Text as Data

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August 16th, 2017

Discovery and Measurement

What is the research process? (Grimmer, Roberts, and Stewart 2017)

- 1) Discovery: a hypothesis or view of the world
- 2) Measurement according to some organization
- 3) Causal Inference: effect of some intervention

Text as data methods assist at each stage of research process

Text as Data Methods for Discovery

Text as Data Methods for Discovery Goal: Automatically Discover Organization (Similar Groups)

Consider a document-term matrix

$$X = \begin{pmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 3 \end{pmatrix}$$

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Suppose documents live in a space \leadsto rich set of results from linear algebra

- Provides a geometry modify with word weighting
 - Natural notions of distance
 - Building block for clustering, supervised learning, and scaling

Doc1 =
$$(1, 1, 3, ..., 5)$$

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Doc2 = $(2, 0, 0, ..., 1)$

$$\begin{array}{rcl} \mathsf{Doc1} & = & (1,1,3,\dots,5) \\ \mathsf{Doc2} & = & (2,0,0,\dots,1) \\ \mathsf{Doc1}, \mathsf{Doc2} & \in & \Re^J \end{array}$$

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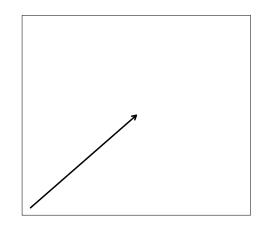
Doc1 · **Doc2** =
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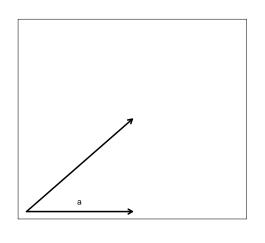
= $1 \times 2 + 1 \times 0 + 3 \times 0 + ... + 5 \times 1$

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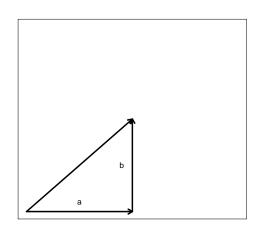
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= $1 \times 2 + 1 \times 0 + 3 \times 0 + ... + 5 \times 1$
= 7

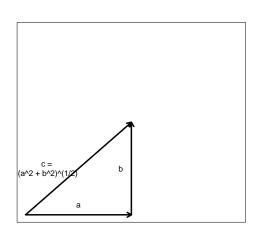




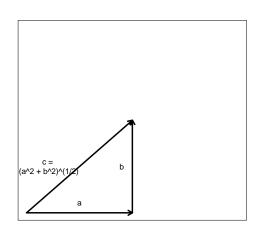
- Pythogorean Theorem: Side with length *a*



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- Side with length *b* and right triangle



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- $c = \sqrt{a^2 + b^2}$



- Pythogorean Theorem: Side with length *a*
- Side with length b and right triangle
- $c = \sqrt{a^2 + b^2}$
- This is generally true

Vector (Euclidean) Length

Definition

Suppose $\mathbf{v} \in \Re^J$. Then, we will define its length as

$$||\mathbf{v}|| = (\mathbf{v} \cdot \mathbf{v})^{1/2}$$

= $(v_1^2 + v_2^2 + v_3^2 + \dots + v_J^2)^{1/2}$

Initial guess \leadsto Distance metrics Properties of a metric: (distance function) $d(\cdot,\cdot)$. Consider arbitrary documents \boldsymbol{X}_i , \boldsymbol{X}_j , \boldsymbol{X}_k

Initial guess → Distance metrics

1)
$$d(\boldsymbol{X}_i, \boldsymbol{X}_j) \geq 0$$

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- 1) $d(X_i, X_j) \ge 0$
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Explore distance functions to compare documents -->

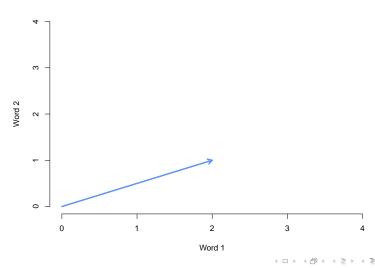
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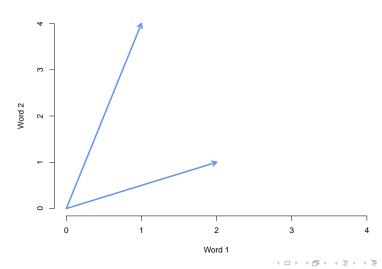
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Explore distance functions to compare documents Do we want additional assumptions/properties?

