} + \psi_2(1+\gamma)\rho a_{i,t}, \tag{A.31}$$

which is model (18). By repeated substitution we write:

$$k_{i,t+1} = \psi_2(1+\gamma)\rho \sum_{s=0}^{\infty} \psi_1^s a_{i,t-s}, \tag{A.32}$$

hence the covariance between capital $k_{i,t+1}$ and fundamental a_{it} becomes:

$$\begin{aligned} \sigma_{ka} &= \mathbb{C}\mathrm{ov}\left(\psi_2(1+\gamma)\rho \sum_{s=0}^{\infty} \psi_1^s a_{i,t-s}, a_{i,t+1}\right) \\ &= \psi_2(1+\gamma)\rho \sum_{s=0}^{\infty} \psi_1^s \,\mathbb{C}\mathrm{ov}\left(a_{i,t-s}, a_{i,t+1}\right) \\ &= \psi_2(1+\gamma)\rho^2 \sum_{s=0}^{\infty} \psi_1^s \rho^s \sigma_a^2 \\ &= \frac{\psi_2(1+\gamma)\rho^2}{1-\psi_1\rho}\sigma_a^2. \end{aligned} \tag{A.33}$$

Substituting σ_a^2, σ_k^2 and σ_{ka} we find the analytical expression for σ_{MRPK}^2 for the specific case that only adjustment costs matter for MRPK dispersion.

C.4 Dynamics of misallocation with heterogeneous determinisic linear trends

In this Section we investigate if there is a deterministic trend in MRPK and MRPL dispersion over time. We augment the equation without deterministic trend in equations (10)–(14) and introduce a deterministic trend $\phi_i t$, i.e:

$$\begin{aligned} y_{i,t} &= \lambda_i + \phi_i t + z_{i,t}, & \text{(A.34)} \\ z_{i,t} &= \rho_i z_{i,t-1} + \varepsilon_{i,t}, & \text{(A.35)} \end{aligned}$$

Combining (A.34) and (A.35) we have:

$$\begin{aligned} y_{i,t} &= (1-\rho_i)\eta_i + \rho_i\phi_i + (1-\rho_i)\phi_i t + \rho_i y_{i,t-1} + \varepsilon_{i,t}, \\ &= \alpha_i + \beta_i t + \rho_i y_{i,t-1} + \varepsilon_{i,t}, & \text{(A.36)} \end{aligned}$$

The deterministic trend is also introduced in the Ng (2008) model, i.e.:

$$V_{t,\infty} = \Lambda_\infty + \theta \cdot t + c, \tag{A.37}$$

We can estimate the fraction of firms with a unit root (θ) as:

$$\hat{\theta} = \frac{1}{T}\sum_{t=1}^{T} \Delta V_{t,N}, \tag{A.38}$$

and the estimate of θ can be obtained from the (time-series) regression:

$$\Delta V_{t,N} = \theta + \beta \Delta t^2 + \eta_{t,N}. \tag{A.39}$$

Because $\beta = var(\phi_i)$ the OLS estimator of β can be used to test the significance of incidental deterministic trends.