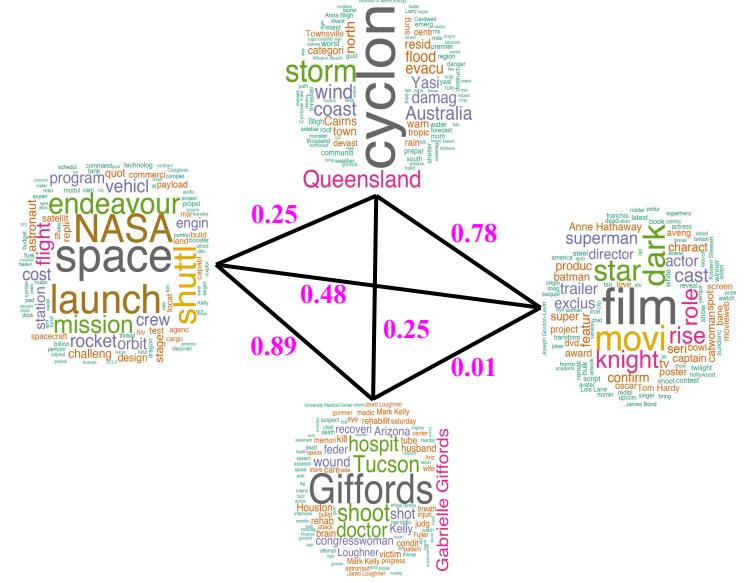
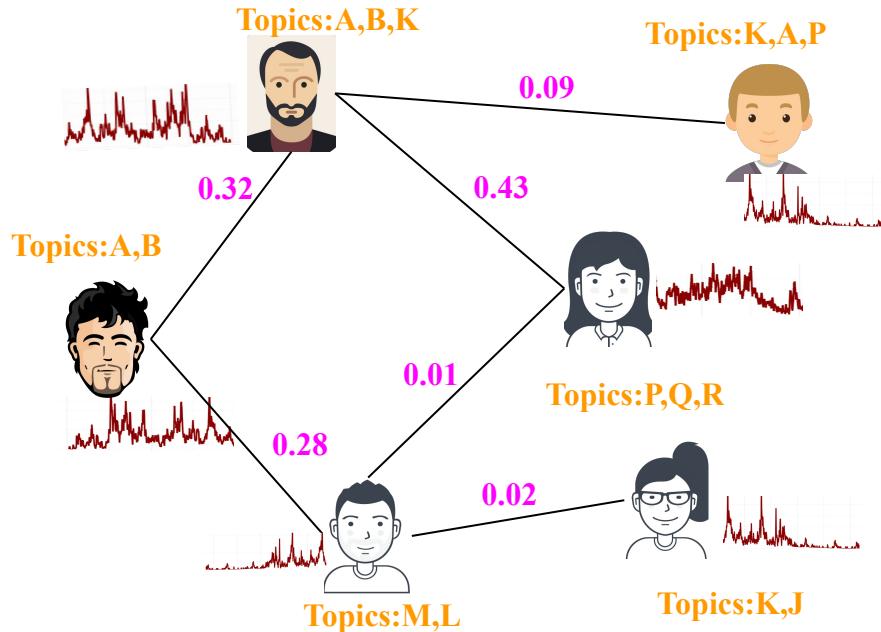


# Discovering Topical Interactions in Text-based Cascades using Hidden Markov Hawkes Process

Srikanta Bedathur (IIT Delhi), Indrajit Bhattacharya (TCS Research),  
**Jayesh Choudhari**, Anirban Dasgupta (IIT Gandhinagar)

# Motivation



- User Temporal Dynamics
- Preferred topics of each user
- Network Strengths (user-user influence)

- Topics
- Topical Interactions

# Data: Network + Time-series of Tweets



Syria Strike Puts U.S.  
Relationship With Russia at Risk



Trump to 'Free NASA' and  
Set Sights on Further Space  
Exploration



Zika: CDC Warns Pregnant  
Women to Avoid Travel to Miami



CDC issues historic travel warning  
over Zika in Miami



Russia and Iran Warn  
U.S. 'Crossed Red Lines  
by Attacking Syria'

Time



Leonardo DiCaprio: We Can't  
Elect a Candidate Who Doesn't  
Believe in Climate Change



Russia denies bombing  
U.S.-backed Syrian rebels



Watch National Geographic's stunning  
climate-change documentary starring  
Leonardo DiCaprio

