Text as Data

Justin Grimmer

Associate Professor
Department of Political Science
University of Chicago

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Discovery and Measurement

What is the research process? (Grimmer, Roberts, and Stewart 2017)

1) Discovery: a hypothesis or view of the world
2) Measurement according to some organization
3) Causal Inference: effect of some intervention

Text as data methods assist at each stage of research process

Measurement

Two approaches to measurement

1) Use an existing classification scheme to categorize documents
2) Simultaneously discover categories and measure prevalence (repurpose discovery methods)

Topic and Mixed Membership Models

Clustering

Document $\rightsquigarrow$ One Cluster

Doc 1

Doc 2

Doc 3

$\vdots$

Doc N

Cluster 1

Cluster 2

$\vdots$

Cluster K

Topic and Mixed Membership Models

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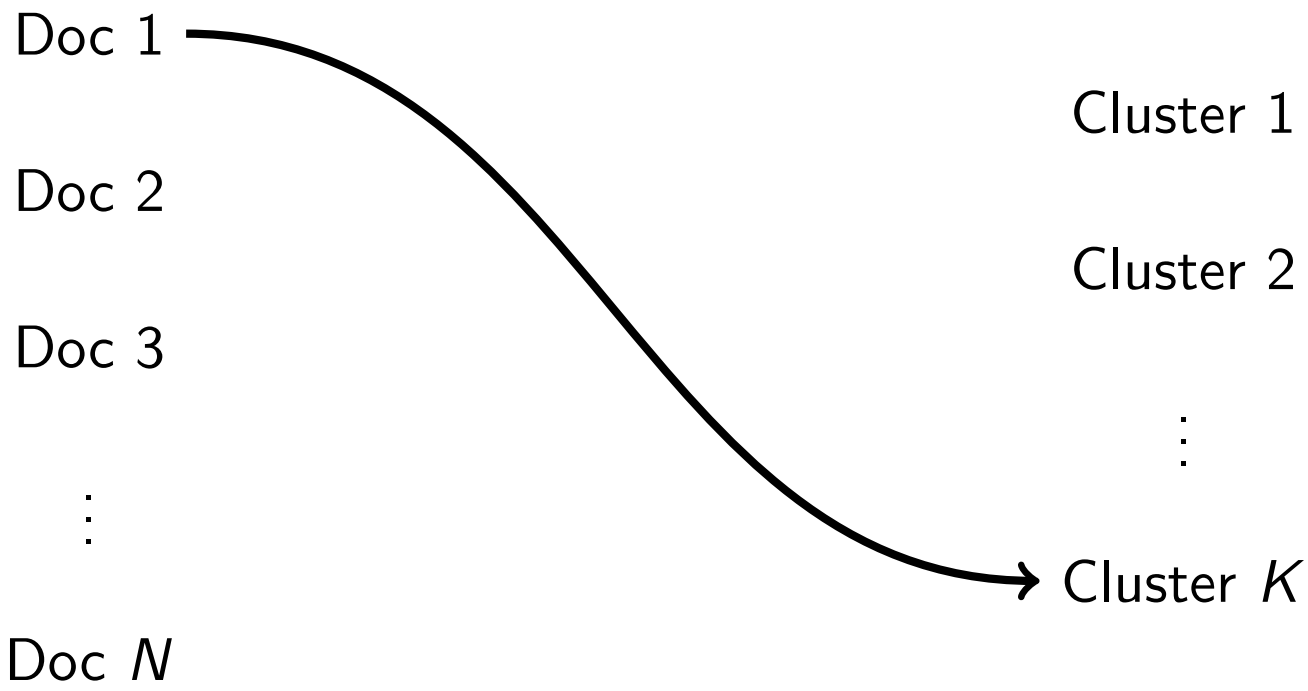

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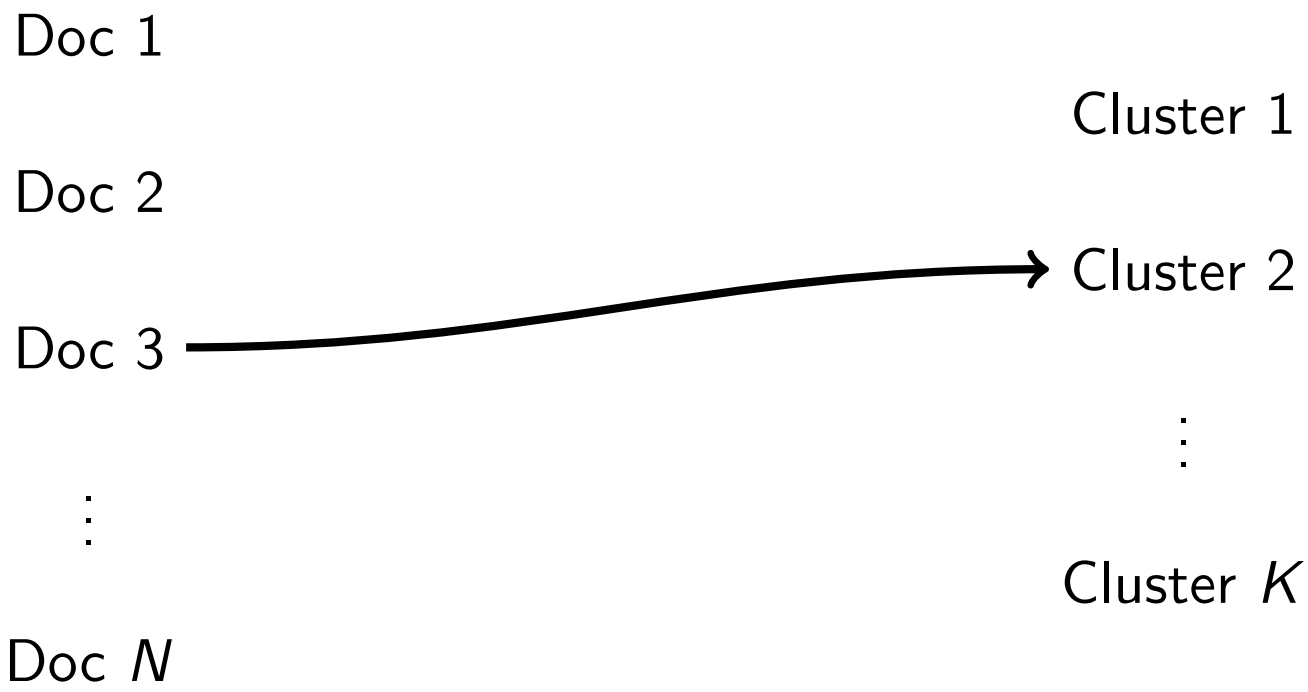

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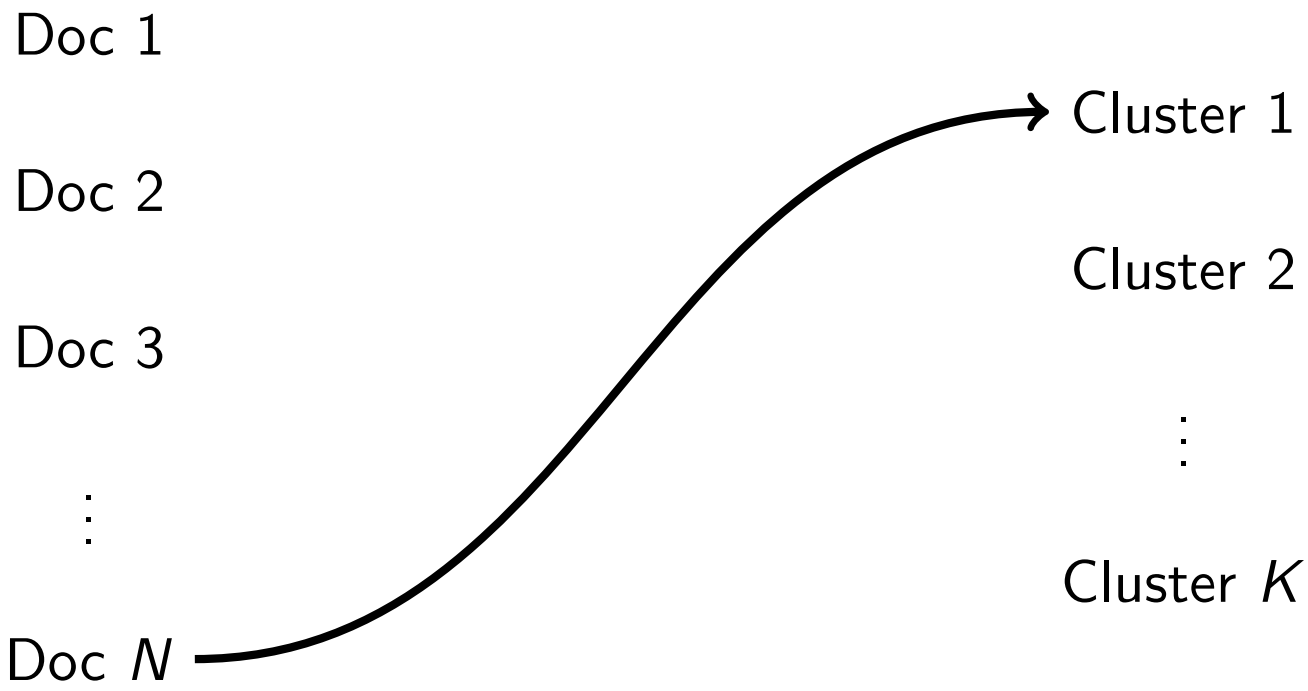

Topic and Mixed Membership Models

Topic Models (Mixed Membership)

Document $\rightsquigarrow$ Many clusters

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Topic and Mixed Membership Models