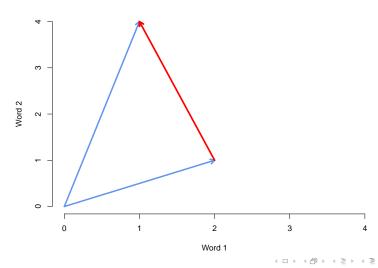
Euclidean Distance



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Euclidean Distance



Definition

The Euclidean distance between documents X_i and X_j as

$$||X_i - X_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

Measuring the Distance Between Documents

Definition

The Euclidean distance between documents X_i and X_j as

$$||X_i - X_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

Suppose $X_i = (1,4)$ and $X_j = (2,1)$. The distance between the documents is:

$$||(1,4) - (2,1)|| = \sqrt{(1-2)^2 + (4-1)^2}$$

= $\sqrt{10}$

What properties should similarity measure have?

- Maximum: document with itself

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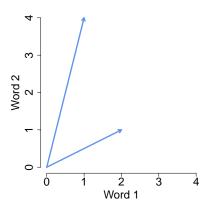
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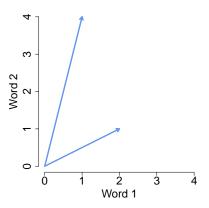
How should additional words be treated?

Measuring Similarity



Measure 1: Inner product

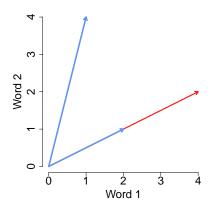
Measuring Similarity

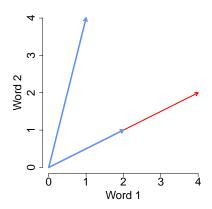


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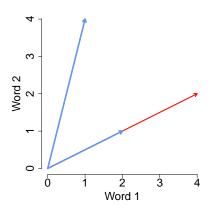
$$(2,1)^{'} \cdot (1,4) = 6$$





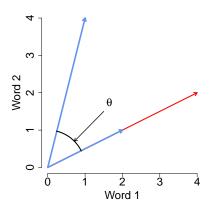


Problem(?): length dependent



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$$(4,2)^{'}(1,4) = 12$$



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 $a \cdot b = ||a|| \times ||b|| \times \cos \theta$

→ロト ←部ト ← 差ト ← 差 → りへで

$$\cos \theta = \left(\frac{a}{||a||}\right) \cdot \left(\frac{b}{||b||}\right)$$

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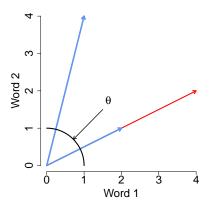
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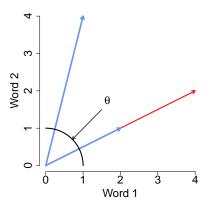
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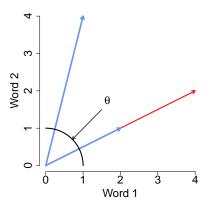
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(0.89, 0.45)'(0.24, 0.97) = 0.65$$



 $\cos \theta$: removes document length from similarity measure



 $\cos\theta$: removes document length from similarity measure Projects texts to unit length representation \leadsto onto sphere



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How to generate weights?

- Assumptions about separating words
- Use training set to identify separating words (Monroe, Ideology measurement)

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 $idf = (idf_1, idf_2, ..., idf_J)$

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Why log?

- Maximum at $n_i = 1$
- Decreases at rate $\frac{1}{n_j} \Rightarrow$ diminishing "penalty" for more common use
- Other functional forms are fine, embed assumptions about penalization of common use

$$\mathbf{X}_{i,\mathrm{idf}} \equiv \underbrace{\mathbf{X}_{i}}_{\mathrm{sf}} \times \mathrm{idf} = (X_{i1} \times \mathrm{idf}_{1}, X_{i2} \times \mathrm{idf}_{2}, \dots, X_{iJ} \times \mathrm{idf}_{J})$$

$$\begin{aligned} \mathbf{X}_{i,\mathrm{idf}} &\equiv \underbrace{\mathbf{X}_i}_{\mathrm{tf}} \times \mathrm{idf} &= (X_{i1} \times \mathrm{idf}_1, X_{i2} \times \mathrm{idf}_2, \dots, X_{iJ} \times \mathrm{idf}_J) \\ \mathbf{X}_{j,\mathrm{idf}} &\equiv \mathbf{X}_j \times \mathrm{idf} &= (X_{j1} \times \mathrm{idf}_1, X_{j2} \times \mathrm{idf}_2, \dots, X_{jJ} \times \mathrm{idf}_J) \end{aligned}$$

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$$\mathbf{X}_{i,\mathrm{idf}} \cdot \mathbf{X}_{j,\mathrm{idf}} = (\mathbf{X}_i \times \mathrm{idf})' (\mathbf{X}_j \times \mathrm{idf})$$

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Inner Product

$$\mathbf{X}_{i,\mathrm{idf}} \cdot \mathbf{X}_{j,\mathrm{idf}} = (\mathbf{X}_i \times \mathbf{idf})'(\mathbf{X}_j \times \mathbf{idf})$$

$$= (\mathrm{idf}_1^2 \times X_{i1} \times X_{j1}) + (\mathrm{idf}_2^2 \times X_{i2} \times X_{j2}) + \dots + (\mathrm{idf}_J^2 \times X_{iJ} \times X_{jJ})$$