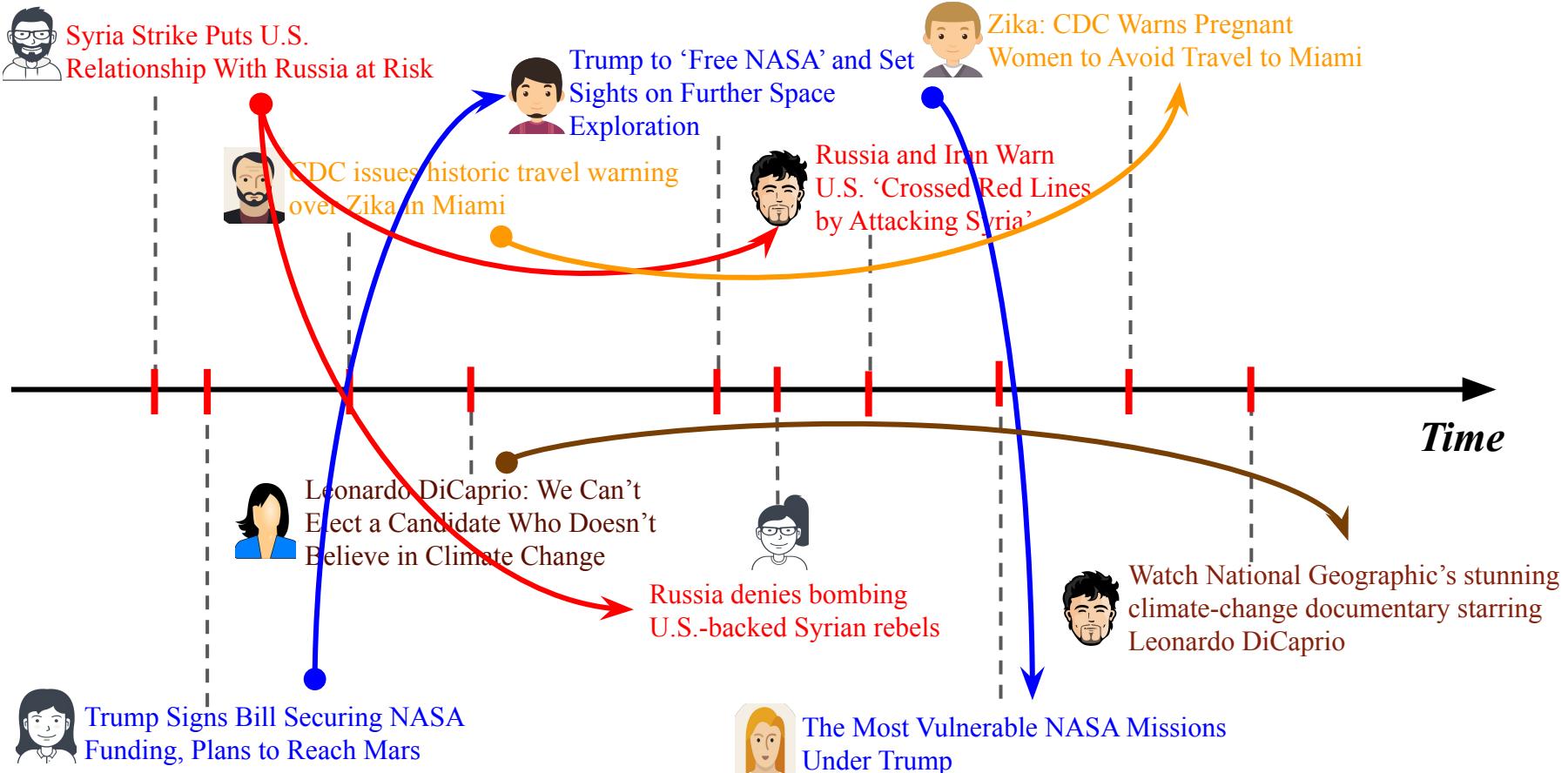


Trump Signs Bill Securing NASA  
Funding, Plans to Reach Mars

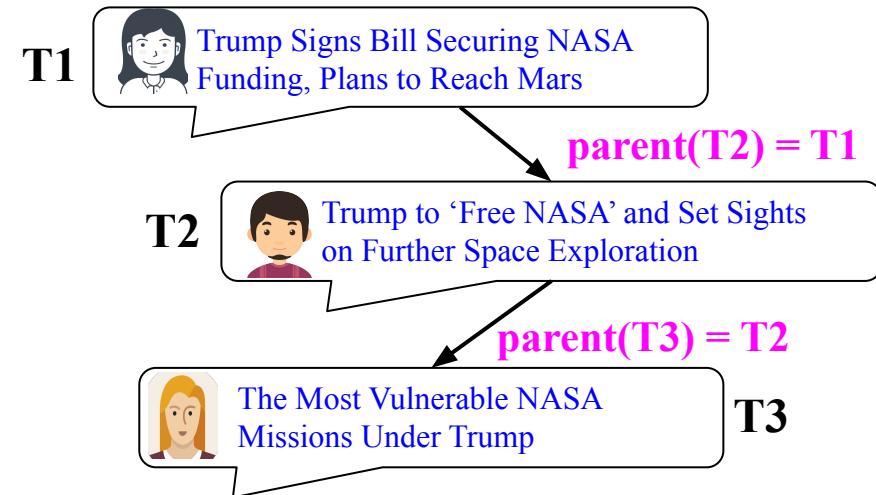
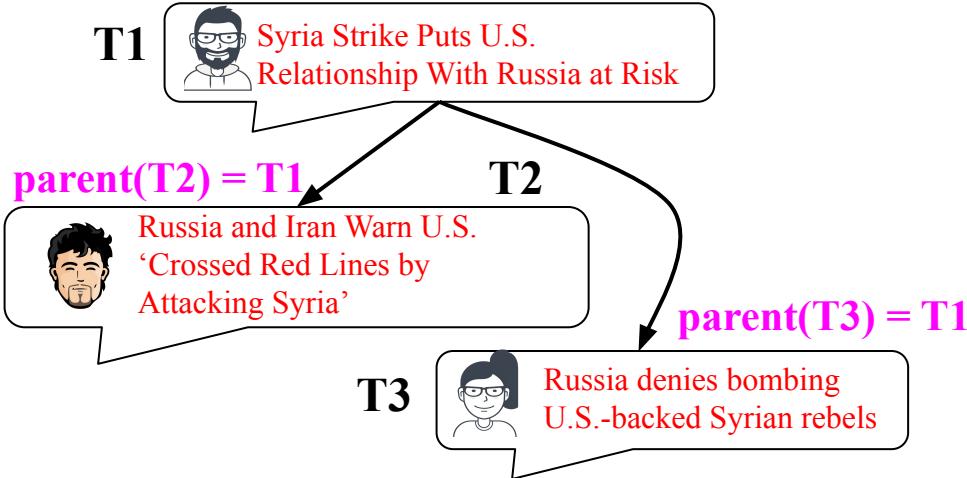


The Most Vulnerable NASA Missions  
Under Trump

# Mixture of Conversations



# Cascades (Separate Conversations)



*Separate these conversations out!!!*

# *Hidden Markov Hawkes Process*

- Coupling of Network (Multivariate) Hawkes Process and the Markov Chain over topics.
- Coupled inference: Collapsed Gibbs sampling

# *Why Topical Interactions?*

## *Parent-Child tweet pair*

Gellman: My definition of whistleblowing: are you shedding light on crucial decision that society should be making for itself. #snowden

Gellman we are living inside a one way mirror; they & big corporations know more and more about us and we know less about them #sxsw

## *Hashtags from top-3 transitioned topics*

agentsofshield, arrow, tvtag, supernatural, chicagoland

**Topic-1:** idol, bbcan2, havesandhavenots, thegamebet  
**Topic-2:** tvtag, houseofcards, agentsofshield, arrow,  
**Topic-3:** soundcloud, hiphop, mastermind, nowplaying

## *Hashtags from a pair of parent-child topics*

steelers, browns, seahawks, fantasyfootball, nfl

mlb, orioles, rays, usmnt, redsox

# *HMHP Generative Process*

- 1) Generate  $(t_e, c_e, z_e)$  for all events according Multivariate Hawkes Process.
- 2) For each topic  $k$ : sample  $\zeta_k \sim Dir_{\mathcal{W}}(\alpha)$
- 3) For each topic  $k$ : sample  $\mathcal{T}_k \sim Dir_K(\beta)$
- 4) For each node  $v$ : sample  $\phi_v \sim Dir_K(\gamma)$
- 5) For each event  $e$  at node  $c_e = v$ :
  - a) i) **if**  $z_e = 0$  (level 0 event):  
draw a topic  $\eta_e \sim Discrete_K(\phi_v)$
  - ii) **else**:  
draw a topic  $\eta_e \sim Discrete_K(\mathcal{T}_{\eta_{z_e}})$
- b) Sample document length  $N_e \sim Poisson(\lambda)$
- c) For  $w = 1 \dots N_e$ : draw word  $x_{e,w} \sim Discrete_{\mathcal{W}}(\zeta_{\eta_e})$

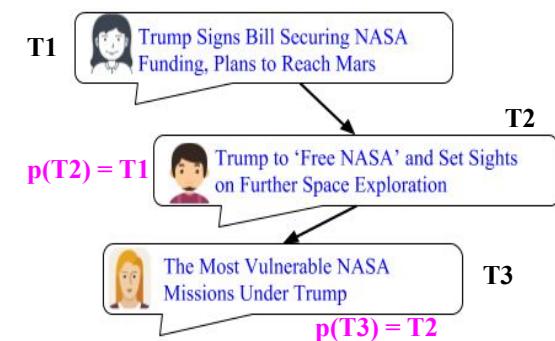
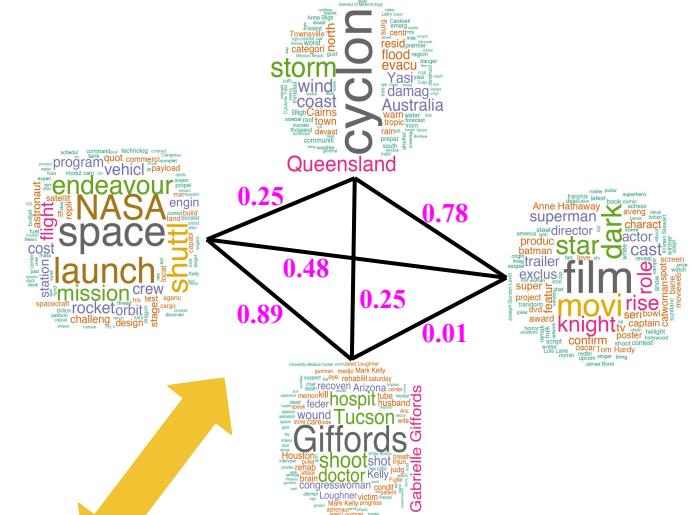
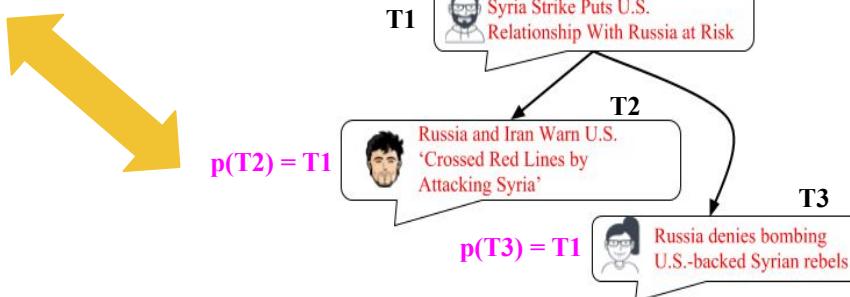
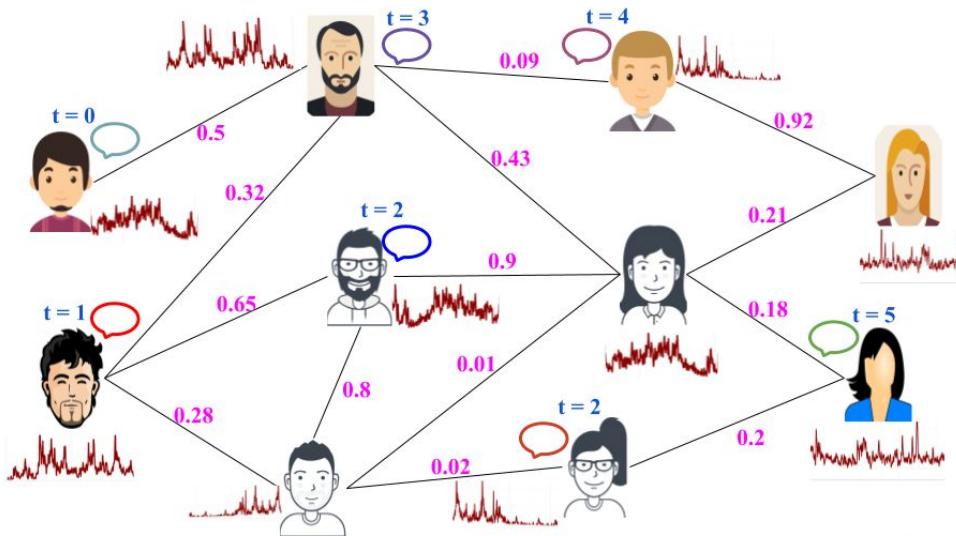
*Temporal Dynamics and  
Network Inference using  
Multivariate Hawkes Process*