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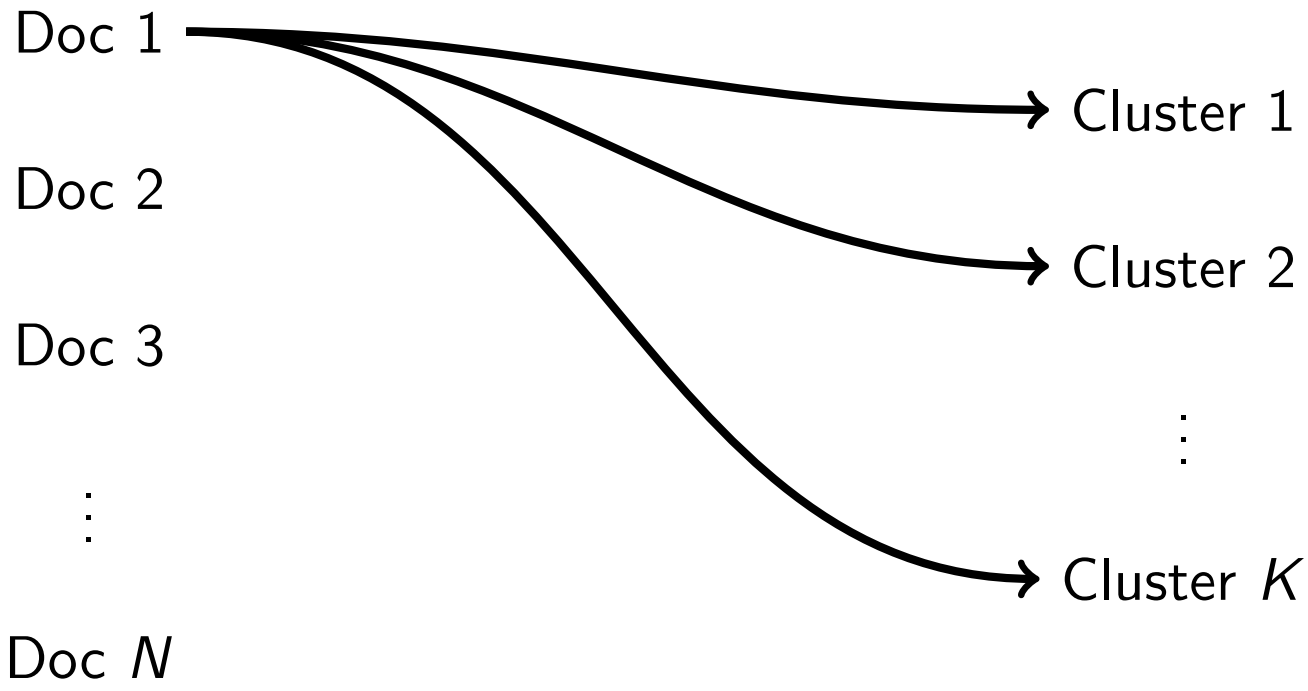
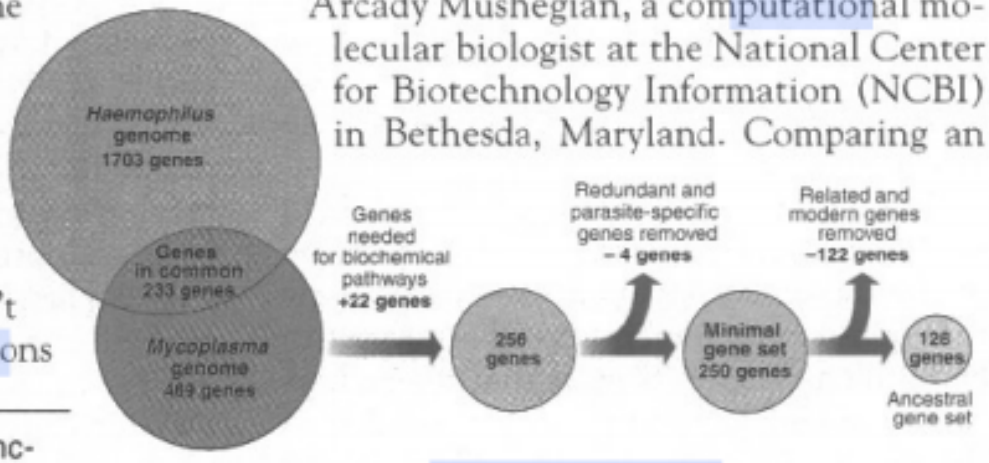


A Statistical Highlighter (With Many Colors)

Seeking Life's Bare (Genetic) Necessities

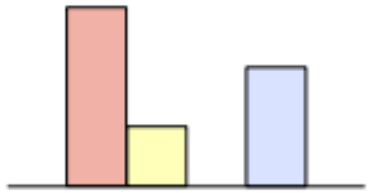
COLD SPRING HARBOR, NEW YORK—How many genes does an organism need to survive? Last week at the genome meeting here,* two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions "are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic numbers game, particularly as more and more genomes are completely mapped and sequenced. "It may be a way of organizing any newly sequenced genome," explains Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an



Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.

* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.



Vanilla Latent Dirichlet Allocation$\rightsquigarrow$ Objective Function

- Consider document i, $(i = 1, 2, \ldots, N)$.

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- Suppose there are M_i total words and $\boldsymbol{x}_i$ is an $M_i \times 1$ vector, where x_{im} describes the m^{th} word used in the document*.

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*Notice: this is a different representation than a document-term matrix. x_{im} is a number that says which of the J words are used. The difference is for clarity and we'll this representation is closely related to document-term matrix

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$$\begin{aligned} \boldsymbol{\pi}_i|\boldsymbol{\alpha} &\sim \text{Dirichlet}(\boldsymbol{\alpha}) \\ \boldsymbol{\tau}_{im}|\boldsymbol{\pi}_i &\sim \text{Multinomial}(1, \boldsymbol{\pi}_i) \end{aligned}$$

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Described in the slides appendix

Running a Topic Model with STM

`to the STM Code`

Why does this work$\rightsquigarrow$ Co-occurrence

Where's the information for each word's topic?

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Reconsider document-term matrix

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Reconsider document-term matrix

	Word_1	Word_2	...	Word_J
Doc_1	0	1	...	0
Doc_2	2	0	...	3
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
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Allows: measure of correlation of term usage across documents

(heuristically: partition words, based on usage in documents)

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Latent Semantic Analysis: Reduce information in matrix using linear algebra (provides similar results, difficult to generalize)

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Allows: measure of correlation of term usage across documents (heuristically: partition words, based on usage in documents)
Latent Semantic Analysis: Reduce information in matrix using linear algebra (provides similar results, difficult to generalize)
Biclustering: Models that partition documents and words simultaneously

Why does this work$\rightsquigarrow$ Co-occurrence logic (h/t Colorado Reed Tutorial)

$$p(\pi, \boldsymbol{T}, \boldsymbol{\Theta}, \boldsymbol{\alpha}|\boldsymbol{X}) \quad \propto \quad p(\boldsymbol{\alpha})p(\pi|\boldsymbol{\alpha})p(\boldsymbol{T}|\pi)p(\boldsymbol{X}|\boldsymbol{\theta}, \boldsymbol{T})$$

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1) $\boldsymbol{\theta} \rightsquigarrow$ Greater weight on terms that occur together

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1) $\boldsymbol{\theta} \rightsquigarrow$ Greater weight on terms that occur together
2) $\boldsymbol{\pi} \rightsquigarrow$ Greater weight on indicators that appear more regularly

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$$p(\pi, \boldsymbol{T}, \boldsymbol{\Theta}, \boldsymbol{\alpha}|\boldsymbol{X}) \quad \propto \quad p(\boldsymbol{\alpha}) \underbrace{p(\pi|\boldsymbol{\alpha})}_{3} \underbrace{p(\boldsymbol{T}|\pi)}_{2} \underbrace{p(\boldsymbol{X}|\boldsymbol{\theta}, \boldsymbol{T})}_{1}$$

1) $\boldsymbol{\theta} \rightsquigarrow$ Greater weight on terms that occur together
2) $\boldsymbol{\pi} \rightsquigarrow$ Greater weight on indicators that appear more regularly
3) $\boldsymbol{\alpha} \rightsquigarrow$ Emphasis on $\boldsymbol{\pi}$ with greater weight

Validation$\rightsquigarrow$ Topic Intrusion

Wednesday$\rightsquigarrow$ discussed several validations

- Labeling paragraphs
  - Identify separating words automatically
  - Label topics manually (read!)
- Statistical methods
  1) Entropy
  2) Exclusivity
  3) Cohesiveness
- Experiment Based Methods
  - Word intrusion$\rightsquigarrow$ topic validity
  - Topic intrusion$\rightsquigarrow$ model fit

Validation$\rightsquigarrow$ Topic Intrusion

1) Ask research assistant to read paragraph
2) Construct experiment
   - For the document, select top three topics
   - Select a fourth topic
   - Show participant, ask her/him to identify intruder

   Higher identification$\rightsquigarrow$ topics are a better model of text

Example 1: Japanese Campaign Manifestos (Catalinac 2011)

- Why is Japan revising its constitution?
- IR question: why is Japan now willing to engage militaristic foreign action?
- One explanation: election reform in 1993, changed electoral incentives
- To answer well: characterize campaigns across 50 + years
  - That sounds hard
  - That sounds impossible
- Determined (relentless) data collection
- Latent Dirichlet Allocation (on japanese texts)

Example 1: Japanese Campaign Manifestos (Catalinac 2011)

Japanese Elections:

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Japanese Elections:

- Election Administration Commission runs elections $\rightarrow$ district level

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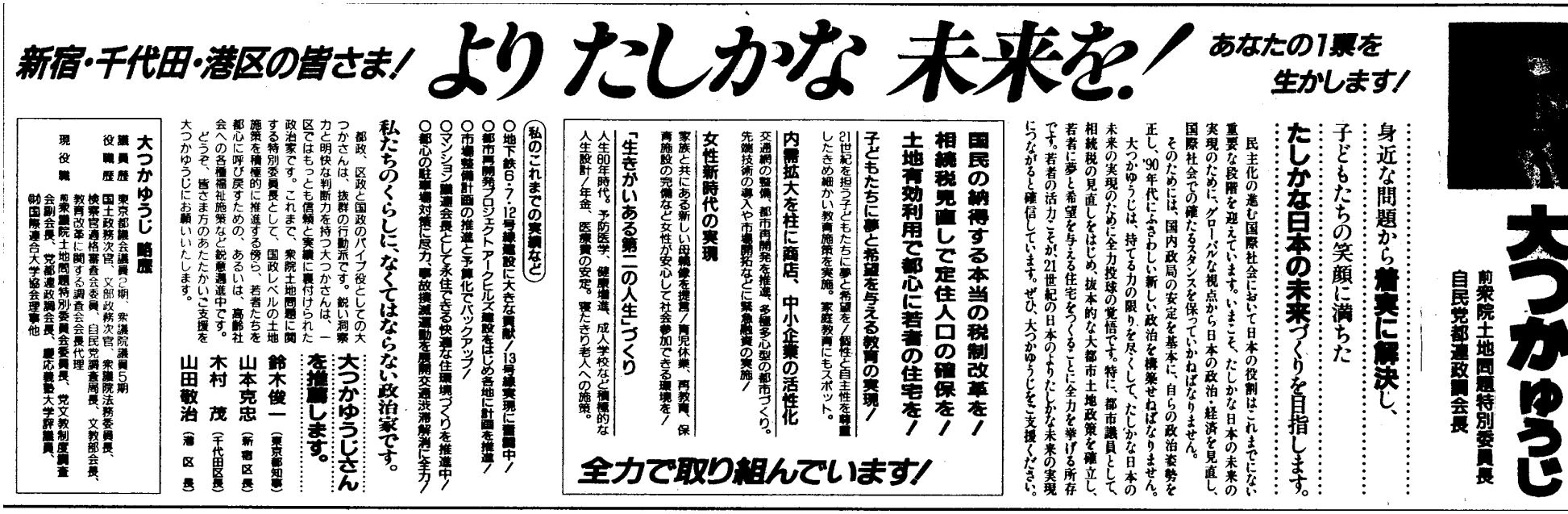
Japanese Elections:

- Election Administration Commission runs elections $\rightarrow$ district level
- Required to submit manifestos for all candidates to National Diet

Example 1: Japanese Campaign Manifestos (Catalinac 2011)

Typical Manifesto:



新宿・千代田・港区の皆さま！

よりたしかな未来を！

あなたの1票を生かします！

大つかゆうじ

前衆院土地問題特別委員長

自民党都連政調会長

身近な問題から着実に解決し、

子どもたちの笑顔に満ちた

たしかな日本の未来づくりを目指します。

民主化の進む国際社会において日本の役割はこれまでにない重要な段階を迎えています。いまこそ、たしかな日本の未来の実現のために、グローバルな視点から日本の政治・経済を見直し、国際社会での確たるスタンスを保っていかねばなりません。

そのためには、国内政局の安定を基本に、自らの政治姿勢を正し、'90年代にふさわしい新しい政治を構築せねばなりません。

大つかゆうじは、持てる力の限りを尽くして、たしかな日本の未来の実現のために全力投球の覚悟です。特に、都市議員として、相続税の見直しをはじめ、抜本的な大都市土地政策を確立し、若者に夢と希望を与える住宅をつくることに全力を挙げる所存です。若者の活力こそが、21世紀の日本のよりたしかな未来の実現につながると確信しています。ぜひ、大つかゆうじをご支援ください。