Define:

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$$\mathbf{\Sigma} = \begin{pmatrix} \mathsf{idf}_1^2 & 0 & 0 & \dots & 0 \\ 0 & \mathsf{idf}_2^2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \mathsf{idf}_J^2 \end{pmatrix}$$

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If we use tf-idf for our documents, then

$$d_2(\boldsymbol{X}_i, \boldsymbol{X}_j) = \sqrt{\sum_{m=1}^{J} (x_{im,idf} - x_{jm,idf})^2}$$
$$= \sqrt{(\boldsymbol{X}_i - \boldsymbol{X}_j)' \boldsymbol{\Sigma} (\boldsymbol{X}_i - \boldsymbol{X}_j)}$$

Final Product

Applying some measure of distance, similarity (if symmetric) yields:

$$\mathbf{D} = \begin{pmatrix} 0 & d(1,2) & d(1,3) & \dots & d(1,N) \\ d(2,1) & 0 & d(2,3) & \dots & d(2,N) \\ d(3,1) & d(3,2) & 0 & \dots & d(3,N) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d(N,1) & d(N,2) & d(N,3) & \dots & 0 \end{pmatrix}$$

Lower Triangle contains unique information N(N-1)/2

Clustering

Fully Automated Clustering

- 1) Distance metric when are documents close?
- 2) Objective function → how do we summarize distances?
- 3) Optimization method \leadsto how do we find optimal clustering?

THERE IS NO A PRIORI OPTIMAL METHOD Computer Assisted Clustering (Grimmer and King, 2011)

- crucial to combine human and computer insights

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 $\theta_k = exemplar for cluster k$

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 $\theta_k = \frac{\mathsf{exemplar}}{\mathsf{exemplar}}$ for cluster k

2) T is an $N \times J$ matrix. Each row is an indicator vector.

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 $\theta_k = exemplar for cluster k$

2) T is an $N \times J$ matrix. Each row is an indicator vector. If observation i is from cluster k, then

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Hard Assignment



Assume squared euclidean distance

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Coordinate descent

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In words: Assign each document x_i to the closest center θ_m^t

$$f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k = \sum_{i=1}^N \tau_{ik}^t \left(\sum_{j=1}^J (x_{ij} - \theta_{jk})^2 \right)$$

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$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_i}{\sum_{i=1}^{N} \tau_{ik}}$$

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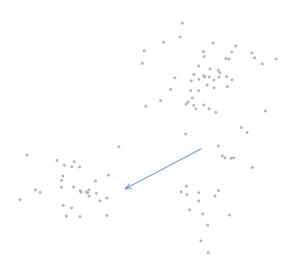
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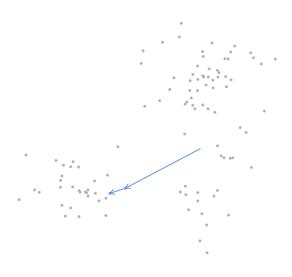
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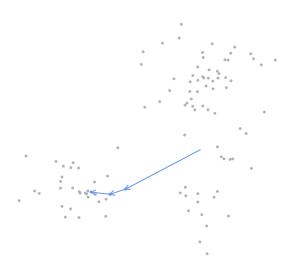
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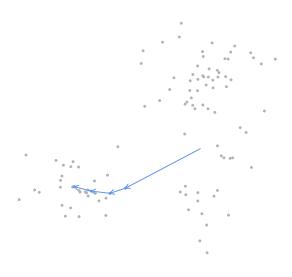
Visual Example