*Cascade reconstruction and  
Topical Interactions coupling  
Multivariate Hawkes Process  
and Topical Markov Chains*

*Topic Model*

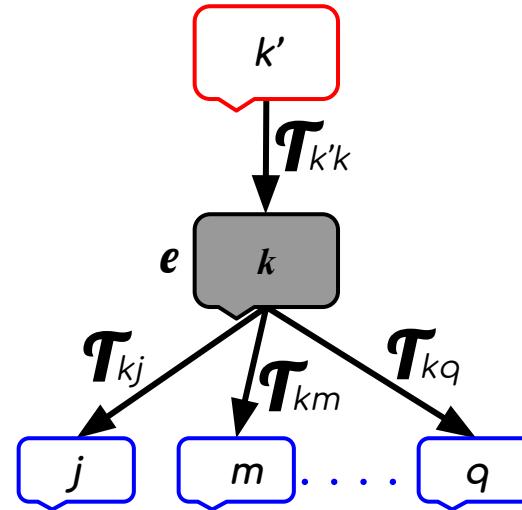
# *Inference*

# *Challenge - Coupled Problems*



# Topic Inference

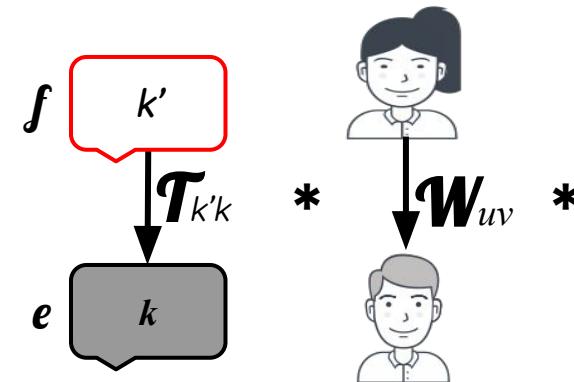
$$\mathbf{P} \left( \begin{array}{l} \text{topic}(e) = k \mid \text{parentStructure}, \text{tweetText}, \\ \{\text{topic}(f) \mid f \neq e\} \end{array} \right) \propto$$



The Most Vulnerable NASA  
Missions Under Trump

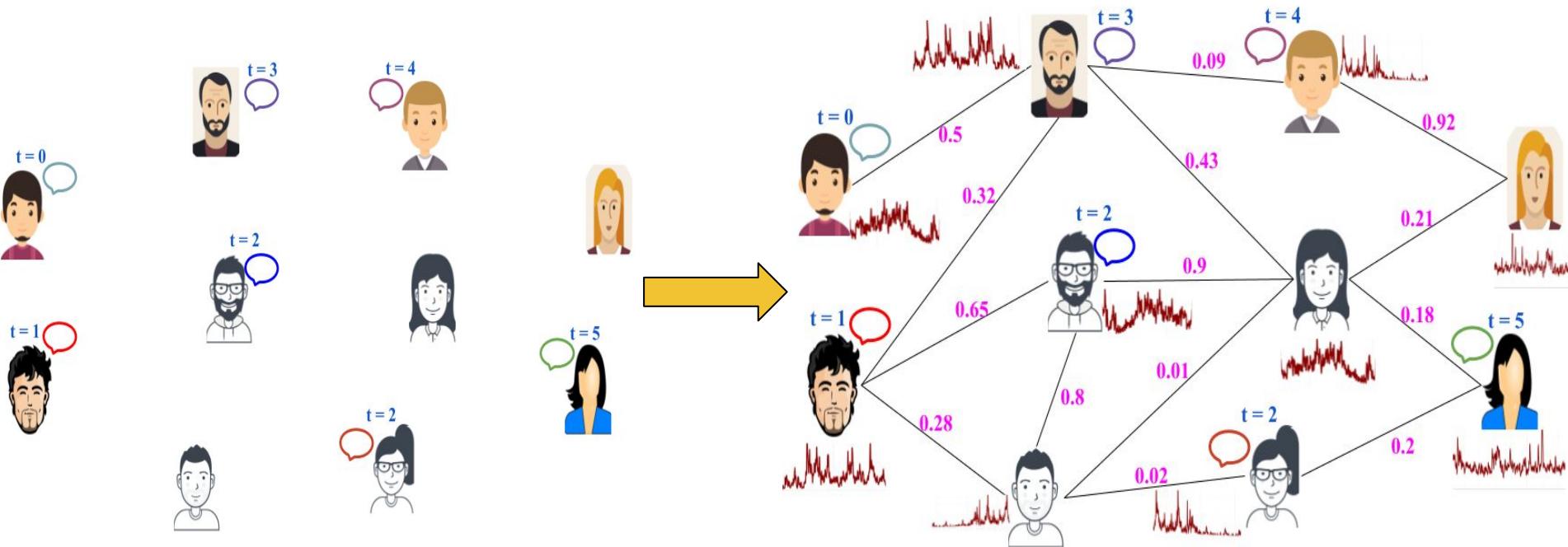
# Cascade Inference

$$\mathcal{P}(\text{par}(e) = f \mid \text{Topics}, \mathcal{W}, \mu, \text{timeStamps}) \propto$$