国民の納得する本当の税制改革を！

相続税見直しで定住人口の確保を！

土地有効利用で都心に若者の住宅を！

子どもたちに夢と希望を与える教育の実現！

21世紀を担う子どもたちに夢と希望を／個性と自主性を尊重したきめ細かい教育施策を実施。家庭教育にもスポット。

内需拡大を柱に商店、中小企業の活性化

交通網の整備、都市再開発を推進。多極多心型の都市づくり。先端技術の導入や市場開拓などに緊急融資の実施／

女性新時代の実現

家族と共にある新しい母親像を提言／育児休業、再教育、保育施設の完備など女性が安心して社会参加できる環境を／

「生きがいある第二の人生」づくり

人生80年時代。予防医学、健康増進、成人学校など積極的な人生設計／年金、医療費の安定。寝たきり老人への施策。

全力で取り組んでいます！

私のこれまでの実績など

○地下鉄6・7・12号線建設に大きな貢献／13号線実現に奮闘中！

○都市再開発プロジェクトアークヒルズ建設をはじめ各地に計画を推進！

○市場整備計画の推進と予算化でバックアップ！

○マンション建設会長として永住できる快適な住環境づくりを推進中！

○都心の駐車場対策に尽力、事故撲滅運動を展開交通渋滞解消に全力！

私たちのくらしに、なくてはならない政治家です。

都政、区政と国政のパイプ役としての大つかさんは、鋭い洞察力と明快な判断力を持つ大つかさんは、一区ではもっとも信頼と実績に裏付けられた政治家です。これまで、衆院土地問題に関する特別委員長として、国政レベルの土地施策を積極的に推進する傍ら、若者たちを都心に呼び戻すための、あるいは、高齢社会への各種福祉施策など鋭意奮進中です。

どうぞ、皆さま方のあたたかいご支援を大つかゆうじにお願いいたします。

大つかゆうじさん

を推薦します。

鈴木俊一（東京都知事）

山本克忠（新宿区長）

木村茂（千代田区長）

山田敬治（港区長）

大つかゆうじ 略歴

議員歴 東京都議会議員2期、衆議院議員5期

役職歴 国土政務次官、文部政務次官、衆議院法務委員長、検察官適格審査会委員、自民党調査局長、文教部会長、教育改革に関する調査会長代理

現役職 衆議院土地問題特別委員長、党文教制度調査会副会長、党都連政調会長、慶応義塾大学評議員、財国際連合大学協会理事他

Example 1: Japanese Campaign Manifestos (Catalinac 2011)

Japanese Elections:

- Election Administration Commission runs elections $\rightarrow$ district level
- Required to submit manifestos for all candidates to National Diet
- Collected from 1950- 2009

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- Harder for Japanese

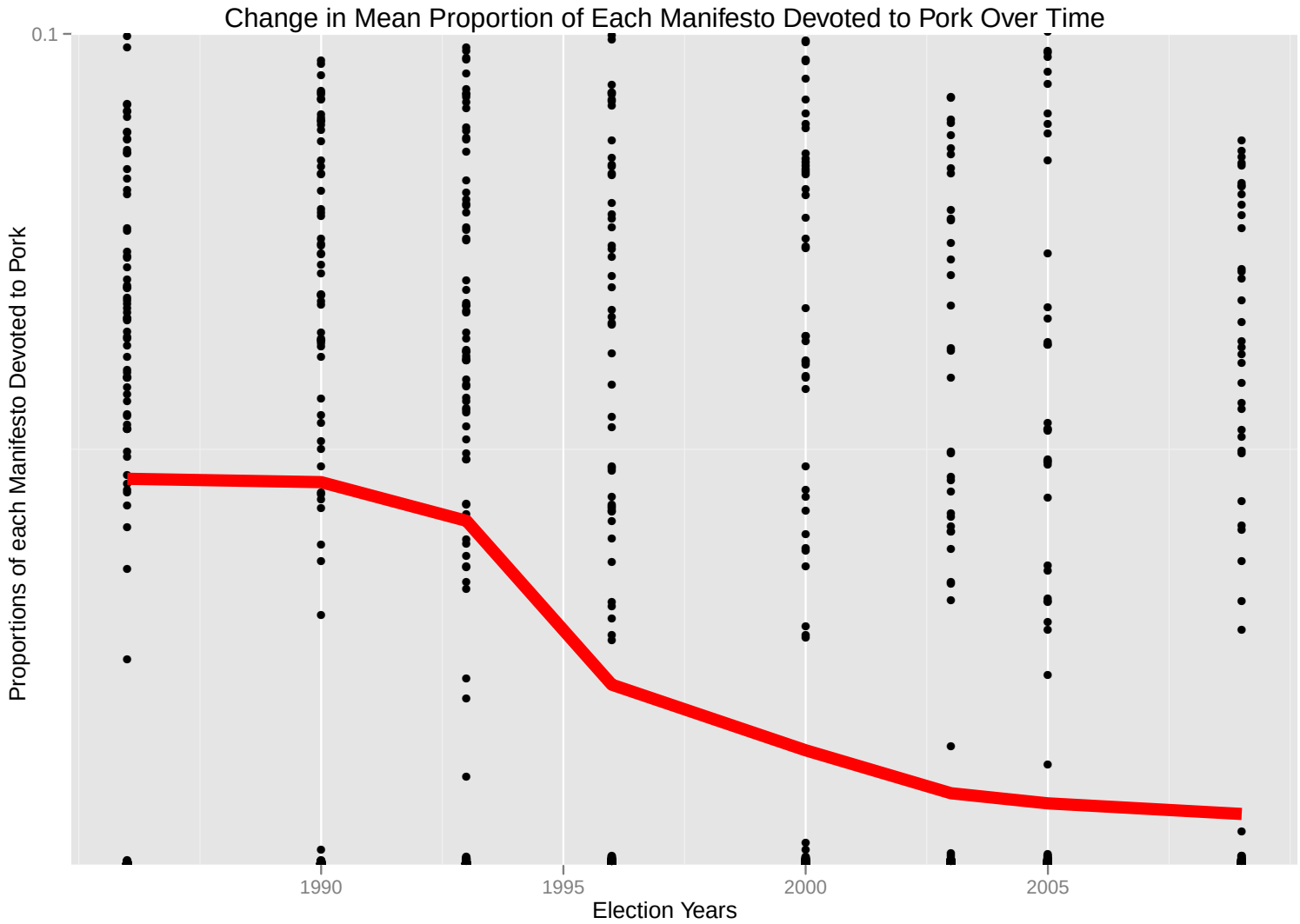
Example 1: Japanese Campaign Manifestos (Catalinac 2014)

- Applies Vanilla LDA (using R Code)
- Output: topics (with Japanese characters)

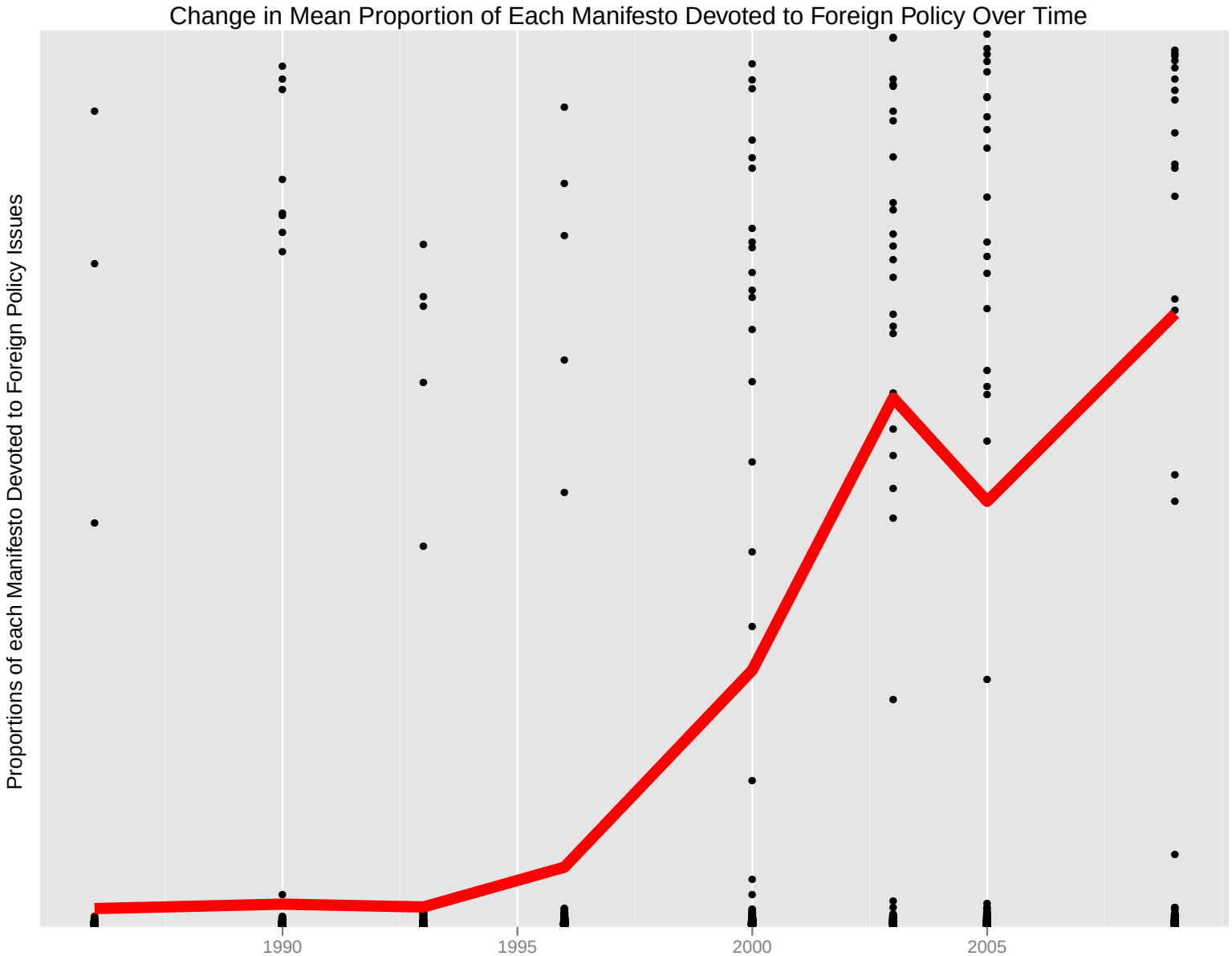
Example 1: Japanese Campaign Manifestos (Catalinac 2011)

