

# Learned Improvements to the Visual Egomotion Pipeline

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Final Oral Examination

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Supervised by Professor Jonathan Kelly



Institute for Aerospace Studies  
UNIVERSITY OF TORONTO

S T A R S  
L A B O R A T O R Y

# Egomotion Estimation

## and ‘Dead’ Reckoning

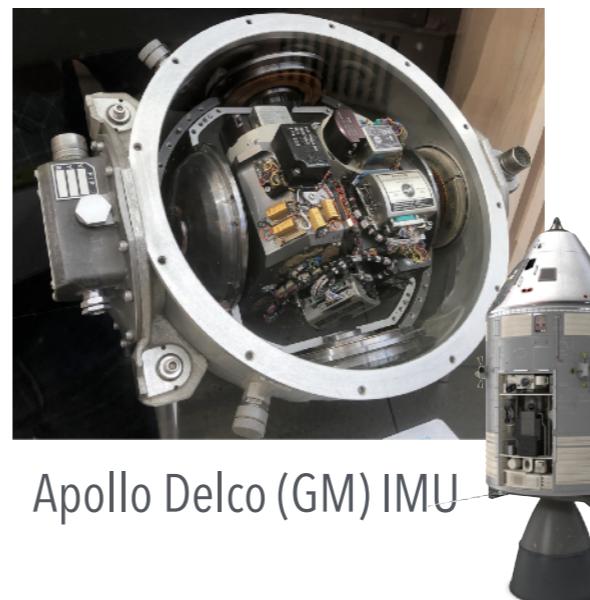


**Egomotion estimation:** the process of estimating the motion of a rigid body using measurements from sensors attached to the body

- ▶ Methods of egomotion estimation:
  1. exteroceptive measurements  
*observing landmarks with known location*
  2. interoceptive measurements  
*integrating rates to infer motion through ‘dead’ reckoning*



Charles Lindbergh used dead reckoning to cross the Atlantic solo in 1927



Apollo Delco (GM) IMU



Concorde Inertial Measurement Unit

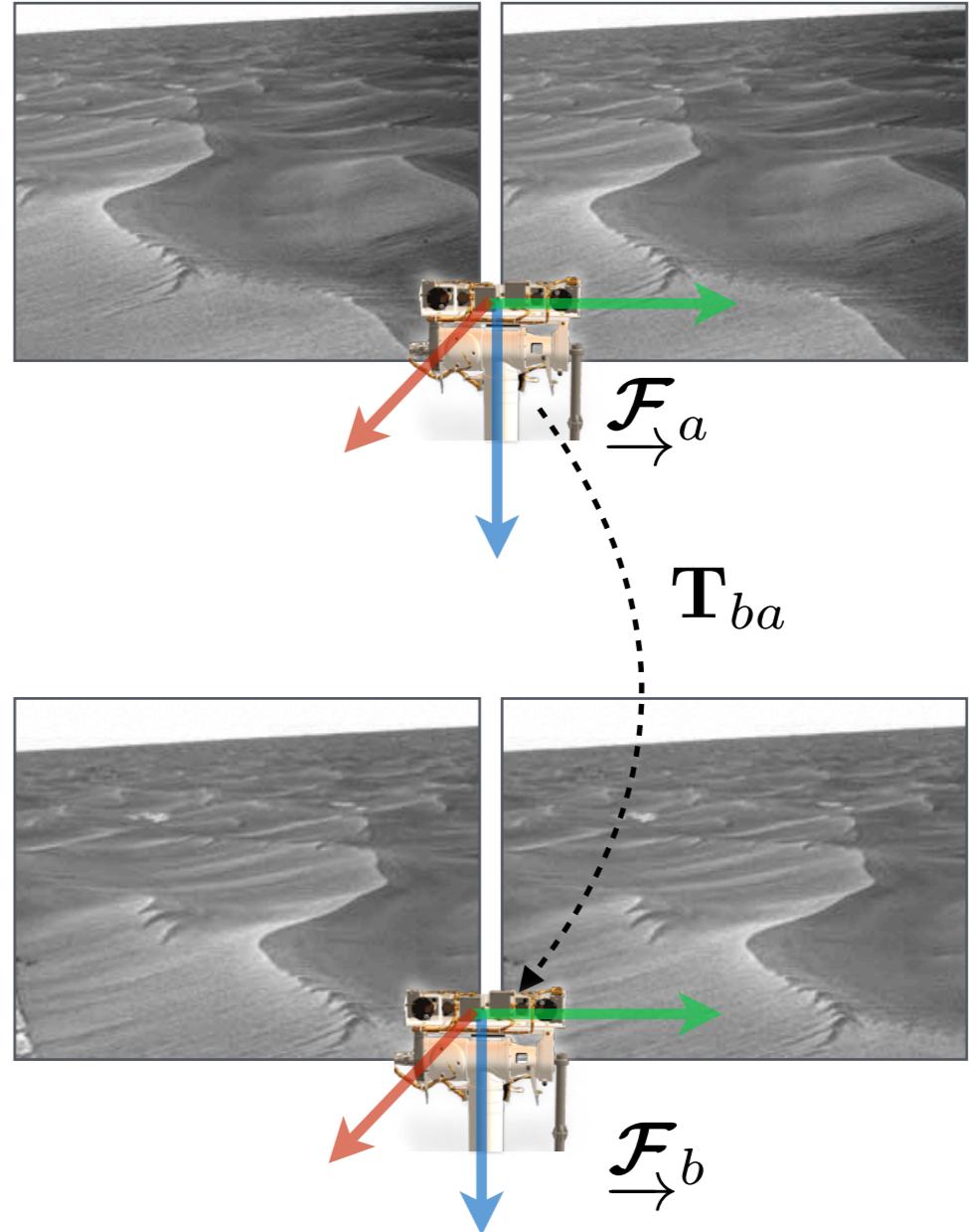
# Visual Egomotion Estimation

or Visual *Odometry*



Hans Moravec

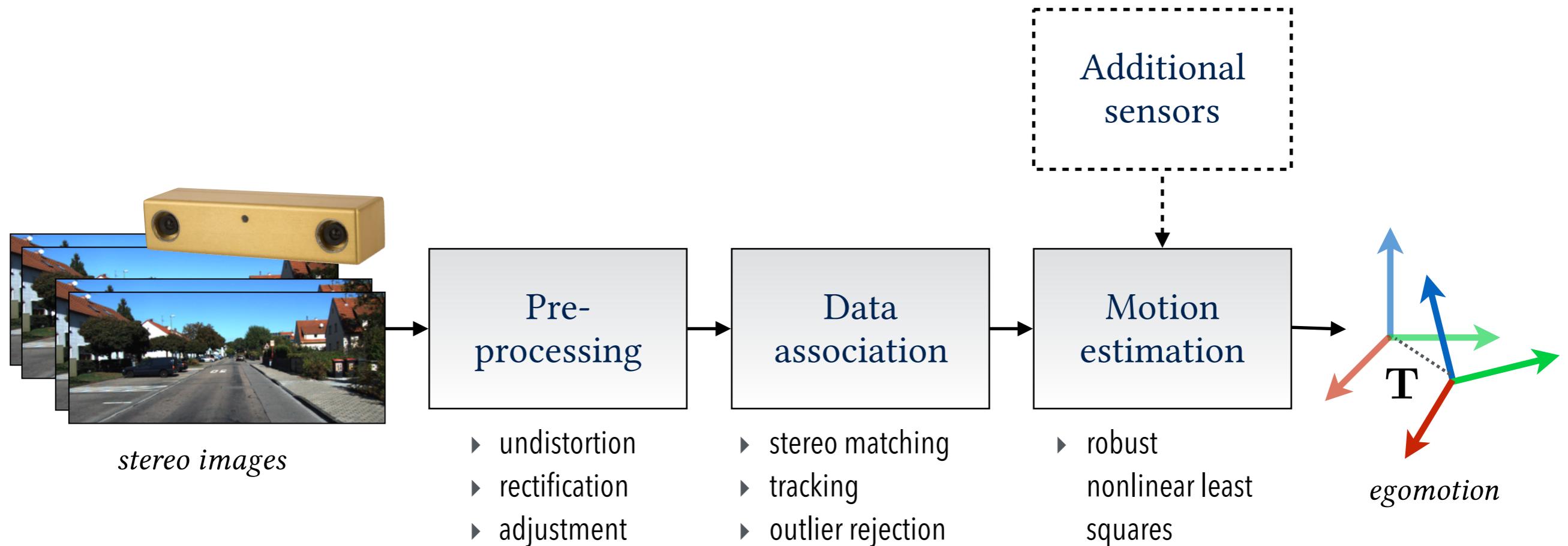
MERs



Scaramuzza and Fraundorfer, "Visual Odometry [Tutorial]," **IEEE Robot. Automat. Mag.** (2011)  
Moravec, "Obstacle avoidance and navigation in the real world by a seeing robot rover," **Ph.D. Thesis** (1980)

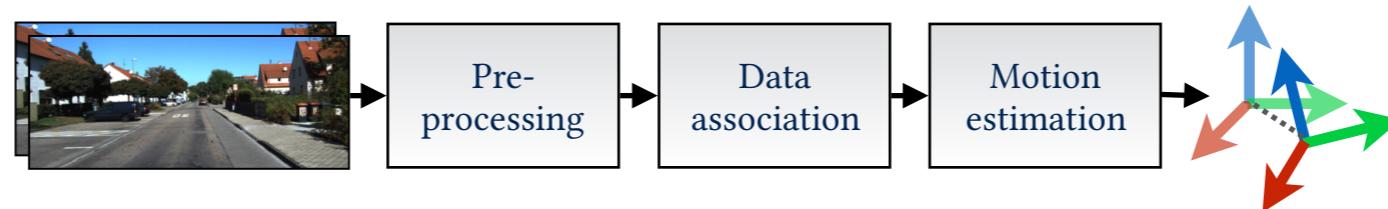


# Visual Egomotion Pipeline

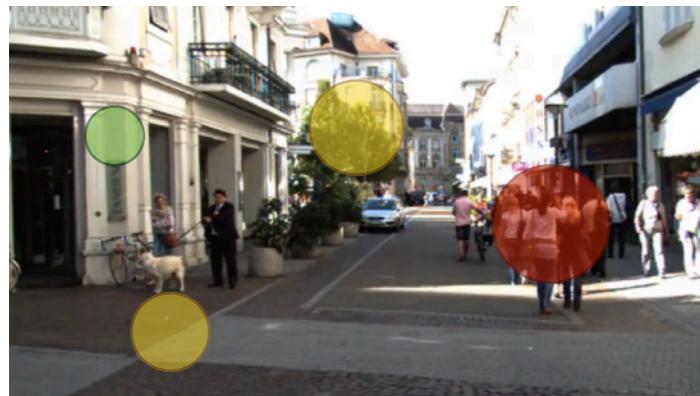


# Visual Egomotion Pipelines

Some Downsides...



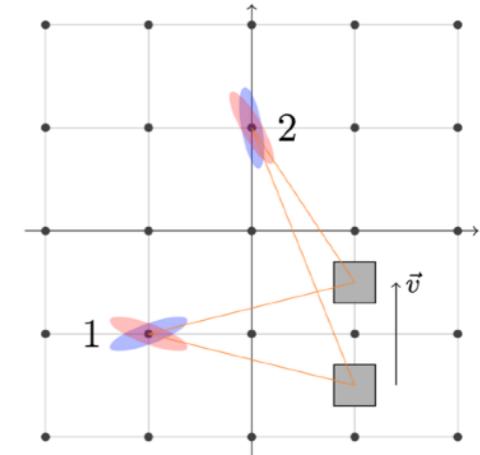
**Basic Uncertainty Quantification**  
(e.g., homoscedastic isotropic uncertainty)



**Low Information Usage**  
(e.g., point features, regions of high gradients)



**Prone to Bias**  
(e.g., imprecise calibration, uncertainty propagation)

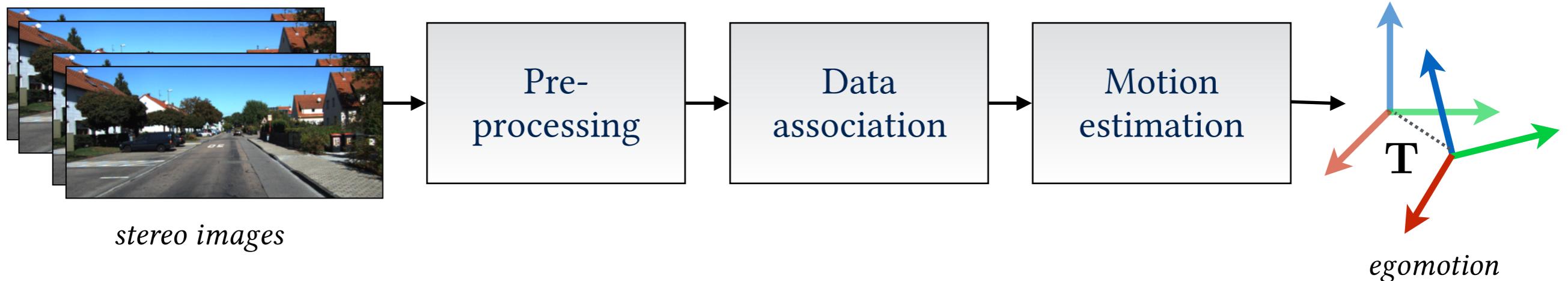


Peretroukhin, Kelly, and Barfoot, "Optimizing Camera Perspective for Stereo Visual Odometry," **CRV** (2014)



# Data-Driven Learning

## A Panacea for Vision-based Autonomy?

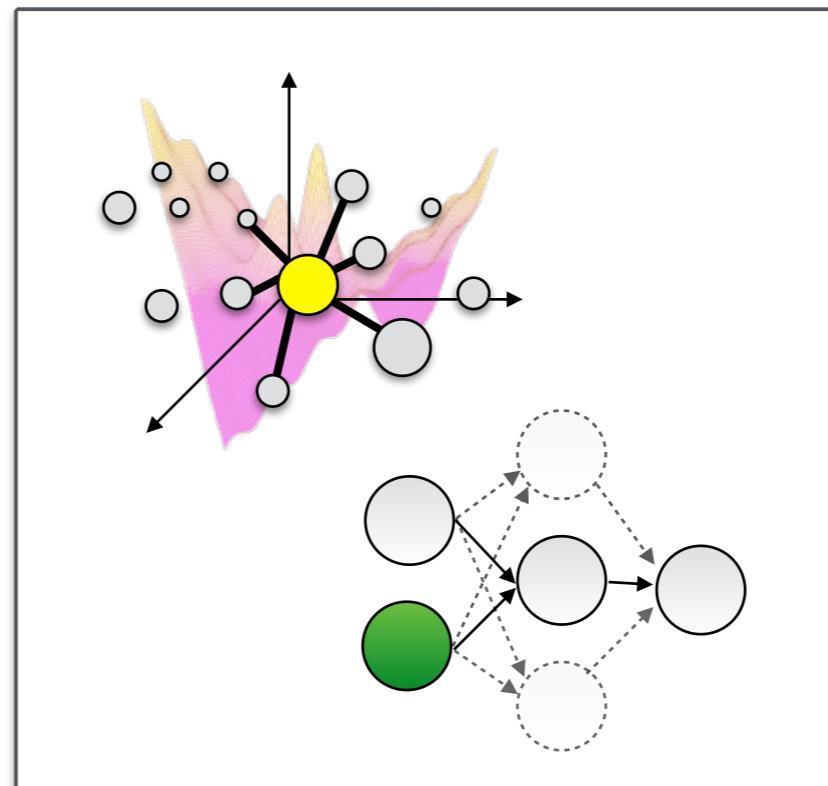


# Data-Driven Learning

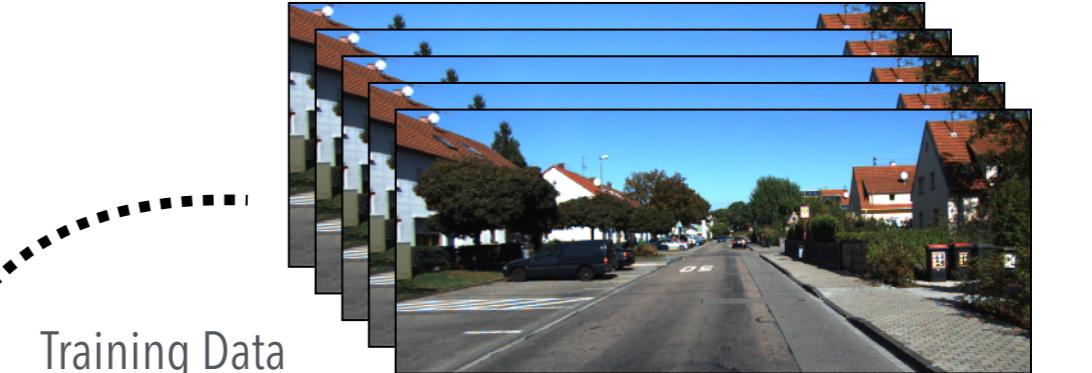
## A Panacea for Vision-based Autonomy?



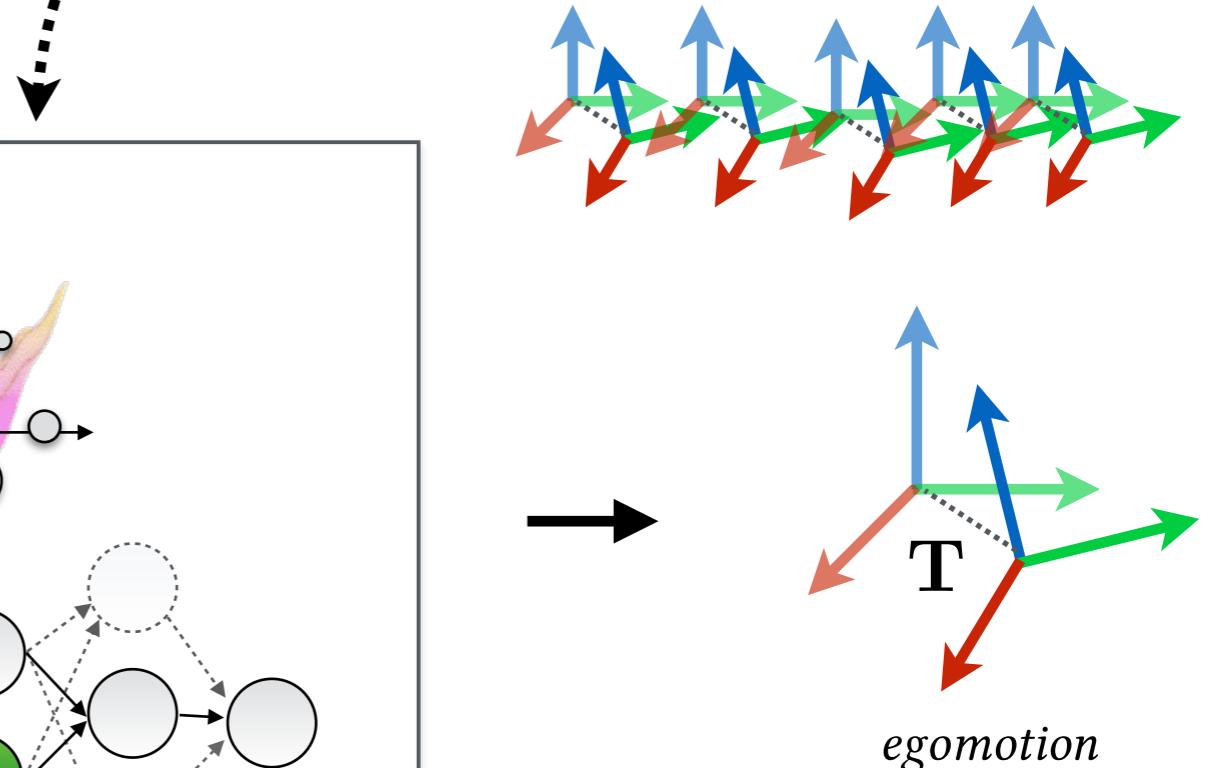
*stereo images*



Learned Model



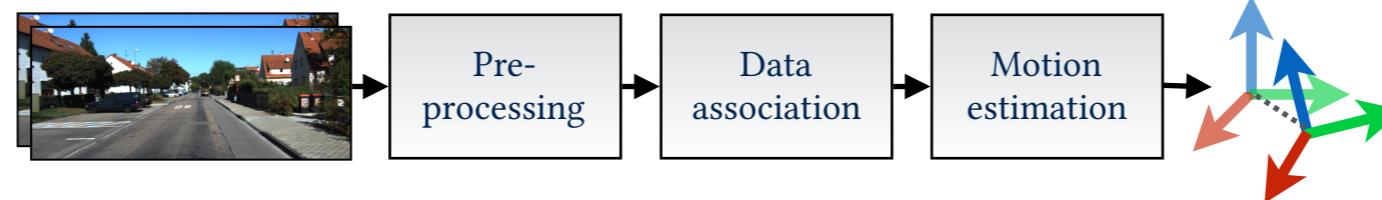
Training Data



*egomotion*

Wang et al., "DeepVO: Towards end-to-end visual odometry with deep Recurrent Convolutional Neural Networks," **ICRA** (2017)  
Zhou et al, "Unsupervised Learning of Depth and Ego-Motion from Video," **CVPR** (2017)

# Benefits of Classical Pipelines



✓ Interpretable & decomposable

✓ Efficient

✓ Probabilistic

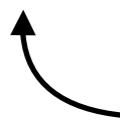
Zhou et al. "Does computer vision matter for action?",  
**Science Robotics** (2019)

Short answer: **Yes!** intermediate 'blocks' improve generalization

✓ Accurate

KITTI Odometry Benchmark Leaderboard (March 2020)

|   | Method   | Setting | Code | Translation | Rotation                     | Runtime | Environment               | Compare                  |
|---|--|---------|------|-------------|------------------------------|---------|---------------------------|--------------------------|
| 1 | V-LOAM   | ROS     |      | 0.55 %      | 0.0013 [deg/m]               | 0.1 s   | 2 cores @ 2.5 Ghz (C/C++) | <input type="checkbox"/> |
|   | J. Zhang and S. Singh: <a href="#">Visual-lidar Odometry and Mapping: Low drift, Robust, and Fast.</a> IEEE International Conference on Robotics and Automation (ICRA) 2015.   |         |      |             | <b>Lidar-based Egomotion</b> |         |                           |                          |
| 2 | LOAM   | ROS     |      | 0.57 %      | 0.0013 [deg/m]               | 0.1 s   | 2 cores @ 2.5 Ghz (C/C++) | <input type="checkbox"/> |
|   | J. Zhang and S. Singh: <a href="#">LOAM: Lidar Odometry and Mapping in Real-time.</a> Robotics: Science and Systems Conference (RSS) 2014.                                     |         |      |             |                              |         |                           |                          |
| 3 | SOFT2  | ROS     |      | 0.65 %      | 0.0014 [deg/m]               | 0.1 s   | 2 cores @ 2.5 Ghz (C/C++) | <input type="checkbox"/> |
|   | I. Cvišić, J. Česić, I. Marković and I. Petrović: <a href="#">SOFT-SLAM: Computationally Efficient Stereo Visual SLAM for Autonomous UAVs.</a> Journal of Field Robotics 2017. |         |      |             |                              |         |                           |                          |

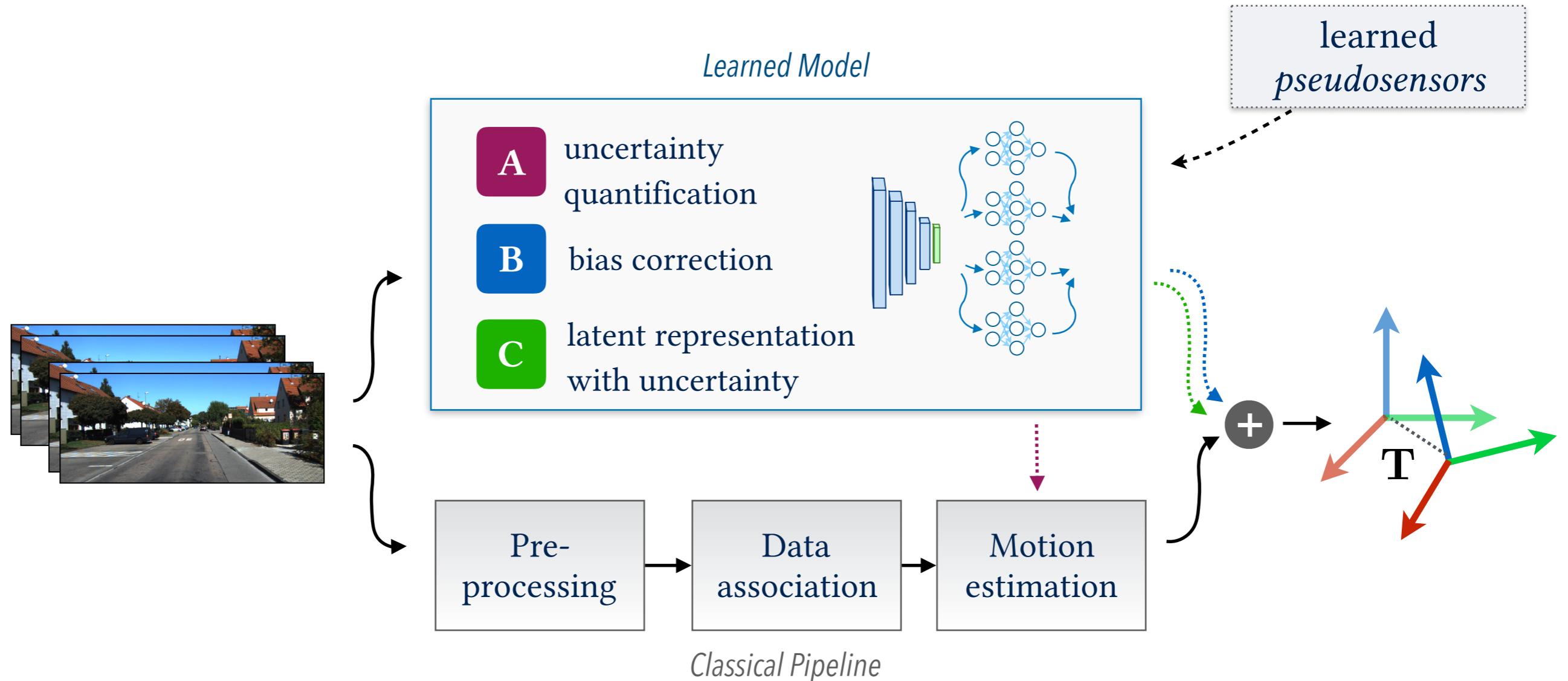


top performing *vision-only* odometry has *no learning*

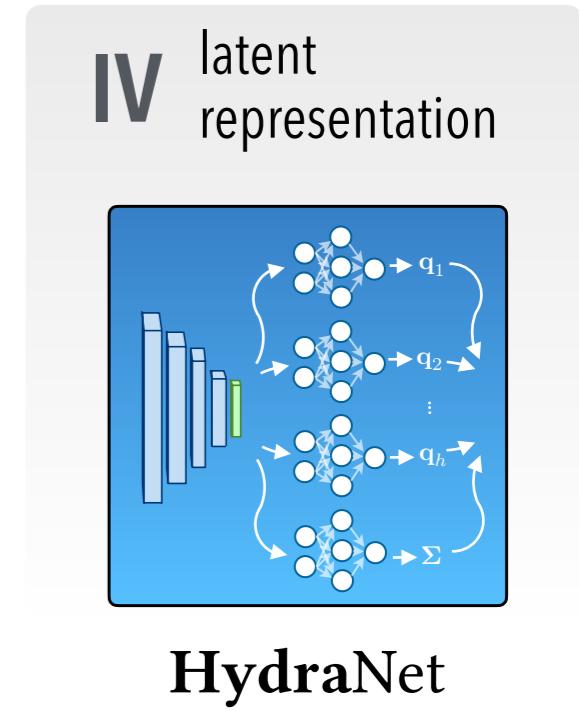
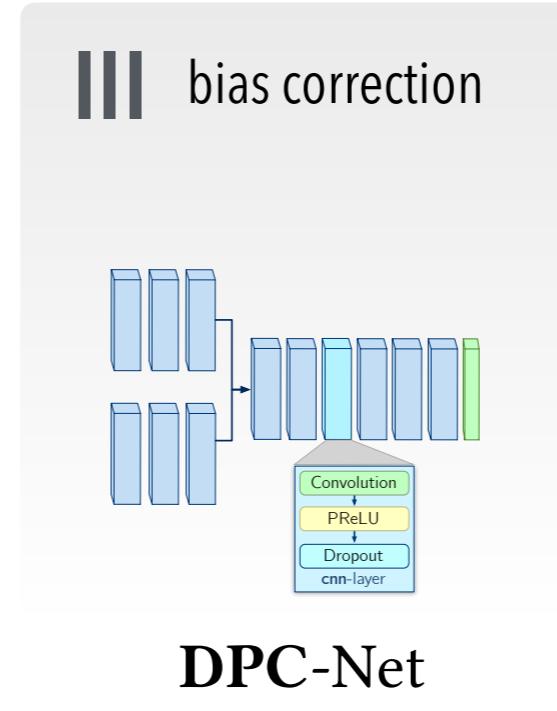
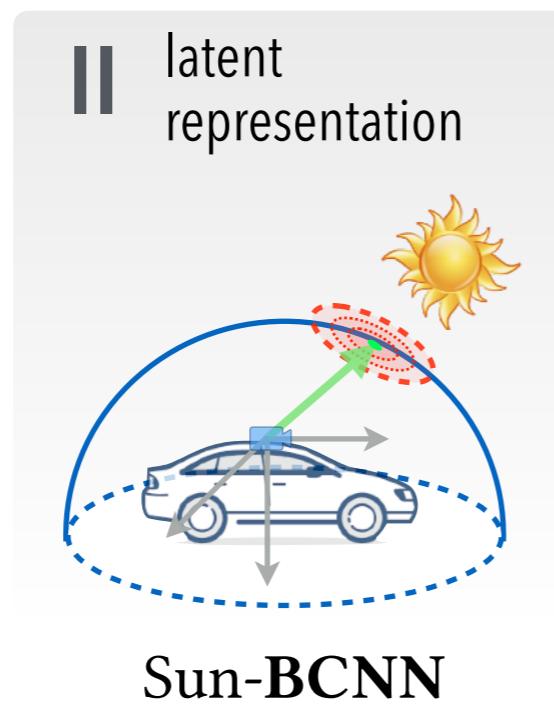
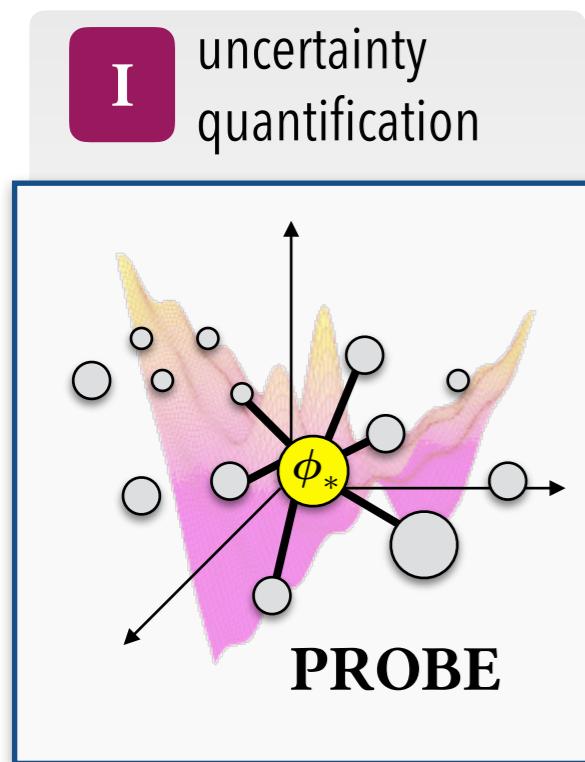