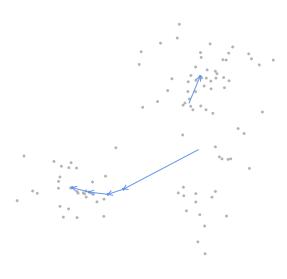


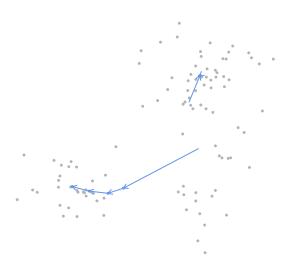


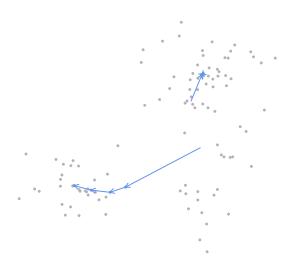


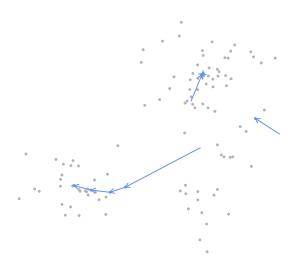


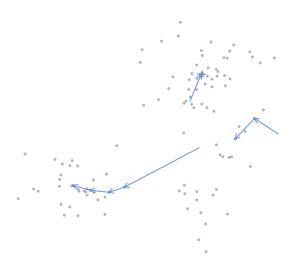


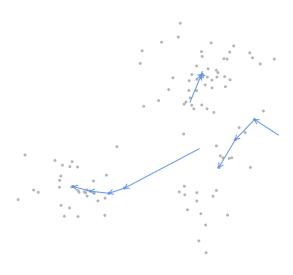


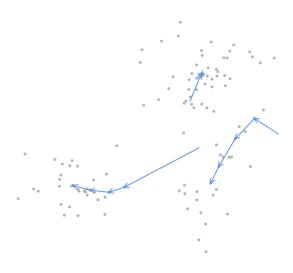


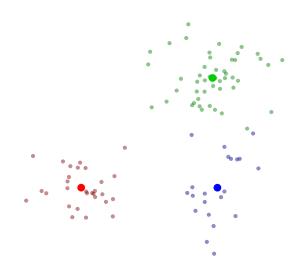












An Example: Jeff Flake

To the R Code!

Unsupervised methods

Unsupervised methods→ low startup costs, high post-model costs

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Think!

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Mixture of von Mises-Fisher (vMF) distributions:

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■ $\tau_i \rightsquigarrow$ Each document's cluster assignment

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Provides:

- $\blacksquare \tau_i \leadsto \mathsf{Each} \; \mathsf{document's} \; \mathsf{cluster} \; \mathsf{assignment}$
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A Motivating Clustering Model → Mixture of von Mises Fisher Distributions

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EM algorithm in slides appendix of Class 10 for my text as data course 🔗

How well does our model perform?

How well does our model perform?→ predict new documents?

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$$\log p(\boldsymbol{x}_{\text{out}}^*|\boldsymbol{\mu},\boldsymbol{\pi},\boldsymbol{X}) = \log \sum_{k=1}^K p(\boldsymbol{x}_{\text{out}}^*,\tau_{ik}|\boldsymbol{\mu}_k,\boldsymbol{\pi},\boldsymbol{X})$$

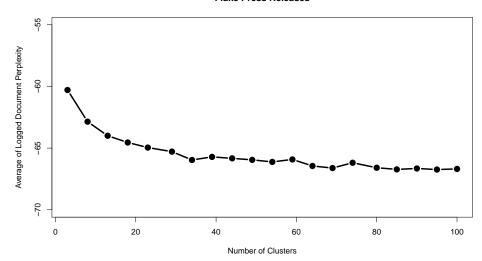
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$$\begin{aligned} \log p(\boldsymbol{x}_{\text{out}}^*|\boldsymbol{\mu}, \boldsymbol{\pi}, \boldsymbol{X}) &= \log \sum_{k=1}^K p(\boldsymbol{x}_{\text{out}}^*, \tau_{ik} | \boldsymbol{\mu}_k, \boldsymbol{\pi}, \boldsymbol{X}) \\ &= \log \sum_{k=1}^K \left[\pi_k \exp(\kappa \boldsymbol{\mu}_k' \boldsymbol{x}_{\text{out}}^*) \right] \end{aligned}$$

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Flake Press Releases



- Prediction → One Task

(Roberts, et al 2017

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Different strategy → measure quality in topics and clusters

- Statistics: measure cohesiveness and exclusivity (Roberts, et al 2017 Forthcoming)
- Experiments: measure topic and cluster quality

Mathematical approaches

Mathematical approaches → suppose we can capture quality with numbers assumes we're in the model → including text representation

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Humans → read texts

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Do humans think the model is performing well?

- 1) Topic Quality
- 2) Cluster Quality

- 1) Take *M* top words for a topic
- 2) Randomly select a top word from another topic
 - 2a) Sample the topic number from I from K-1 (uniform probability)
 - 2b) Sample word j from the M top words in topic l
 - 2c) Permute the words and randomly insert the intruder:
 - List:

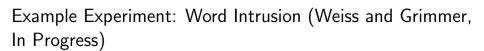
test =
$$(v_{k,3}, v_{k,1}, v_{l,j}, v_{k,2}, v_{k,4}, v_{k,5})$$

bowl, flooding, olympic, olympics, nfl, coach

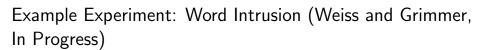
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stocks, investors, fed, guns, trading, earning

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Higher rate of intruder identification → more exclusive/cohesive topics



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Deploy on Mechanical Turk

Cluster Quality (Grimmer and King 2011)

Assessing Cluster Quality with experiments

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Design to assess cluster quality

- Estimate clusterings

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- Select clustering with highest cluster quality
- Can be used to compare any clusterings, regardless of source

How do we Choose K?