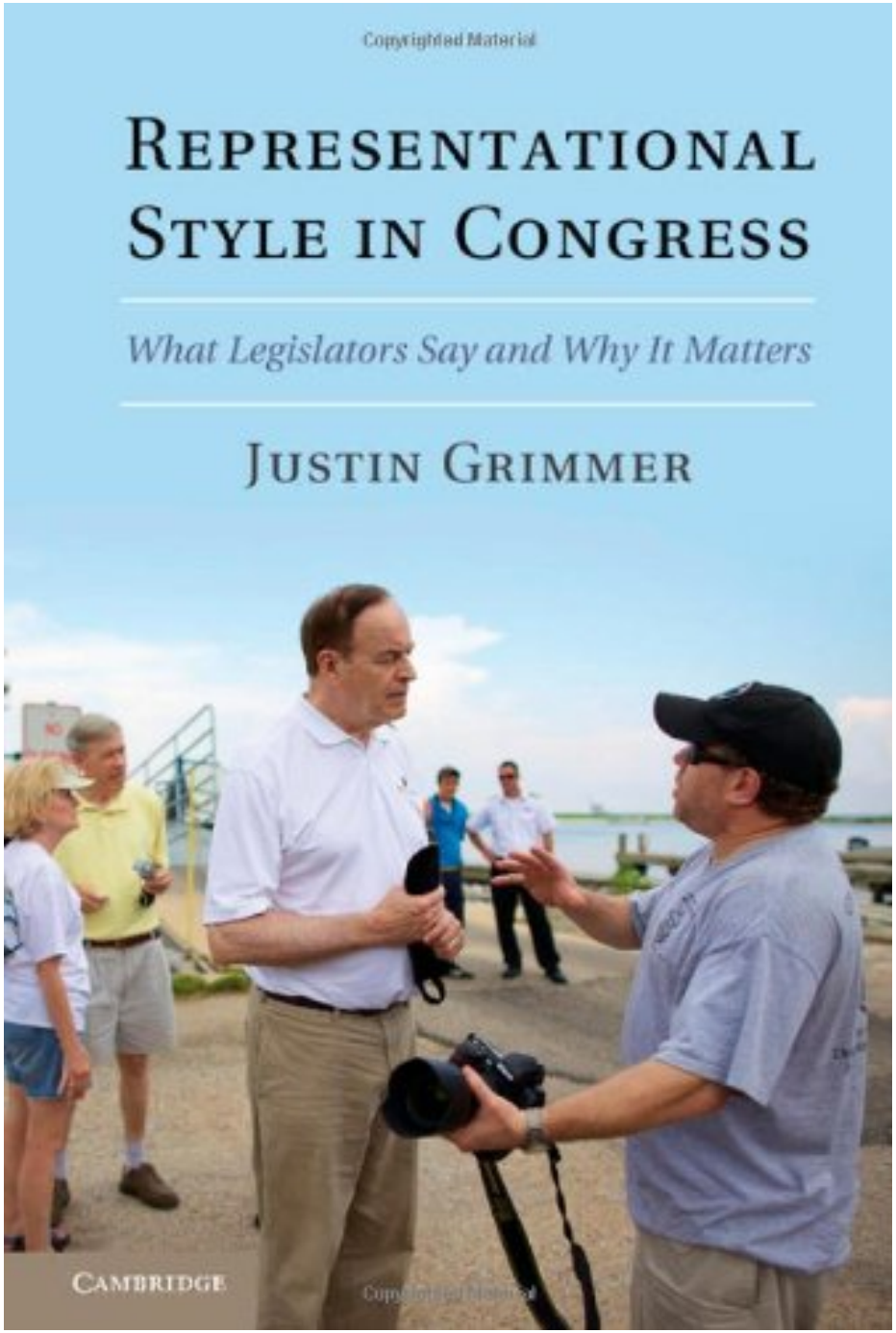
Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic
改革	年金	推進	区	政治	日本
郵政	円	整備	政策	改革	国
民営	廃止	図る	地域	国民	外交
小泉	改革	つとめる	まち	企業	国家
構造	兆	社会	鹿児島	自民党	社会
政府	実現	対策	全力	日本	国民
官	無駄	振興	選挙	共産党	保障
推進	日本	充実	国政	献金	安全
民	増税	促進	作り	金権	地域
自民党	削減	安定	横浜	党	拉致
日本	一元化	確立	対策	選挙	経済
制度	政権	企業	中小	禁止	守る
民間	子供	実現	発電	憲法	問題
年金	地域	中小	推進	腐敗	北朝鮮
実現	ひと	育成	エネルギー	団体	教育
進める	サラリーマン	制度	企業	区	責任
断行	制度	政治	声	ソ連	力
地方	議員	地域	実現	守る	創る
止める	金	福祉	活性	平和	安心
保障	民主党	事業	自民党	円	目指す
財政	年間	改革	地方	反対	誇り
作る	一掃	確保	尽くす	真	憲法
賛成	郵政	強化	商店	是正	可能
社会	道路	教育	いかす	一掃	道
国民	交代	施設	全国	悪政	未来
公務員	社会保険庁	生活	政党	抜本	ひと
力	月額	支援	ひと	定数	再生
経済	手当	環境	支援	政党	将来
国	談合	発展	経済	金丸	解決
安心	支援	施策	福祉	改悪	基本
Postal privatization	Reducing Wasteful Public Spending	Pork for the District	Policies for the district	Political Reform	Nation

Example 1: Japanese Campaign Manifestos (Catalinac 2011)



Example 1: Japanese Campaign Manifestos (Catalinac 2011)




Copyrighted Material
REPRESENTATIONAL STYLE IN CONGRESS
What Legislators Say and Why It Matters
JUSTIN GRIMMER
CAMBRIDGE

Example 2: Automated Literature Reviews

Recall: literature reviews are hard to conduct
LDA: developed (in part) to help structure JSTOR database
Use JSTOR's research service to obtain data to analyze
Question: How do scholars use classic text: Home Style
Analysis: all articles that cite Home Style in JSTOR's data

Example 2: Automated Literature Reviews

Output: topic estimates

- Obtain $\log \theta_k$ from model
- One method to summarize a topic:
  - $\exp(\log \theta_k)$ (select 10-20 biggest words)
  - $\exp(\log \theta_k) - \text{Average}_{j \neq k} \exp(\log \theta_j)$ (select 10-20 biggest words)

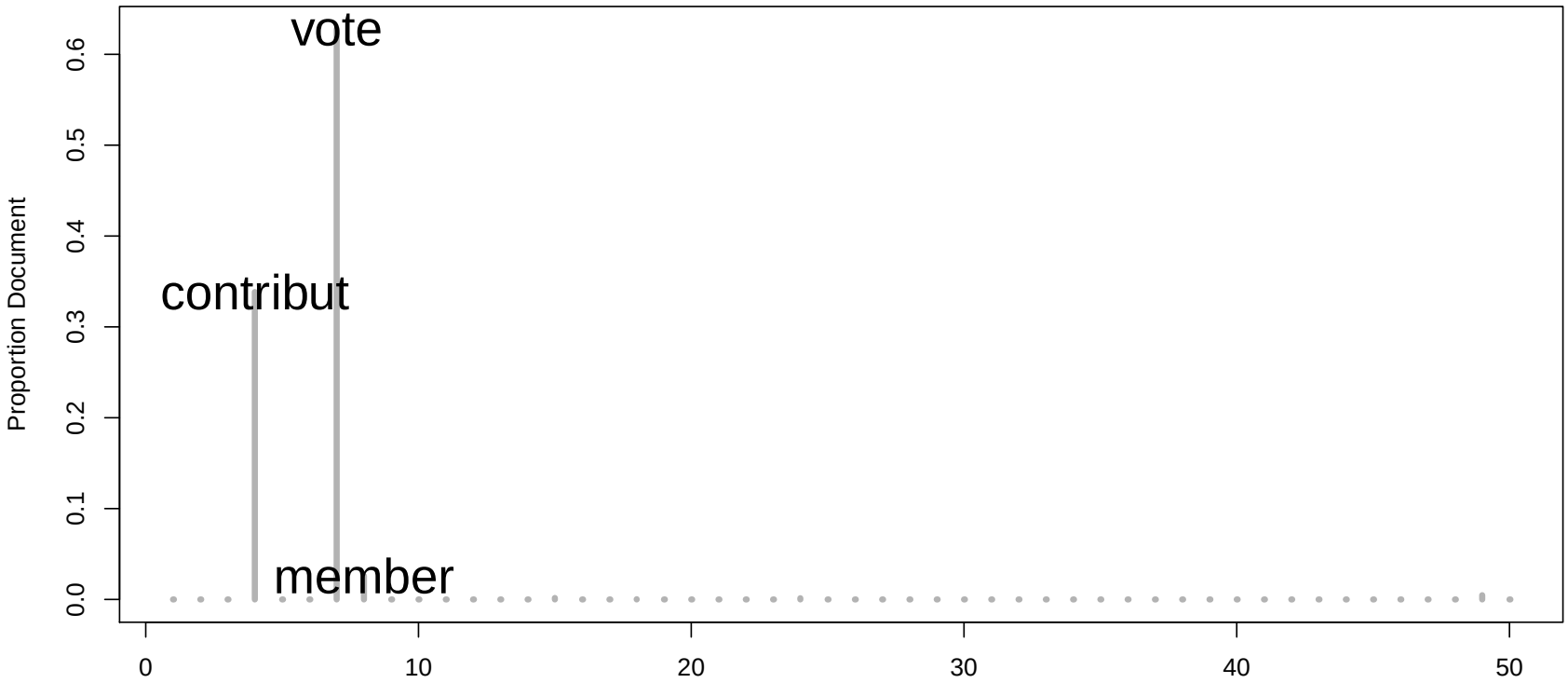
Example 2: Automated Literature Reviews

Example topics:

Label	Stems	Proportic
Life Style	member,district,attent,congress,time,cohort,retir	0.03
Comp.Home	constitu,mp,member,parti,role,local,british	0.02
Casework	casework,district,constitu,variabl,staff,congression,fiorina	0.03
Votes	vote,variabl,model,estim,measur,legisl,constitu	0.04
Id. Shirk	ideolog,vote,shirk,constitu,parti,senat,voter	0.03
C. letters	mail,govern, activ,respond,commun,offic	0.02

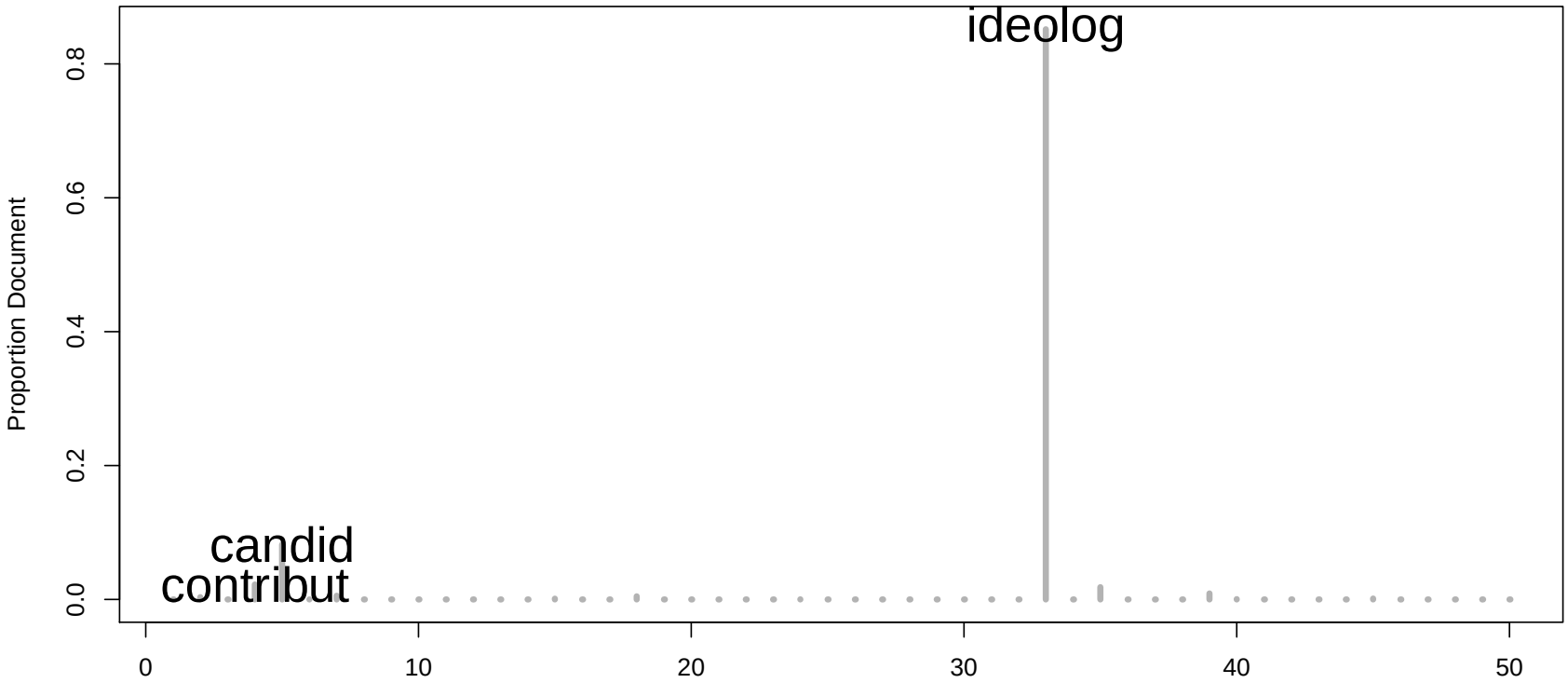
Example Document

Wawro (2001) "A Panel Probit Analysis of Campaign Contributions and Roll Call Votes"