# My Doctoral Work

## Learned Improvements to the Visual Egomotion Pipeline



# Learned Improvements



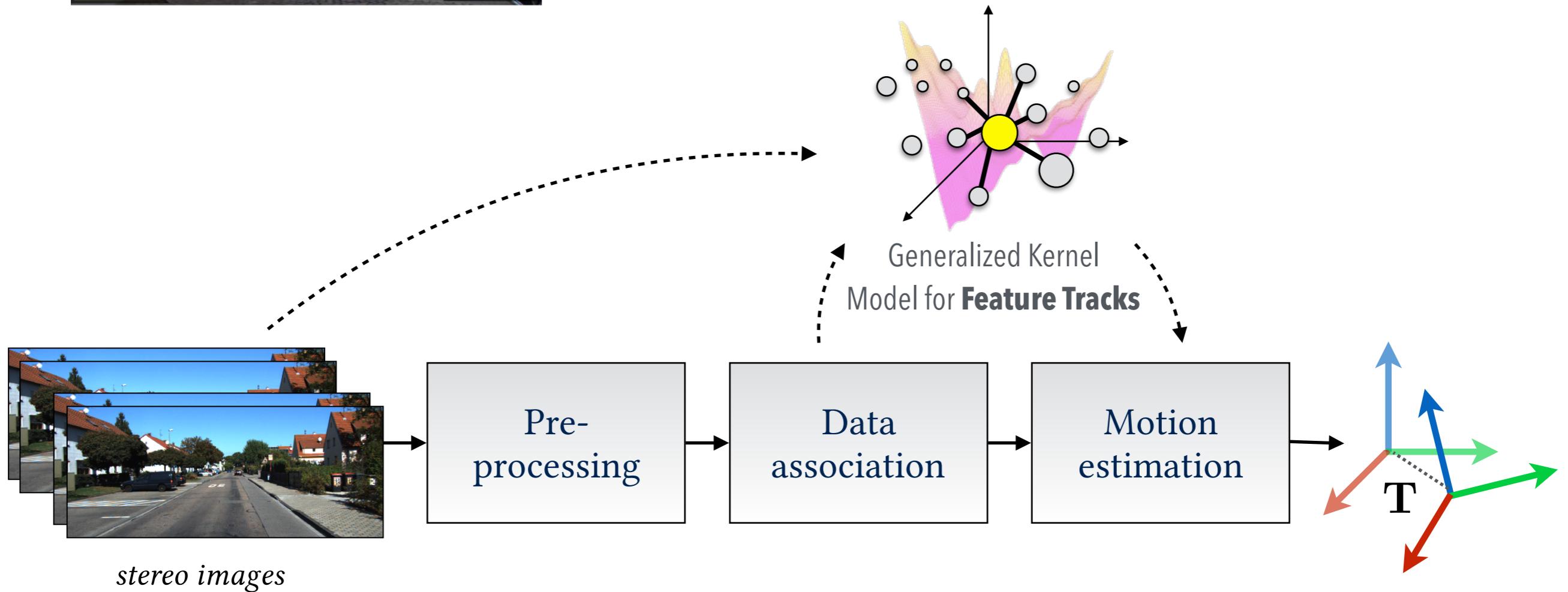
Predictive Robust  
Estimation

IROS 2015  
ICRA 2016

# Predictive Robust Estimation (PROBE)



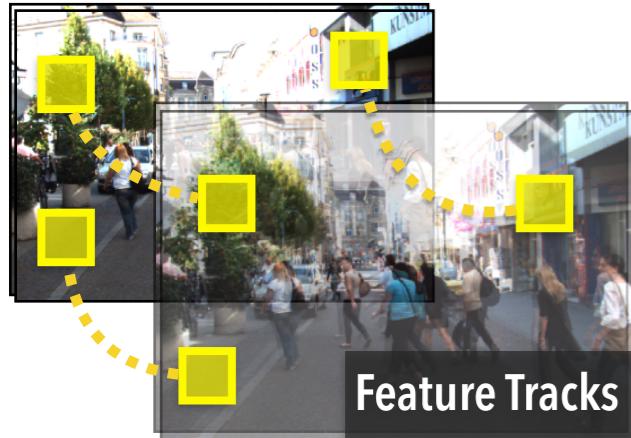
Not all feature matches contain the same information.  
Can we incorporate a **Bayesian model for stereo reprojection errors** into an egomotion pipeline?



W. R. Vega-Brown, M. Doniec, and N. G. Roy, "Nonparametric Bayesian inference on multivariate exponential families," **NeurIPS** (2014)  
V. Peretroukhin, W. Vega-Brown, N. Roy, and J. Kelly, "PROBE-GK: Predictive Robust Estimation using Generalized Kernels," **ICRA** (2016)

# Predictive Robust Estimation

Classical Visual Odometry

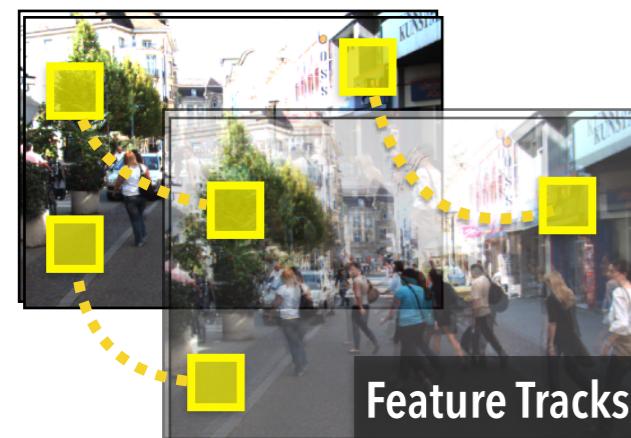


Errors modelled with **stationary** point uncertainty:  
 $\mathbf{e}_i \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$

MAP Estimator

$$\arg \min_{\mathcal{T} \in \text{SE}(3)} \sum_{i=1}^N \mathbf{e}_i^\top \mathbf{R}^{-1} \mathbf{e}_i$$

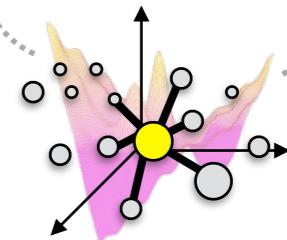
Visual Odometry with  
**PROBE-GK**



Errors  $\mathbf{e}_i$  modelled with **Bayesian**, kernel-based covariance density  
 $p(\mathbf{R}_i) = \text{IW}(\mathbf{R}; \Psi_i, \nu_i)$

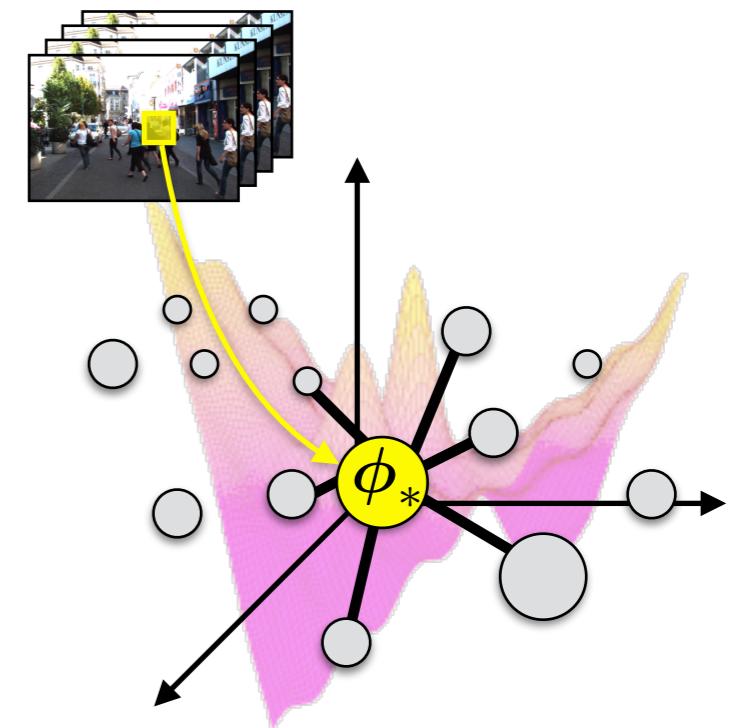
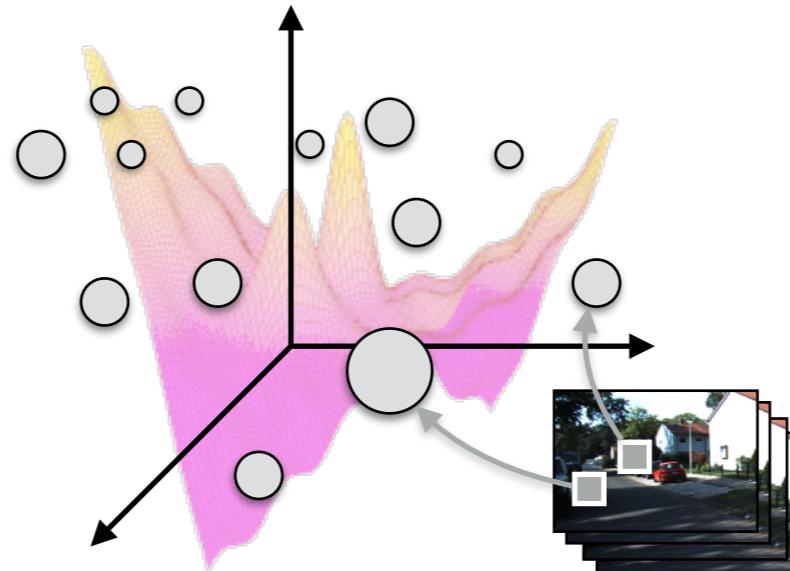
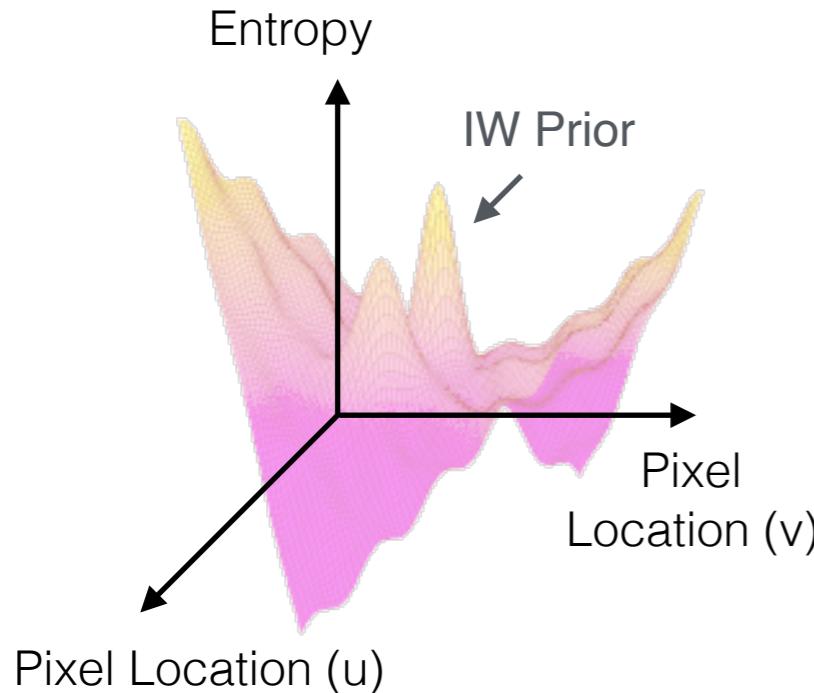
**Predictively Robust** Estimator

$$\arg \min_{\mathcal{T} \in \text{SE}(3)} \sum_{i=1}^N (\nu_i + 1) \log (1 + \mathbf{e}_i^\top \Psi_i^{-1} \mathbf{e}_i)$$



# Uncertainty Model

## Bayesian Predictive Model for Covariance



1. **Define:** Inverse-Wishart prior on covariance matrices at all locations within prediction space  $\phi$
3. b) Marginalize out the covariance, solve robust cost.

2. **Train:** Populate space with training data (empirical errors). Ground truth not required.

3. **Test:** a) Compute Inverse-Wishart posterior using technique of Generalized Kernels.

**Predictively  
Robust Cost**

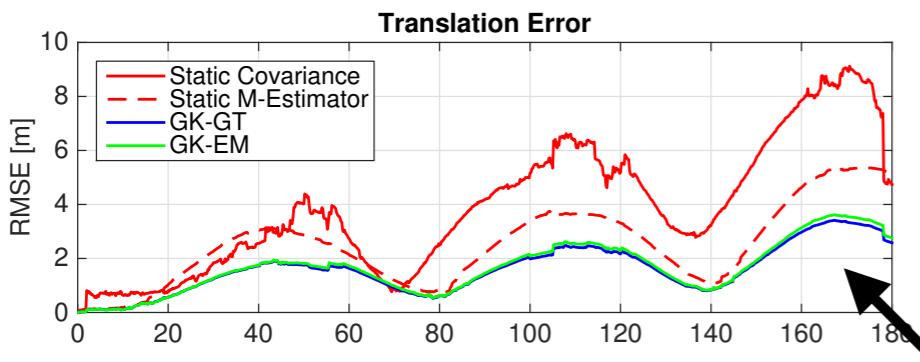
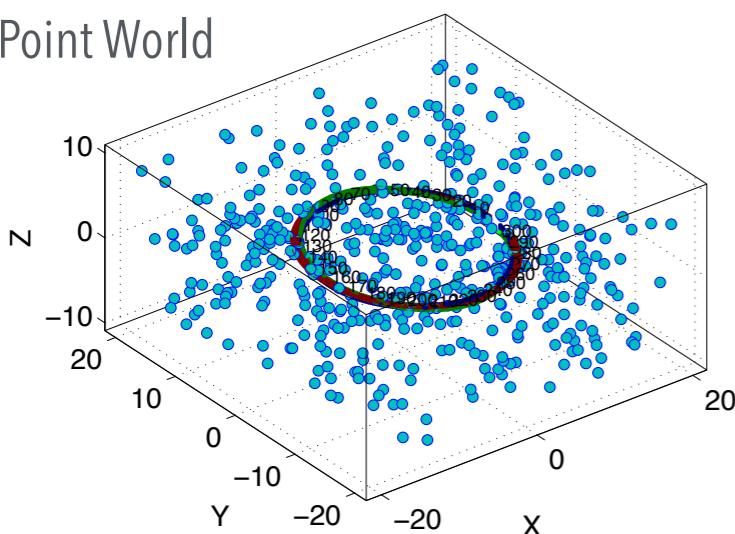
$$\arg \min_{\mathcal{T} \in \text{SE}(3)} \sum_{i=1}^N (\nu_i + 1) \log (1 + e_i^\top \Psi_i^{-1} e_i)$$

# Using PROBE to Improve VO

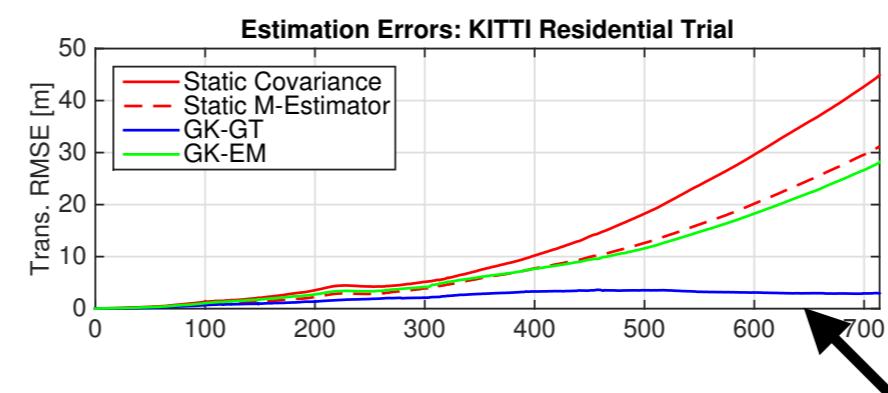
Synthetic, KITTI and UTIAS Experiments

## Synthetic

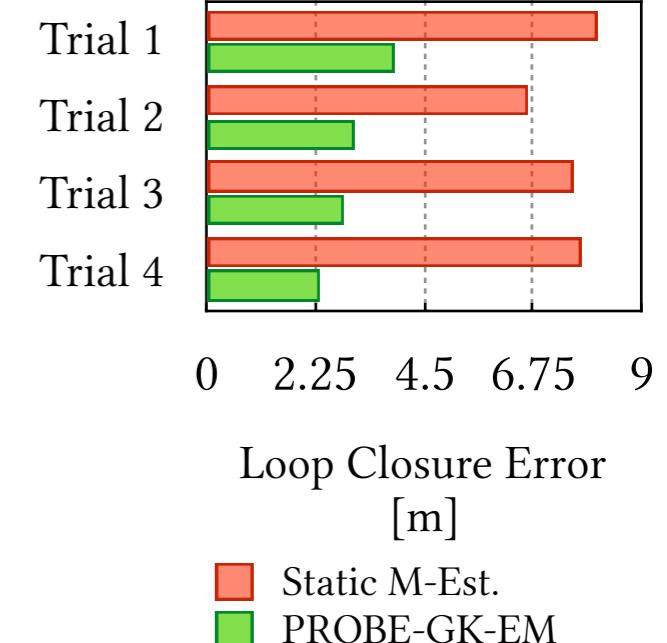
Point World



## KITI Odometry Dataset

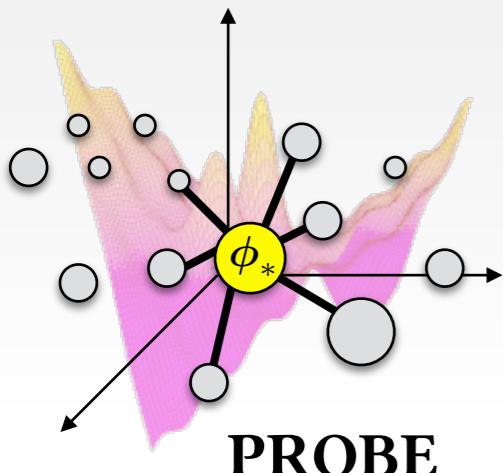


## UTIAS Mars Dome



# Learned Improvements

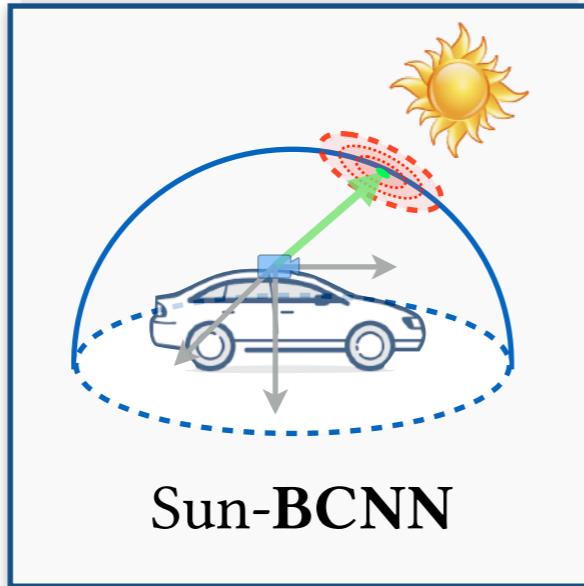
I uncertainty quantification



PROBE  
Predictive Robust  
Estimation

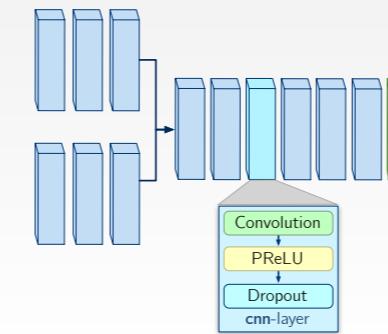
IROS 2015  
ICRA 2016

II latent representation



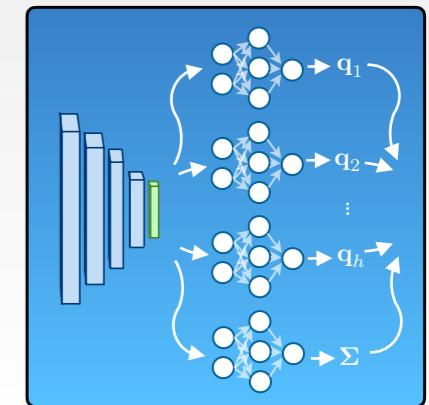
Learning Sun  
Direction with  
Uncertainty  
ISER 2017, ICRA 2017,  
IJRR 2018

