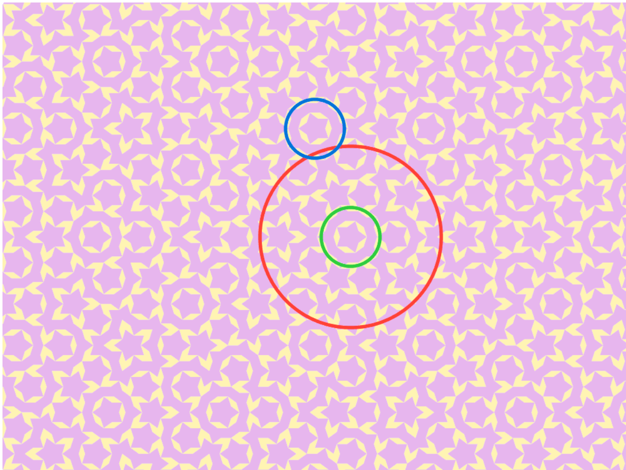

Figure 1: A pattern contained inside (green) circle with diameter d will begin repeating itself within $\frac{\sqrt[3]{\phi}}{2}d$ from its perimeter. This upper bound is shown in red. As predicted, the pattern (blue) begins repeating in this bound.

0.1 DENSITY OF PATTERNS

Graphical representations of Penrose tilings are considered by many to be beautiful, or at the very least aesthetic. This is largely because they apparently have patterns, or finite tile configurations, which repeat regularly throughout the tiling. What's more, once we recognize a pattern, we inevitably find repetitions of it again and again. John Conway showed us that this is no coincidence.

Property 1. *Any pattern or tiling configuration contained inside a circle of radius, d will begin repeating itself within $\frac{\sqrt[3]{\phi}}{2}d$ from the perimeter of this circle. [?]*

We can see in Fig.1, if we consider any pattern inside a Penrose tiling, we are guaranteed that this pattern will begin repeating itself with relative proximity given by Conway's upper bound.

Gardner refers to these patterns as 'towns' [**?**]. He describes a scenario where we fully explore a region of the tiling, called our home town, as it is familiar to us. Perhaps we venture outside of our hometown. Conway's upper bound shows us that, if we walk in the right direction, we are guaranteed to find another town exactly matching ours within a relatively small distance from the perimeter of our town.

Corollary 1. *No point in a Penrose tiling is more than $\frac{\sqrt[3]{\phi}}{2}d$ away from a pattern with diameter d.*

Consider, again, Gardner's towns. If we are transported from our home town to another region in our tiling. Or worse yet, if we are transported wholly into another Penrose tiling altogether. We know from Conway's upper bound, and from the local isomorphism of all Penrose tilings, that we are no more than a relatively short walk from a town which exactly matches our familiar hometown.

Gardner provides us with another analogy to emphasize the remarkable density of the repeating tiles [**?**]. Consider, for contrast, the digits of an unpatterned number like π. It is conjectured that if we choose an arbitrarily long, finite sequence of digits, that we will find it somewhere within π. Consider, for simplicity, a sequence of only 10 digits long. If we begin searching for such a sequence within the digits π we suspect that we will eventually find it. However, this process has no upper bound, so we could be searching for an arbitrarily long amount of time. If these digits behaved like Penrose tiles, we would be guaranteed to find a our finite sequence within some relatively small distance from any random starting location in the decimal expansion. Further, we would also be guaranteed to have that sequence repeat again within another relatively short distance. Of course, this distance depends on the length of the finite sequence we're considering.

So while it is impressive that patterns within Penrose tiles are guaranteed to repeat themselves, what's truly remarkable is the density of these repetitions.