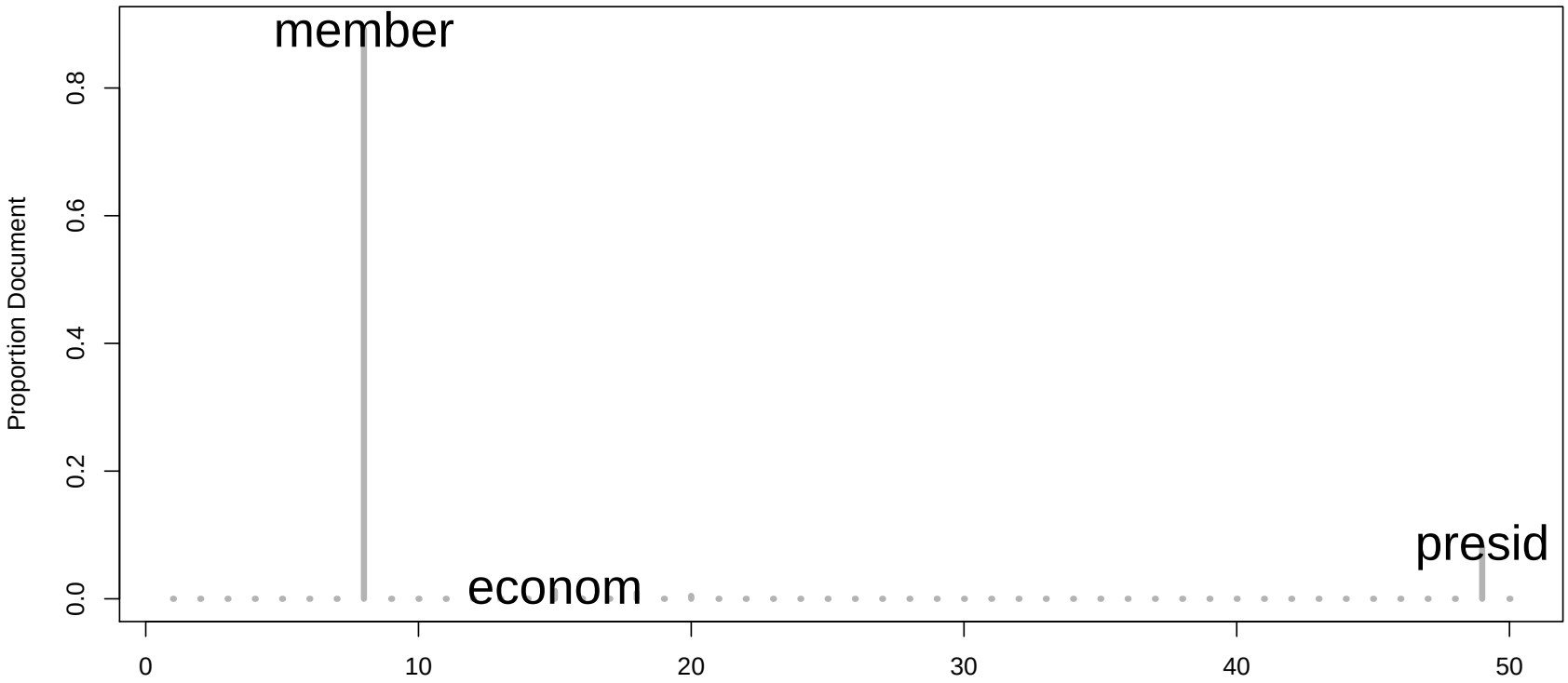
Example Document

Bender (1996) "Legislator Voting and Shirking A Critical Review of the Literature"



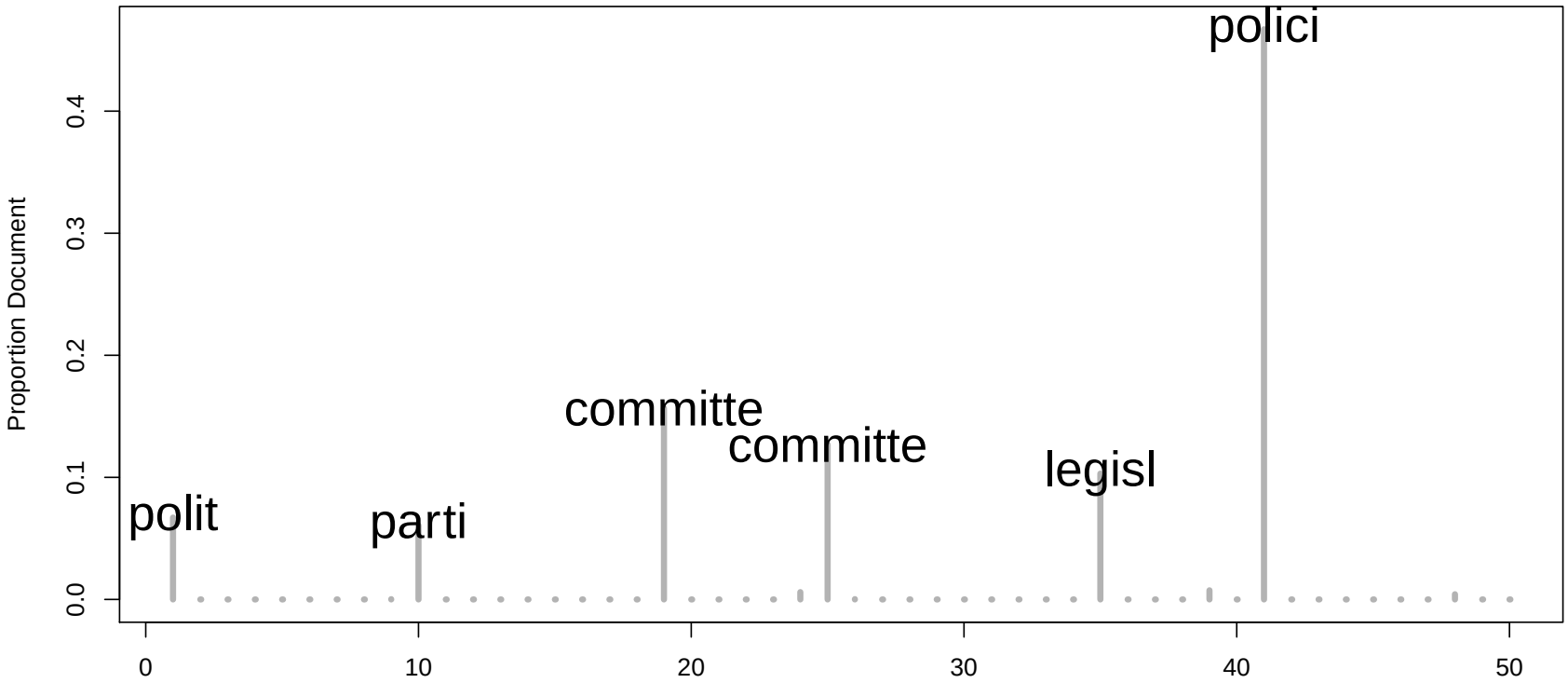
Example Document

Parker (1980) "Cycles in Congressional District Attention"

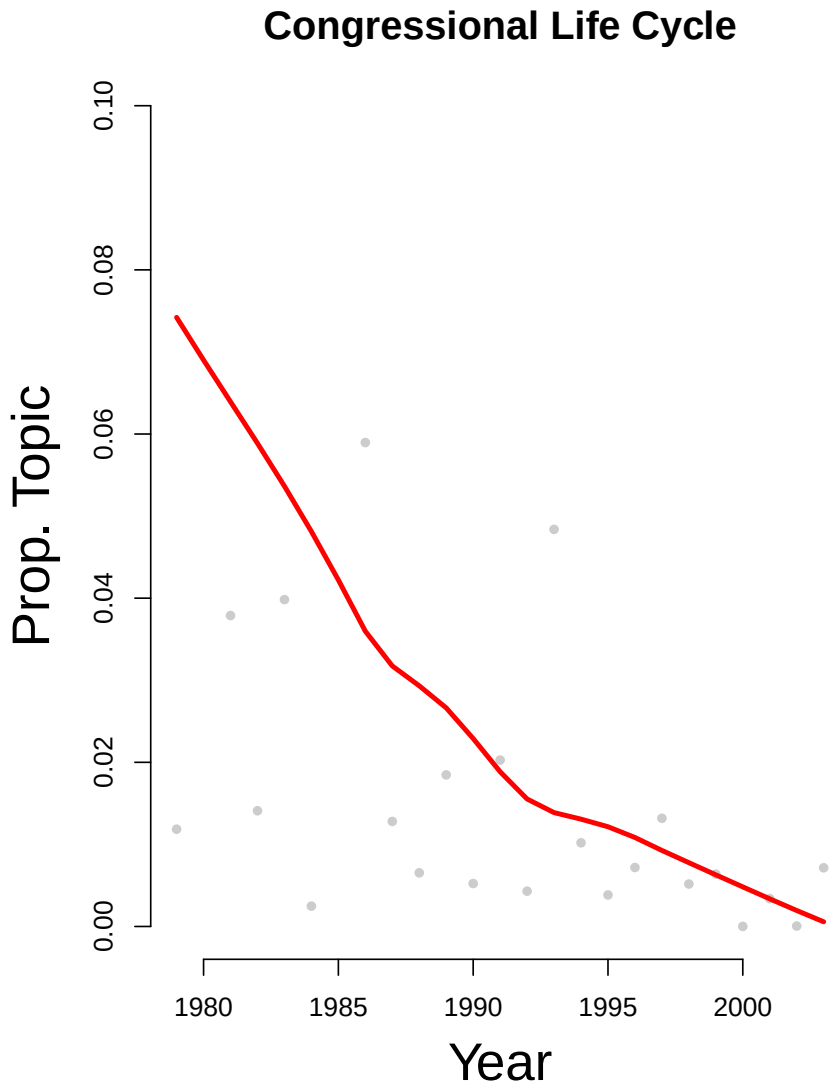


Example Document

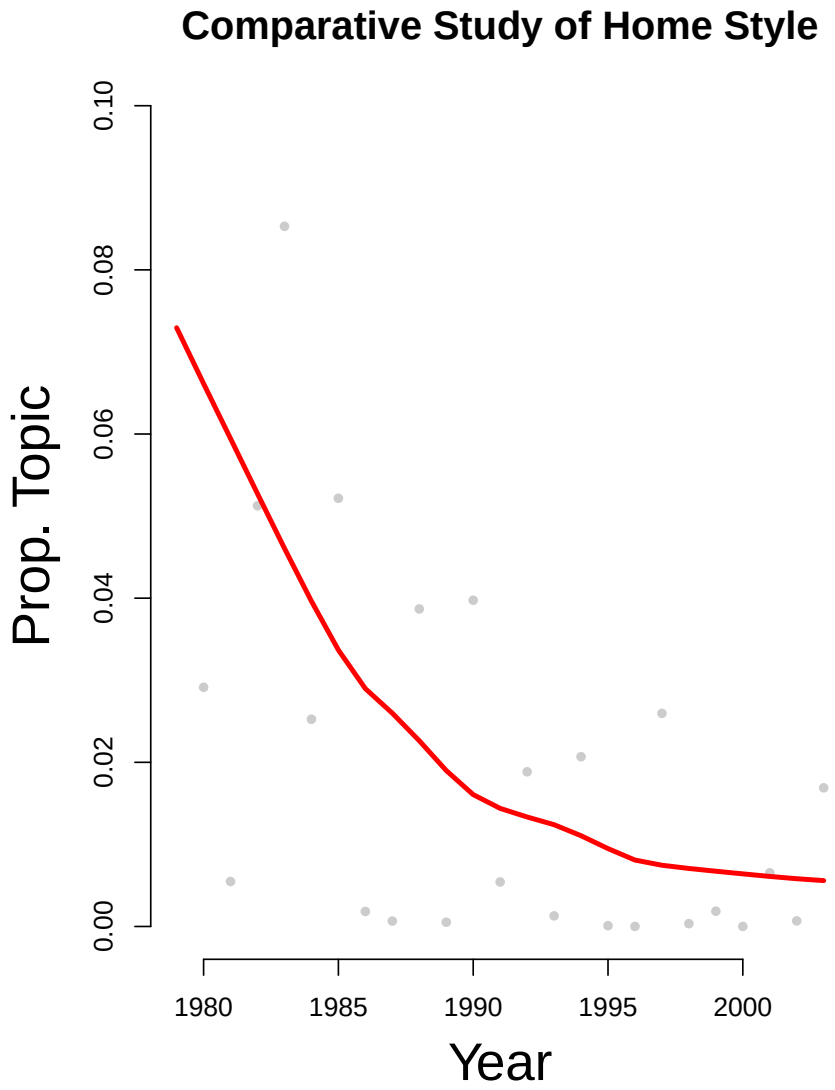
Shepsle (1985) "Policy Consequences of Government by Congressional Subcommittees"