III bias correction



DPC-Net

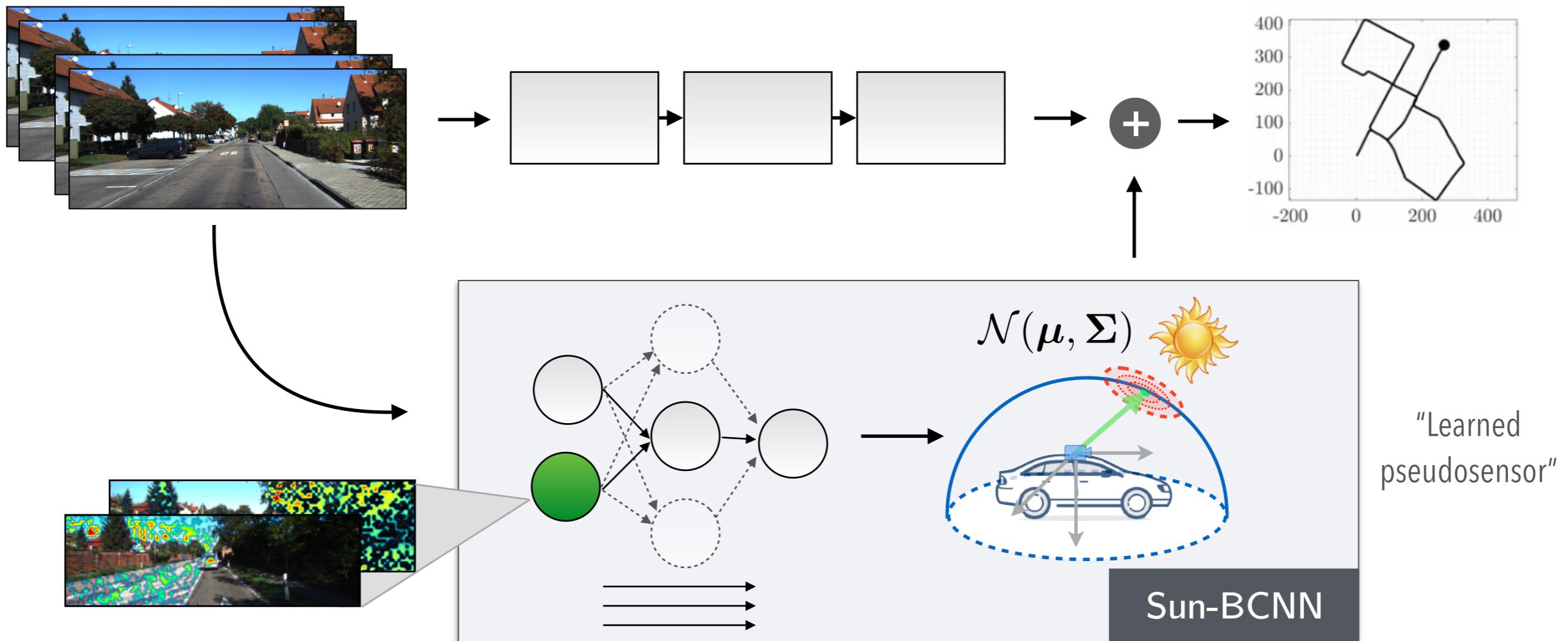
IV latent representation



HydraNet

# Sun-BCNN: A Virtual Sun Sensor

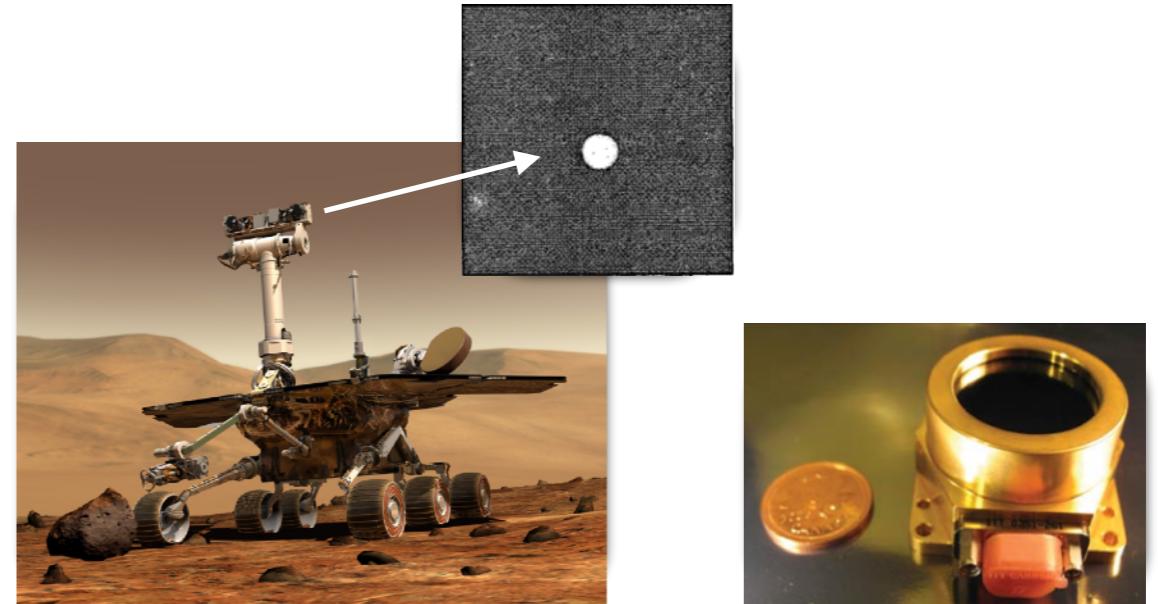
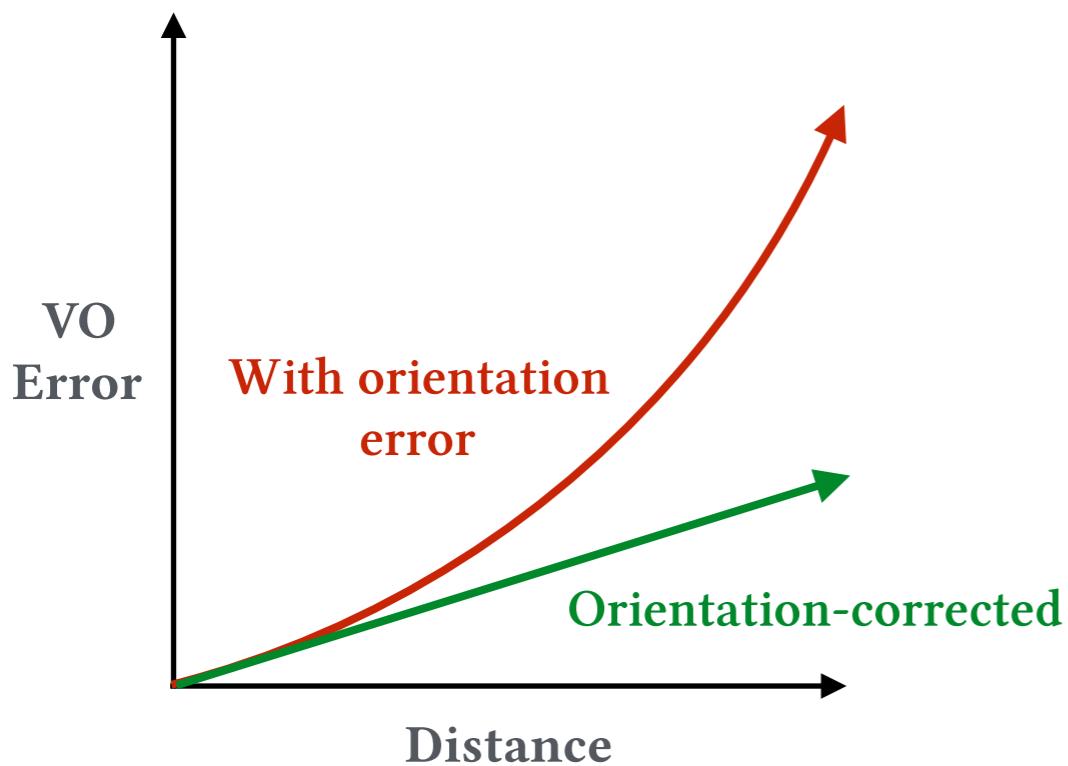
Can we use deep learning to infer the direction of the sun (with *uncertainty*) and use it to improve egomotion estimates?



Peretroukhin, Clement, and Kelly, "Reducing Drift in Visual Odometry by Inferring Sun Direction Using a Bayesian Convolutional Neural Network," **ICRA** (2017)  
Peretroukhin, Clement, and Kelly, "Inferring sun direction to improve visual odometry: A deep learning approach," **IJRR** (2018)

# Sun-aided Visual Odometry

Visual odometry is a dead-reckoning technique and suffers from **super-linear error growth**, largely due to accumulated orientation error



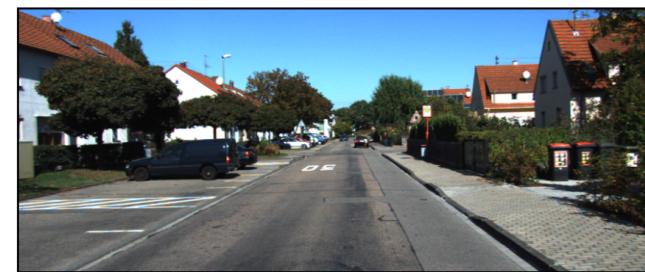
Specially oriented camera  
(e.g., MERs)

Specialized sun sensor

Drift can be reduced using **absolute orientation information** (e.g., observing the Sun)

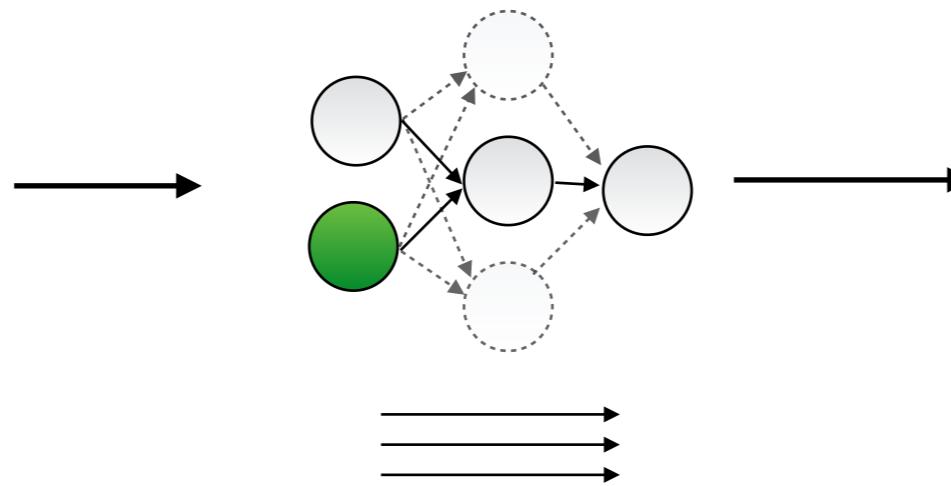
# Sun-BCNN

## A Bayesian CNN for Finding the Sun



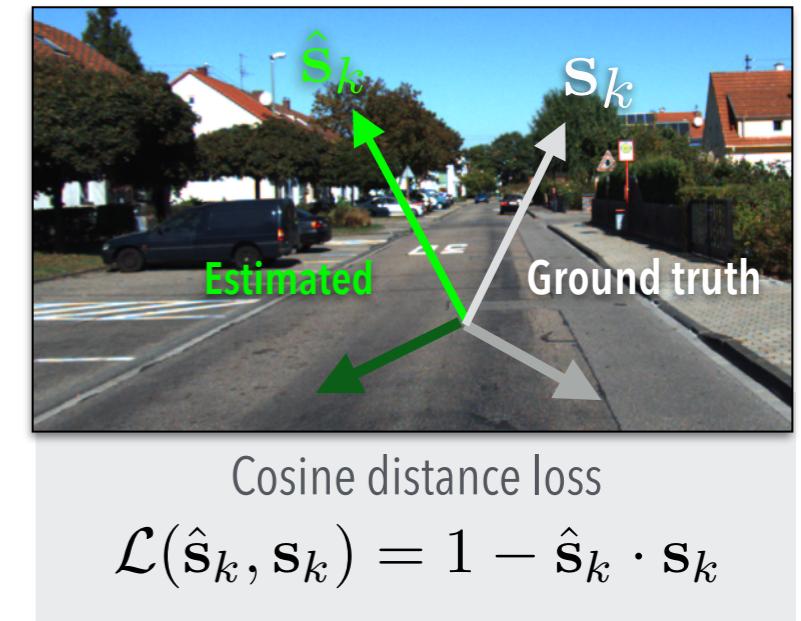
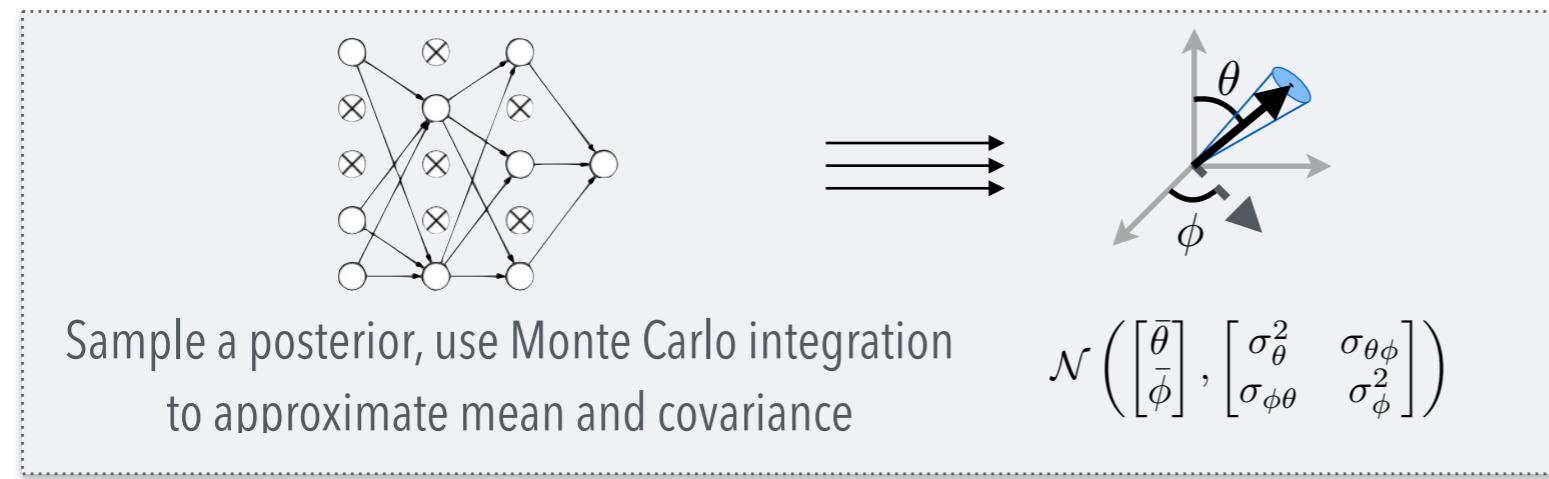
RGB image

Bayesian GoogLeNet



3D sun vector  
with uncertainty

'Monte Carlo' dropout

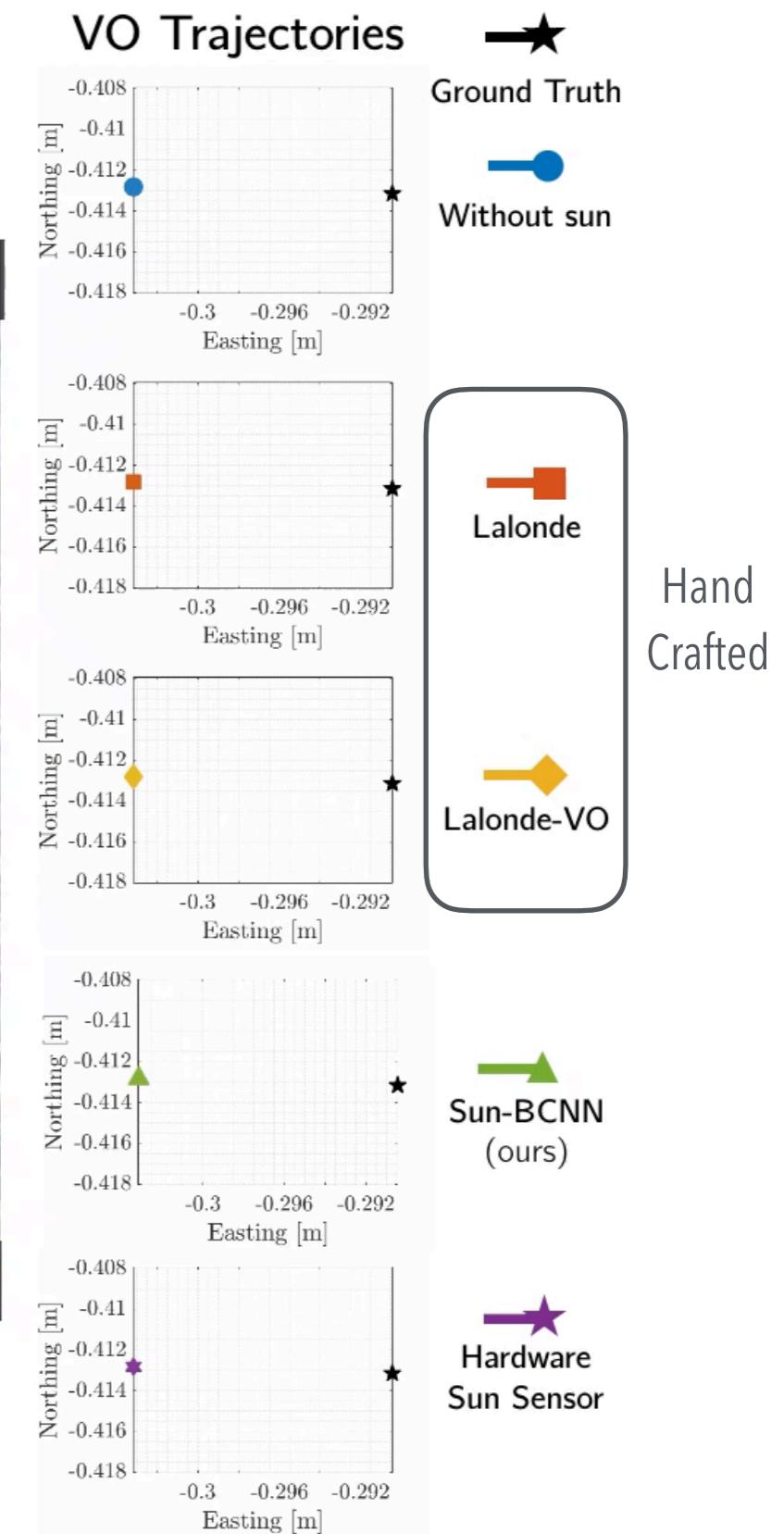
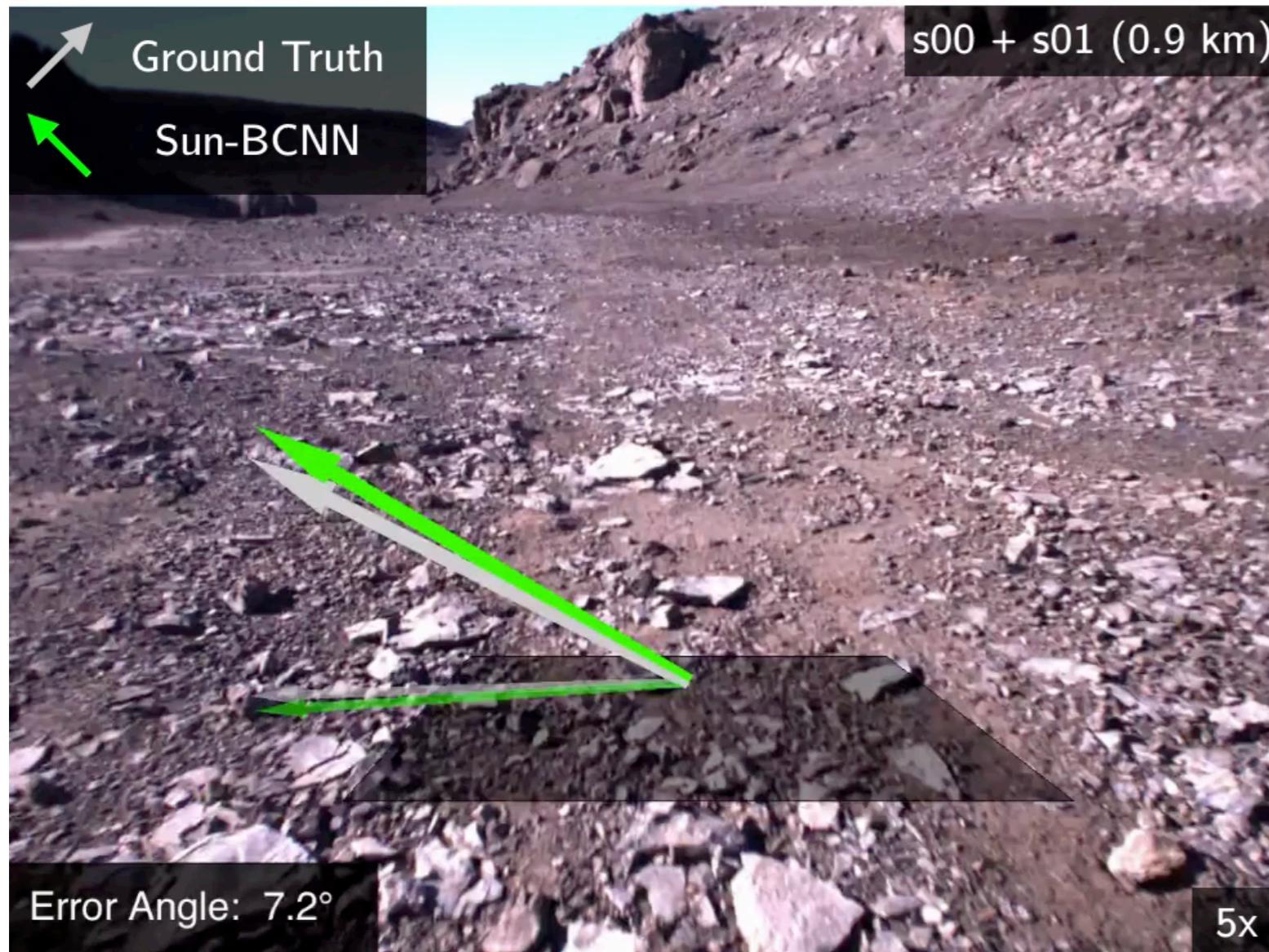


Gal and Ghahramani, "Dropout as a Bayesian Approximation: Representing Model Uncertainty in Deep Learning" [ICML\(2016\)](#)



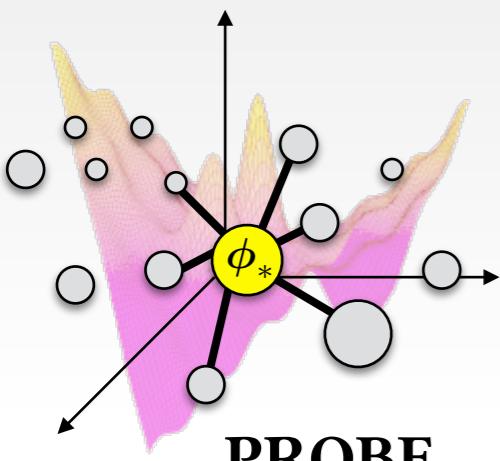
# Sun-BCNN Testing

## Devon Island



# Learned Improvements

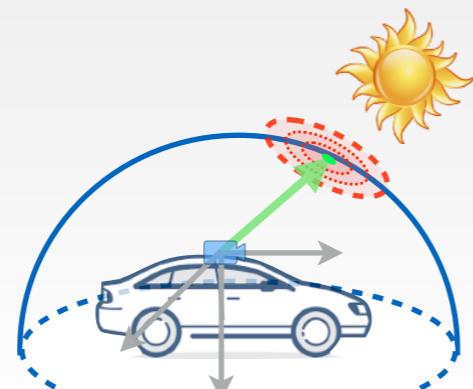
I uncertainty quantification



Predictive Robust  
Estimation

IROS 2015  
ICRA 2016

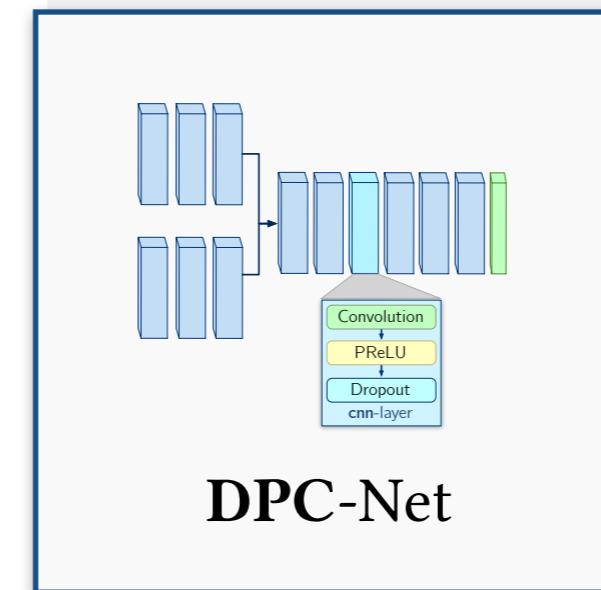
II latent representation



Learning Sun  
Direction with  
Uncertainty

ISER 2017, ICRA 2017,  
IJRR 2018

III bias correction

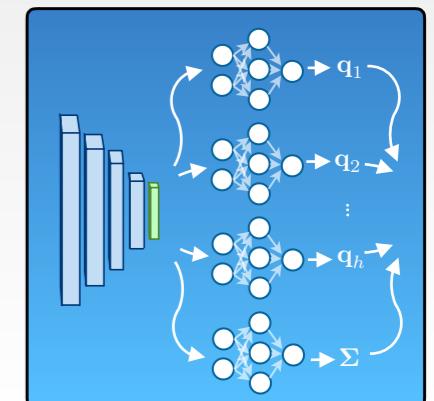


DPC-Net

Learning Estimator  
Bias through Deep  
Pose Correction

ICRA / RA-L 2018  
ICRA 2020

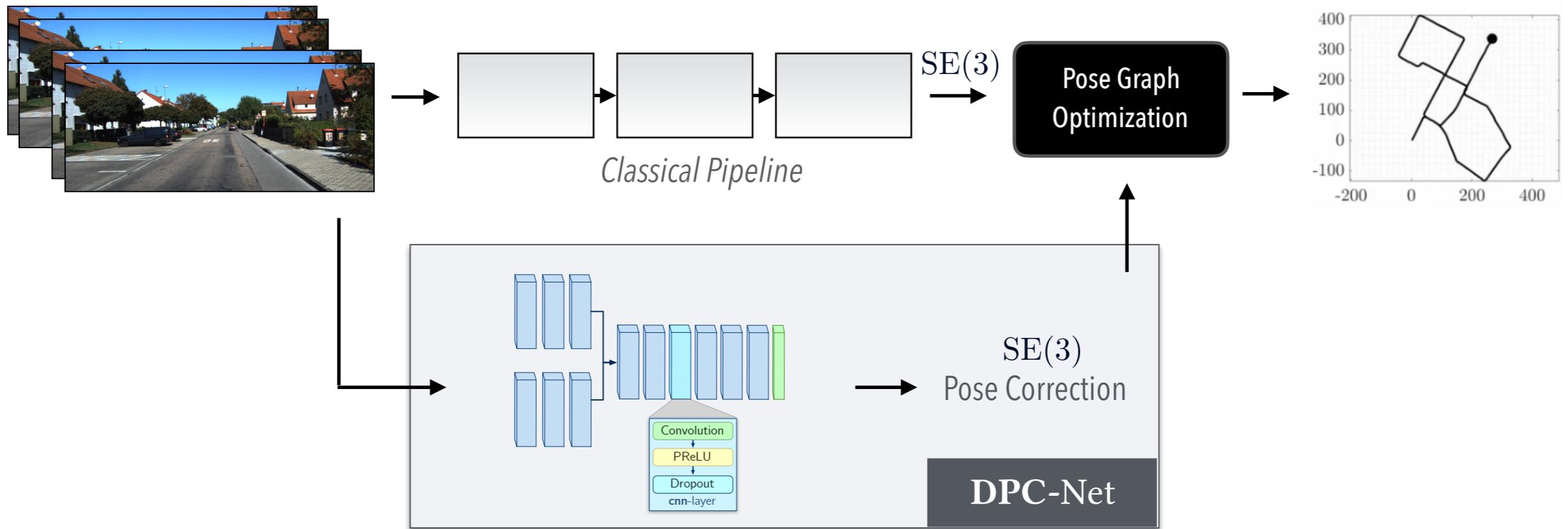
IV latent representation



HydraNet

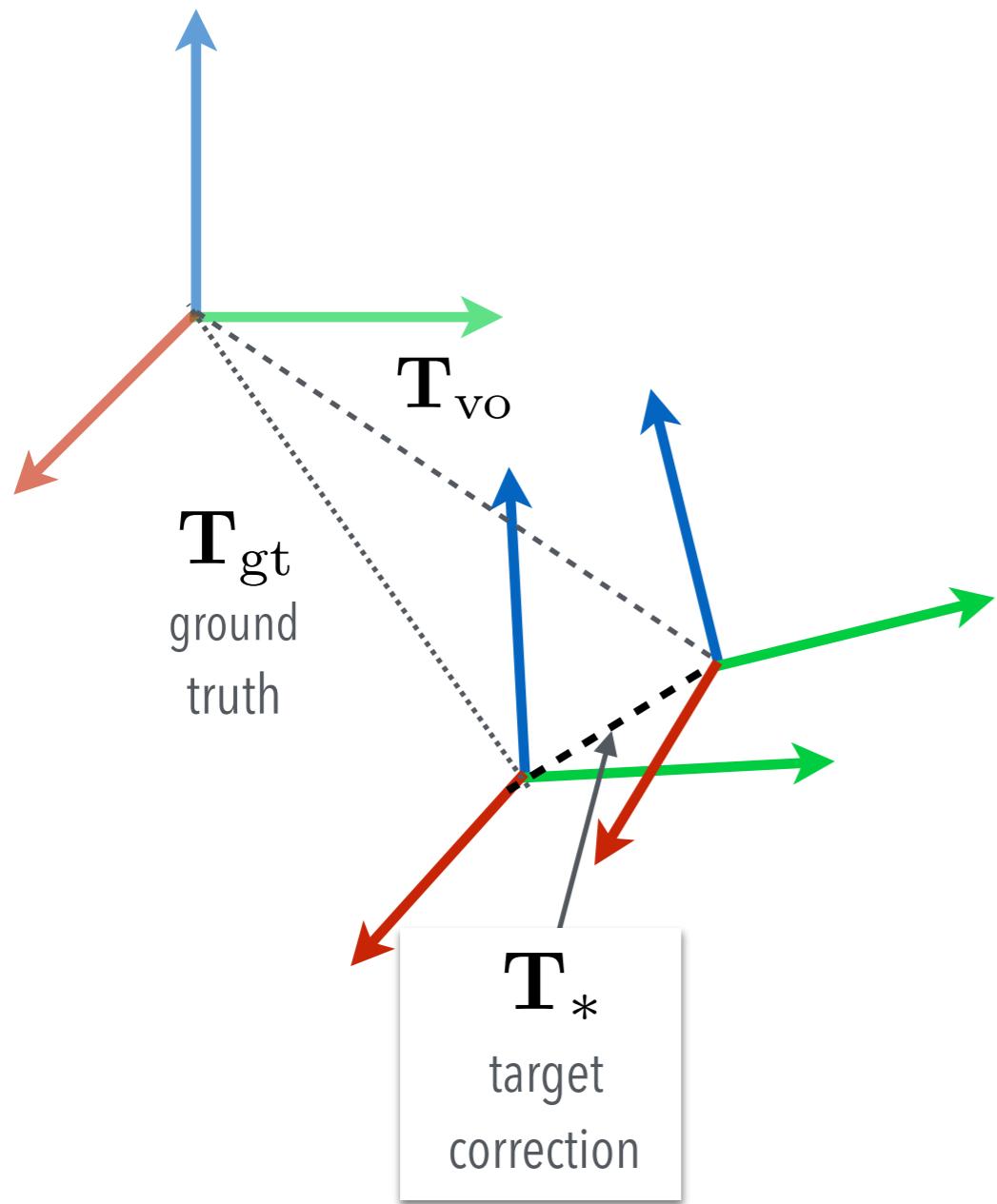
# Deep Pose Corrections

Can we generalize Sun-BCNN to learn **SE(3) pose residuals** to correct estimator bias?