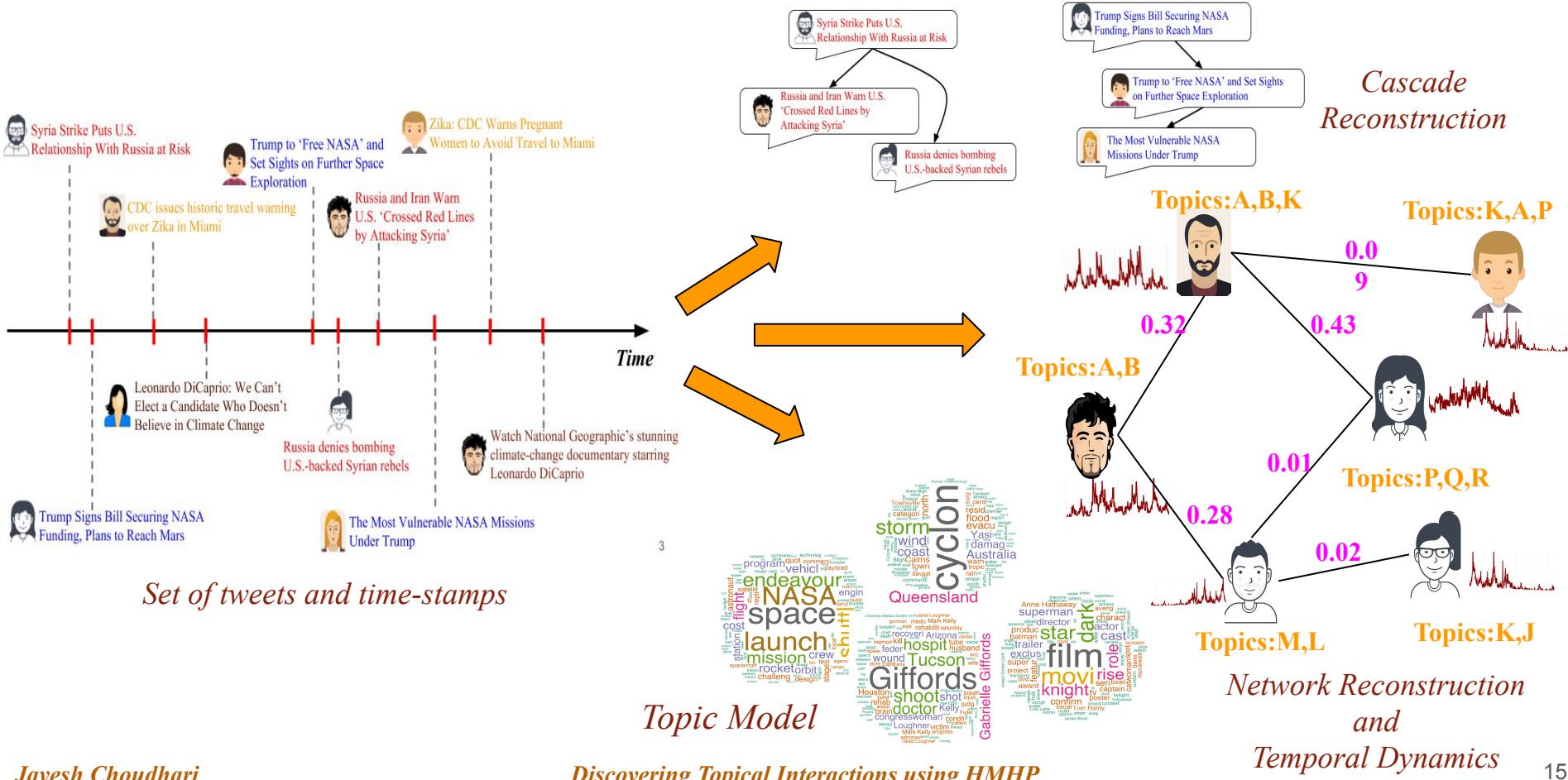
# *Existing Models*

# *Network Hawkes Model*



***Does not model (textual) content of events / tweets***

# Hawkes Topic Model (HTM) [He et al. '15]



# *Missing Topical Interactions in HTM*

[#MASalert] Statement By Our Group CEO,  
Ahmad Jauhari Yahya on MH370 Incident.  
Released at 9.05am/8 Mar 2014

Missing #MalaysiaAirlines flight carrying  
227 passengers (including 2 infants) of 13  
nationalities and 12 crew members.

Repeating patterns in the topics of the  
parent and child events

## *Generation of Topic of child event in HTM*

If event  $e$  is not spontaneous, then  
 $\text{Topic}(e) \sim \text{Normal}(\text{Topic}(\text{parent}(e)), \sigma^2 I)$

v/S

## *Generation of Topic of child event in HMHP*

If event  $e$  is not spontaneous, then  
 $\text{Topic}(e) \sim \zeta(\text{Topic}(\text{parent}(e)))$

where,  $\zeta$  is Topical Interaction Distribution

**Note:** These parent-child pairs are neither retweets nor does twitter provide any signal to know any relation about these pairs

# *Results*

# *Datasets*

## ***Twitter (Real Data):***

- *500K tweets corresponding to top 5K hashtags from the most prolific 1M users generated in a contiguous part of March 2014*

## ***Semi-Synthetic:***

- *Retain the underlying set of nodes and the follower graph from a sample of Twitter Data.*
- *Estimate the parameters required for our model from the data.*
- *Generate 5 different samples of 1M events using HMHP model.*

# *Baselines*

- **$HWK + DIAG$ :**
  - *Simplified HMHP with diagonal topical interactions*
- **$HWK \times LDA$ :**
  - *Network Hawkes model for cascade structure and time-stamps*
  - *LDA mixture model for the textual content*
- **$HTM$  (*Hawkes Topic Model*)**

# *Reconstruction Accuracy (Semi-Synthetic Dataset)*

	<b>HMHP</b>	<b>HWK+Diag</b>	<b>HWK×LDA</b>
<i>Mean APE</i>	<b>0.448</b>	0.565	0.552
<i>Median APE</i>	<b>0.255</b>	0.283	0.287

## *Network Reconstruction Error*

*Mean Error* :- ~18% lower

*Median Error* :- ~10% lower

	<b>HMHP</b>	<b>HWK+Diag</b>	<b>HWK×LDA</b>
<i>Accuracy</i>	<b>0.581</b>	0.362	0.37
<i>Recall@1</i>	<b>0.595</b>	0.373	0.38
<i>Recall@3</i>	<b>0.778</b>	0.584	0.589

## *Cascade Reconstruction Accuracy*

*Acc/Recall@1* :- ~57% better

*Recall@3* :- ~32% better

	<b>HMHP</b>	<b>HWK+Diag</b>	<b>HWK×LDA</b>
<i>Precision</i>	<b>0.893</b>	0.123	0.781
<i>Recall</i>	<b>0.746</b>	0.367	0.752
<i>F1</i>	<b>0.811</b>	0.18	0.765

## *Topic Identification*

*HMHP* performs ~5-6% better

# *Generalization Performance* (Twitter Dataset)

## *Heldout Log-Likelihood*

#Topics	Log-Likelihood	HMHP	HWK + Diag	HWK x LDA
25	<i>Content</i>	-30499278	-33356945	-30532938
	<i>Time</i>	-4236958	-4042903	-4299630
	<i>Total</i>	<b>-34736237</b>	-37399849	-34832568
50	<i>Content</i>	-30141081	-33427354	-30089733
	<i>Time</i>	-4288438	-4510072	-4343571
	<i>Total</i>	<b>-34429519</b>	-37937426	-34433305
75	<i>Content</i>	-29860909	-33433922	-29861050
	<i>Time</i>	-4285293	-4510535	-4373736
	<i>Total</i>	<b>-34146202</b>	-37944457	-34234787

*HMHP performs ~5% better than the baselines*

# *Comparison with HTM [He et al. '15]*

## *Synthetic events generated using HMHP model*

Window Length	1000	2000	3000	4000	5000
HTM	2.811	1.982	1.464	1.292	1.351
HMHP	1.297	0.925	0.677	0.646	0.657

**Network Inference (TAE) (*lower the better*)**

Window Length	1000	2000	3000	4000	5000
HTM	0.681	0.687	0.712	0.716	0.708
HMHP	0.926	0.924	0.95	0.94	0.935

**Parent Identification (Accuracy) (*higher the better*)**

# *Comparison with HTM [He et al. '15]*

*Synthetic events (short documents) generated using HTM model*

Window Length	1000	2000	3000	4000	5000
HTM	3.167	2.377	2.014	1.964	1.519
HMHP	1.696	1.200	1.168	1.396	1.243

**Network Inference (TAE) (*lower the better*)**

Window Length	1000	2000	3000	4000	5000
HTM	0.575	0.588	0.61	0.618	0.628
HMHP	0.716	0.730	0.736	0.730	0.748

**Parent Identification (Accuracy) (*higher the better*)**

# *Generative Model*

# *What all to model?*

- Temporal Dynamics for each user
- User Network Strengths
- Topics
- Topical Interactions
- Topic preference for each user

# *Modeling Time + Network: Hawkes Process*

$$\lambda_v(t) = \mu_v(t) + \sum_{n=1}^{|\mathcal{H}_{t^-}|} h_{c_n, v}(t - t_n)$$

# *Modeling Time + Network: Multivariate Hawkes Process*

$$\lambda_v(t) = \mu_v(t) + \sum_{n=1}^{|\mathcal{H}_{t^-}|} h_{c_n, v}(t - t_n)$$

The diagram illustrates the components of the Hawkes process formula. At the top, the equation  $\lambda_v(t) = \mu_v(t) + \sum_{n=1}^{|\mathcal{H}_{t^-}|} h_{c_n, v}(t - t_n)$  is shown. The term  $\mu_v(t)$  is enclosed in a red square and connected by a vertical line to a box labeled "Base Intensity". The entire summation term  $\sum_{n=1}^{|\mathcal{H}_{t^-}|} h_{c_n, v}(t - t_n)$  is also enclosed in a red rounded rectangle and connected by a vertical line to a box labeled "Excitation because of History (Neighbors)".