Generate many candidate models

- 1) Assess using numerical values
- 2) Use experiments
- 3) Read
- 4) Final decision --> combination

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Deep problem in cluster analysis literature: full automation requires more information

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- Our answer: a geography of clusterings

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- 8) (Or, our new strategy: represent entire Bell space directly; no need to examine document contents)

Crosas, Grimmer, King, and Stewart (2017) → Consilience

Consilience.com example (email me for assignment + access)

- Paper (Grimmer and King 2011): introduce new evaluation methods (like Cluster Quality)

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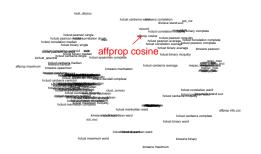
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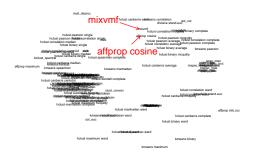
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- Apply our method (relying on many clustering algorithms)





Each point is a clustering Affinity Propagation-Cosine (Dueck and Frey 2007)



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Close to:

Mixture of von Mises-Fisher distributions (Banerjee et. al. 2005)

⇒ Similar clustering of documents



Space between methods:



Space between methods:



Space between methods: local cluster ensemble





Found a region with clusterings that all reveal the same important insight



Mixture:



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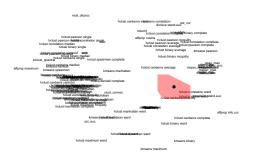
0.39 Hclust-Canberra-McQuitty

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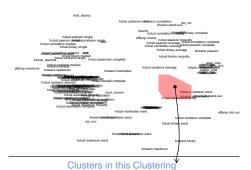
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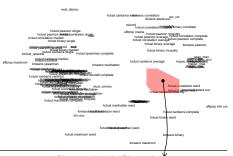
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Clusters in this Clustering



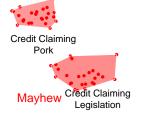
Credit Claiming, Pork:

"Sens. Frank R. Lautenberg (D-NJ) and Robert Menendez (D-NJ) announced that the U.S. Department of Commerce has awarded a \$100,000 grant to the South Jersey Economic Development District"

Mayhew

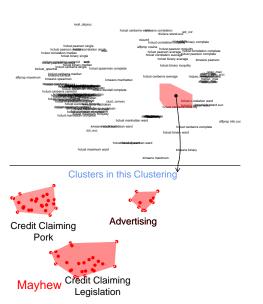


Clusters in this Clustering



Credit Claiming, Legislation:

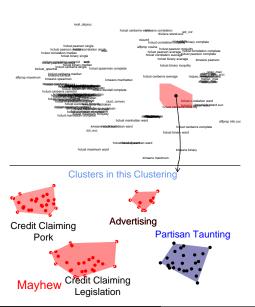
"As the Senate begins its recess, Senator Frank Lautenberg today pointed to a string of victories in Congress on his legislative agenda during this work period"



Advertising:

"Senate Adopts Lautenberg/Menendez Resolution Honoring Spelling Bee Champion from New Jersey"

Example Discovery: Partisan Taunting



Partisan Taunting:

"Republicans Selling Out Nation on Chemical Plant Security"

Important Concept Overlooked in Mayhew's (1974) typology



Sen. Lautenberg on Senate Floor 4/29/04 "Senator Lautenberg Blasts Republicans as 'Chicken Hawks' " [Government Oversight]

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Consequences for representation: Deliberative, Polarization, Policy



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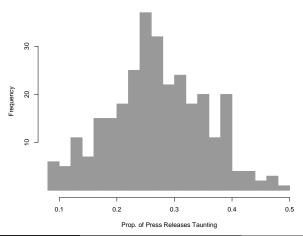
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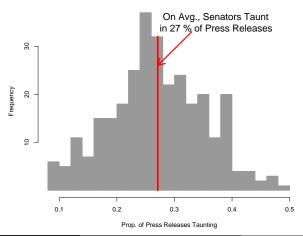
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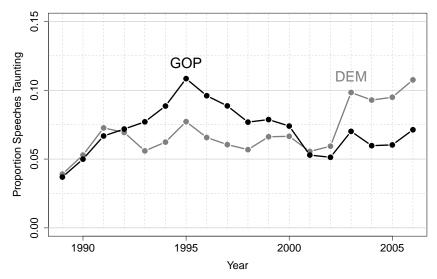
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Over Time Tauting Rates in Speeches



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 - Observe group with most productivity 20-30 years later
 - To identify limits of methods, when to use which approach, need evaluations for the usefulness of conceptualizations

Stylometry Who Wrote Disputed Federalist Papers?

Federalist papers → Mosteller and Wallace (1963)

- Persuade citizens of New York State to adopt constitution
- Canonical texts in study of American politics
- 77 essays
 - Published from 1787-1788 in Newspapers
 - And under the name Publius, anonymously

Who Wrote the Federalist papers?

- Jay wrote essays 2, 3, 4,5, and 64
- Hamilton: wrote 43 papers
- Madison: wrote 12 papers

Disputed: Hamilton or Madison?