Peretroukhin and Kelly, "DPC-Net: Deep Pose Correction for Visual Localization," **ICRA / RA-L** (2018)

# SE(3) Corrections for Visual Odometry



We learn SE(3) corrections  $T_*$  such that

$$T_{gt} = T_* T_{vo}$$

What do we correct for?

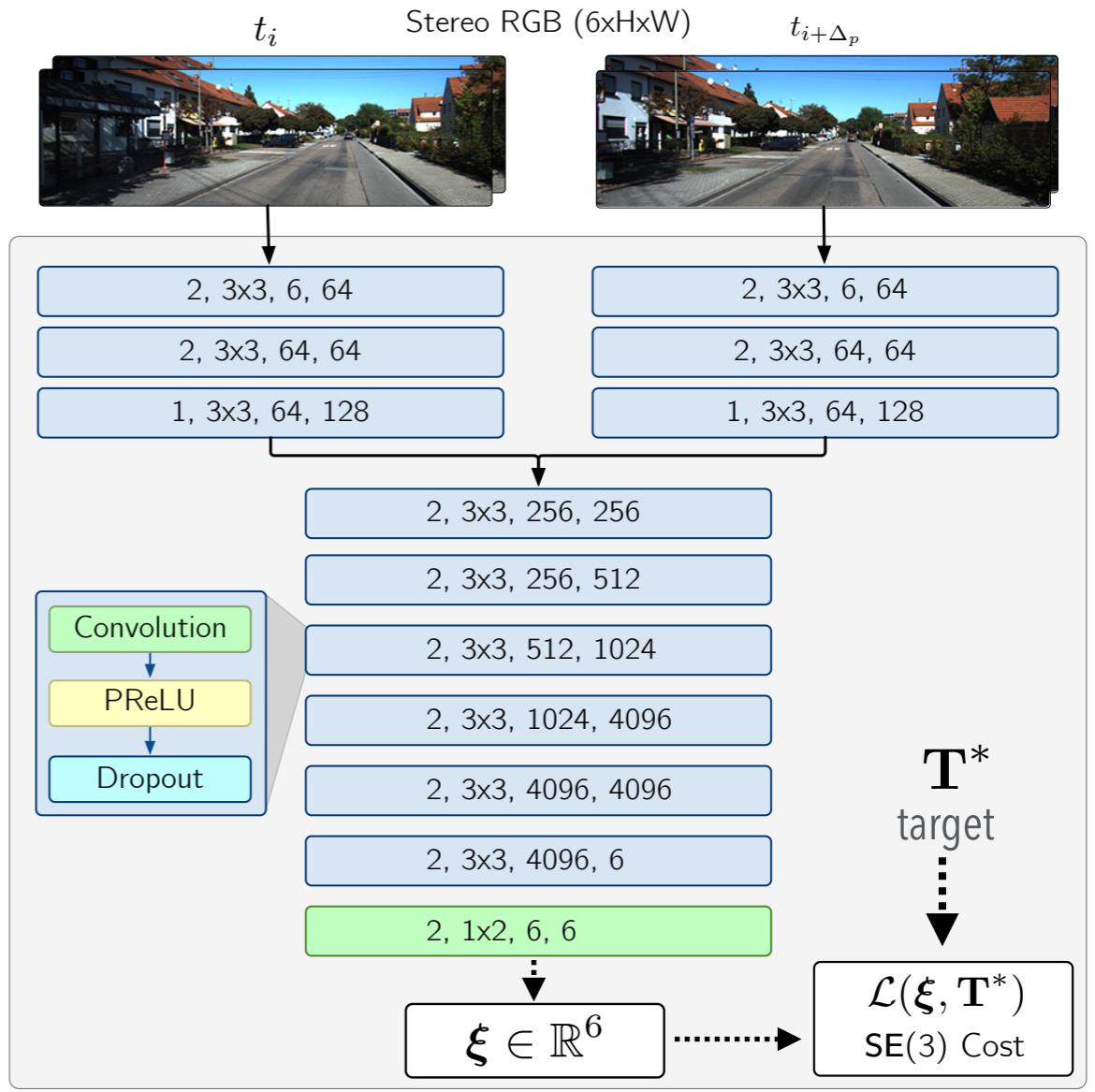


- ▶ **Estimator biases** (e.g., due to feature distribution or errors in stereo triangulation)
- ▶ Intrinsic / extrinsic **mis-calibration**
- ▶ **Poor feature tracking** due to blur, localized texture, non-Gaussian residuals

# DPC-Net

## Structure and Loss

- ▶ DPC-Net is composed of convolutional layers
- ▶ The network takes images as input and outputs an unconstrained vector in the tangent space of identity,  $\xi$
- ▶ Although the output is unconstrained vector, we store the target corrections,  $\mathbf{T}^*$ , in matrix form
- ▶ We derive a novel loss based on the the **geodesic distance**:



$$\mathcal{L}(\xi) = \frac{1}{2} g(\xi)^T \Sigma^{-1} g(\xi) \quad \text{where} \quad g(\xi) \triangleq \log \left( \exp (\xi^\wedge) \mathbf{T}^{*-1} \right)^\vee$$

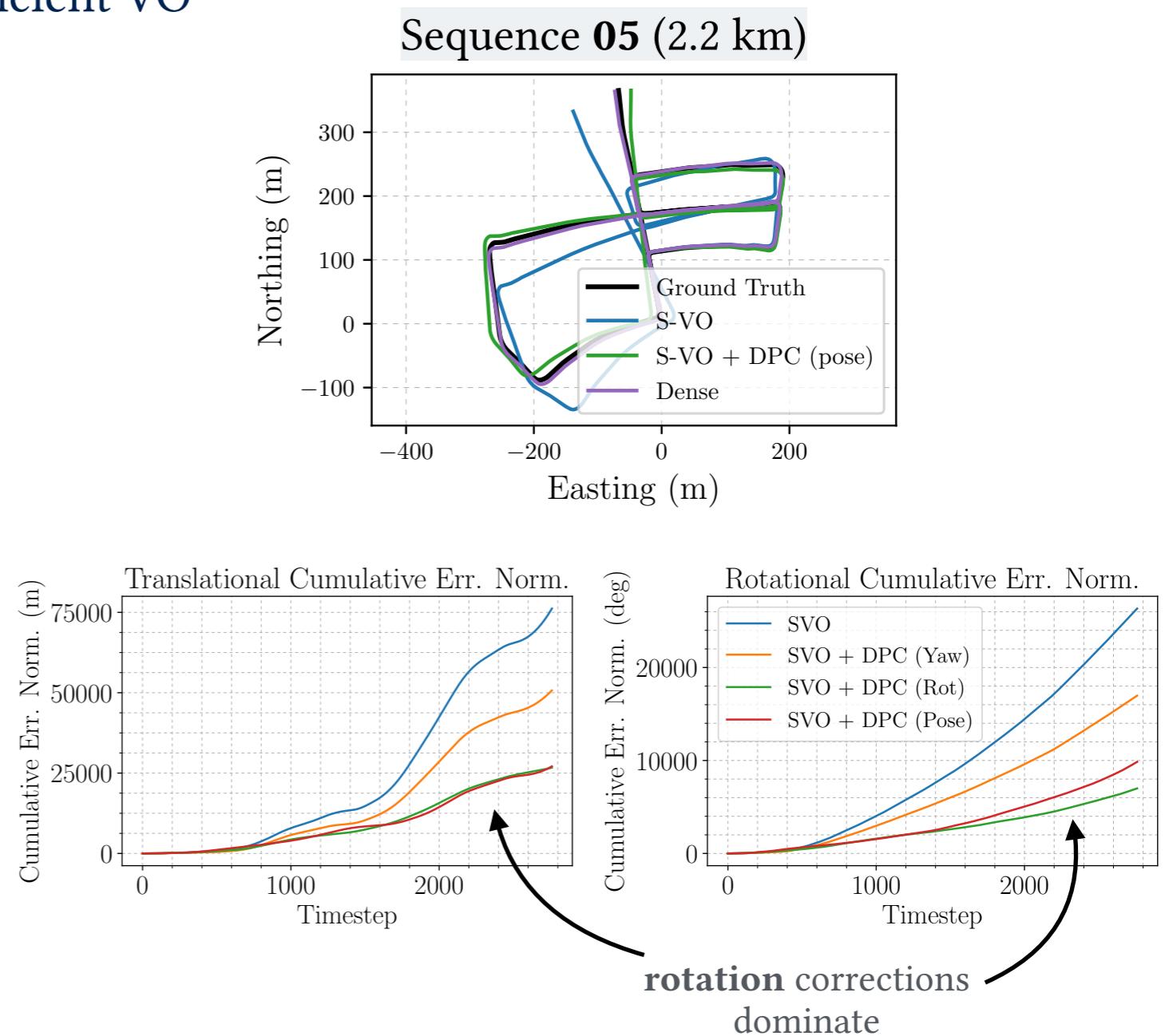
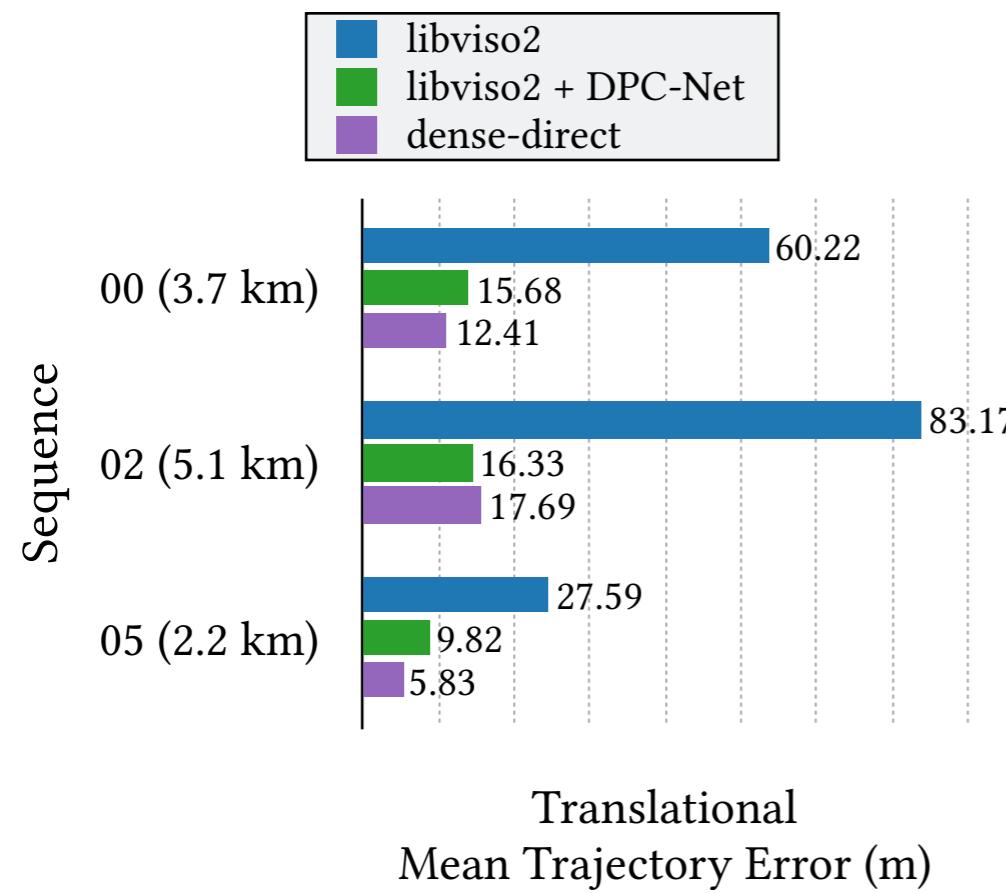
weighs rotation and translation terms  
based on training data



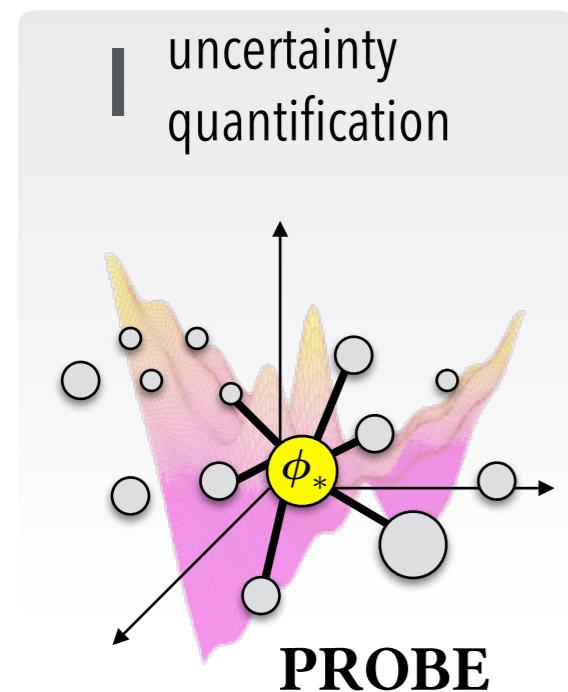
# DPC-Net | Correcting libviso2



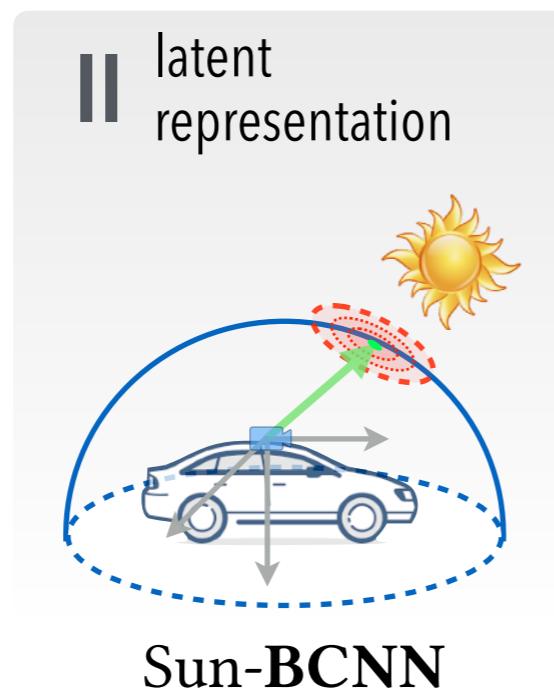
- We evaluated DPC-Net on the KITTI odometry benchmark and train it to correct an efficient VO estimator based on libviso2



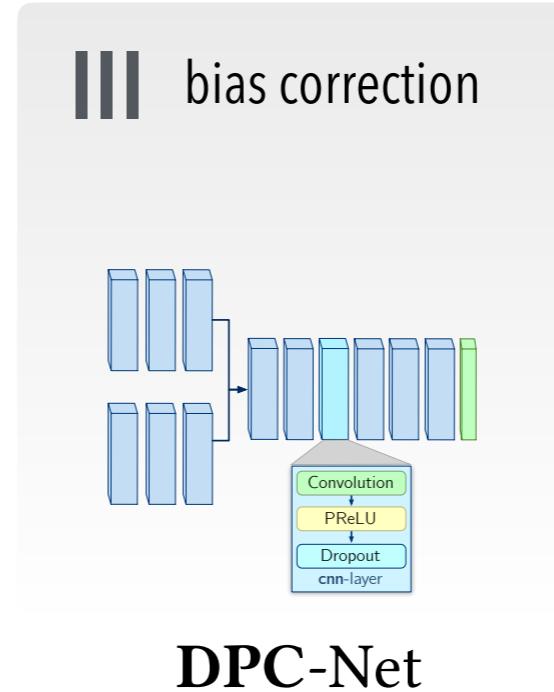
# Learned Improvements



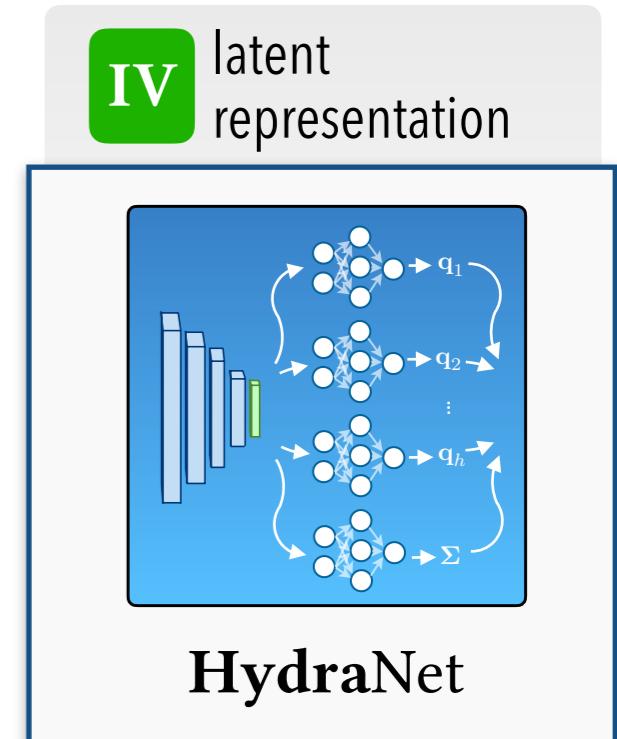
Predictive Robust Estimation  
IROS 2015  
ICRA 2016



Learning Sun Direction with Uncertainty  
ISER 2017, ICRA 2017, IJRR 2018



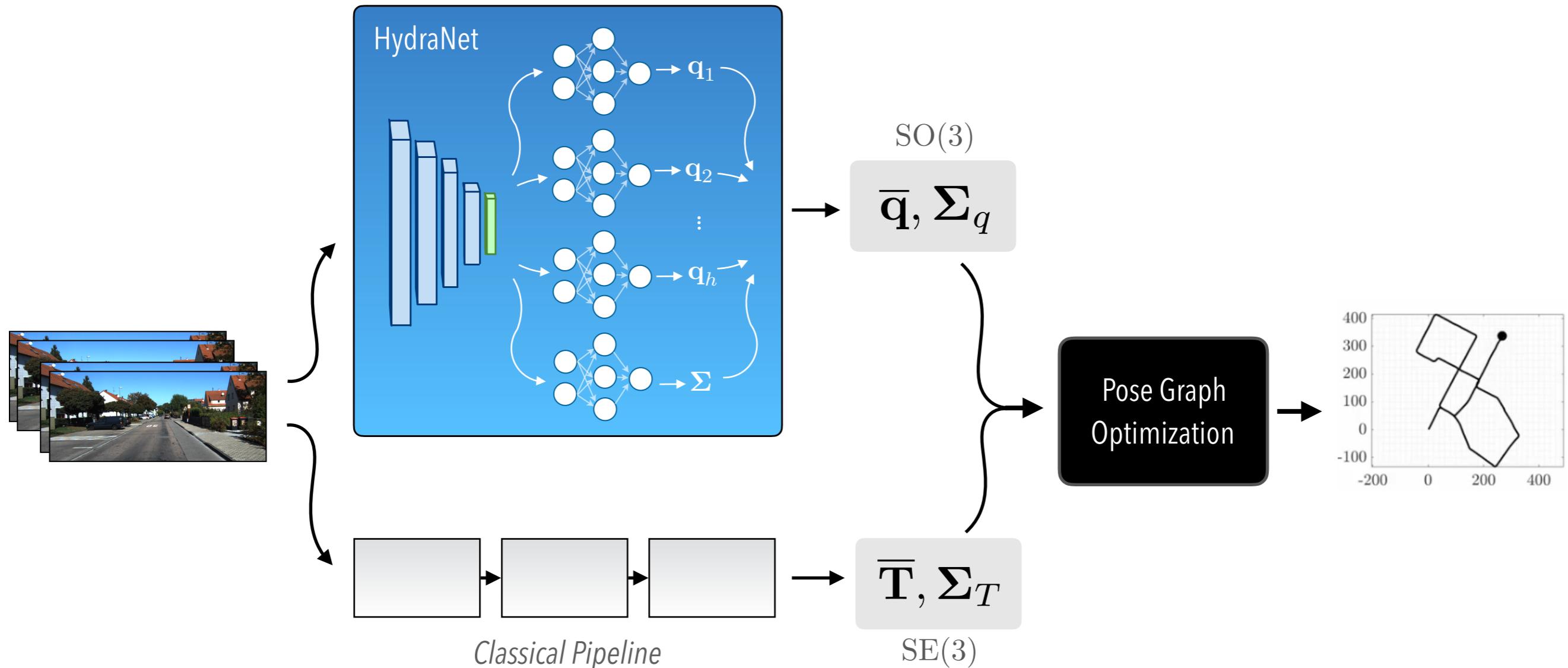
Learning Estimator Bias through Deep Pose Correction  
ICRA / RA-L 2018  
ICRA 2020



Learning Rotation With Uncertainty  
CVPR Workshops 2019

# Deep Rotation Regression with Uncertainty

Can learned estimates of camera rotation (with *uncertainty*) improve visual egomotion?



Kneip, Siegwart, and Pollefeys, "Finding the Exact Rotation between Two Images Independently of the Translation," **ECCV** (2012)  
Peretroukhin, Wagstaff and Kelly, "Deep Probabilistic Regression of Elements of  $SO(3)$ ," **CVPR, Workshop on Uncertainty and Robustness in Deep Visual Learning** (2019)



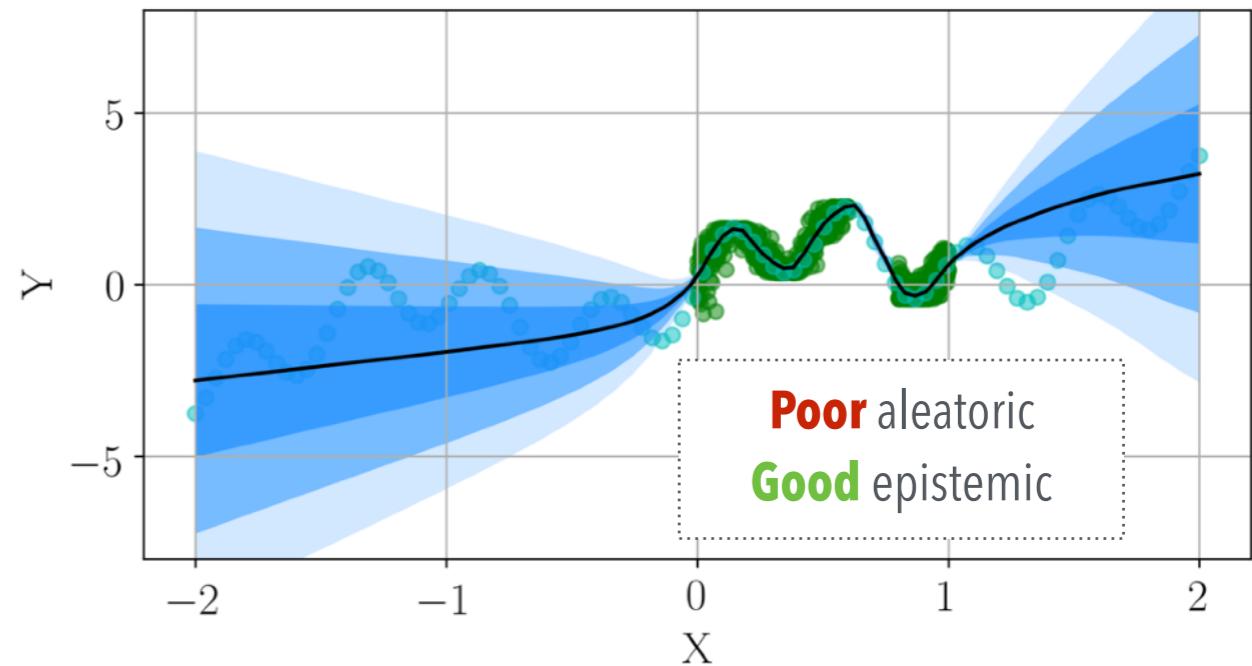
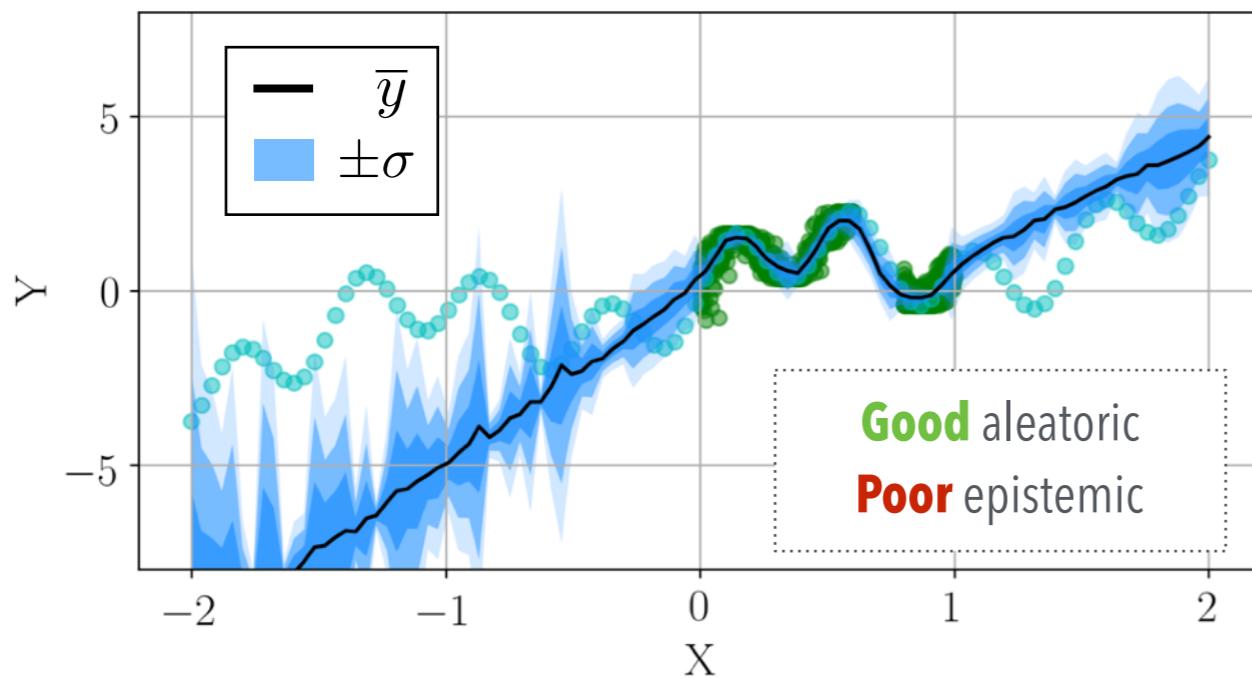
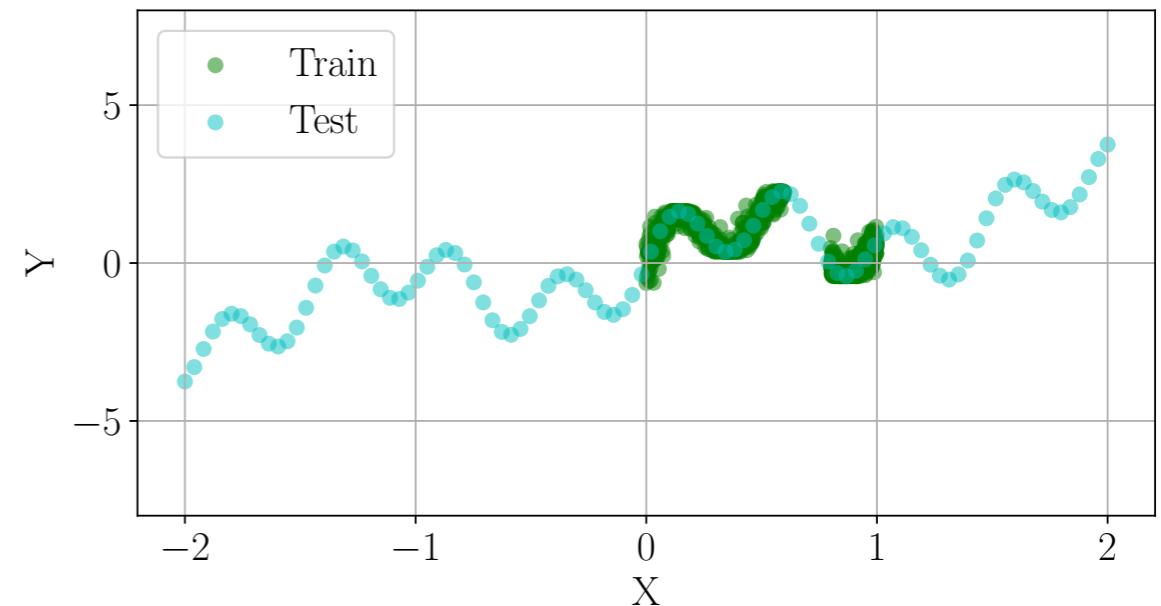
# Sources of Uncertainty