$$h_{u,v}(\Delta t) = W_{u,v} f(\Delta t)$$

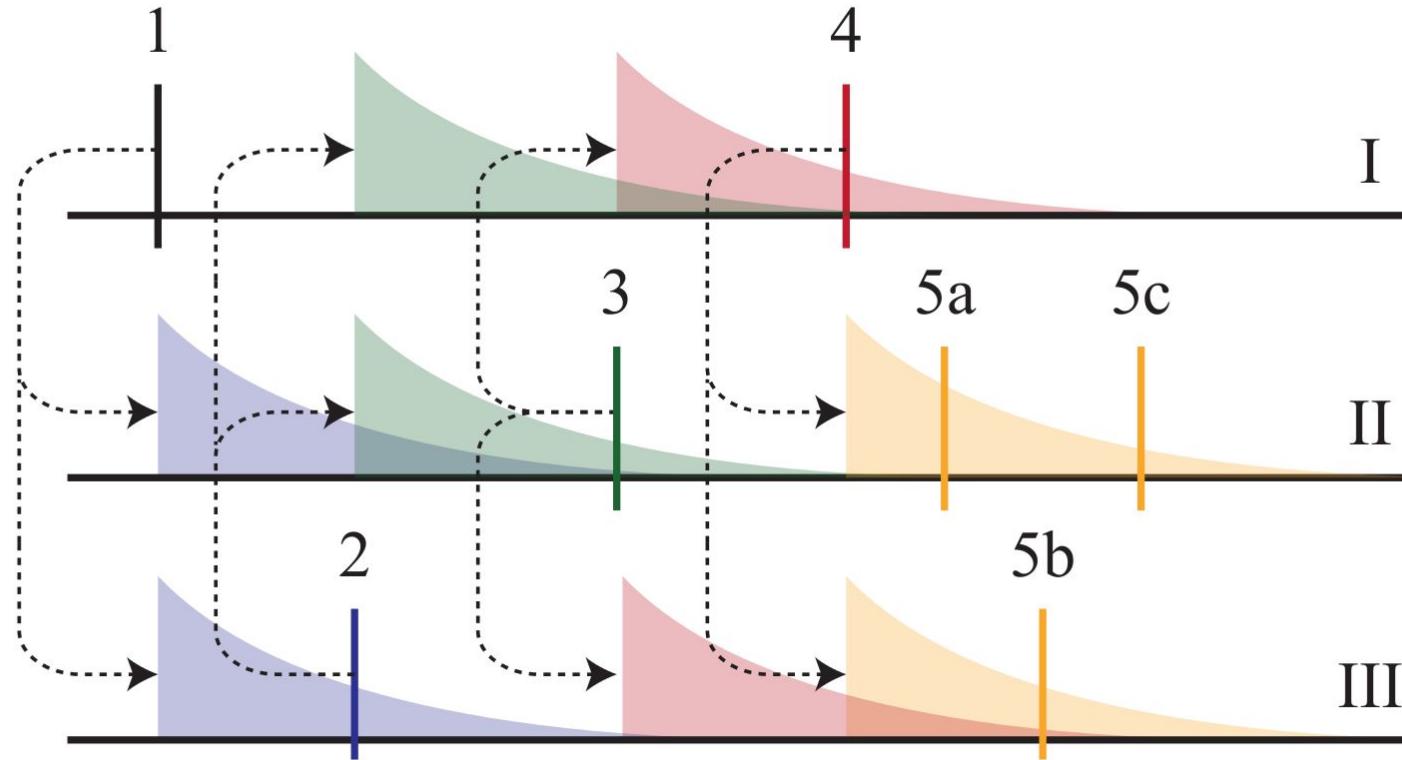
## *Level wise event generation [A. Simma 2010]*

- Draw (spontaneous) events for each user with the base intensity -- (*Level-0 events*).
- Subsequent events are drawn using the following non-homogenous Poisson process

$$\Pi_l \sim Poisson \left( \sum_{(t_n, c_n, z_n) \in \Pi_{l-1}} h_{c_n, \cdot}(t, t_n) \right)$$

*Level-i event can be anywhere on the timeline, it's just that the timestamps of level-i events is greater than the timestamps of level-(i-1) events*

# *Modeling Time + Network: Multivariate Hawkes Process*



# *HMHP Generative Process*

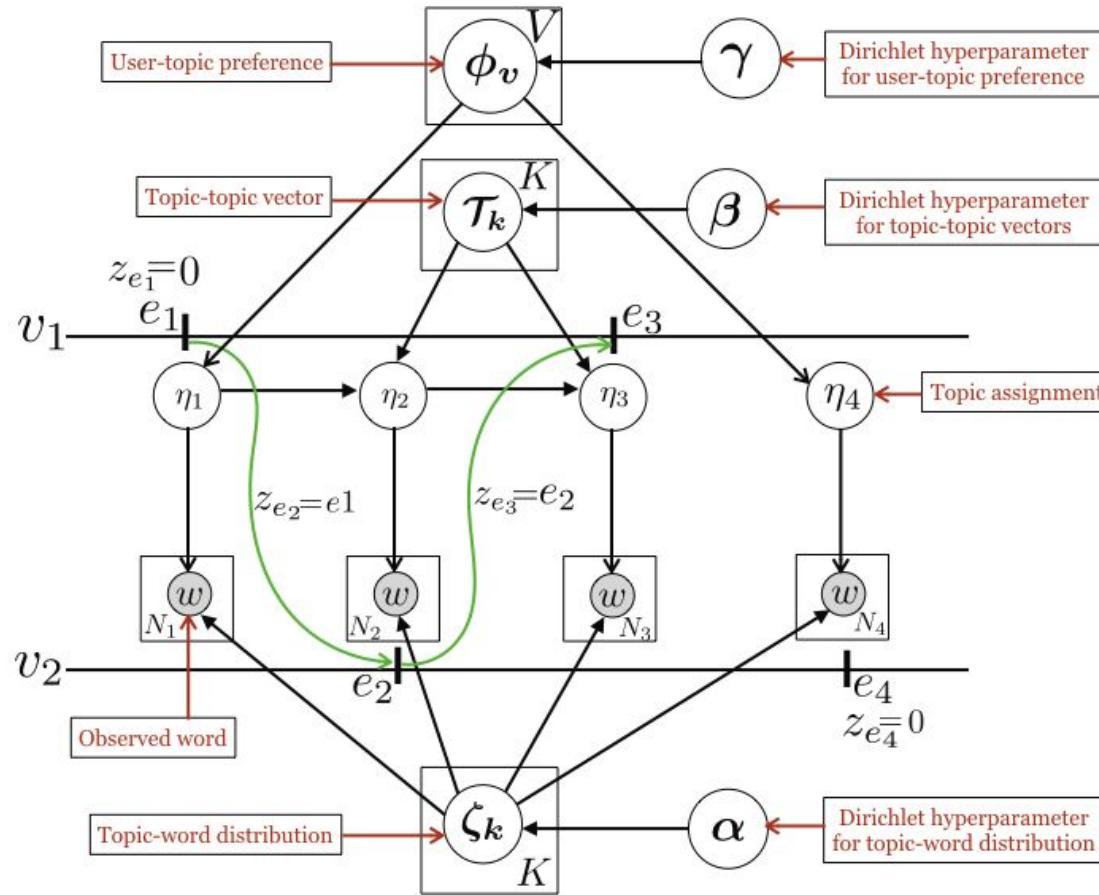
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*Temporal Dynamics and  
Network Inference using  
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*Cascade reconstruction and  
Topical Interactions coupling  
Multivariate Hawkes Process  
and Topical Markov Chains*

*Topic Model*

# Generative Model



# *Inference*

# Joint Probability

$$\begin{aligned}
& P(E, \Phi, \mathcal{T}, \zeta, \eta, z \mid \alpha, \beta, \gamma, W, \mu) = \\
& \prod_{v \in V} P(\phi_v \mid \gamma) \times \prod_{k=1}^K P(\zeta_k \mid \alpha) \times \prod_{k=1}^K P(\mathcal{T}_k \mid \beta) \\
& \times \prod_{e \in E} \left\{ \left[ \prod_{e': t_{e'} < t_e} P(\eta_e | \mathcal{T}_{\eta_{z_e}})^{\delta_{z_e, e'}} \right] P(\eta_e | \phi_v)^{\delta_{z_e, 0}} \right\} \\
& \quad \times \prod_{e \in E} \left[ \prod_{w=1}^{N_e} P(x_{e,w} | \eta_e, \zeta_{\eta_e}) \right] \\
& \quad \times \prod_{v \in V} \left[ \exp \left( - \int_0^T \mu_v(\tau) d\tau \right) \prod_{e \in E} \mu_v(t_e)^{\delta_{c_e, v} \delta_{z_e, 0}} \right] \\
& \quad \times \prod_{e \in E} \prod_{v \in V} \left[ \exp \left( - \int_{t_e}^T h_{c_e, v}(\Delta\tau) d\tau \right) \prod_{e' \in E} h_{c_e, c_{e'}}(\Delta(t_{e'}))^{\delta_{c_{e'}, v} \delta_{z_{e'}, e}} \right]
\end{aligned}$$