- Essays: 49-58, 62, and 63
- Joint Essays: 18-20

Task: identify authors of the disputed papers.

Task: Classify papers as Hamilton or Madison using dictionary methods

Setting up the Analysis

Training → papers Hamilton, Madison are known to have authored Test → unlabeled papers Preprocessing:

- Hamilton/Madison both discuss similar issues
- Differ in extent they use stop words
- Focus analysis on the stop words

Setting up the Analysis

- $\mathbf{Y} = (Y_1, Y_2, ..., Y_N) = (Hamilton, Hamilton, Madison, ..., Hamilton)$ $N \times 1$ matrix with author labels
- Define the number of words in federalist paper i as num $_i$

$$\mathbf{X} = \begin{pmatrix} \frac{1}{\mathsf{num}_1} & \frac{2}{\mathsf{num}_1} & \frac{0}{\mathsf{num}_1} & \cdots & \frac{3}{\mathsf{num}_1} \\ \frac{0}{\mathsf{num}_2} & \frac{1}{\mathsf{num}_2} & \frac{0}{\mathsf{num}_2} & \cdots & \frac{0}{\mathsf{num}_2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{0}{\mathsf{num}_N} & \frac{0}{\mathsf{num}_N} & \frac{1}{\mathsf{num}_N} & \cdots & \frac{0}{\mathsf{num}_N} \end{pmatrix}$$

 $N \times J$ counting stop word usage rate

-
$$\theta = (\theta_1, \theta_2, \dots, \theta_J)$$

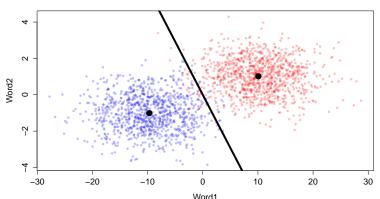
Word weights.

Objective Function

Heuristically: find $\theta^* = (\theta_1^*, \theta_2^*, \dots, \theta_I^*)$ used to create score

$$p_i = \sum_{j=1}^J \theta_j^* X_{ij}$$

that maximally discriminates between categories



Objective Function

Define:

$$oldsymbol{\mu}_{\mathsf{Madison}} = rac{1}{N_{\mathsf{Madison}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) oldsymbol{\chi}_i$$
 $oldsymbol{\mu}_{\mathsf{Hamilton}} = rac{1}{N_{\mathsf{Hamilton}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) oldsymbol{\chi}_i$

Objective Function

We can then define functions that describe the "projected" mean and variance for each author

$$g(\theta, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison}) = \frac{1}{N_{\mathsf{Madison}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) \boldsymbol{\theta}' \boldsymbol{X}_i = \boldsymbol{\theta}' \boldsymbol{\mu}_{\mathsf{Madison}}$$

$$g(\theta, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Hamilton}) = \frac{1}{N_{\mathsf{Hamilton}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) \boldsymbol{\theta}' \boldsymbol{X}_i = \boldsymbol{\theta}' \boldsymbol{\mu}_{\mathsf{Hamilton}}$$

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Objective Function --> Optimization

$$\begin{split} f(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}) &= \frac{\left(g(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Hamilton}) - g(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison})\right)^2}{s(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Hamilton}) + s(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison})} \\ &= \frac{\left(\boldsymbol{\theta}'(\boldsymbol{\mu}_{\mathsf{Hamilton}} - \boldsymbol{\mu}_{\mathsf{Madison}})\right)^2}{\mathsf{Scatter}_{\mathsf{Hamilton}} + \mathsf{Scatter}_{\mathsf{Madison}}} \end{split}$$

Optimization \rightsquigarrow find θ^* to maximize $f(\theta, X, Y)$, assuming independence across dimensions.

(Fisher's) Linear Discriminant Analysis

Optimization >>> Word Weights

For each word j, construct weight θ_j^* ,

$$\begin{array}{ll} \mu_{j,\mathsf{Hamilton}} & = & \frac{\sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) X_{ij}}{\sum_{j=1}^{J} \sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) X_{ij}} \\ \mu_{j,\mathsf{Madison}} & = & \frac{\sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) X_{ij}}{\sum_{j=1}^{J} \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) X_{ij}} \\ \sigma_{j,\mathsf{Hamilton}}^2 & = & \mathsf{Var}(X_{i,j} | \mathsf{Hamilton}) \\ \sigma_{j,\mathsf{Madison}}^2 & = & \mathsf{Var}(X_{i,j} | \mathsf{Madison}) \end{array}$$

We can then generate weight θ_i^* as

$$\theta_{j}^{*} = \frac{\mu_{j, \text{Hamilton}} - \mu_{j, \text{Madison}}}{\sigma_{j, \text{Hamilton}}^{2} + \sigma_{j, \text{Madison}}^{2}}$$

Optimization \sim Trimming the Dictionary

- Trimming weights: Focus on discriminating words (very simple regularization)
- Cut off: For all $|\theta_i^*| < 0.025$ set $\theta_i^* = 0$.