## 1. Aleatoric ('observation' noise)

- ▶ A result of the underlying process (e.g., *sensor noise*)

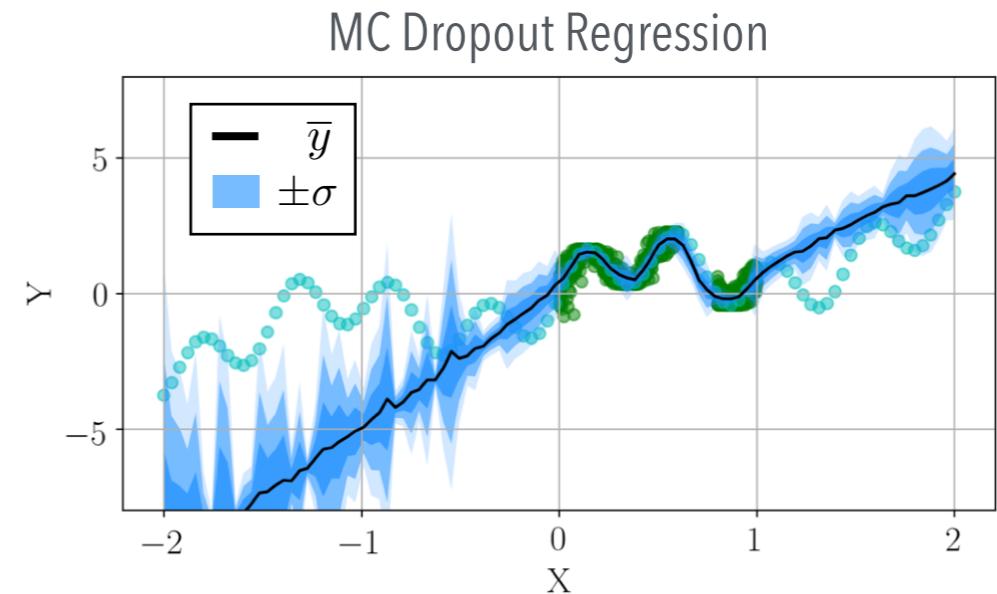
## 2. Epistemic ('model' uncertainty)

- ▶ A result of a 'distance' between training data and test data

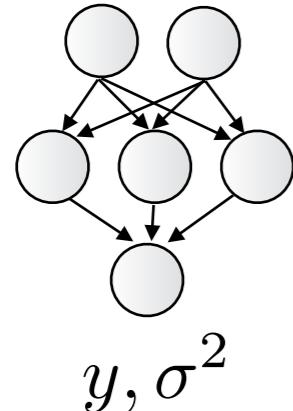


# Uncertainty in Neural Networks

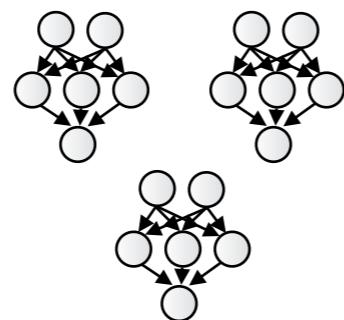
- ▶ Monte Carlo dropout (technique used by Sun-BCNN) also often **poorly captures epistemic uncertainty**
- ▶ Some proposed alternatives:



**Direct Covariance Learning**

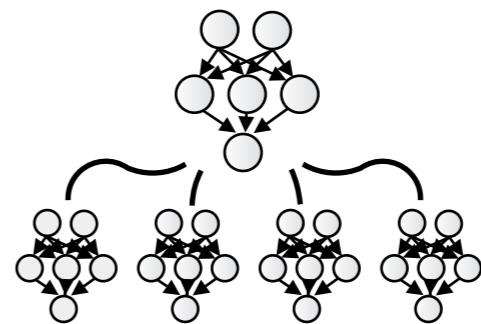
$$\mathcal{L}(y, \sigma^2, y_t) = \frac{1}{\sigma^2} (y - y_t)^2 + \log \sigma^2$$


**Ensembles**  
(Bootstrap Aggregating)



$$\bar{y}, \text{var}(\{y_i\})$$

**HydraNet**



$$\bar{y}, \text{var}(\{y_i\}) + \sigma^2$$



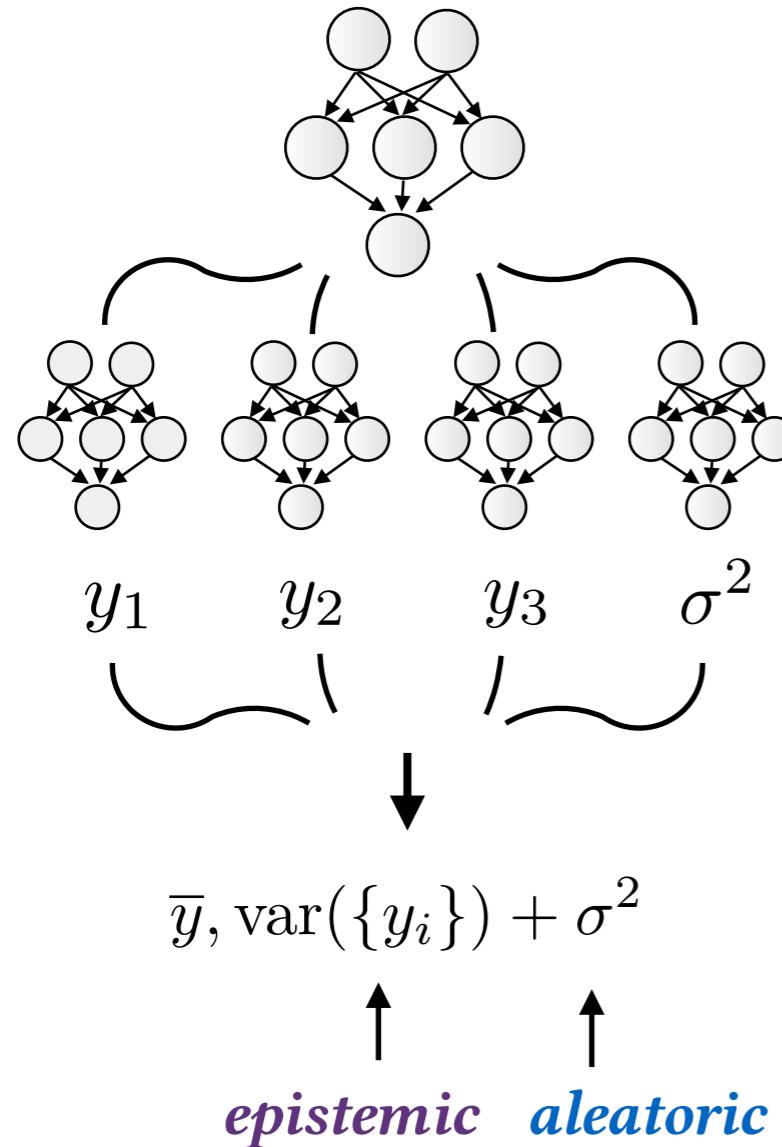
Lernaean Hydra from  
Greek Mythology

Liu, Ok et al., "Deep Inference for Covariance Estimation...", **ICRA** (2018)

Lakshminarayanan et al., "Simple and scalable predictive uncertainty estimation using deep ensembles," **NeurIPS** (2017)

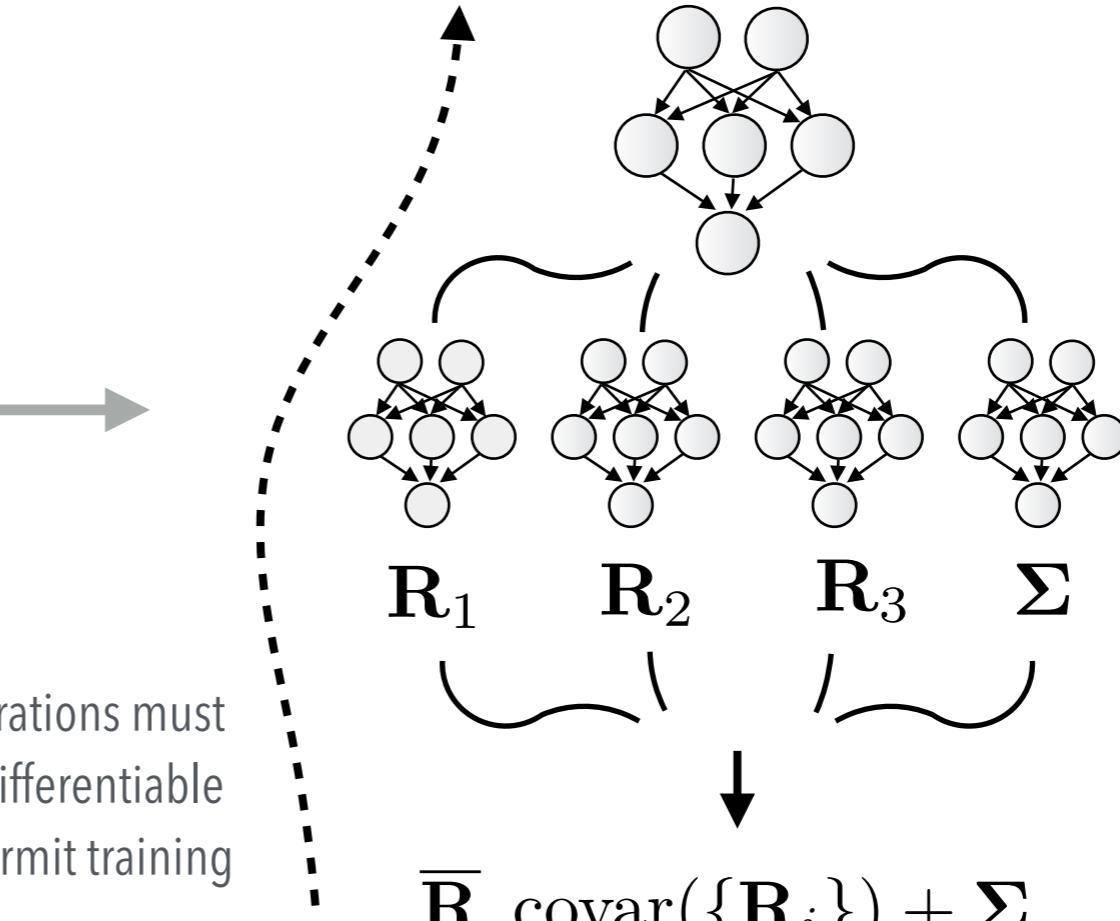
# HydraNet

## Epistemic and Aleatoric Uncertainty

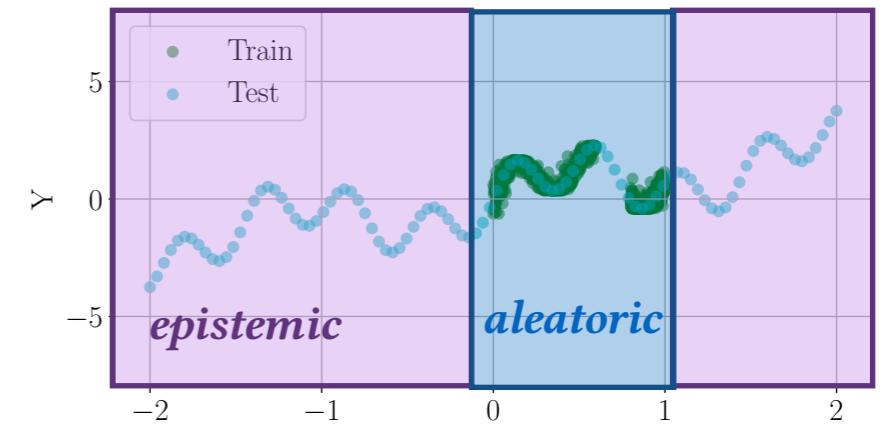


Euclidian targets

Operations must  
be differentiable  
to permit training



SO(3) targets

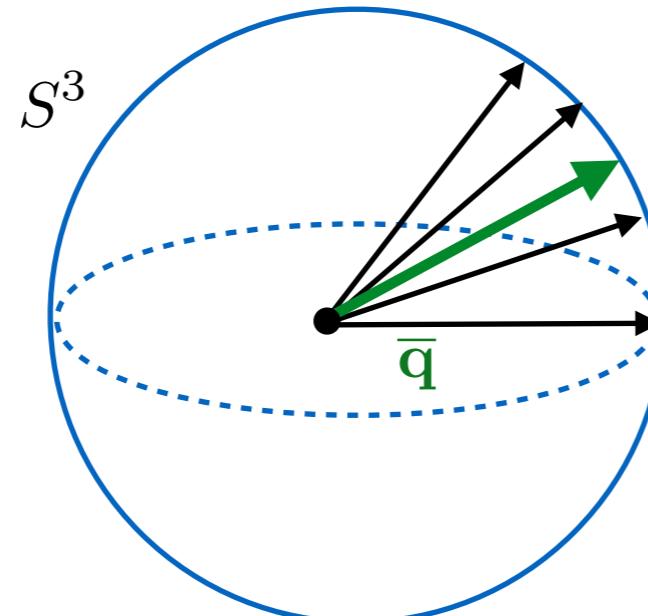
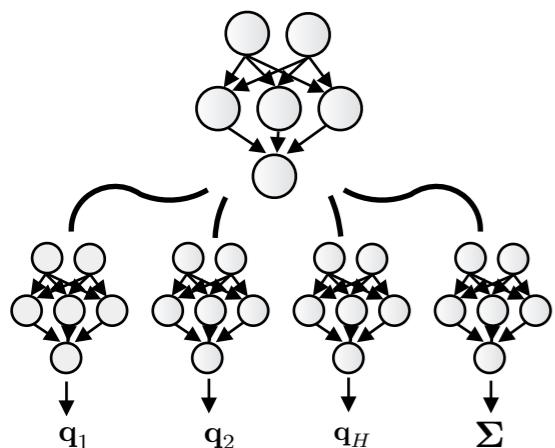


# Rotation Averaging

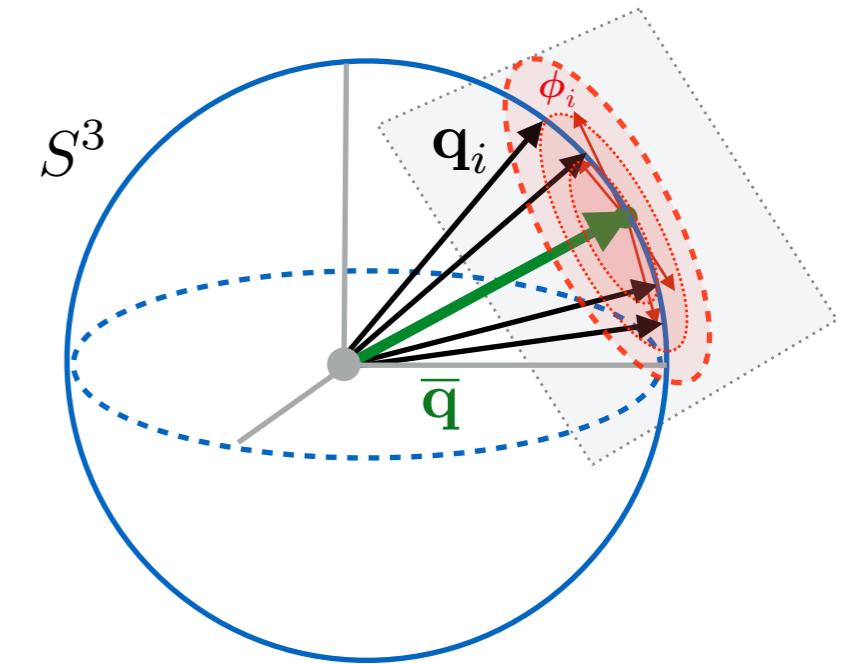
$$\bar{\mathbf{R}} = \underset{\mathbf{R} \in \text{SO}(3)}{\operatorname{argmin}} \sum_{i=1}^n d(\mathbf{R}_i, \mathbf{R})^2$$



$$\bar{\mathbf{q}} = \underset{\mathbf{q} \in \mathbb{S}^3}{\operatorname{argmin}} \sum_{i=1}^n d_{\text{quat}}(\mathbf{q}_i, \mathbf{q})^2$$



$$\bar{\mathbf{q}} = \frac{\sum_{i=1}^H \mathbf{q}_i}{\left\| \sum_{i=1}^H \mathbf{q}_i \right\|}$$



$$\text{covar}(\{\mathbf{q}_i\}) = \frac{1}{H-1} \sum_{i=1}^H \boldsymbol{\phi}_i \boldsymbol{\phi}_i^\top$$

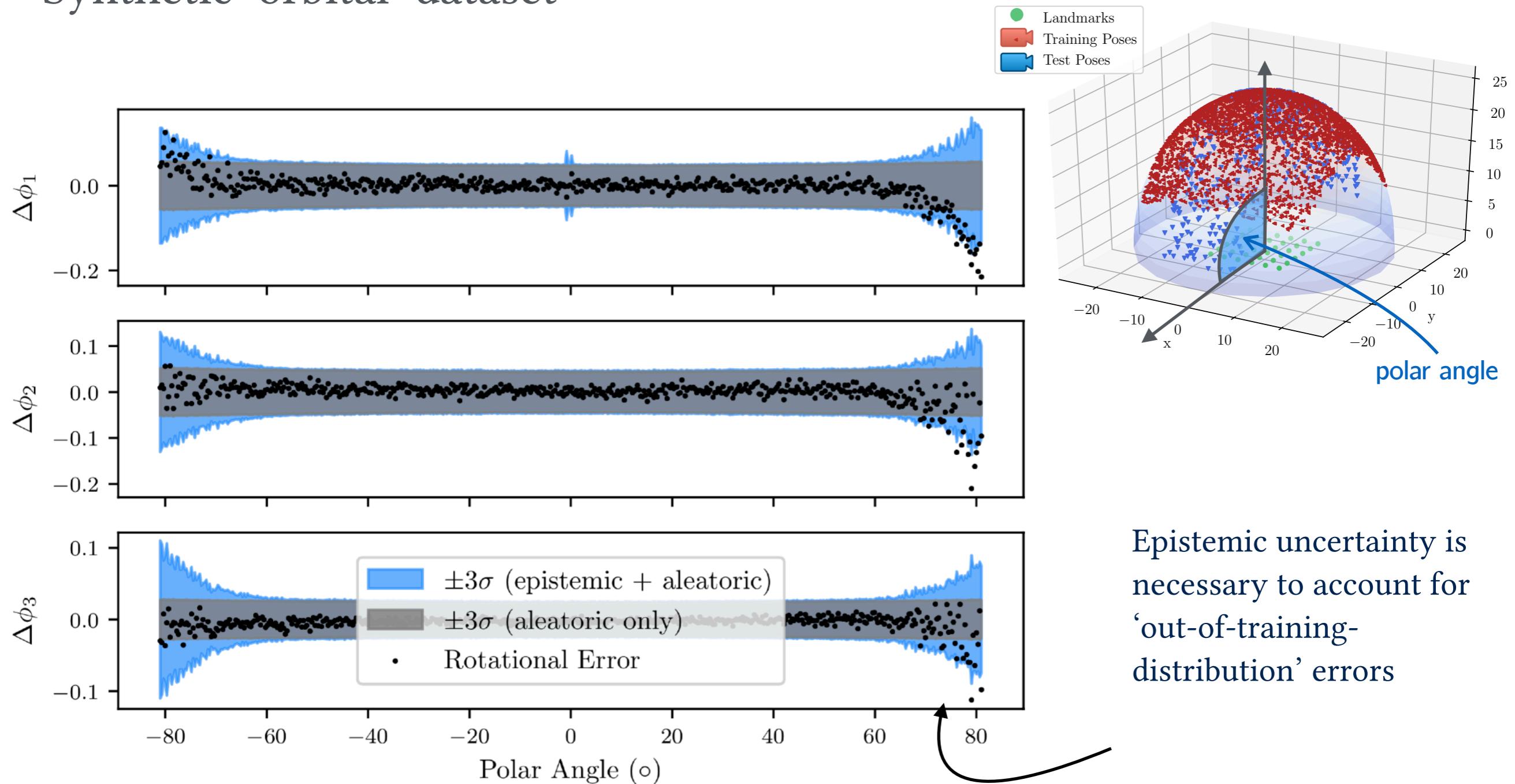
Hartley et al., "Rotation Averaging", [IJCV](#) (2013)

Sola et al., "A micro Lie theory for State Estimation in Robotics", [arXiv](#) (2019)



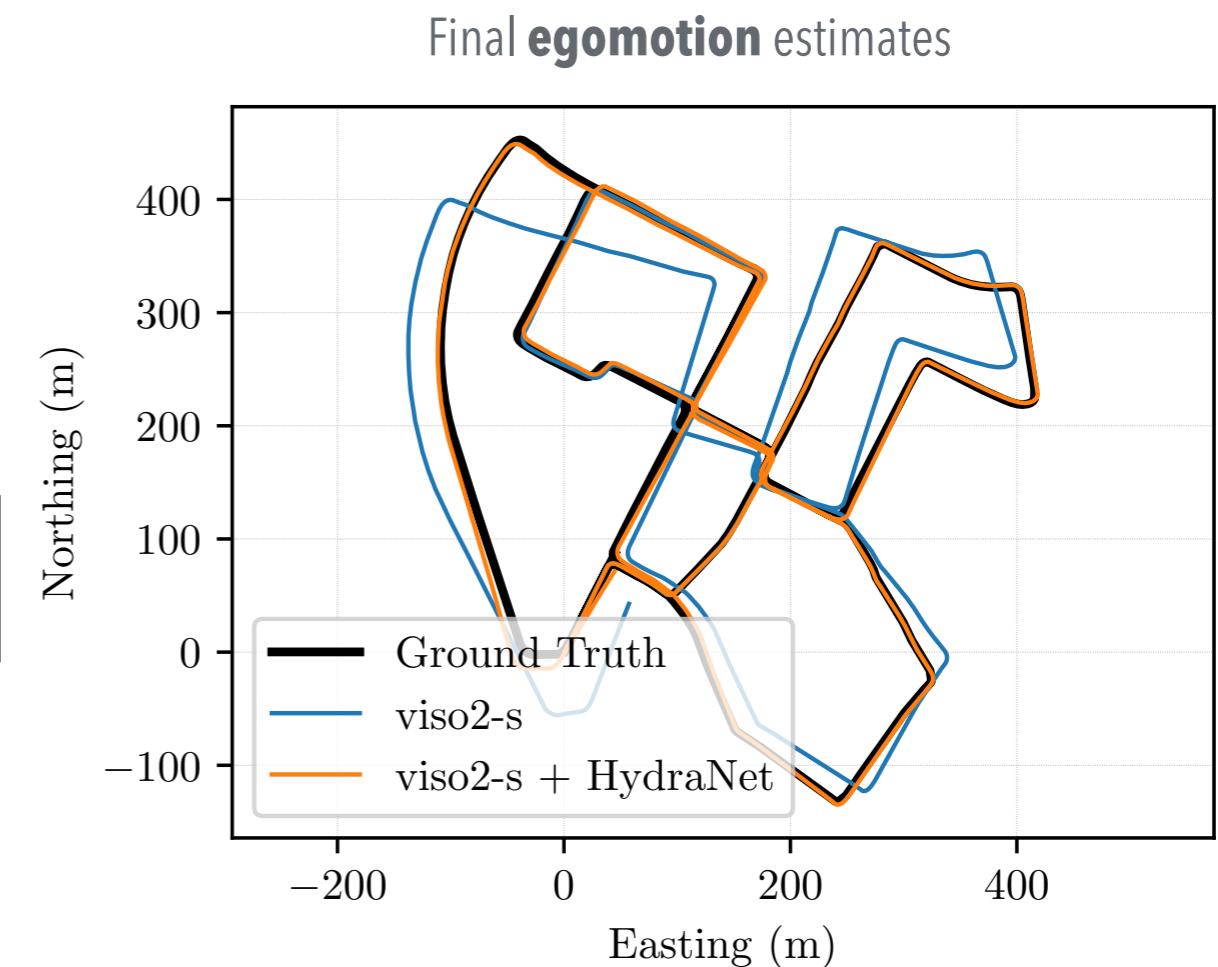
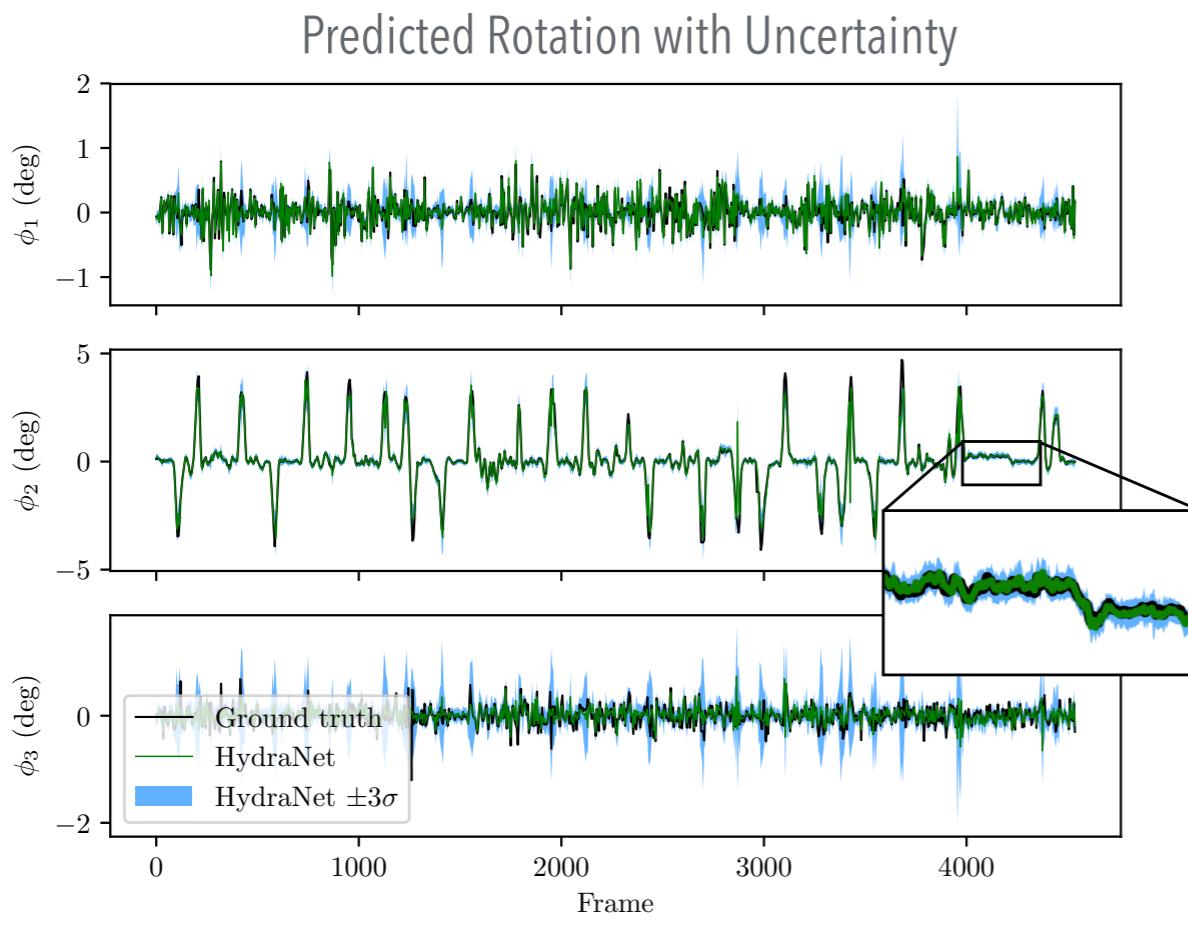
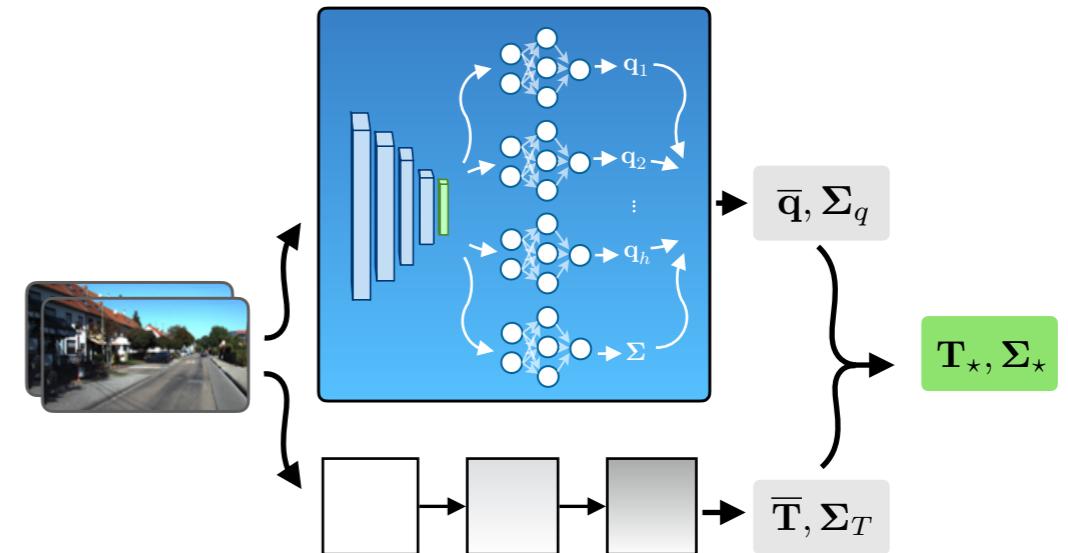
# Importance of Epistemic Uncertainty