# Joint Probability

$$P(E, \Phi, \mathcal{T}, \zeta, \eta, z | \alpha, \beta, \gamma, W, \mu) =$$

$$\begin{aligned}
& \prod_{v \in V} P(\phi_v | \gamma) \times \prod_{k=1}^K P(\zeta_k | \alpha) \times \prod_{k=1}^K P(\mathcal{T}_k | \beta) \quad \bullet \text{ Priors} \\
& \times \prod_{e \in E} \left\{ \left[ \prod_{e': t_{e'} < t_e} P(\eta_e | \mathcal{T}_{\eta_{z_e}})^{\delta_{z_e, e'}} \right] P(\eta_e | \phi_v)^{\delta_{z_e, 0}} \right\} \quad \bullet \text{Topic Transitions/Interactions} \\
& \quad \times \prod_{e \in E} \left[ \prod_{w=1}^{N_e} P(x_{e,w} | \eta_e, \zeta_{\eta_e}) \right] \quad \bullet \text{Topics for spontaneous events} \\
& \quad \times \prod_{v \in V} \left[ \exp \left( - \int_0^T \mu_v(\tau) d\tau \right) \prod_{e \in E} \mu_v(t_e)^{\delta_{c_e, v} \delta_{z_e, 0}} \right] \quad \bullet \text{Generating words for each doc} \\
& \times \prod_{e \in E} \prod_{v \in V} \left[ \exp \left( - \int_{t_e}^T h_{c_e, v}(\Delta\tau) d\tau \right) \prod_{e' \in E} h_{c_e, c_{e'}}(\Delta(t_{e'}))^{\delta_{c_{e'}, v} \delta_{z_{e'}, e}} \right] \quad \bullet \text{Time (spontaneous events)} \\
& \quad \bullet \text{Time (Influenced Events)}
\end{aligned}$$

# Topic Inference

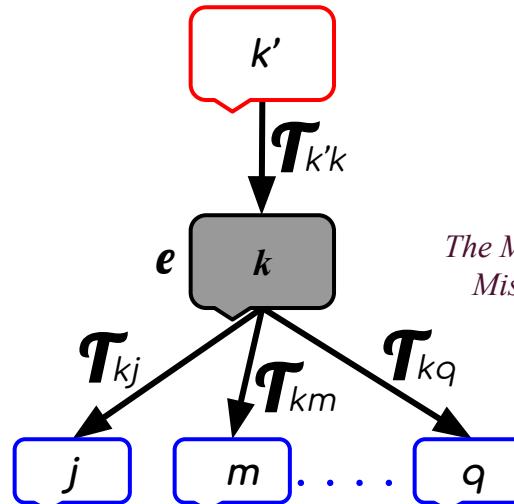
$$\begin{aligned}
 \mathcal{P} \left[ \begin{array}{l} \text{topic}(e) = k \mid \text{parentStructure}, \text{tweetText}, \\ \{ \text{topic}(f) \mid f \neq e \} \end{array} \right] \propto & \frac{\beta_k + N_{k',k}^{(\neg(z_e,e))}}{(\sum_l \beta_l) + N_{k'}^{(\neg(z_e,e))}} \times \frac{\prod_{w \in d_e} \prod_{i=0}^{N_e^w - 1} (\alpha_w + \mathfrak{T}_{k,w}^{\neg e} + i)}{\prod_{i=0}^{N_e - 1} ((\sum_{w \in \mathcal{W}} \alpha_w) + \mathfrak{T}_k^{\neg e} + i)} \\
 & \times \frac{\prod_{l'=1}^K \prod_{i=0}^{N_{k,l'}^{(C_e)} - 1} (\beta_{l'} + N_{k,l'}^{(\neg C_e)} + i)}{\prod_{i=0}^{N_k^{(C_e)} - 1} ((\sum_{l'} \beta_{l'}) + N_k^{\neg C_e} + i)}
 \end{aligned}$$

# Topic Inference

$$\mathcal{P} \left[ \begin{array}{l} \text{topic}(e) = k \mid \text{parentStructure, tweetText,} \\ \{\text{topic}(f) \mid f \neq e\} \end{array} \right] \propto$$

$$\frac{\beta_k + N_{k',k}^{(\neg(z_e,e))}}{(\sum_l \beta_l) + N_{k'}^{(\neg(z_e,e))}} \times \frac{\prod_{w \in d_e} \prod_{i=0}^{N_e^w - 1} (\alpha_w + \mathfrak{T}_{k,w}^{\neg e} + i)}{\prod_{i=0}^{N_e - 1} ((\sum_{w \in \mathcal{W}} \alpha_w) + \mathfrak{T}_k^{\neg e} + i)}$$

$$\times \frac{\prod_{l'=1}^K \prod_{i=0}^{N_{k,l'}^{(C_e)} - 1} (\beta_{l'} + N_{k,l'}^{(\neg C_e)} + i)}{\prod_{i=0}^{N_k^{(C_e)} - 1} ((\sum_{l'} \beta_{l'}) + N_k^{\neg C_e} + i)}$$



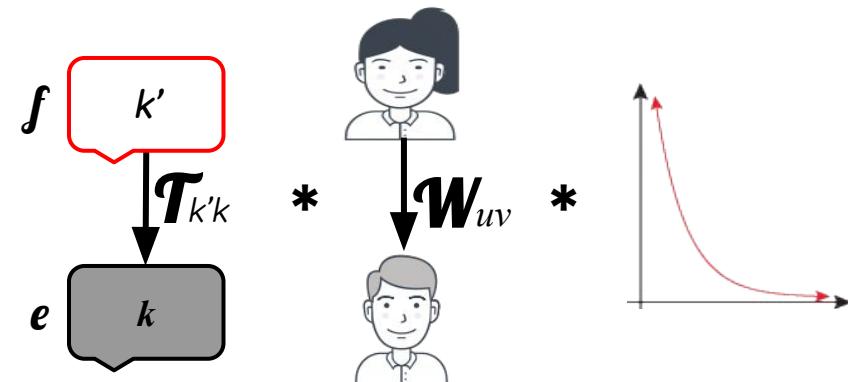
# *Cascade Inference*

$$\mathcal{P}(\text{par}(e) = f | \text{Topics}, \mathcal{W}, \mu, \text{timeStamps}) \propto \frac{(\beta_k + N_{k',k} - 1)}{((\sum_{k=1}^K \beta_k) + N_{k'} - 1)} \times h_{u_{e'}, u_e}(t_e - t_{e'})$$

# Cascade Inference

$$\mathcal{P}(\text{par}(e) = f | \text{Topics}, \mathcal{W}, \mu, \text{timeStamps}) \propto \frac{(\beta_k + N_{k',k} - 1)}{((\sum_{k=1}^K \beta_k) + N_{k'} - 1)} \times h_{u_{e'}, u_e}(t_e - t_{e'})$$

$$\mathcal{P}(\text{par}(e) = f | \text{Topics}, \mathcal{W}, \mu, \text{timeStamps}) \propto$$



## *Network Inference*

$$P(W_{u,v} = x \mid E_t^{(u,v)}, z) \propto x^{\alpha_1} \exp(-x\beta_1)$$

*where,*

$$\alpha_1 = (N_{u,v} + \alpha - 1)$$

$$\beta_1 = (N_u + \frac{1}{\beta})^{-1}$$

# *Summary*

- *Generative model for textual time-series from user networks having topical interactions*
- *Couples Topical Markov Chains and Multivariate Hawkes Processes*
- *Scalable collectively inference using collapsed Gibbs Sampling*
- *More accurate cascade reconstruction, topic identification and network reconstruction and better generalization for test data*
- *Derive insights about topical interactions that the existing models cannot*