Classification → Determining Authorship

For each disputed document i, compute discrimination statistic

$$p_i = \sum_{j=1}^J \theta_j^* X_{ij}$$

 $p_i \rightsquigarrow \text{classification (linear discriminator)}$

- Above midpoint in training set \rightarrow Hamilton text
- Below midpoint in training set \rightarrow Madison text

Findings: Madison is the author of the disputed federalist papers.

 ${\sf Classification} {\leadsto} \ {\sf Custom} \ {\sf Dictionaries}$

Classification → Custom Dictionaries

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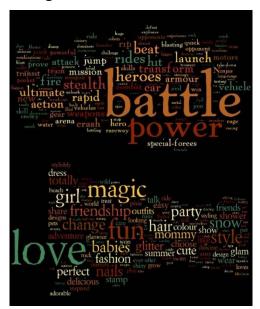
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Vague and Difficult to derive before hand

Congressional Press Releases and Floor Speeches

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- Partial answer: identify words that distinguish press releases and floor speeches

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 - Minimum : All documents in one category
- Conditional uncertainty (X_j) (conditional entropy)
 - Condition on presence of word X_i
 - Randomly sample a press release
 - Guess press release/floor statement
 - Word presence reduces uncertainty
 - Unrelated: Conditional uncertainty = uncertainty
 - Perfect predictor: Conditional uncertainty = 0
- Mutual information(X_j): uncertainty conditional uncertainty (X_j)
 - Maximum: Uncertainty $\rightarrow X_i$ is perfect predictor
 - Minimum: $0 \rightarrow X_i$ fails to separate speeches and floor statements

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- log₂? Encodes bits
- Maximum: Pr(Press) = Pr(Speech) = 0.5
- Minimum: $Pr(Press) \rightarrow 0 \text{ (or } Pr(Press) \rightarrow 1)$

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- Maximum: X_j unrelated to Press Releases/Floor Speeches
- Minimum: X_j is a perfect predictor of press release/floor speech

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$$(X_j) = H(Doc) - H(Doc|X_j)$$

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Bigger mutual information \Rightarrow better discrimination

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Bigger mutual information \Rightarrow better discrimination

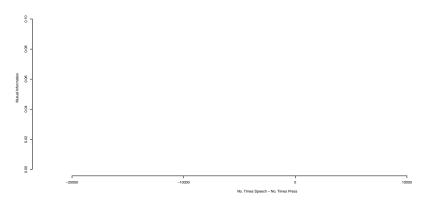
Objective function and optimization \leadsto estimate probabilities that we then place in mutual information

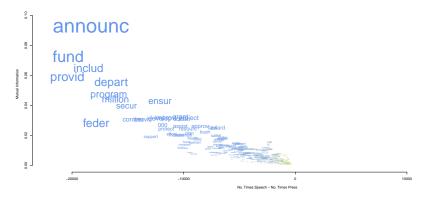
Formula for mutual information (based on ML estimates of probabilities)

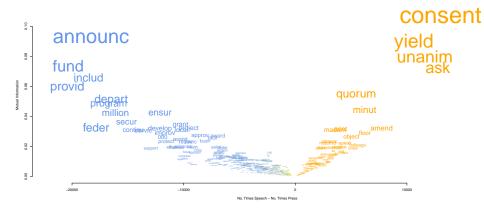
```
n_p = Number Press Releases
  n_s = Number of Speeches
   D = n_p + n_s
  n_j = \sum_{i=1}^D X_{i,j} (No. docs X_j appears)
 n_{-i} = No. docs X_i does not appear
 n_{i,p} = No. press and X_i
 n_{i,s} = No. speech and X_i
n_{-i,p} = No. press and not X_i
n_{-i,s} = No. speech and not X_i
```

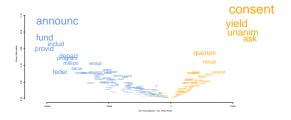
Formula for Mutual Information

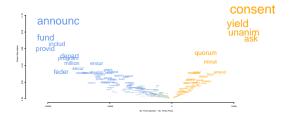
$$MI(X_{j}) = \frac{n_{j,p}}{D} \log_{2} \frac{n_{j,p}D}{n_{j}n_{p}} + \frac{n_{j,s}}{D} \log_{2} \frac{n_{j,s}D}{n_{j}n_{s}} + \frac{n_{-j,p}}{D} \log_{2} \frac{n_{-j,p}D}{n_{-j}n_{p}} + \frac{n_{-j,s}}{D} \log_{2} \frac{n_{-j,s}D}{n_{-j}n_{s}}.$$





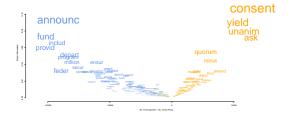




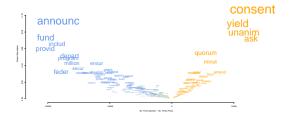


What's Different?