History of Home Style



History of Home Style



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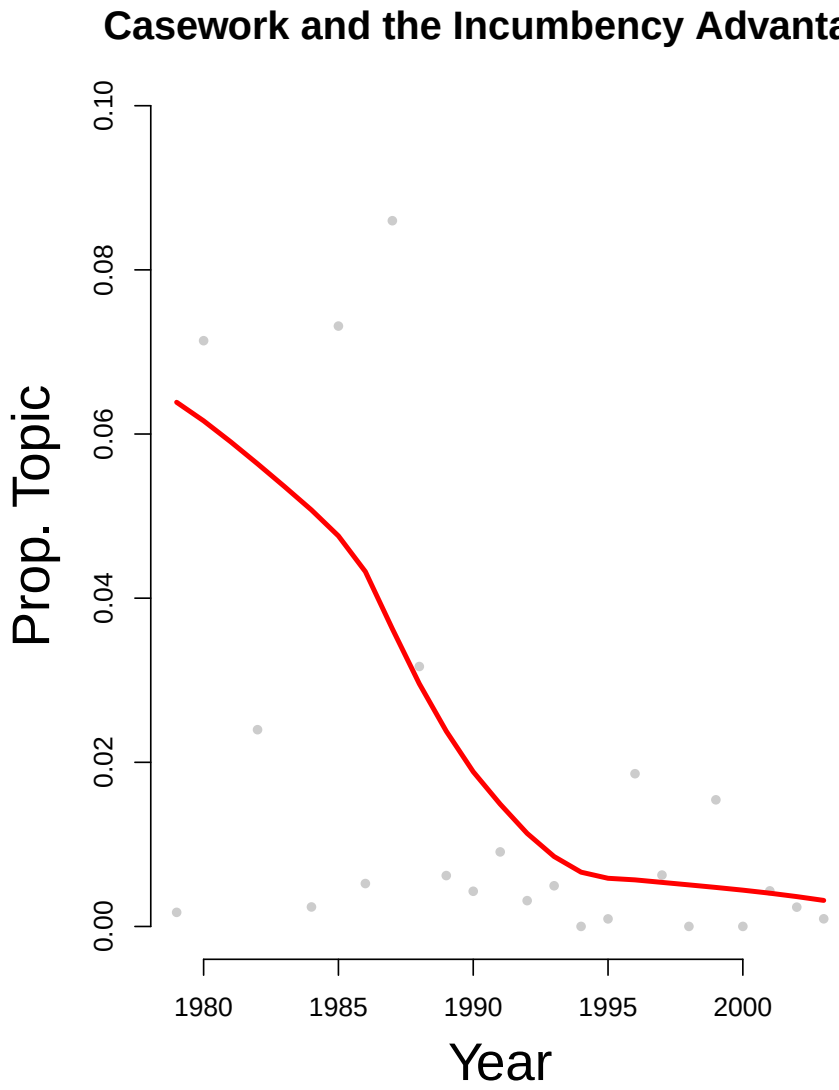

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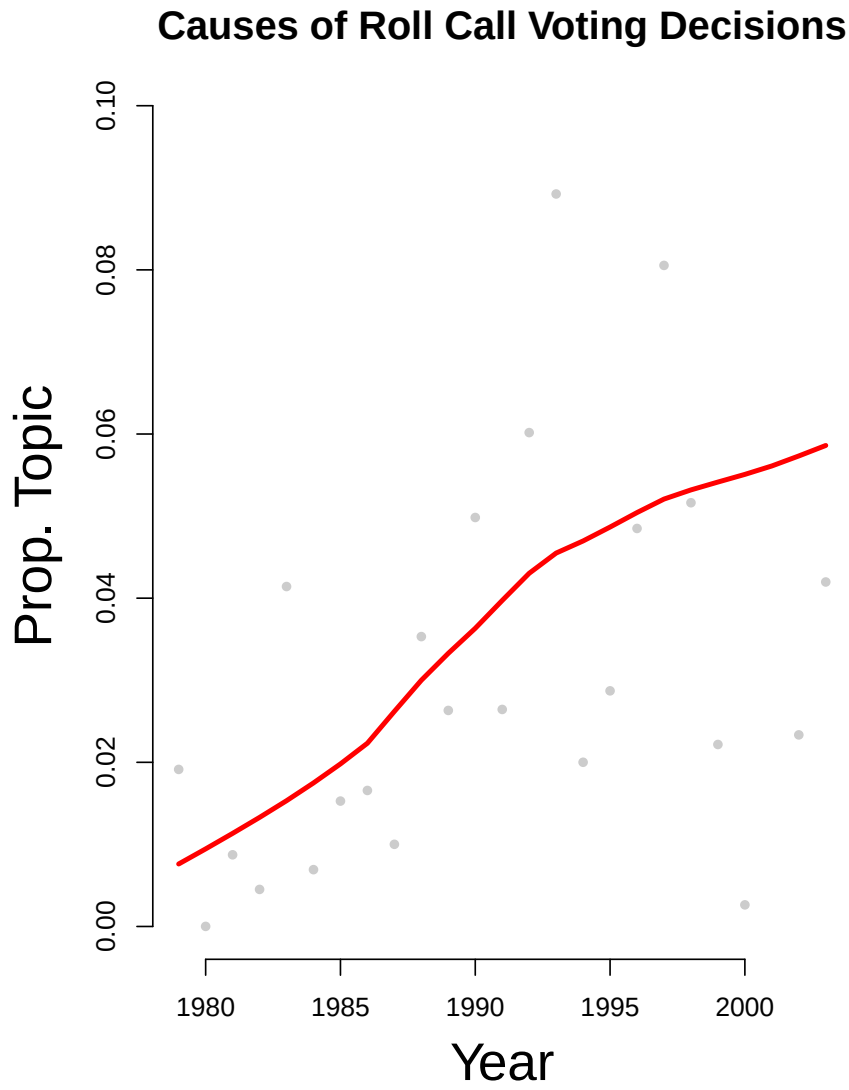

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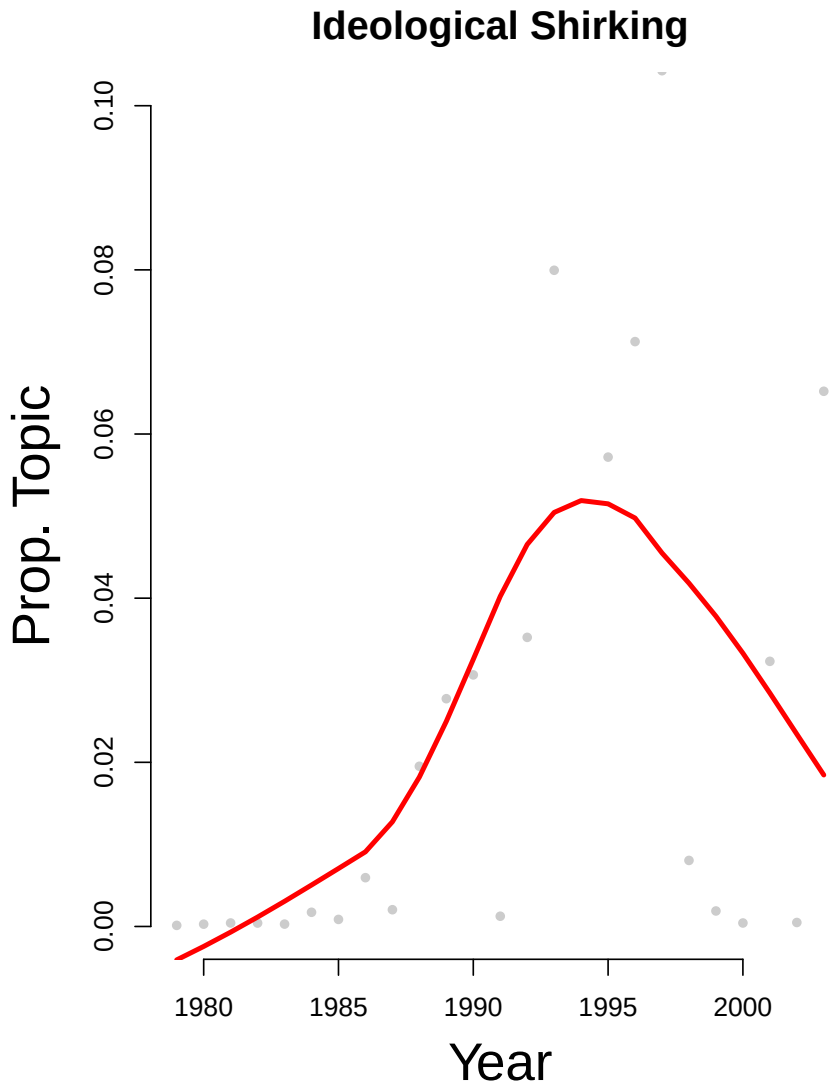