Synthetic ‘orbital’ dataset



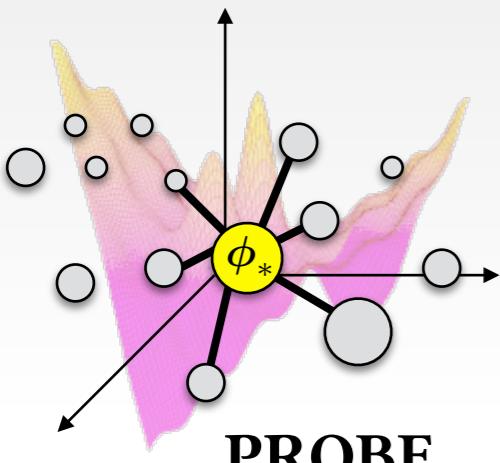
# Sliding Window VO

libviso2 SE(3) (*with uncertainty*) +  
HydraNet SO(3) (*with uncertainty*)



# Learned Improvements

I uncertainty quantification

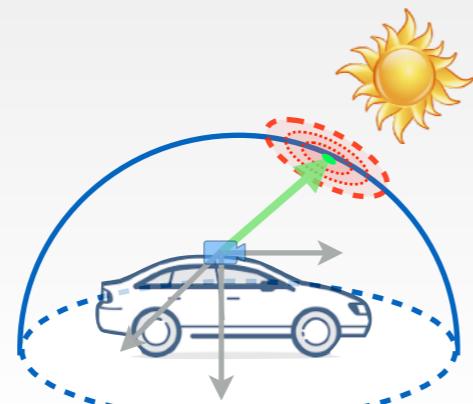


**PROBE**

Predictive Robust Estimation

IROS 2015  
ICRA 2016

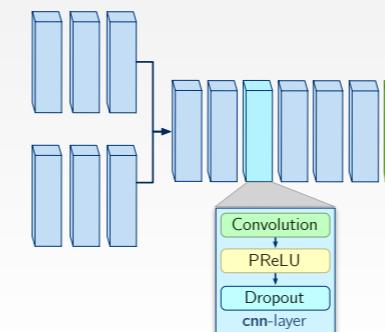
II latent representation



**Sun-BCNN**

Learning Sun Direction with Uncertainty  
ISER 2017, ICRA 2017,  
IJRR 2018

III bias correction

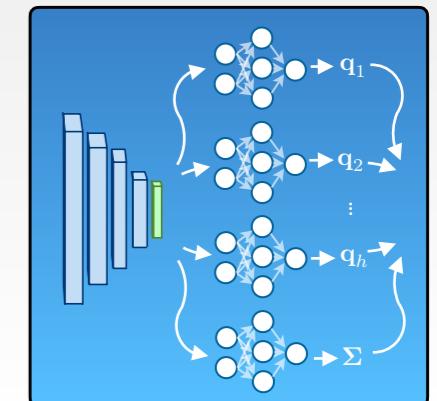


**DPC-Net**

Learning Estimator Bias through Deep Pose Correction

ICRA / RA-L 2018  
ICRA 2020

IV latent representation

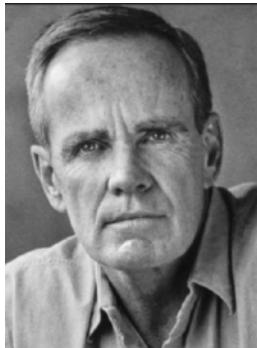


**HydraNet**

Learning Rotation With Uncertainty  
CVPR Workshops 2019

# Concluding Remarks

## On the Synthesis of Learning and Classical Modelling



...in this world more things exist without our knowledge than with it and the order in creation which you see is that which you have put there, like a string in a maze...

**Cormac McCarthy**, Blood Meridian, or the Evening Redness in the West (1985)

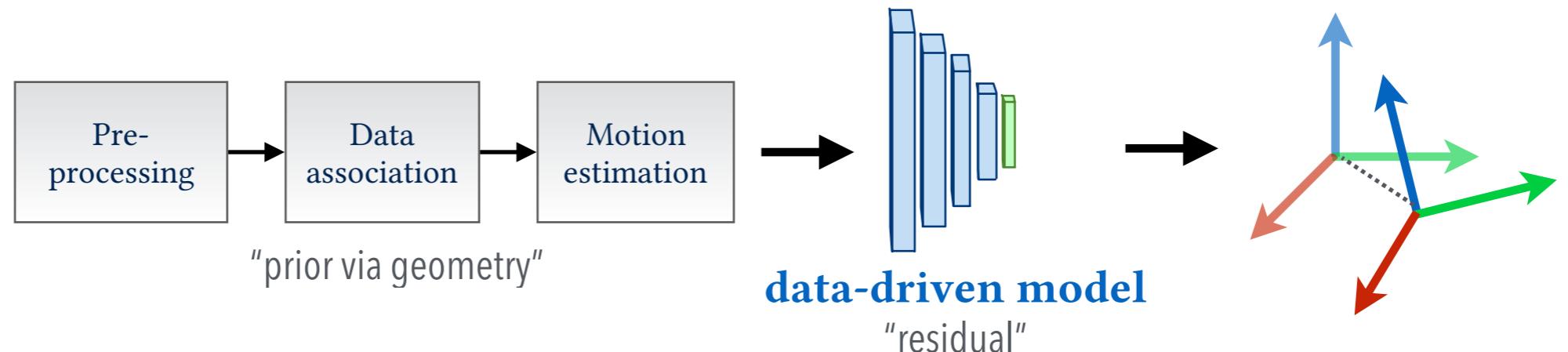


The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.

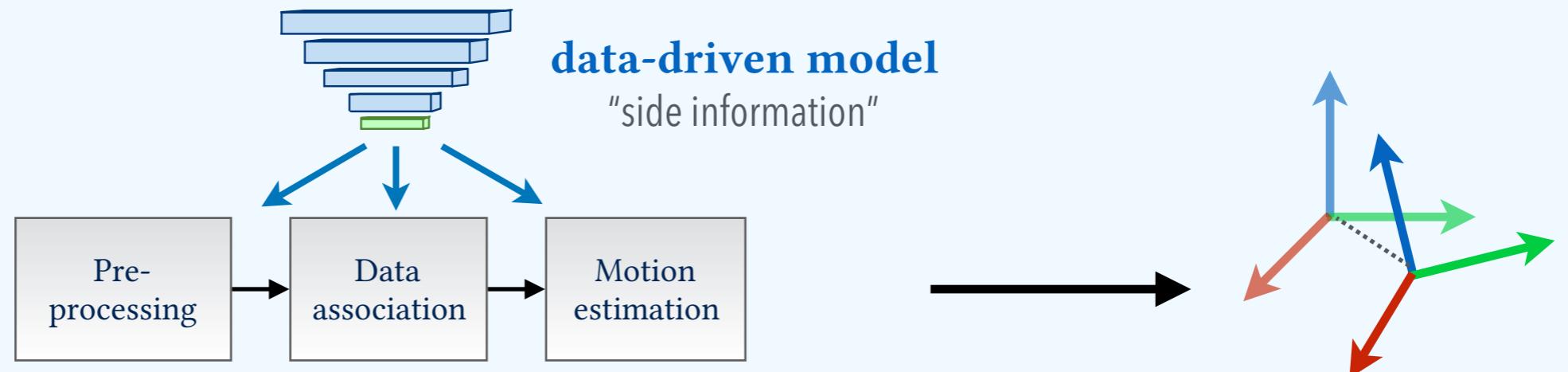
**E.P. Wigner**, The Unreasonable Effectiveness of Mathematics in the Natural Sciences (1960)

# Three Methods of Synthesis

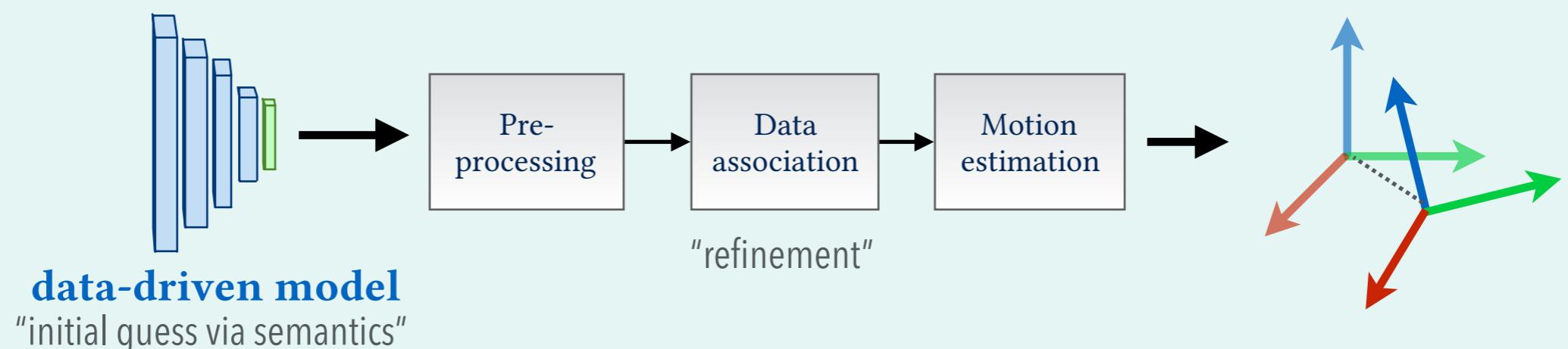
I  
Correction



II  
Augmentation



III  
Initialization



# A heartfelt *thank you* to...

my lab mates



my advisors



Jonathan  
Kelly



Angela  
Schoellig

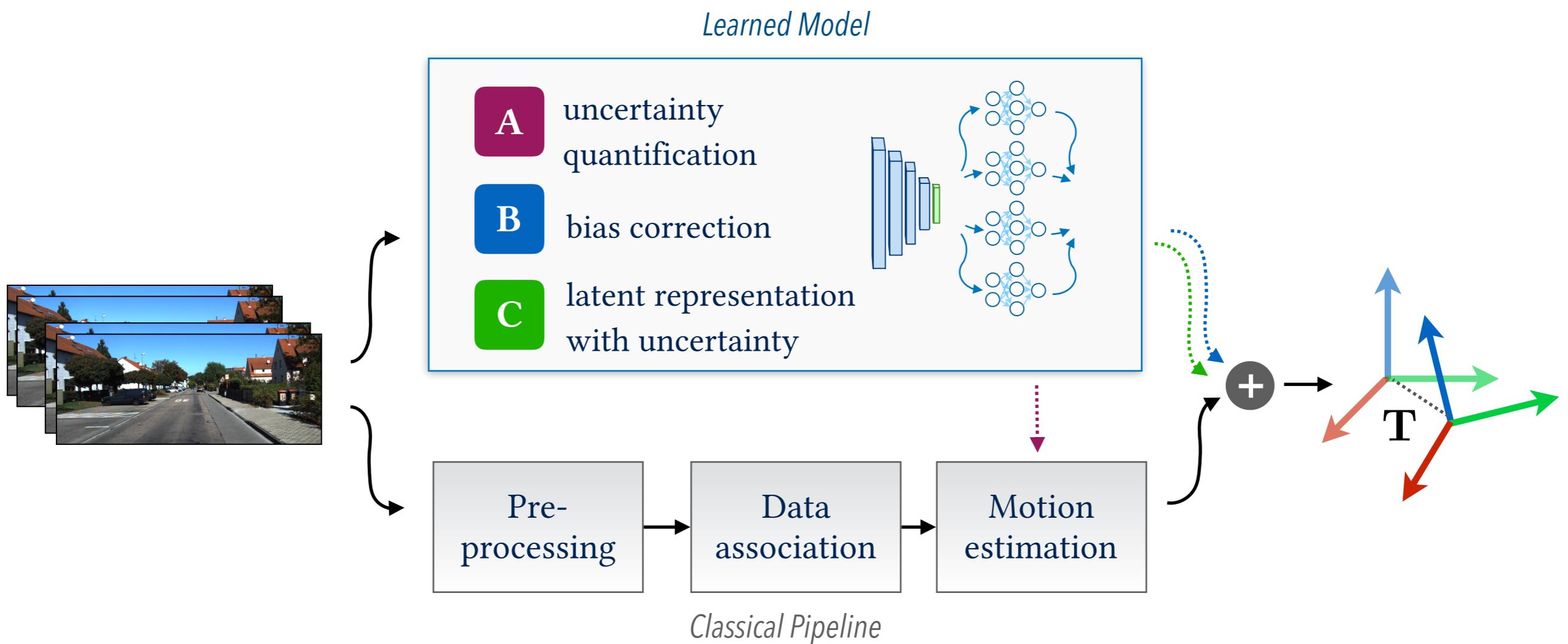


Tim  
Barfoot

and my friends and family 😊

# Learned Improvements to the Visual Egomotion Pipeline

On the Synthesis of Learning and Classical Modelling

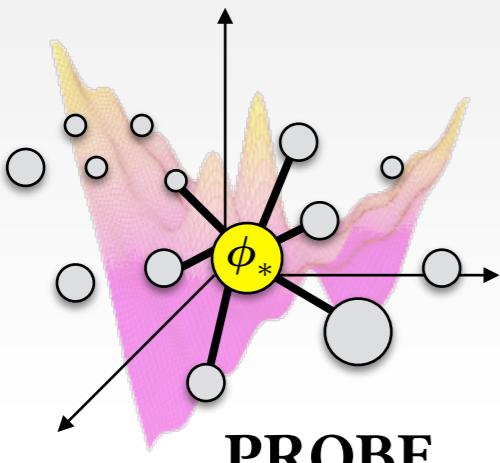


# Additional Slides



# Learned Improvements

I uncertainty quantification

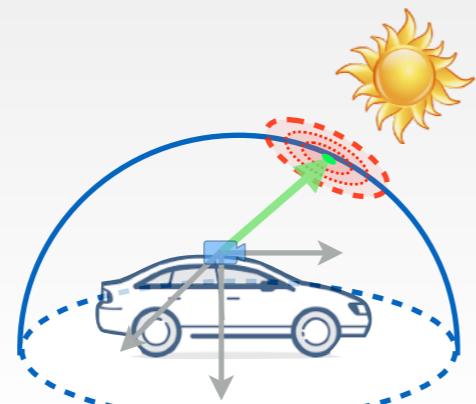


**PROBE**

Predictive Robust Estimation

IROS 2015  
ICRA 2016

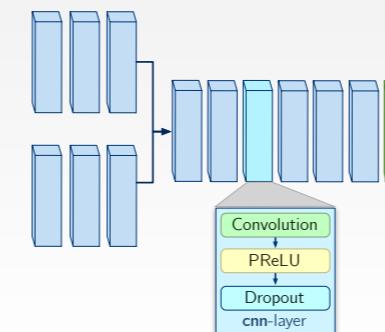
II latent representation



**Sun-BCNN**

Learning Sun Direction with Uncertainty  
ISER 2017, ICRA 2017,  
IJRR 2018

III bias correction

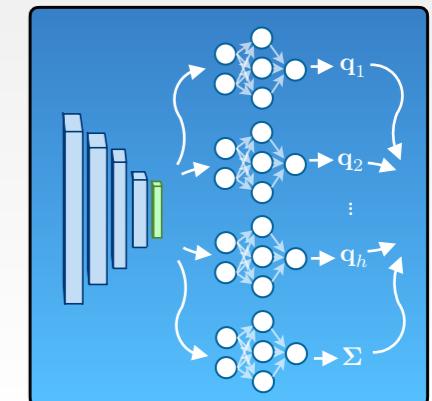


**DPC-Net**

Learning Estimator Bias through Deep Pose Correction

ICRA / RA-L 2018  
ICRA 2020

IV latent representation

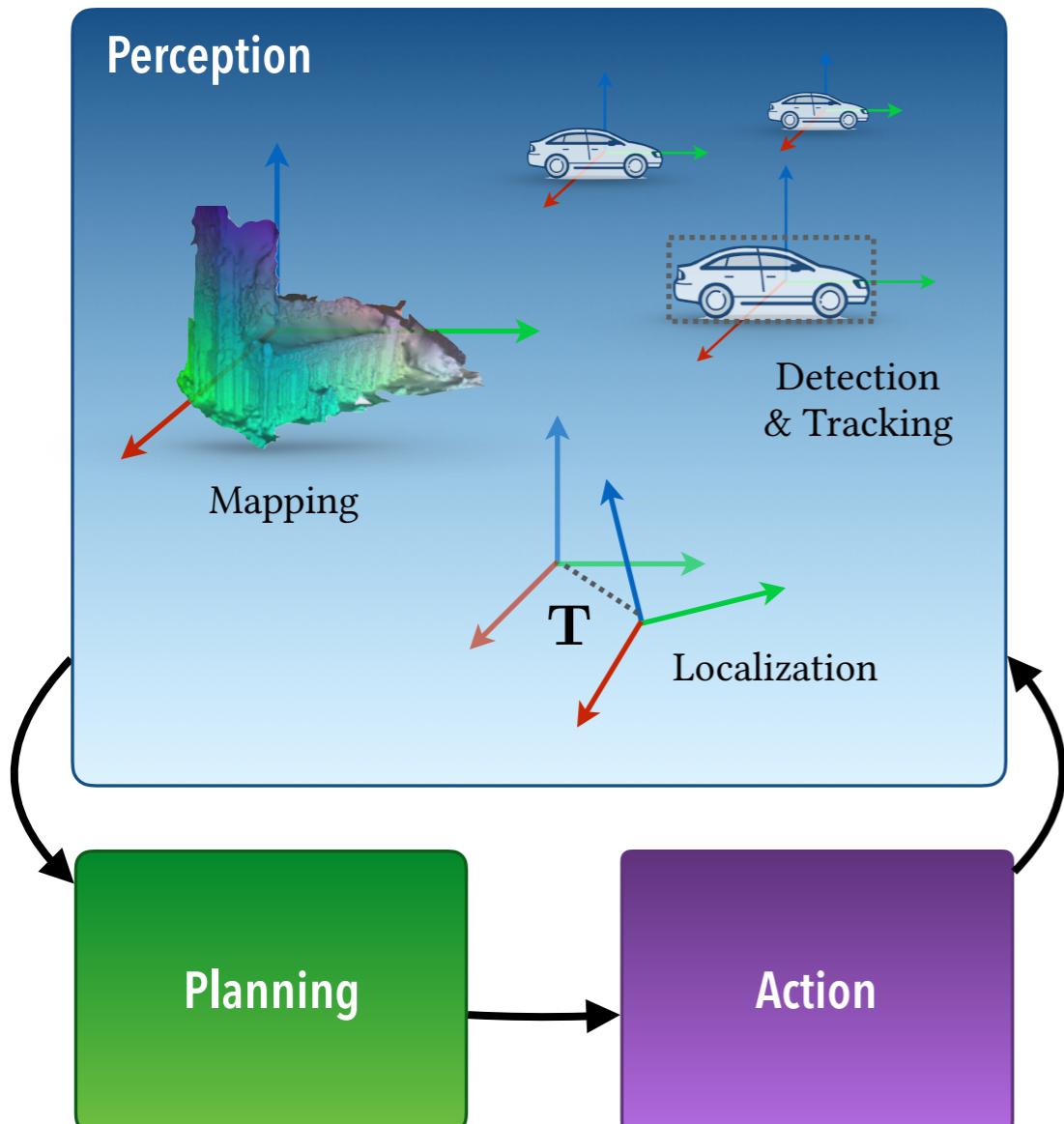


**HydraNet**

Learning Rotation With Uncertainty  
CVPR Workshops 2019

# Visual Egomotion

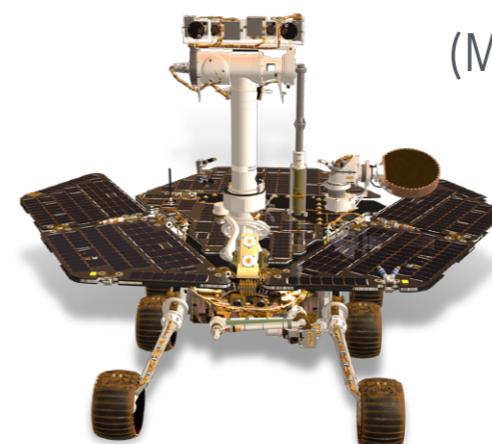
A Core Component of Vision-based Autonomy



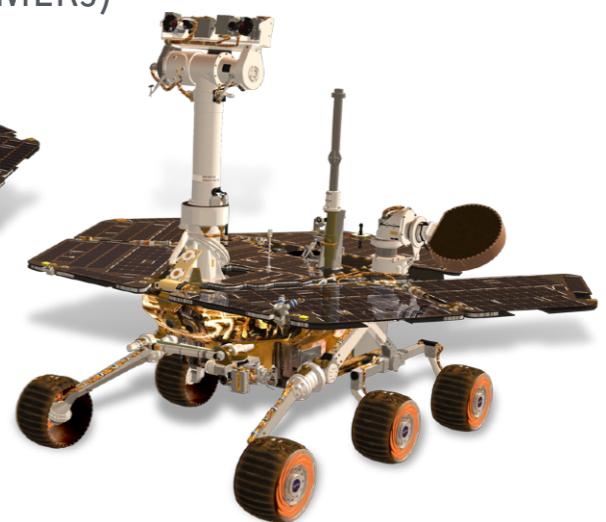
**Aerial autonomy**  
(Skydio 2 Drone)



