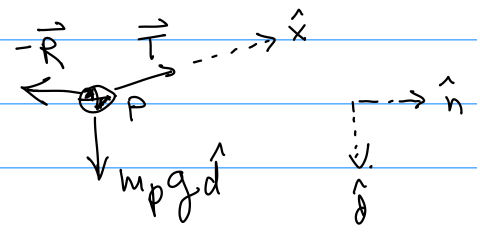
Rocket Equations of Motion w/ Moving Center of Mass

Assumptions.

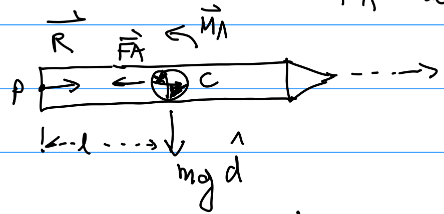
1. Rocket motor is treated as a point mass.
2. Simple variable mass rocket model: $\vec{T} = \mathring{m} u_e$ $\quad \Sigma \vec{F} = m\vec{a}$

FBD Motor



FBD Rocket

$\vec{FA}$: aero forces



$\vec{MA}$: aero moments

$$\vec{T} = \mathring{m}_p u_e \hat{x}$$

$$m_p \overset{e}{\vec{V}}{}^p = m_p\left(\overset{b}{\vec{V}}{}^p + \vec{\omega} \times \vec{V}^p\right)$$

$$= \mathring{m}_p u_e \hat{x} + m_p g \hat{d} - \vec{R}$$

$$m \overset{e}{\vec{V}}{}^c = m\left(\overset{b}{\vec{V}}{}^c + \vec{\omega} \times \vec{V}^c\right)$$

$$= \vec{R} + mg\hat{d} + \vec{FA}$$

$$\frac{{}^e d}{dt}\left(J \vec{\omega}_b\right) = J \mathring{\vec{\omega}}_b + \vec{\omega} \times J . \vec{\omega}_b$$

$$= \vec{MA}_b + (-l\hat{x}) \times \vec{R}_b$$

note $J = J_b^{c}$ $\leftarrow$ @ c $\leftarrow$ in frame b

$$\vec{r}^{\,op} = \vec{r}^{\,oc} + \vec{r}^{\,cp}$$

$$\overset{e}{\vec{r}}{}^{op} \equiv \vec{V}^p = \vec{V}^c + \vec{\omega} \times \vec{r}^{\,cp}$$

$$\overset{b}{\vec{V}}{}^p = \overset{b}{\vec{V}}{}^c + \overset{b}{\vec{\omega}} \times \vec{r}^{\,cp} + \vec{\omega} \times \cancel{\overset{b}{\vec{r}}{}^{cp}}^{\,0}$$

Choose b as working frame.

$$\mathring{\vec{V}}{}_b^p = \mathring{\vec{V}}{}_b^c + \mathring{\vec{\omega}}_b \times \vec{r}_b^{\,cp}$$

$\mathring{\omega}_b$ would cause a coupling & be more difficult to solve EOM's

small, ignore

small, ignore

$$\boxed{\begin{aligned} m_p\left(\mathring{\vec{V}}{}_b^c + \mathring{\vec{\omega}}_b \times \vec{r}_b^{\,cp} + \vec{\omega}_b \times \vec{V}_b^c + \vec{\omega}_b \times \vec{\omega}_b \times \vec{r}_b^{\,cp}\right) \\ = \mathring{m}_p u_e \hat{x} + m_p g \hat{d} - \vec{R}_b \end{aligned}}$$

$\vec{R}_b$ dominated by $\mathring{m}_p u_e$ & $m_p g \hat{d}$

$$\boxed{m\left(\mathring{\vec{V}}{}_b^c + \vec{\omega}_b \times \vec{V}_b^c\right) = \vec{R}_b + mg\hat{d} + \vec{FA}_b}$$

$$\boxed{J \mathring{\vec{\omega}}_b + \vec{\omega}_b \times J \vec{\omega}_b = \vec{MA}_b + (-l\hat{x}) \times \vec{R}_b}$$

$\Downarrow$

solve for $\mathring{\vec{\omega}}_b$ & $\mathring{\vec{V}}{}_b^c$

$$\vec{R}_b \approx m_p u_e \hat{x} + m_p g \hat{d} - m_p\left(\mathring{\vec{V}}_b^{\,c} + \vec{\omega}_b \times \vec{V}_b^{\,c}\right)$$

$$m\left(\mathring{\vec{V}}_b^{\,c} + \vec{\omega}_b \times \vec{V}_b^{\,c}\right) = (m+m_p) g \hat{d} + m_p u_e \hat{x} - m_p\left(\mathring{\vec{V}}_b^{\,c} + \vec{\omega}_b \times \vec{V}_b^{\,c}\right) + \vec{F_{A_b}}$$

m_T

$$(m+m_p)\left(\mathring{\vec{V}}_b^{\,c} + \vec{\omega}_b \times \vec{V}_b^{\,c}\right) = (m+m_p) g \hat{d} + m_p u_e \hat{x} + \vec{F_{A_b}}$$

$$\boxed{\mathring{\vec{V}}_b^{\,c} = g\hat{d} + \frac{m_p u_e}{m_T}\hat{x} + \frac{\vec{F_{A_b}}}{m_T} - \vec{\omega}_b \times \vec{V}_b^{\,c}}$$

$$\mathring{\vec{\omega}}_b = J^{-1}\left[\vec{M_{A_b}} + \left[m_p u_e \hat{x} + m_p g \hat{d} - m_p\left(\mathring{\vec{V}}_b^{\,c} + \vec{\omega}_b \times \vec{V}_b^{\,c}\right)\right] \times (l\hat{x}) - \vec{\omega}_b \times J\vec{\omega}_b\right]$$

$$\boxed{\mathring{\vec{\omega}}_b = J^{-1}\left[\vec{M_{A_b}} - \frac{m_p \vec{F_{A_b}}}{(m+m_p)} \times (l\hat{x}) - \vec{\omega}_b \times J\vec{\omega}_b\right]}$$

makes sense, gravity never rotates vehicle, but aerodynamics can after c.g. moves.