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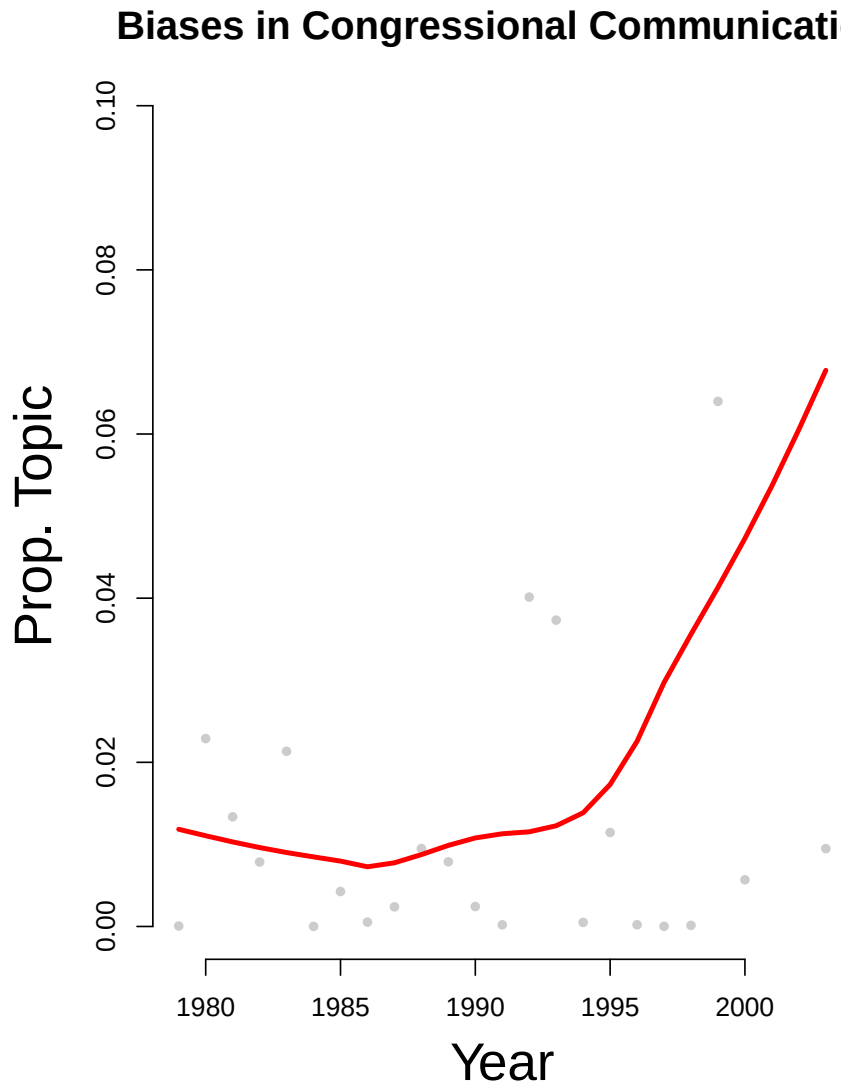
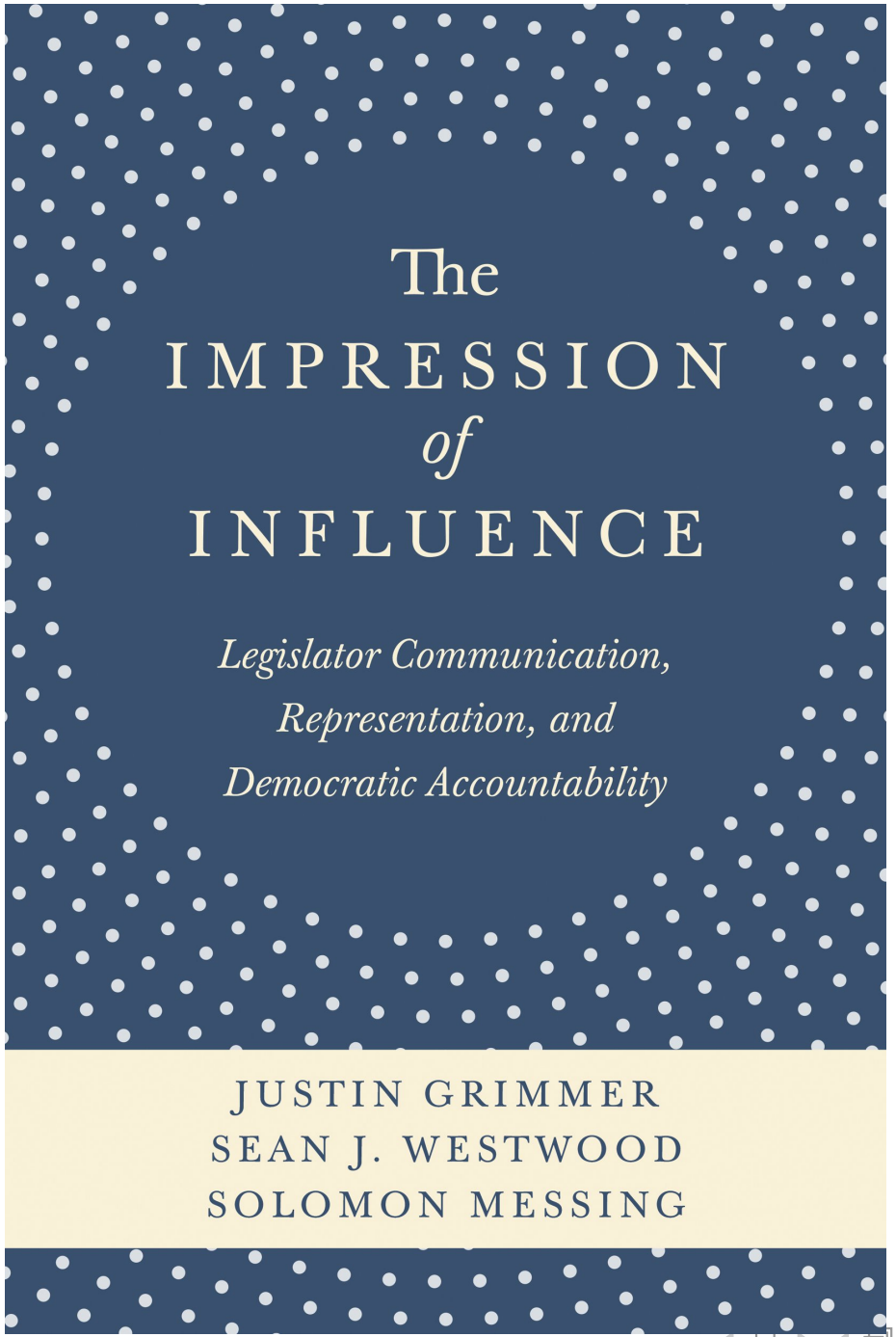



The
IMPRESSION
of
INFLUENCE
Legislator Communication,
Representation, and
Democratic Accountability
JUSTIN GRIMMER
SEAN J. WESTWOOD
SOLOMON MESSING

What legislators claim (Grimmer, Westwood, Messing 2014)

What legislators claim (Grimmer, Westwood, Messing 2014) $\rightsquigarrow$ LDA credit claiming press releases

Labels	Key Words	Proportion

What legislators claim (Grimmer, Westwood, Messing 2014) $\rightsquigarrow$ LDA credit claiming press releases

Labels	Key Words	Proportion
Requested appropriations	bill,funding,house,million,appropriations	0.08

"Dave Camp announced today that he was able to secure $2.5 million for widening M-72 from US-31 easterly 7.2 miles to Old M-72. The bill will now head to the Senate for consideration...We have two more hurdles to clear to make sure the money is in the bill when it hits the President's desk: a vote in the Senate and a conference committee" (Camp, 2005)

What legislators claim (Grimmer, Westwood, Messing 2014) $\rightsquigarrow$ LDA

credit claiming press releases

Labels	Key Words	Proportion
Requested appropriations	bill,funding,house,million,appropriations	0.08

"Congressman Doc Hastings has boosted federal funding for work on the Columbia Basin water supply for next year. Hastings has added $400,000 for work on the Odessa Subaquifer, which when combined with the funding in the President's budget request, totals $1 million for Fiscal Year 2009"..."Hastings' funding for the Odessa Subaquifer and Potholes Reservoir was included in the Fiscal Year 2009 Energy and Water Appropriations bill which was approved today by the full House Appropriations Committee. (Hastings, 2008)"

What legislators claim (Grimmer, Westwood, Messing 2014) $\rightsquigarrow$ LDA credit claiming press releases

Labels	Key Words	Proportion
Requested appropriations	bill,funding,house,million,appropriations	0.08
Fire department grants	fire,grant,department,program,firefighters	0.08

"Maurice Hinchey (D-NY) today announced that the West Endicott Fire Company has been awarded a $17,051 federal grant to purchase approximately 10 sets of protective clothing, as well as radio equipment and air packs for its volunteer firefighters" (Hinchey, 2008)

What legislators claim (Grimmer, Westwood, Messing 2014) $\rightsquigarrow$ LDA credit claiming press releases

Labels	Key Words	Proportion
Requested appropriations	bill,funding,house,million,appropriations	0.08
Fire department grants	fire,grant,department,program,firefighters	0.08

"Congressman Pete Visclosky today announced that the Crown Point Fire Department will receive a $16,550 Department of Homeland Security (DHS) grant to purchase a modular portable video system" (Visclosky, 2008)

What legislators claim (Grimmer, Westwood, Messing 2014) $\rightsquigarrow$ LDA

credit claiming press releases

Labels	Key Words	Proportion
Requested appropriations	bill,funding,house,million,appropriations	0.08
Fire department grants	fire,grant,department,program,firefighters	0.08
Stimulus	recovery,funding,jobs,information, act,	0.06

What legislators claim (Grimmer, Westwood, Messing 2014) $\rightsquigarrow$ LDA credit claiming press releases

Labels	Key Words	Proportion
Requested appropriations	bill,funding,house,million,appropriations	0.08
Fire department grants	fire,grant,department,program,firefighters	0.08
Stimulus	recovery,funding,jobs,information, act,	0.06
Transportation	transportation,project,airport,transit,million	0.06

Correlated Topic Models

Dirichlet distribution$\rightsquigarrow$ Assumes negative covariance between topics
Logistic Normal Distribution$\rightsquigarrow$ Allows some positive covariance between topics

$$
\begin{aligned}
\boldsymbol{\theta}_k &\sim \text{Dirichlet}(\mathbf{1}) \\
\boldsymbol{\eta}_i | \boldsymbol{\mu}, \boldsymbol{\Sigma} &\sim \text{Multivariate Normal}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \\
\boldsymbol{\pi}_i &= \frac{\exp(\boldsymbol{\eta}_i)}{\sum_{k=1}^{K} \exp(\eta_{ik})} \\
\boldsymbol{\tau}_{im} | \boldsymbol{\pi}_i &\sim \text{Multinomial}(1, \boldsymbol{\pi}_i) \\
x_{im} | \boldsymbol{\theta}_k, \tau_{imk} = 1 &\sim \text{Multinomial}(1, \boldsymbol{\theta}_k)
\end{aligned}
$$

LDA Revisited

$$
\begin{aligned}
\boldsymbol{\theta}_k &\sim \text{Dirichlet}(\mathbf{1}) \\
\boldsymbol{\pi}_i|\boldsymbol{\alpha} &\sim \text{Dirichlet}(\boldsymbol{\alpha}) \\
\boldsymbol{\tau}_{im}|\boldsymbol{\pi}_i &\sim \text{Multinomial}(1, \boldsymbol{\pi}_i) \\
x_{im}|\boldsymbol{\theta}_k, \tau_{imk} = 1 &\sim \text{Multinomial}(1, \boldsymbol{\theta}_k)
\end{aligned}
$$

LDA Revisited

$$
\begin{aligned}
\textbf{Unigram Model}_k &\sim \text{Dirichlet}(\mathbf{1}) \\
\textbf{Doc. Prop}_i &\sim \text{Dirichlet}(\textbf{Pop. Proportion}) \\
\textbf{Word Topic}_{im} &\sim \text{Multinomial}(1, \textbf{Doc. Prop}_i) \\
\text{Word}_{im} &\sim \text{Multinomial}(1, \textbf{Unigram Model}_k)
\end{aligned}
$$