**Assistive Driving**  
(Tesla Roadster)

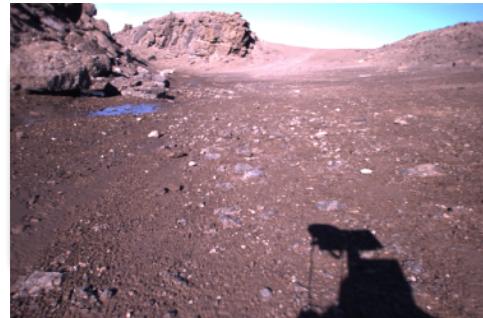


**Planetary Rovers**  
(MERs)



# Sun-BCNN Testing

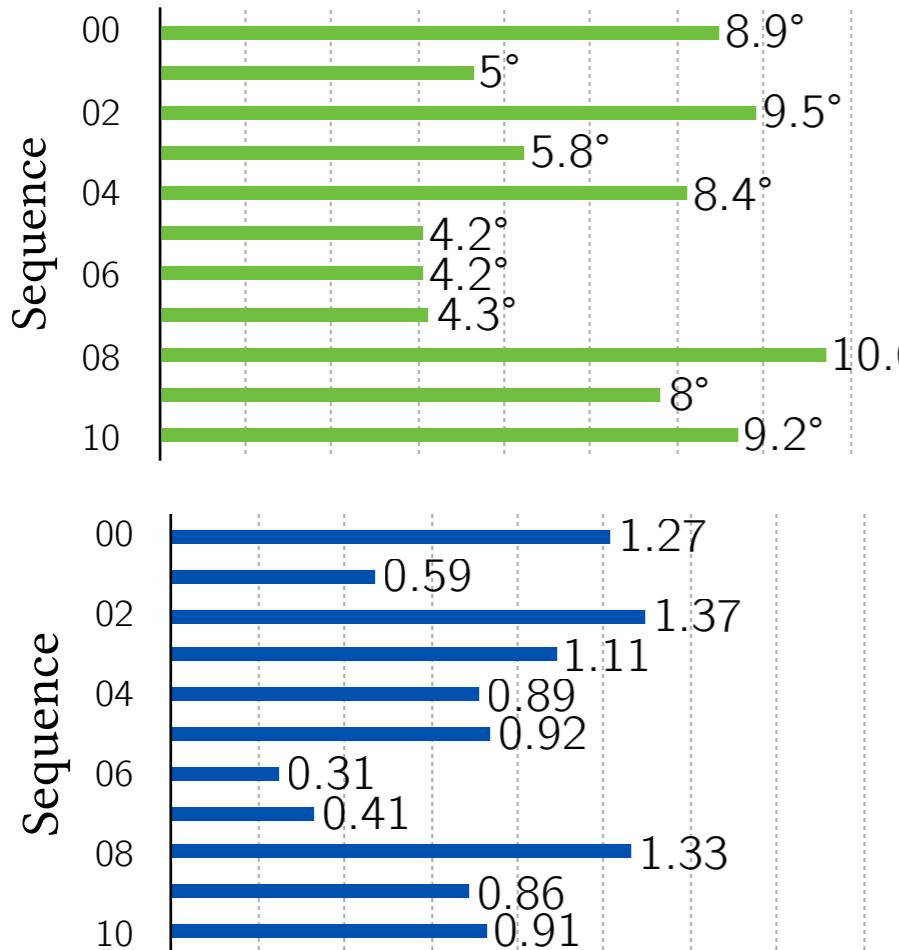
Devon Island and KITTI



**Ground truth**

Sinclair

Interplanetary  
digital sun sensor



ANEES

Median  
Error

ANEES

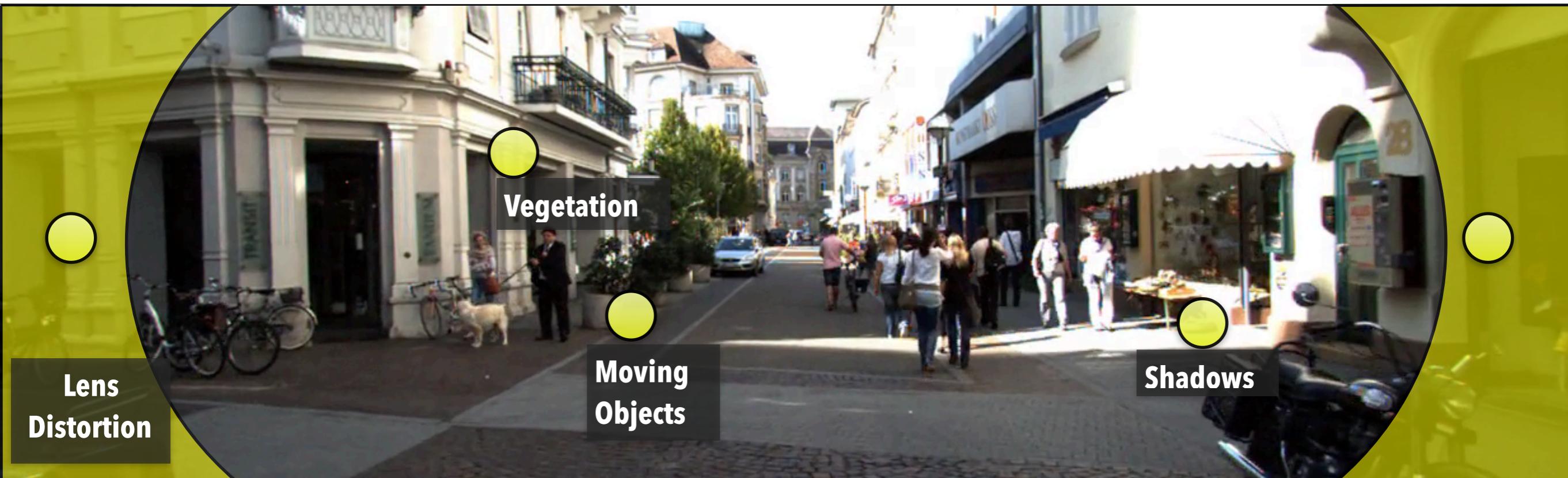


**Ground truth**

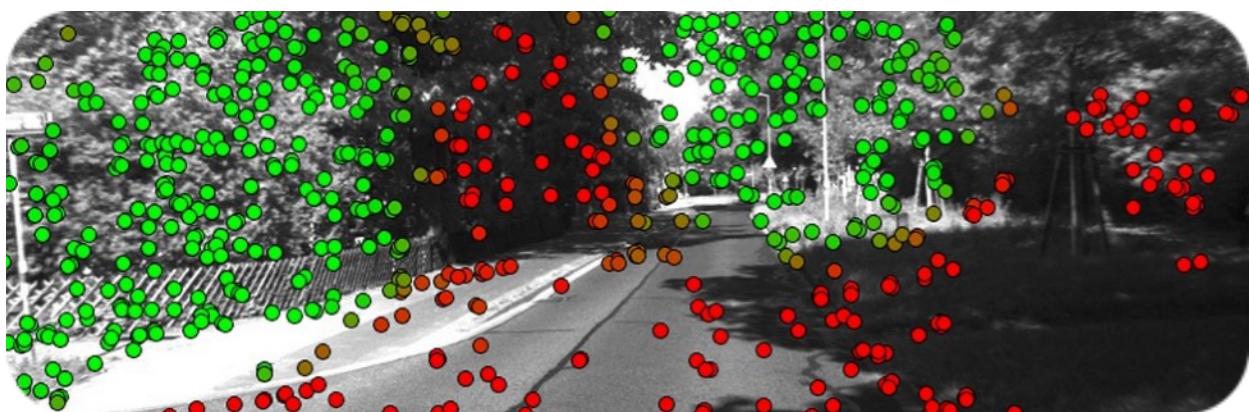
6-DoF GPS-INS and a solar ephemeris model (based on GPS timestamp)



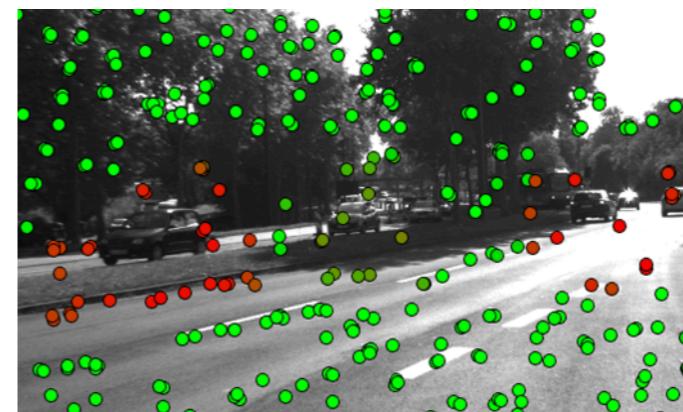
# Predictive Robust Estimation



Latent Predictive Factors



Entropy



Optical Flow Variance

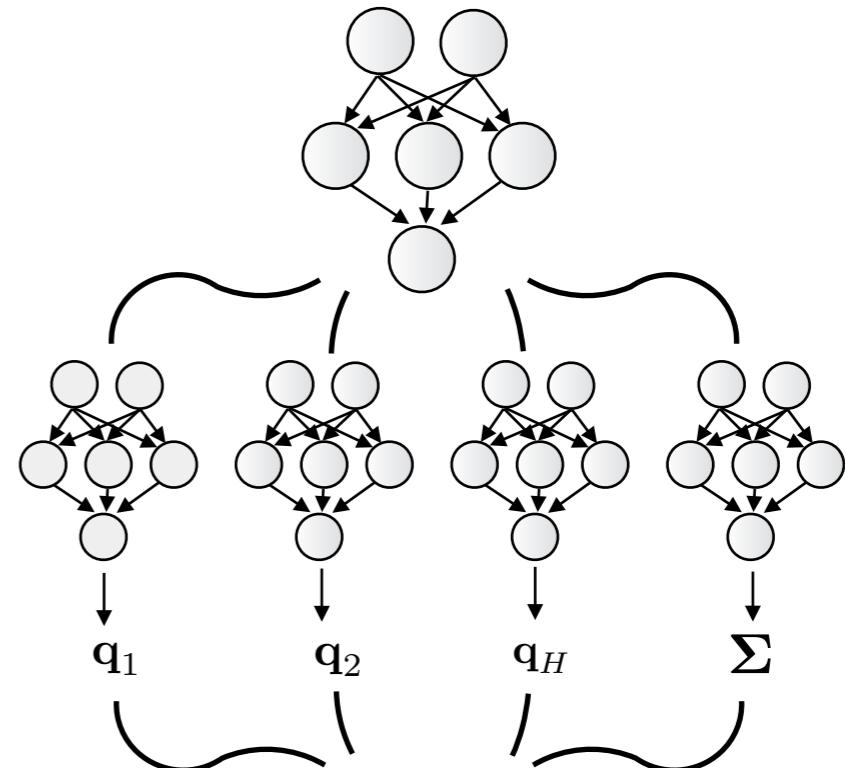
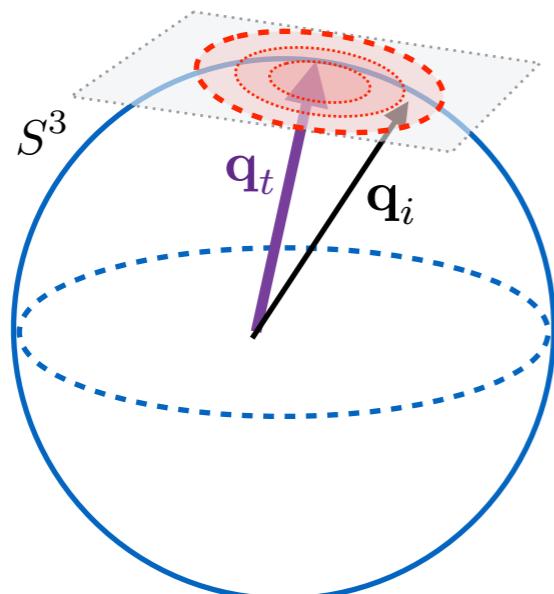


# Covariance-aware Loss

- In order to learn a covariance matrix we define a supervised loss based on the tangent space of each target rotation:

$$\mathcal{L}_{\text{NLL}}(\mathbf{q}, \mathbf{q}_t, \Sigma) = \boldsymbol{\phi}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\phi} + \log \det (\boldsymbol{\Sigma})$$

with  $\boldsymbol{\phi} = \text{Log} (\mathbf{q} \otimes \mathbf{q}_t^{-1})$



$$\mathcal{L} = \sum_{h=1}^H \mathcal{L}_{\text{NLL}}(\mathbf{q}_h, \mathbf{q}_t, \Sigma)$$

Hu and Kantor, "Parametric Covariance Prediction for Heteroscedastic Noise", **IROS** (2015)  
Forster et al., "IMU Preintegration on Manifold...", **RSS** (2015)

# Residual Learning in Robotics

## Helicopter Dynamics

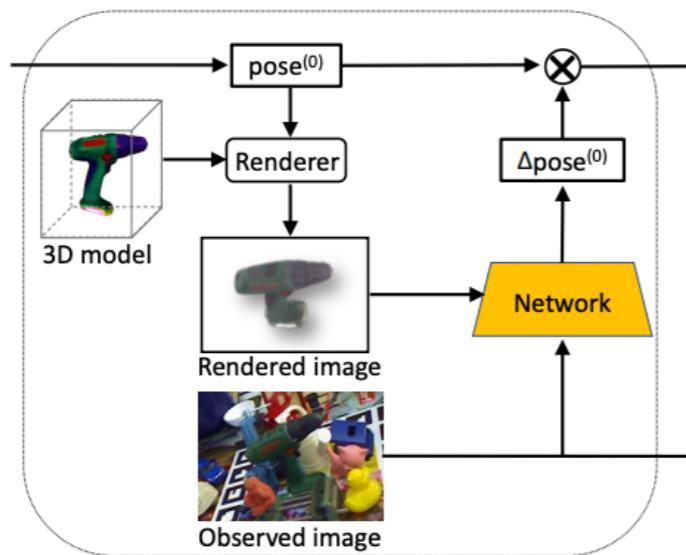


Punjani and Abbeel, Deep Learning Helicopter Dynamics Models. [ICRA \(2015\)](#).

## Throwing Objects



## Object Pose Regression



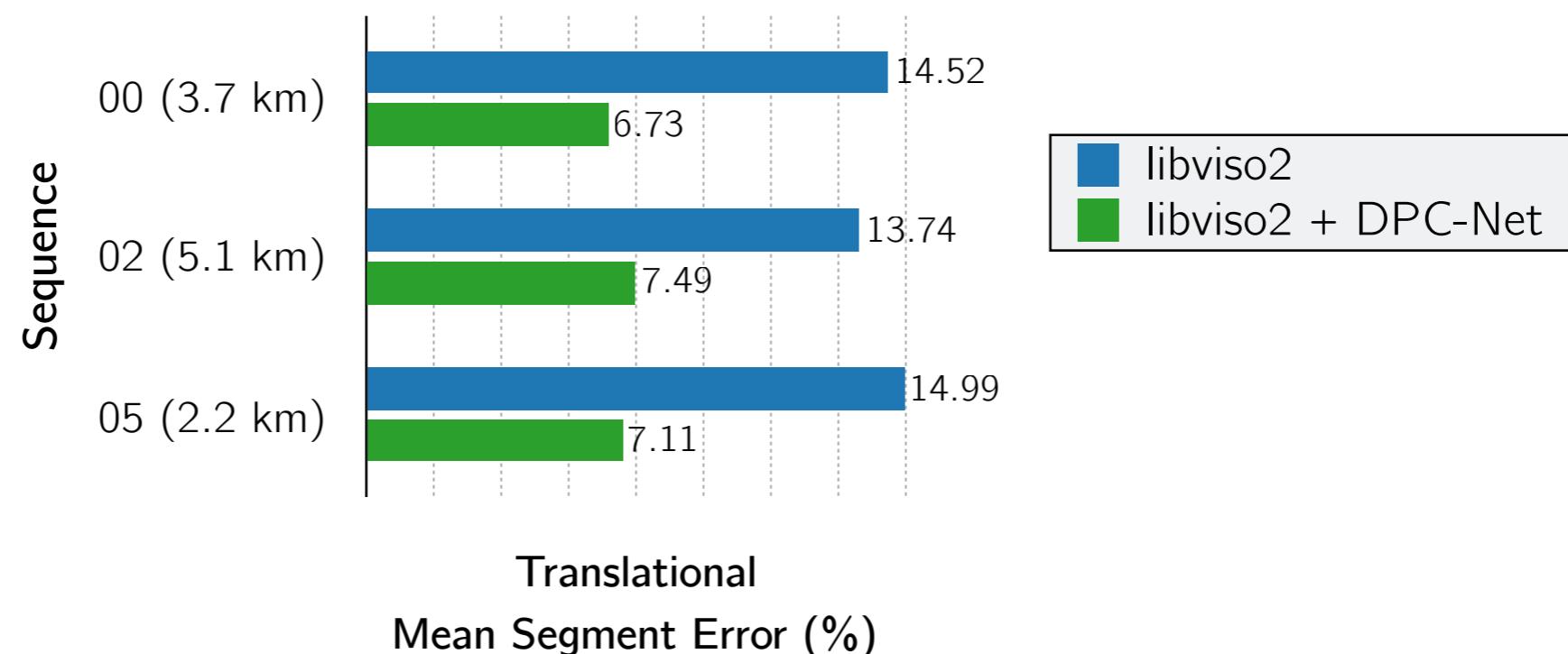
Li et al., DeepIM: Deep Iterative Matching for 6D Pose Estimation, [ECCV \(2018\)](#).

Zeng et al., TossingBot: Learning to Throw Arbitrary Objects with Residual Physics. [RSS \(2019\)](#).



# DPC-Net | Correcting for Lens Distortion

$$\begin{bmatrix} x_d \\ y_d \end{bmatrix} = (1 + \kappa_1 r^2 + \kappa_2 r^4 + \kappa_3 r^6) \begin{bmatrix} x_n \\ y_n \end{bmatrix}$$



# SO(3) and SE(3)

Rigid-body rotations form the matrix Lie group SO(3) - the Special Orthogonal group:

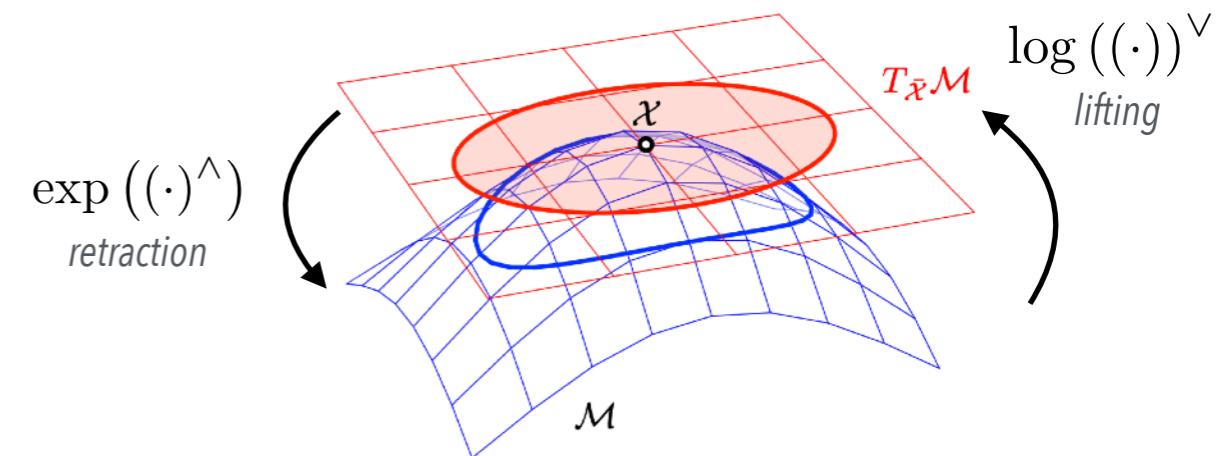
$$\text{SO}(3) = \{\mathbf{R} \in \mathbb{R}^{3 \times 3} \mid \mathbf{R}^T \mathbf{R} = \mathbf{1}, \det \mathbf{R} = 1\}$$

$$\mathbf{R} = \exp(\boldsymbol{\phi}^\wedge) = \sum_{n=0}^{\infty} \frac{1}{n!} (\boldsymbol{\phi}^\wedge)^n \quad \boldsymbol{\phi} \in \mathbb{R}^3$$

Rigid-body *transformations* form the matrix Lie group SE(3) - the Special Euclidian group:

$$\text{SE}(3) = \{\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \in \mathbb{R}^{4 \times 4} \mid \mathbf{R} \in \text{SO}(3), \mathbf{t} \in \mathbb{R}^3\}$$

$$\mathbf{T} = \exp(\boldsymbol{\xi}^\wedge) = \sum_{n=0}^{\infty} \frac{1}{n!} (\boldsymbol{\xi}^\wedge)^n \quad \boldsymbol{\xi} \in \mathbb{R}^6$$



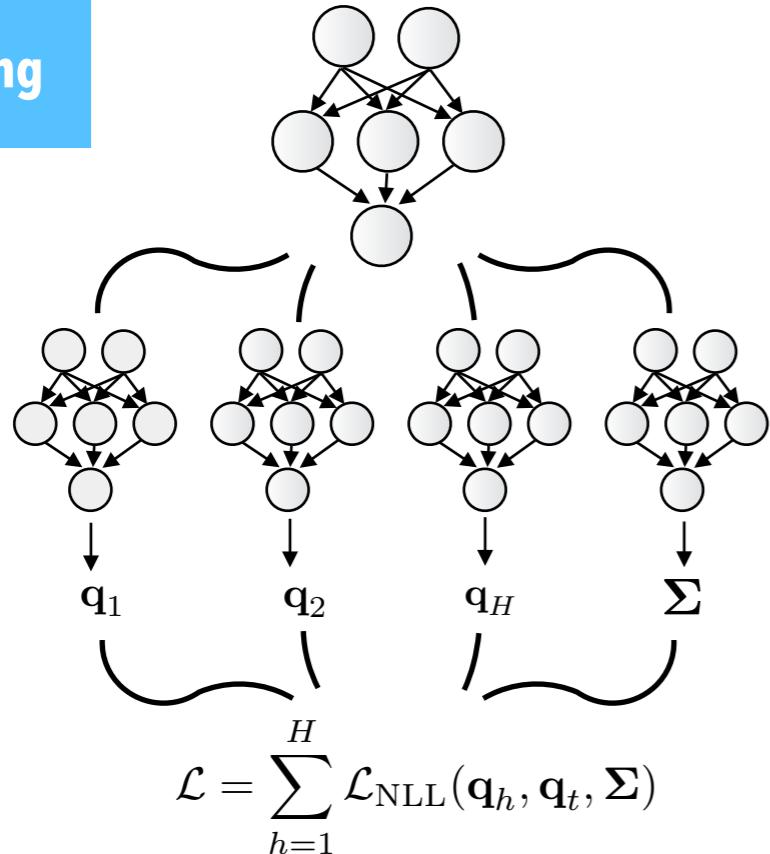
$\boldsymbol{\phi}, \boldsymbol{\xi}$  are unconstrained, but  
 $\exp((\cdot)^\wedge)$  is **surjective**

Sola et al., "A micro Lie theory for State Estimation in Robotics", arXiv (2019)

Barfoot., "State Estimation for Robotics." Cambridge University Press (2017)

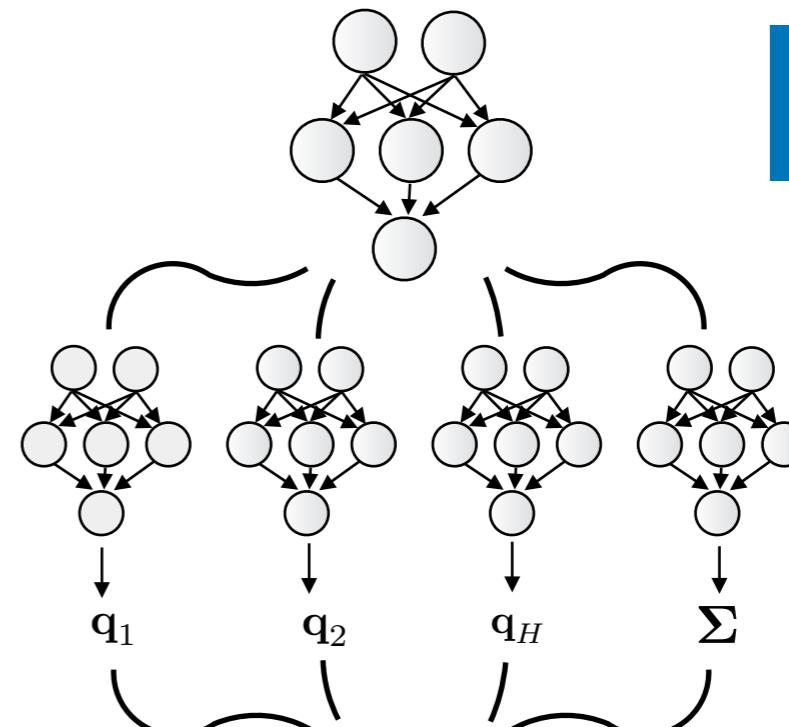
# Deep Probabilistic Regression of Rotations

Training



$$\begin{aligned}\mathcal{L}_{\text{NLL}}(\mathbf{q}, \mathbf{q}_t, \Sigma) &= \boldsymbol{\phi}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\phi} + \log \det(\boldsymbol{\Sigma}) \\ \boldsymbol{\phi} &= \text{Log}(\mathbf{q} \otimes \mathbf{q}_t^{-1})\end{aligned}$$

Testing



1 Compute mean rotation

2 Compute residuals

3 Compute covariance as

*epistemic*

$$\frac{1}{H-1} \sum_{h=1}^H \boldsymbol{\phi}_h \boldsymbol{\phi}_h^T + \Sigma$$

*aleatoric*

# DPC-Net | Loss

We use a Mahalanobis-like norm loss,

$$\mathcal{L}(\boldsymbol{\xi}) = \frac{1}{2} g(\boldsymbol{\xi})^T \boldsymbol{\Sigma}^{-1} g(\boldsymbol{\xi}) \quad \text{where} \quad g(\boldsymbol{\xi}) \triangleq \log \left( \exp (\boldsymbol{\xi}^\wedge) \mathbf{T}^{*-1} \right)^\vee$$

$\boldsymbol{\xi} \in \mathbb{R}^6$   
network output  
 $\mathbf{T}^* \in \text{SE}(3)$   
target

Our loss has a covariance-based metric tensor that **naturally balances** rotation and translation terms:

$$\boldsymbol{\Sigma} = \frac{1}{N-1} \sum_{i=1}^N \left( \boldsymbol{\xi}_i^* - \bar{\boldsymbol{\xi}}^* \right) \left( \boldsymbol{\xi}_i^* - \bar{\boldsymbol{\xi}}^* \right)^T \quad \text{where} \quad \boldsymbol{\xi}_i^* \triangleq \log (\mathbf{T}_i^*)^\vee$$

training targets

compare to..

$$\mathcal{L} = \|\hat{\mathbf{x}} - \mathbf{x}\| + \beta \left\| \hat{\mathbf{q}} - \frac{\mathbf{q}}{\|\mathbf{q}\|} \right\|$$

Kendall et al., PoseNet, [ICRA \(2016\)](#)

We derive an **analytic gradient** based on middle perturbations:

$$\frac{\partial \mathcal{L}(\boldsymbol{\xi})}{\partial \boldsymbol{\xi}} = g(\boldsymbol{\xi})^T \boldsymbol{\Sigma}^{-1} \frac{\partial g(\boldsymbol{\xi})}{\partial \boldsymbol{\xi}} \quad \frac{\partial g(\boldsymbol{\xi})}{\partial \boldsymbol{\xi}} = \mathcal{J}(g(\boldsymbol{\xi}))^{-1} \mathcal{J}(\boldsymbol{\xi}) \quad \text{where} \quad \mathcal{J}(\cdot)$$

left SE(3) Jacobian

# Monte Carlo Dropout Revisited

- To make connection between dropout and Bayesian NNs, Gal turns to **variational inference** to approximate posterior over weights:

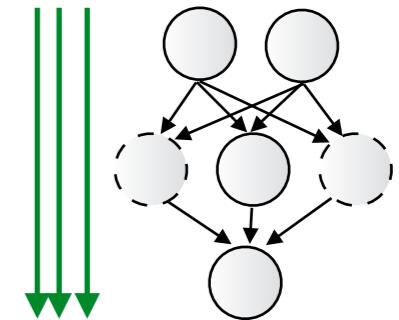
$$q(\mathbf{w}) \sim p(\mathbf{w} | \mathbf{X}, \mathbf{S})$$

(matrix with  $K_i$  weights for layer  $i$ )

$$q(\mathbf{w}_i) = \mathbf{M}_i \operatorname{diag} \left\{ \{b_j^i\}_{j=1}^{K_i} \right\},$$
$$b_j^i \in \operatorname{Bernoulli}(p_i)$$

(dropout probability)

**Monte Carlo Dropout**  
(Variational Inference)



$$\bar{y}, \operatorname{var}(\{y_i\}) + \tau^{-1}$$

Gal, "Uncertainty in Deep Learning" **Ph.D. Thesis** (2016)

- Gal shows that the **dropout training loss is equivalent to minimizing the KL divergence** between the posterior and this variational distribution

$$D_{\text{KL}} ( p(\mathbf{w} | \mathbf{X}, \mathbf{S}) || q(\mathbf{w}) )$$

- However**, for linear networks, one can show that this posterior **does not concentrate with more data**, and requires careful tuning of hyperparameters

Osband, "Risk versus uncertainty in deep learning: Bayes, bootstrap and the dangers of dropout," **NeurIPS** (2017)

# Sun-BCNN Training

## KITTI & Devon Island

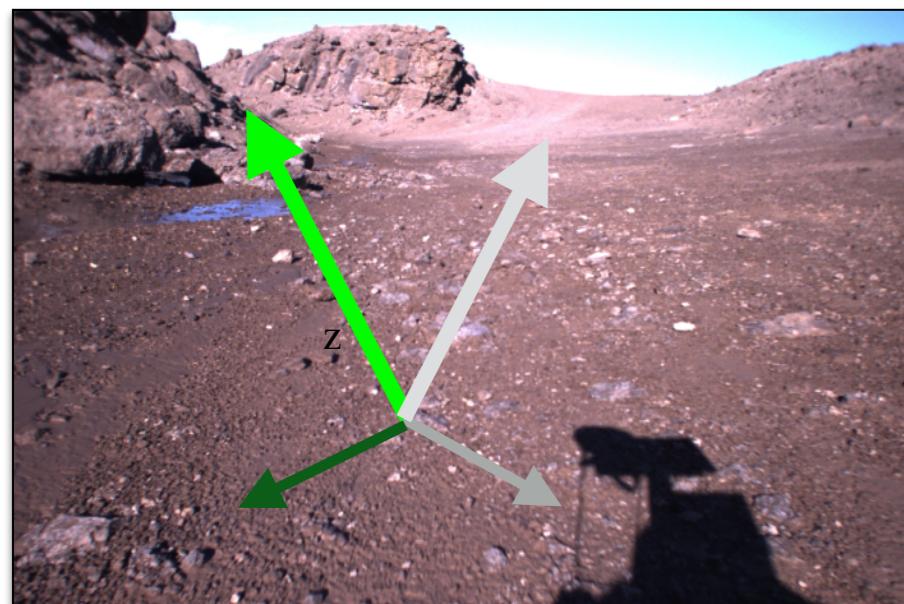


KITTI Vision Benchmark



### Ground truth

6-DoF GPS-INS and a solar ephemeris model (based on GPS timestamp)