# *Knowledge in HMHP*

# *Topical Structure*

Tonight, we heard from two candidates -- but  
only one president. **#ImWithHer**

8:13 AM - 27 Sep 2016

7,060 Retweets 20,150 Likes



"Hillary won big time. It was a shut out." --  
**@HardballChris #debatenight**

8:14 AM - 27 Sep 2016

4,313 Retweets 12,374 Likes



# *Topical Structure*

Tonight, we heard from two candidates -- but  
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US Presidential Candidate

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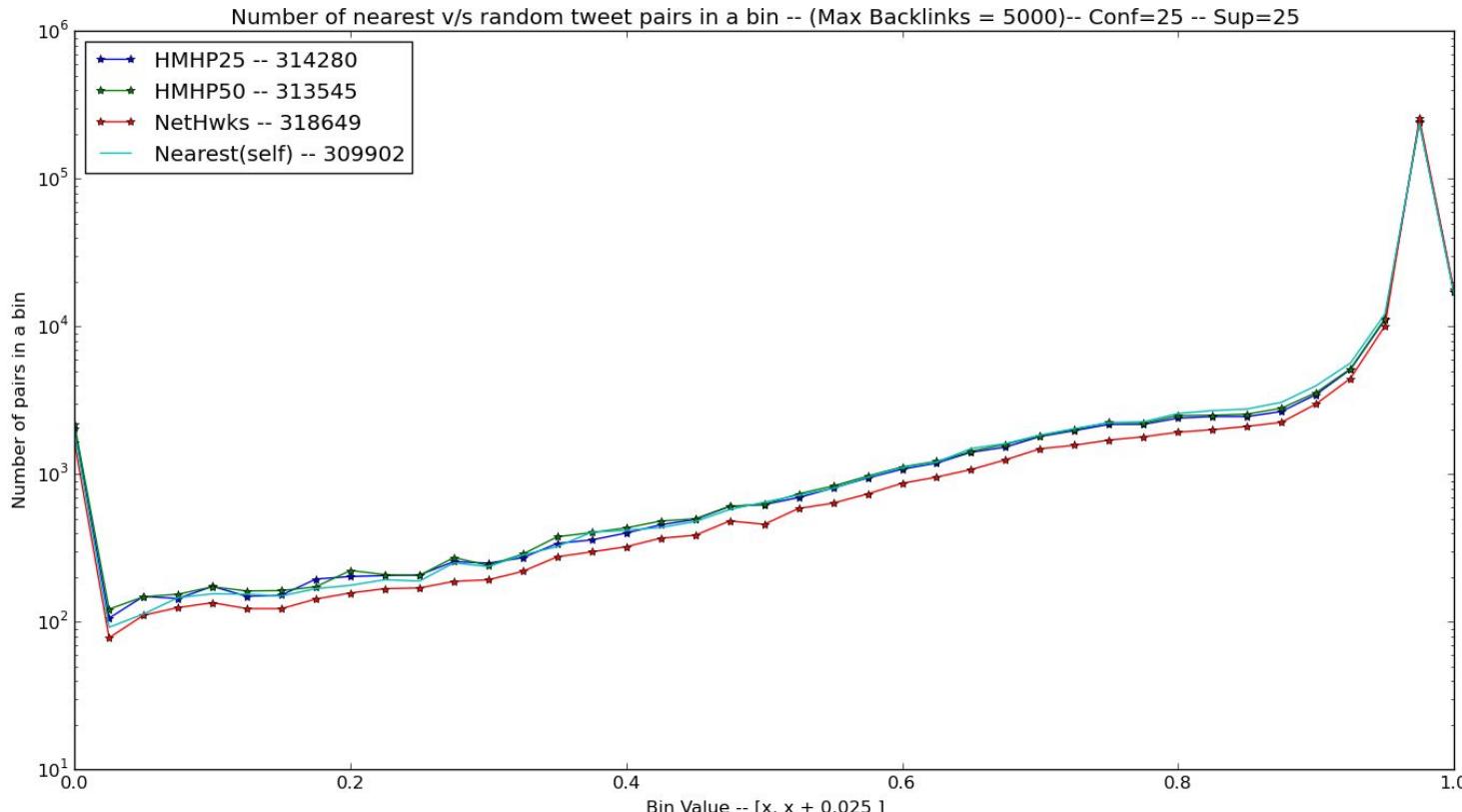
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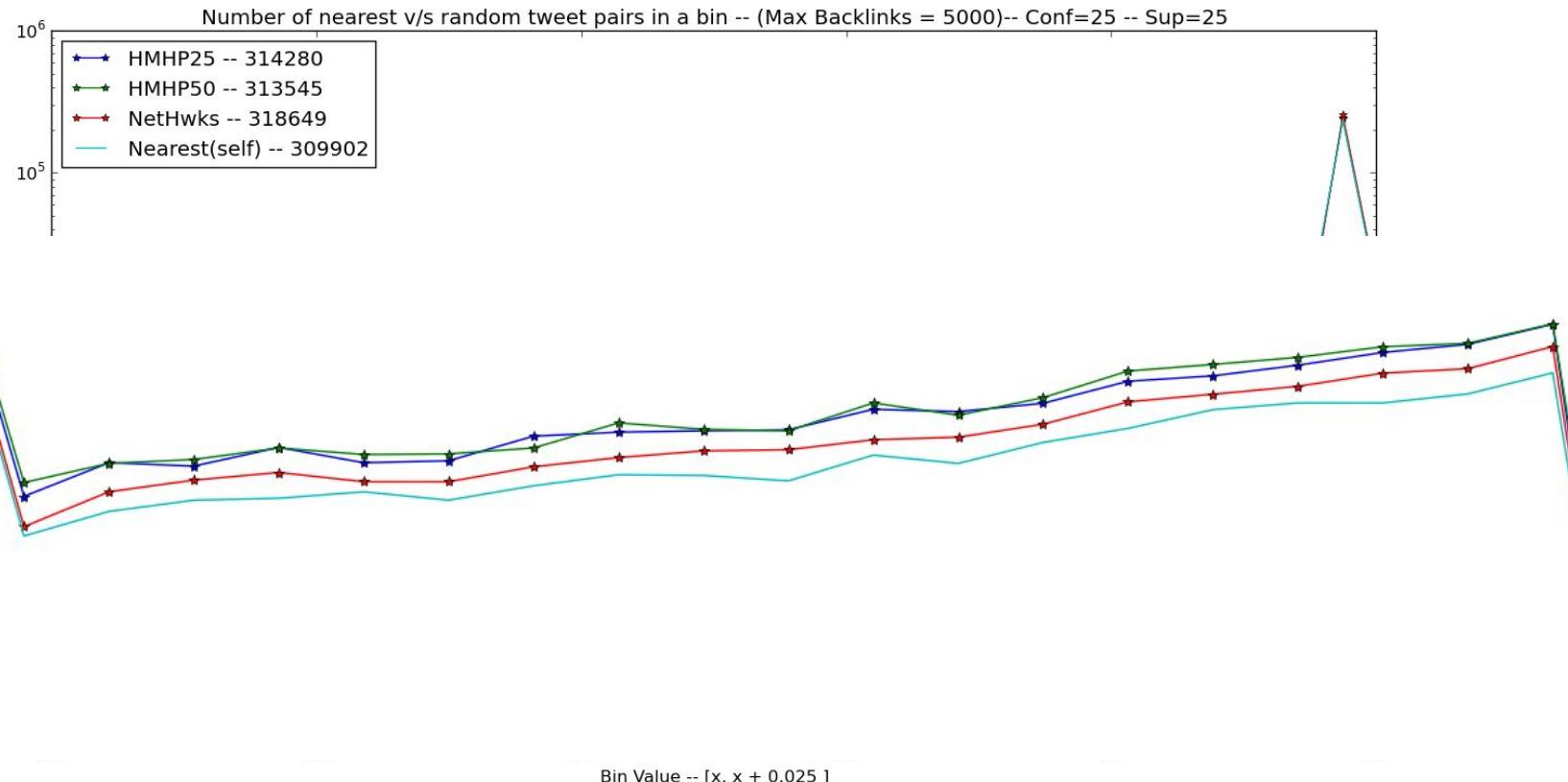
*Entities in parent-child tweet pairs are “closer” on Wiki?*