- Press Releases: Credit Claiming



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- Floor Speeches: Procedural Words

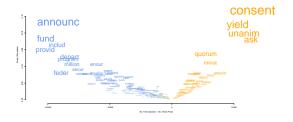


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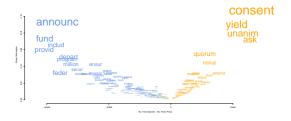
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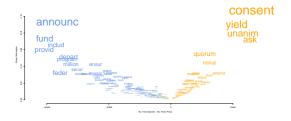
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- Sample 500 Press Releases, 500 Floor Speeches
- Credit Claiming: 36% Press Releases, 4% Floor Speeches

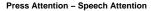
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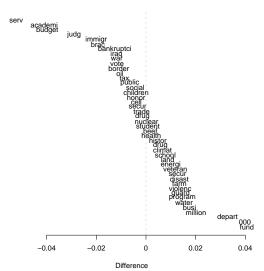


What's Different?

- Press Releases: Credit Claiming
- Floor Speeches: Procedural Words
- Validate: Manual Classification
- Sample 500 Press Releases, 500 Floor Speeches
- Credit Claiming: 36% Press Releases, 4% Floor Speeches
- Procedural: 0% Press Releases, 44% Floor Speeches

What's Different About Press Releases





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Strategy Construct objective function on *proportions* (and then calculate log-odds)

Suppose we're interested in how a word separates partisan speech.

 $\mathbf{Y} = (Republican, Republican, Democrat, \dots, Republican)$

X =Unnormalized matrix of word counts $N \times J$ Define

$$\mathbf{x}_{\mathsf{Republican}} = (\sum_{i=1}^{N} I(Y_i = \mathsf{Republican}) X_{i1}, \sum_{i=1}^{N} I(Y_i = \mathsf{Republican}) X_{i2}, \dots, \sum_{i=1}^{N} I(Y_i = \mathsf{Republican}) X_{iJ})$$

with $N_{Republican} = Total$ number of Republican words

 $\pi_{\mathsf{Republican}} \ \sim \ \mathsf{Dirichlet}(lpha)$

```
m{\pi}_{\mathsf{Republican}} \sim \mathsf{Dirichlet}(m{lpha}) \ m{x}_{\mathsf{Republican}} | m{\pi}_{\mathsf{Republican}} \sim \mathsf{Multinomial}(m{N}_{\mathsf{Republican}}, m{\pi}_{\mathsf{Republican}})
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$$p(\pmb{\pi}|\pmb{lpha},\pmb{X},\pmb{Y}) \; \propto \; p(\pmb{\pi}|\pmb{lpha})p(\pmb{x}_{\mathsf{Republican}}|\pmb{\pi}\pmb{lpha},\pmb{Y})$$

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Calculating Log Odds Ratio

Define log Odds Ratio; as

$$\log \mathsf{Odds} \; \mathsf{Ratio}_j \;\; = \;\; \log \left(\frac{\pi_{\mathsf{Republican},j}}{1 - \pi_{\mathsf{Republican},j}} \right) - \log \left(\frac{\pi_{\mathsf{Democratic},j}}{1 - \pi_{\mathsf{Democratic},j}} \right)$$

$$Var(\log Odds \ Ratio_j) \approx \frac{1}{x_{jD} + \alpha_j} + \frac{1}{x_{jR} + \alpha_j}$$

$$Std. \ Log \ Odds_j = \frac{\log Odds \ Ratio_j}{\sqrt{Var(\log Odds \ Ratio_j)}}$$

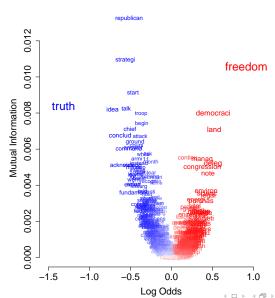
Applying the Model

https://gist.github.com/thiagomarzagao/5851207 How do Republicans and Democrats differ in debate? Condition on topic and examine word usage

- Press Releases (64,033)
- Topic Coded
- Given press release is about topic, what are the features that distinguish Republican and Democratic language?

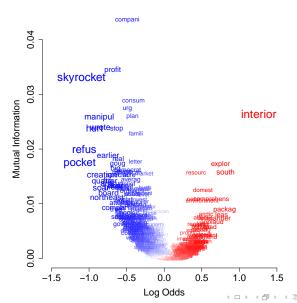
Mutual Information, Standardized Log Odds

Iraq War, Partisan Words



Mutual Information, Standardized Log Odds

Gas Prices, Partisan Words



Gentzkow, Shapiro, and Taddy (2017): Rhetorical Polarization

Figure 3: Average partisanship of speech, penalized estimates