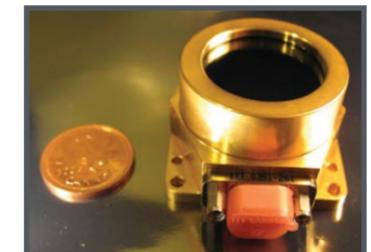


Devon Island Rover Navigation



### Ground truth

Sinclair  
Interplanetary  
digital sun  
sensor



# Rotation Averaging

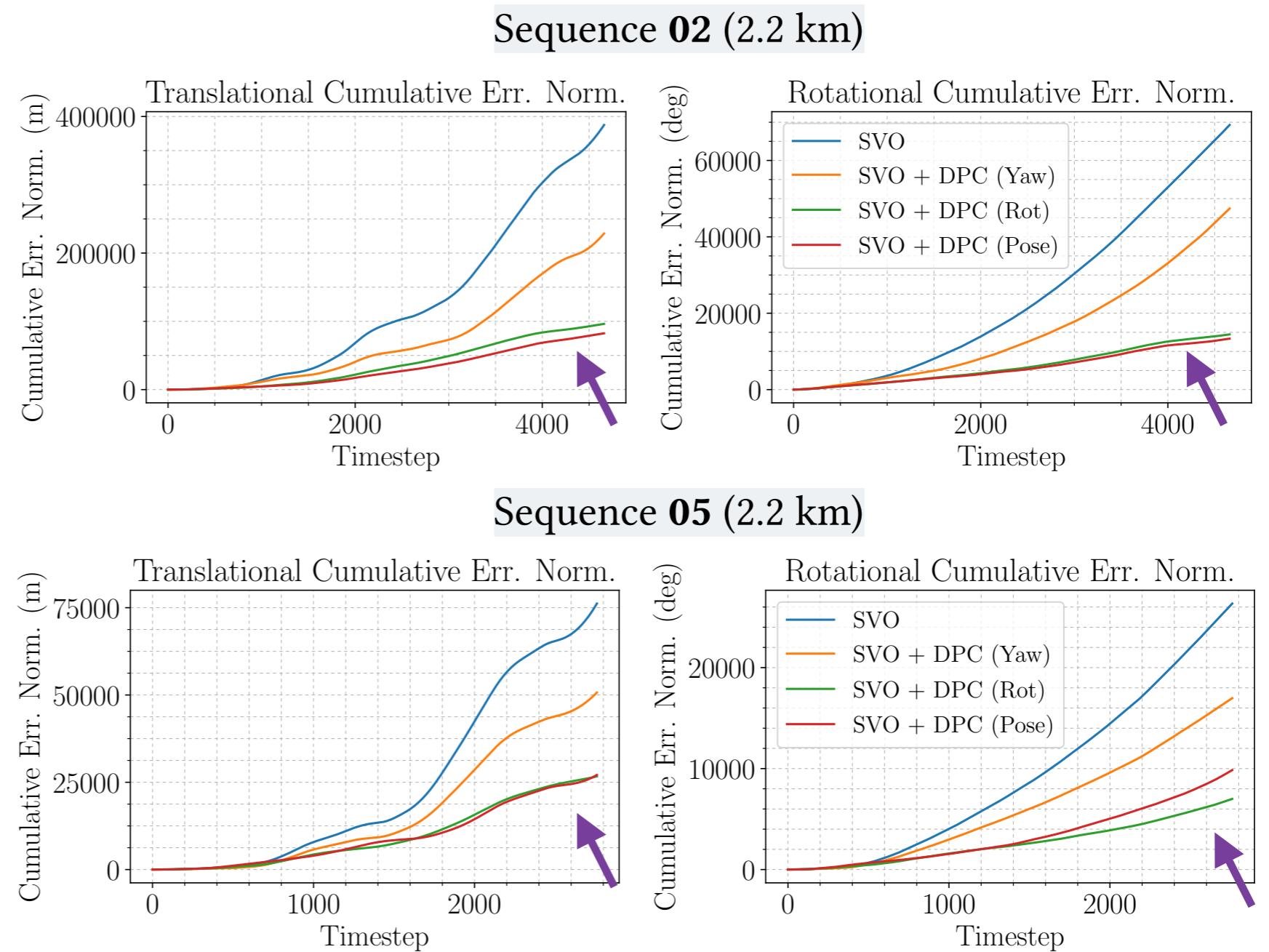
$$\bar{\mathbf{R}} = \operatorname{argmin}_{\mathbf{R} \in \text{SO}(3)} \sum_{i=1}^n d(\mathbf{R}_i, \mathbf{R})^2$$

|                               | Metric  | Resulting Mean   |
|-------------------------------|---|--|
| <i>Angular<br/>(Geodesic)</i> | $d_{\text{ang}}(\mathbf{R}_a, \mathbf{R}_b) = \left\  \text{Log} \left( \mathbf{R}_a \mathbf{R}_b^T \right) \right\ _2 = \theta$                      | Karcher mean<br><b>(requires iteration)</b>  |
| <i>Chordal</i>                | $d_{\text{chord}}(\mathbf{R}_a, \mathbf{R}_b) = \ \mathbf{R}_a - \mathbf{R}_b\ _{\text{Frob}} = 2\sqrt{2} \sin \frac{\theta}{2}$                      | Euclidian mean in $\mathbb{R}^9$<br>projected onto $\text{SO}(3)$<br><b>(requires SVD)</b> |
| <i>Quaternion</i>             | $d_{\text{quat}}(\mathbf{q}_a, \mathbf{q}_b) = \min (\ \mathbf{q}_a - \mathbf{q}_b\ _2, \ \mathbf{q}_a + \mathbf{q}_b\ _2) = 2 \sin \frac{\theta}{4}$ | Arithmetic mean<br>projected onto unit sphere<br><b>(simple analytic expression)</b>       |

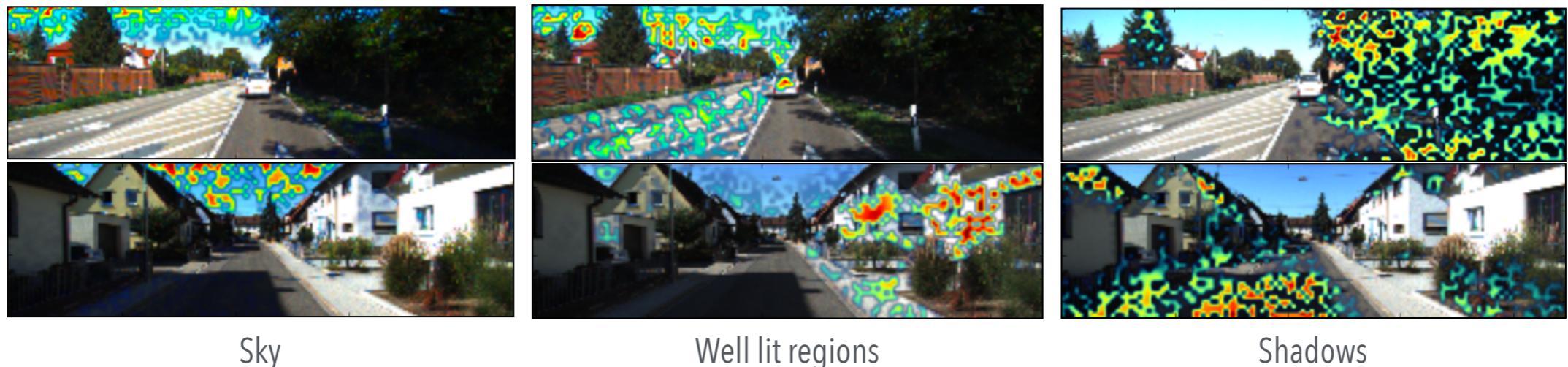
Hartley et al., "Rotation Averaging", IJCV (2013)

# DPC-Net | Rotation Corrections Only?

- ▶ Correcting **rotation only** can be nearly as good as correcting full pose corrections
- ▶ Metric translation information is **difficult to generalize**

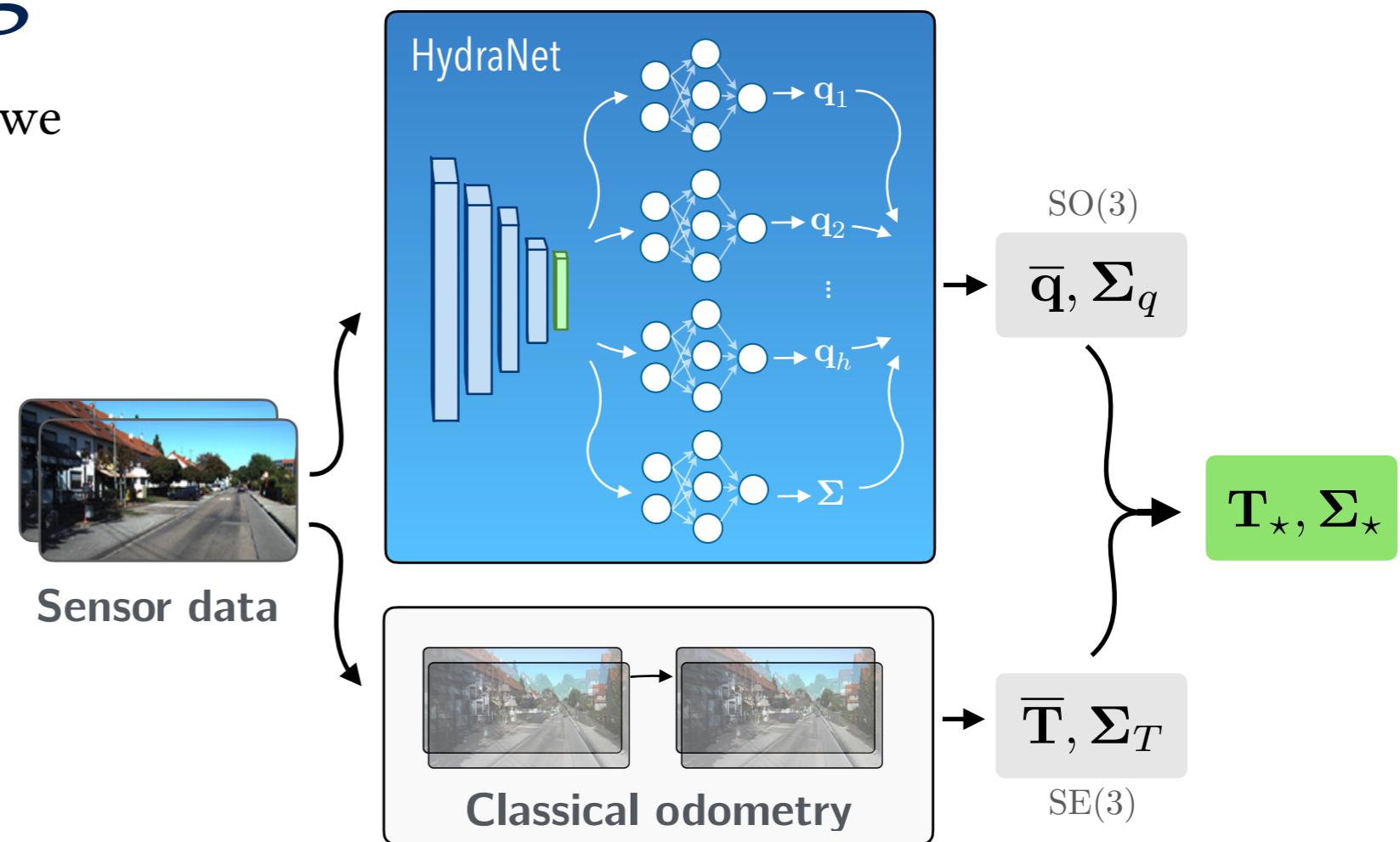
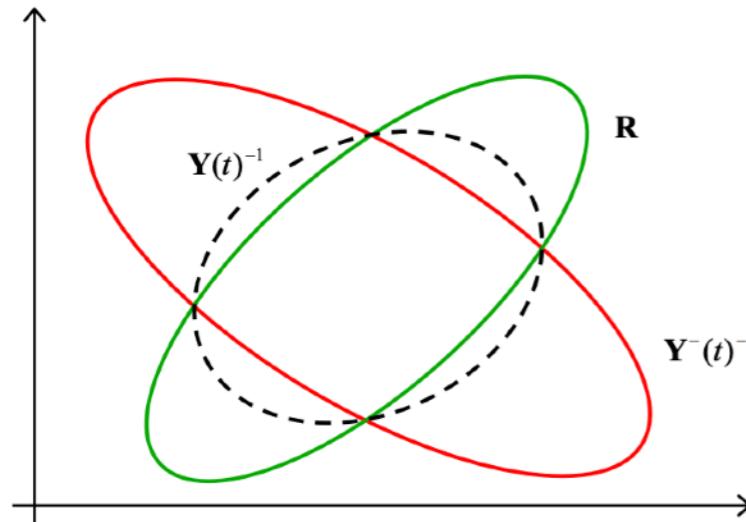


# Sun-BCNN ‘activations’

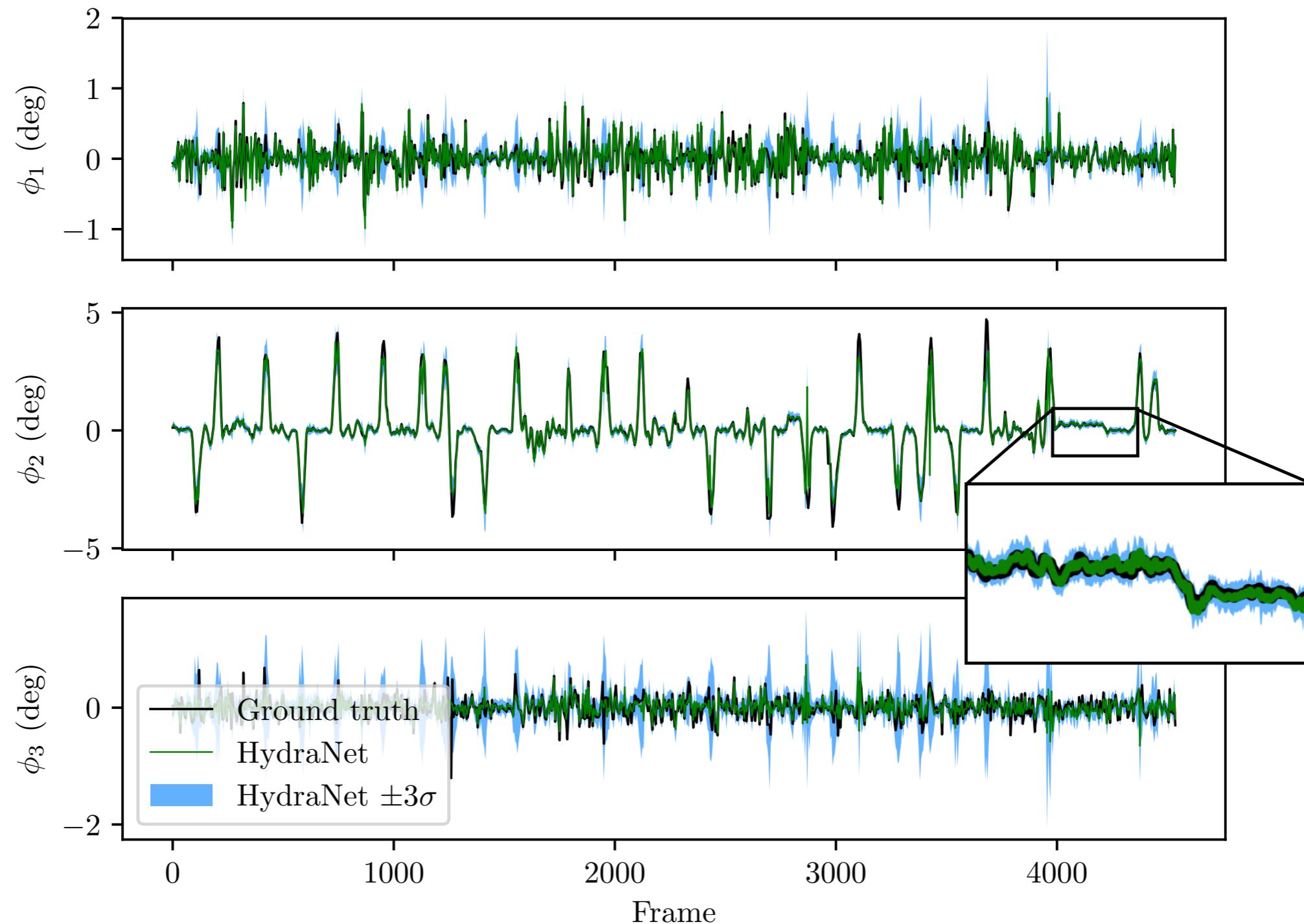


# Double Counting

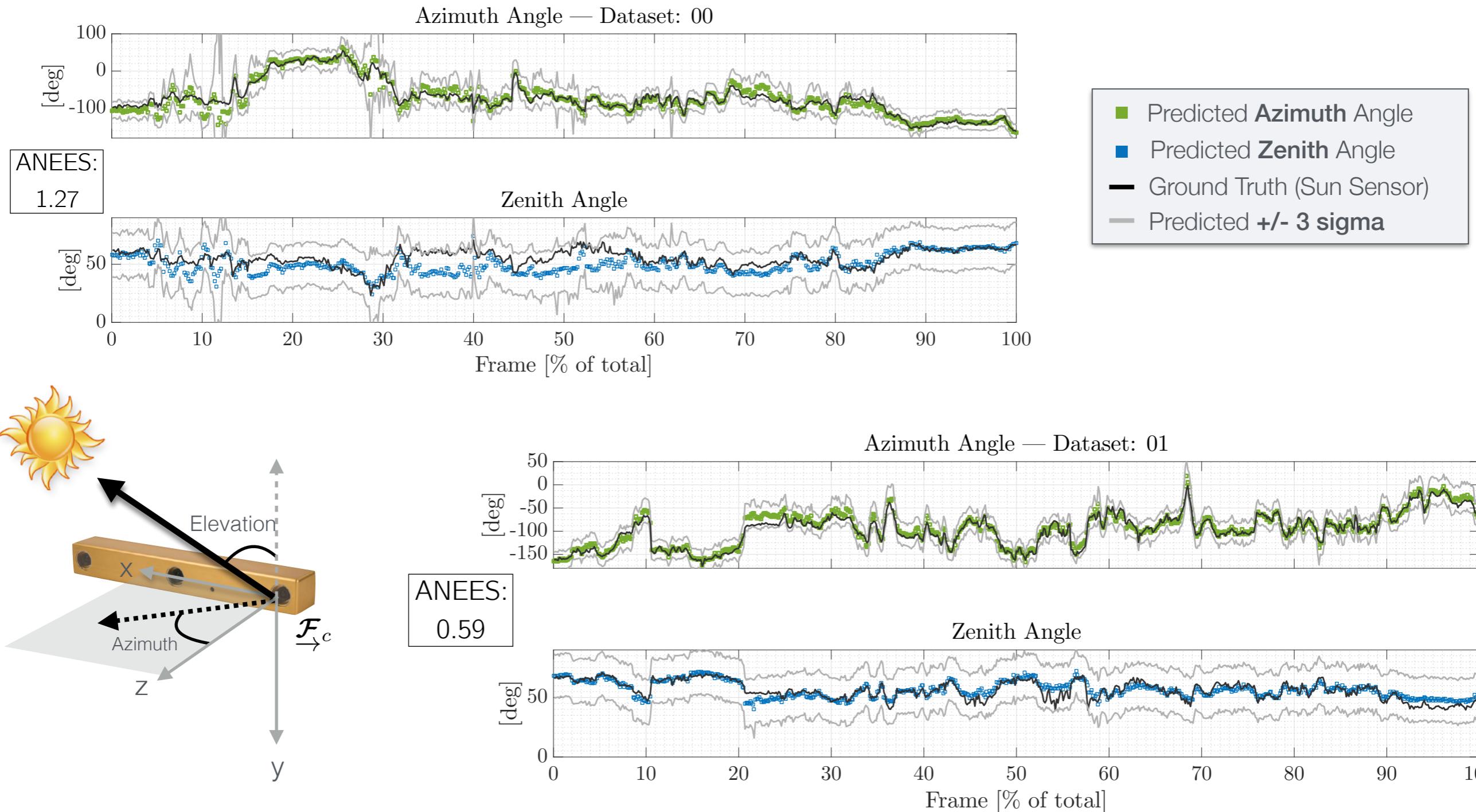
- In order to mitigate correlations, we are investigating to *Covariance Intersection*



# HydraNet Predictions

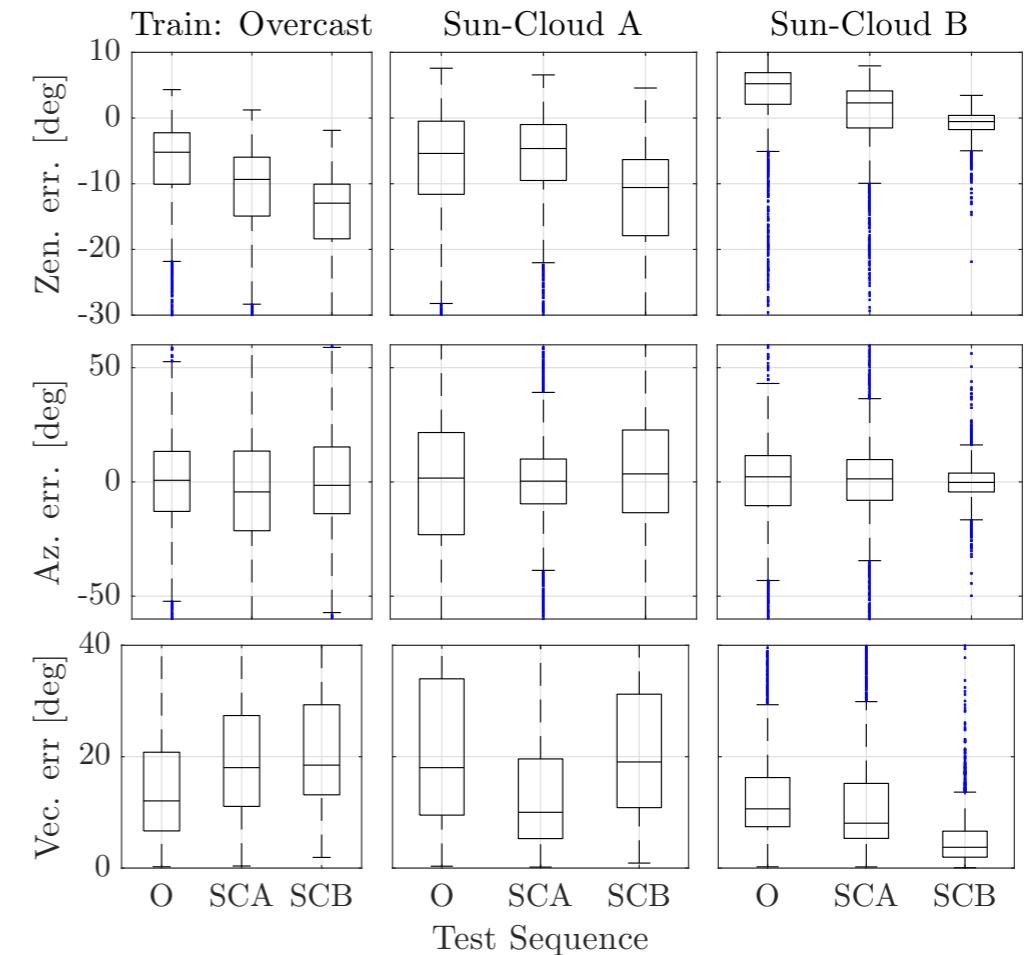
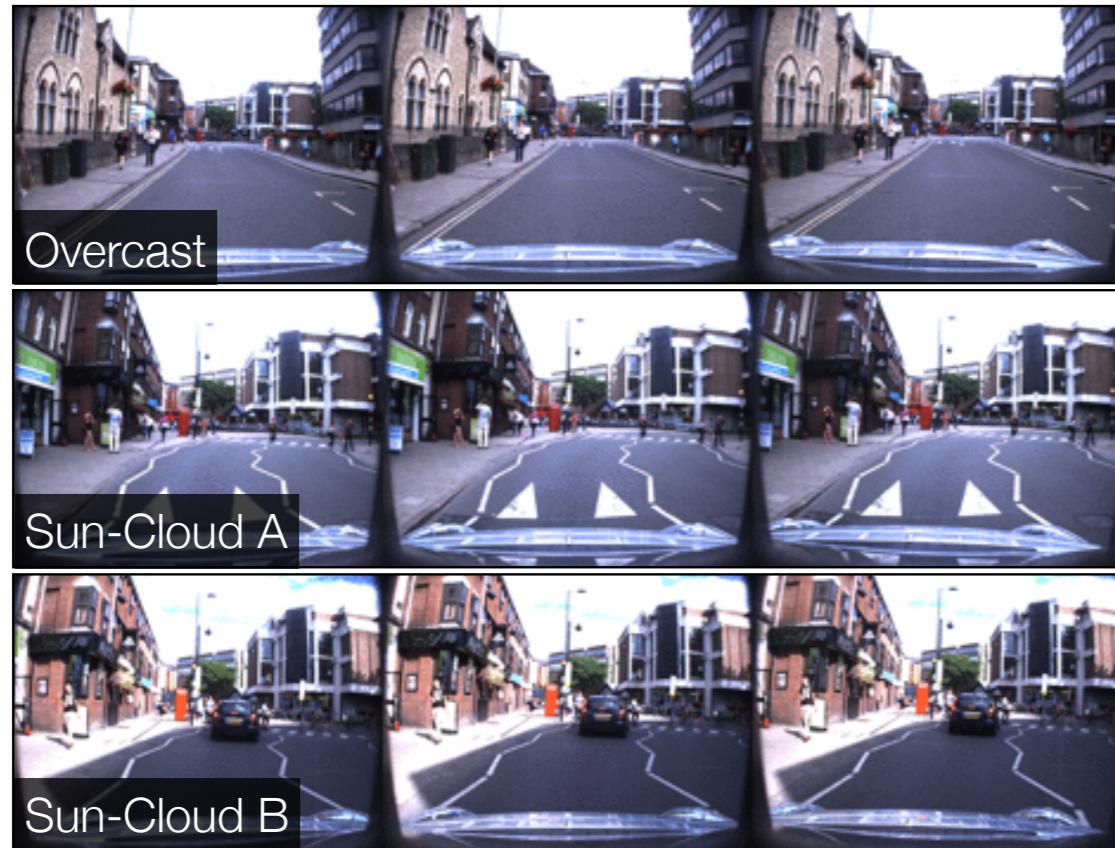


# Visual sun sensing: Devon Island



# Sensitivity analysis

We tested the effect of cloud cover using the **Oxford Robotcar Dataset**



Sun-BCNN works in cloudy conditions, especially when trained in sunny conditions

Maddern et al., 1 year, 1000 km: The Oxford RobotCar dataset, **IJRR (2016)**



# DPC-Net | Improving stereo VO

- We reduce the m-ATE (mean Absolute Trajectory Error) of our estimator by up to 75%