# *Jaccard Distance between Parent-Child Tweets*



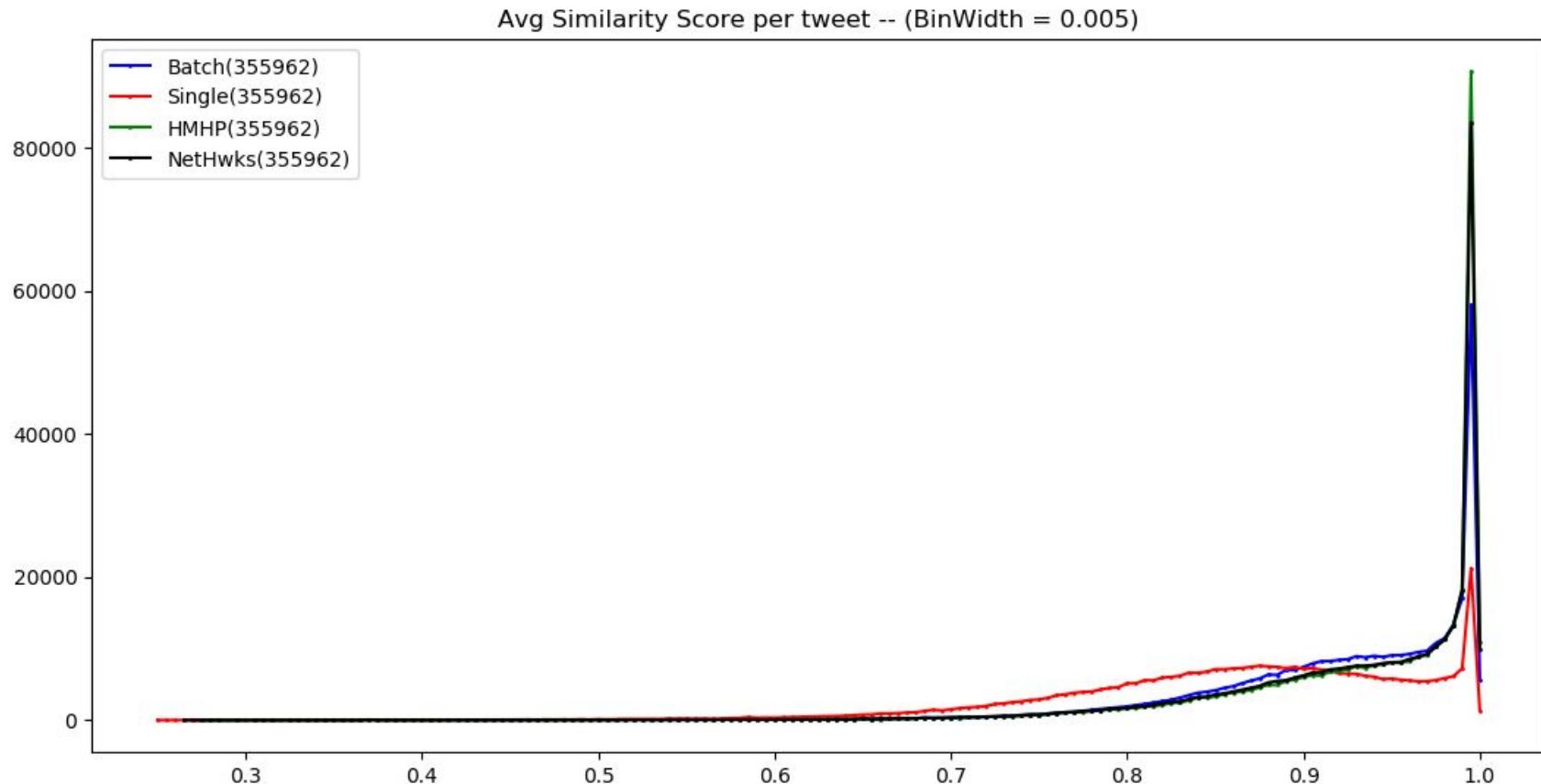
*Note: Annotation is done using DBpedia Spotlight*

# *Jaccard Distance between Parent-Child Tweets*



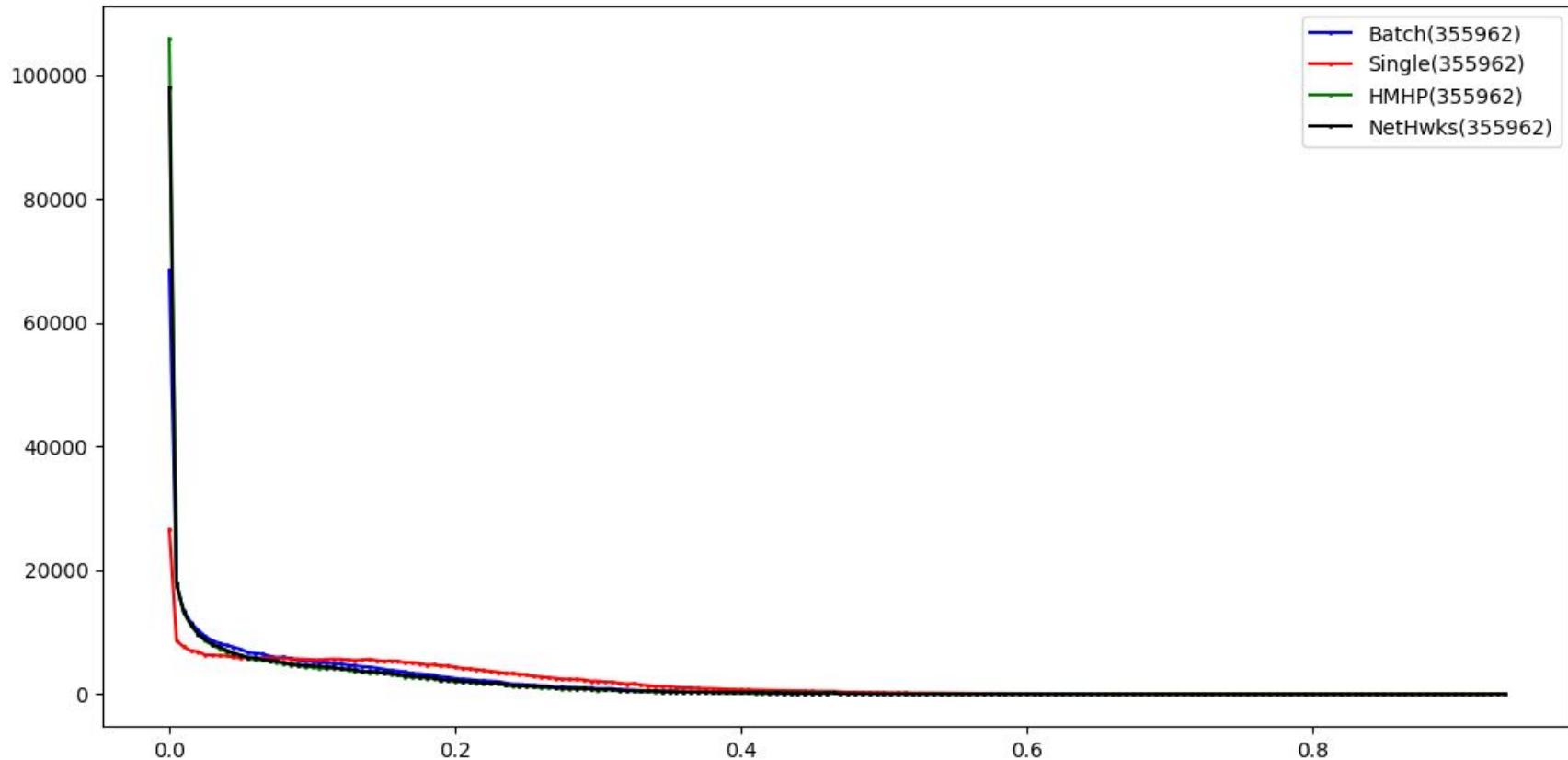
# *Better Cascades Better Annotation?*

# *Avg. Similarity Score for Cascades*



# *Avg. Percentage Second Rank for Cascades*

Avg Perc Second Rank per tweet -- (BinWidth = 0.005)



# *Coupling Cascades and Entity Identification*

- *Better parent-child identification (cascade construction) can help in better annotation (entity identification)*
- *Better annotation can help in better parent-child identification (cascade construction)?*

## Goal

*A Generative model for textual time-series data from user networks  
having topical interactions along with structure among topics*

# *Thank You*