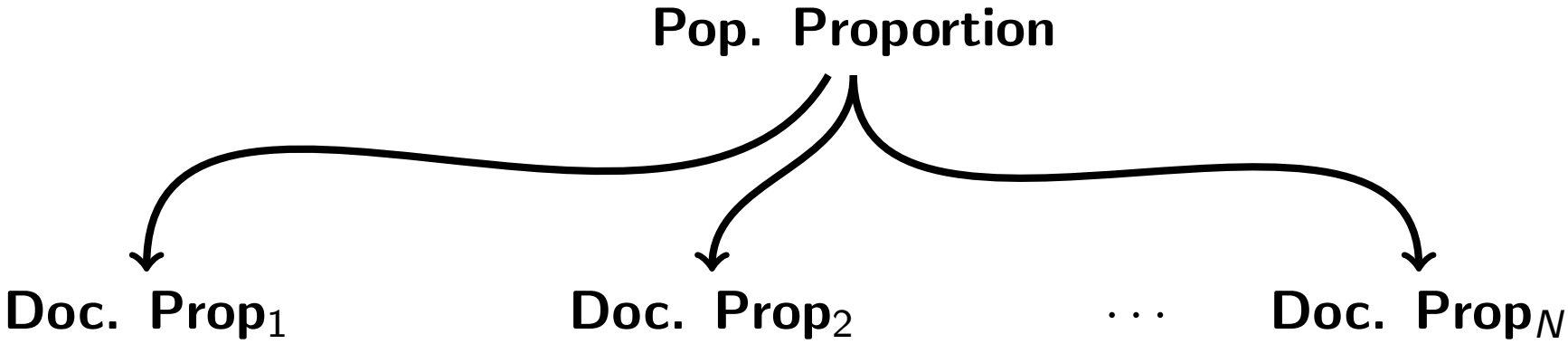
A General Hierarchical Structure

LDA:

Pop. Proportion

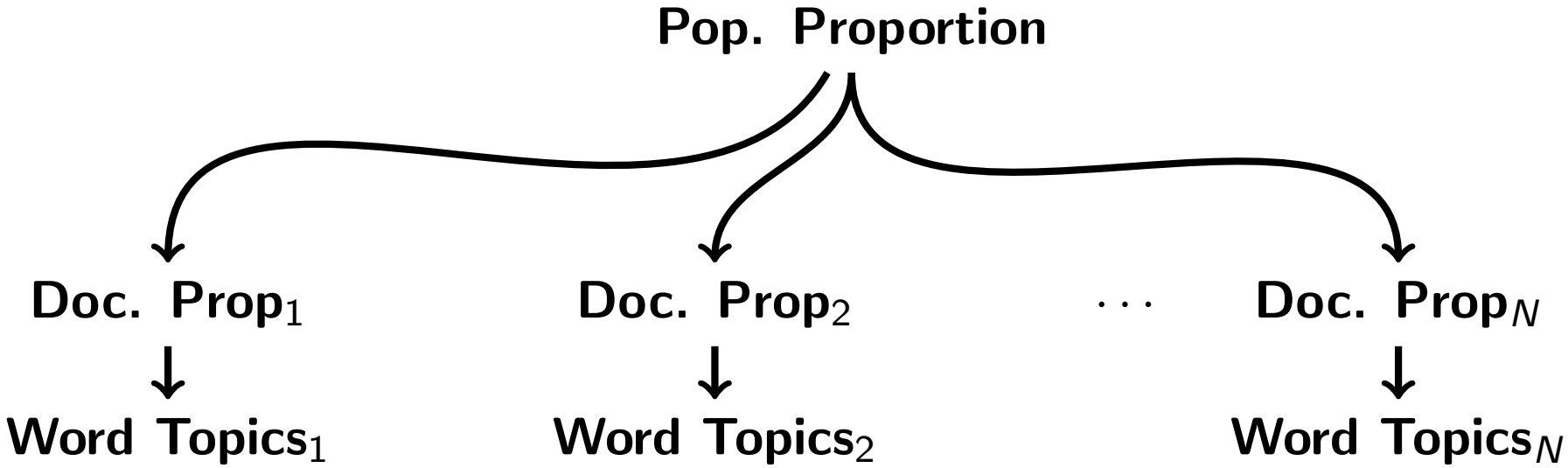
A General Hierarchical Structure

LDA:



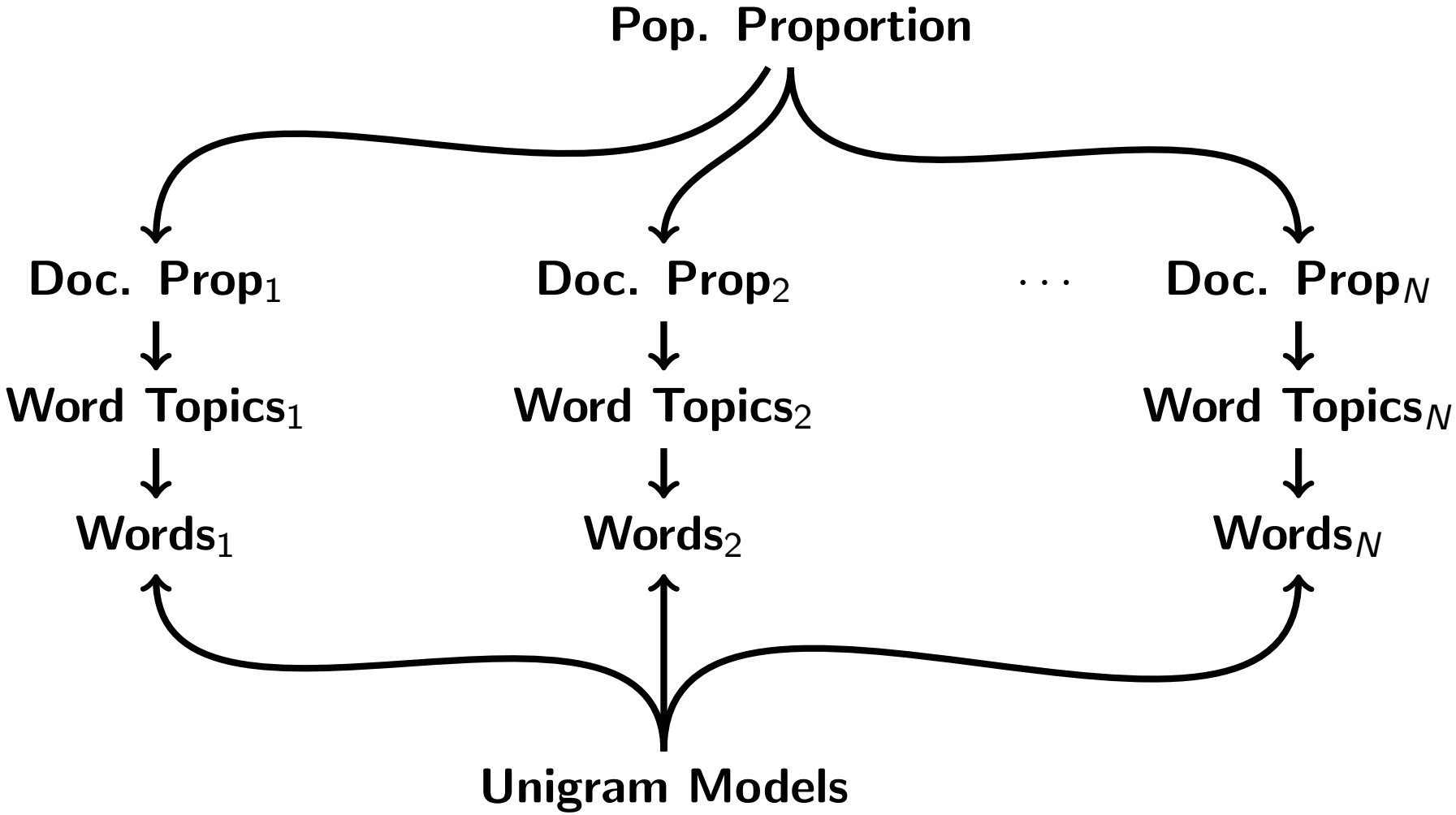
A General Hierarchical Structure

LDA:



A General Hierarchical Structure

LDA:



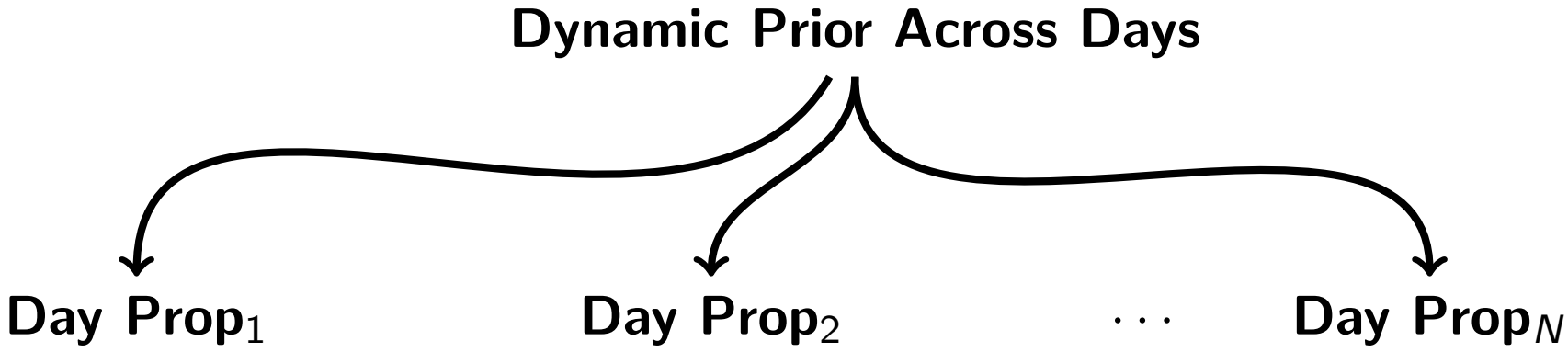
A General Hierarchical Structure

Dynamic Topic Model (Quinn et al 2010)

Dynamic Prior Across Days

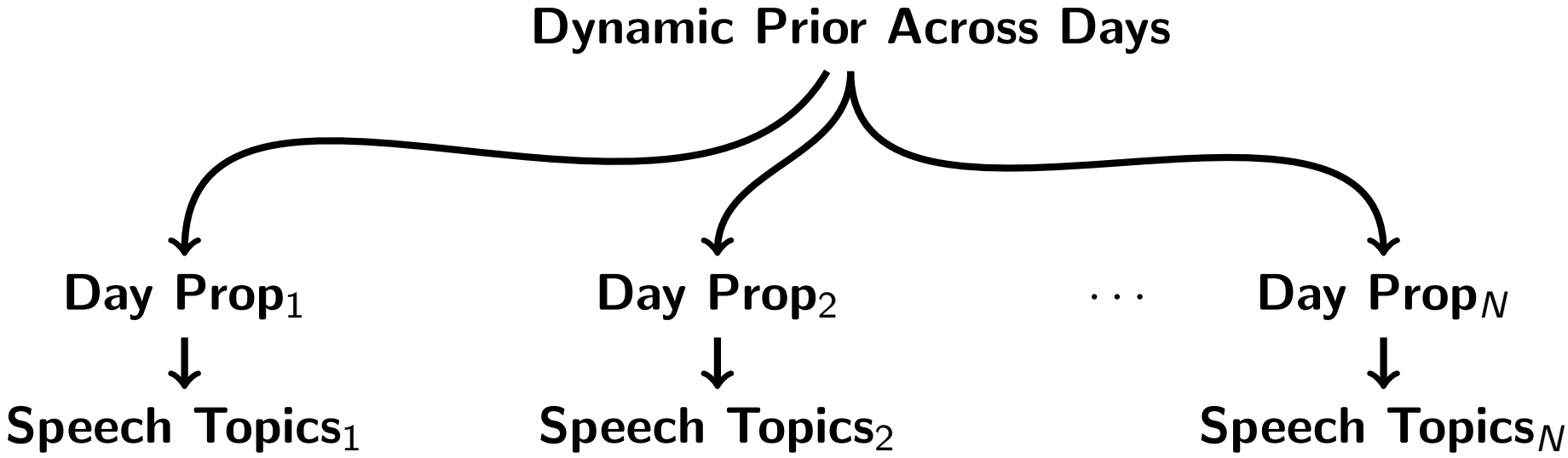
A General Hierarchical Structure

Dynamic Topic Model (Quinn et al 2010)



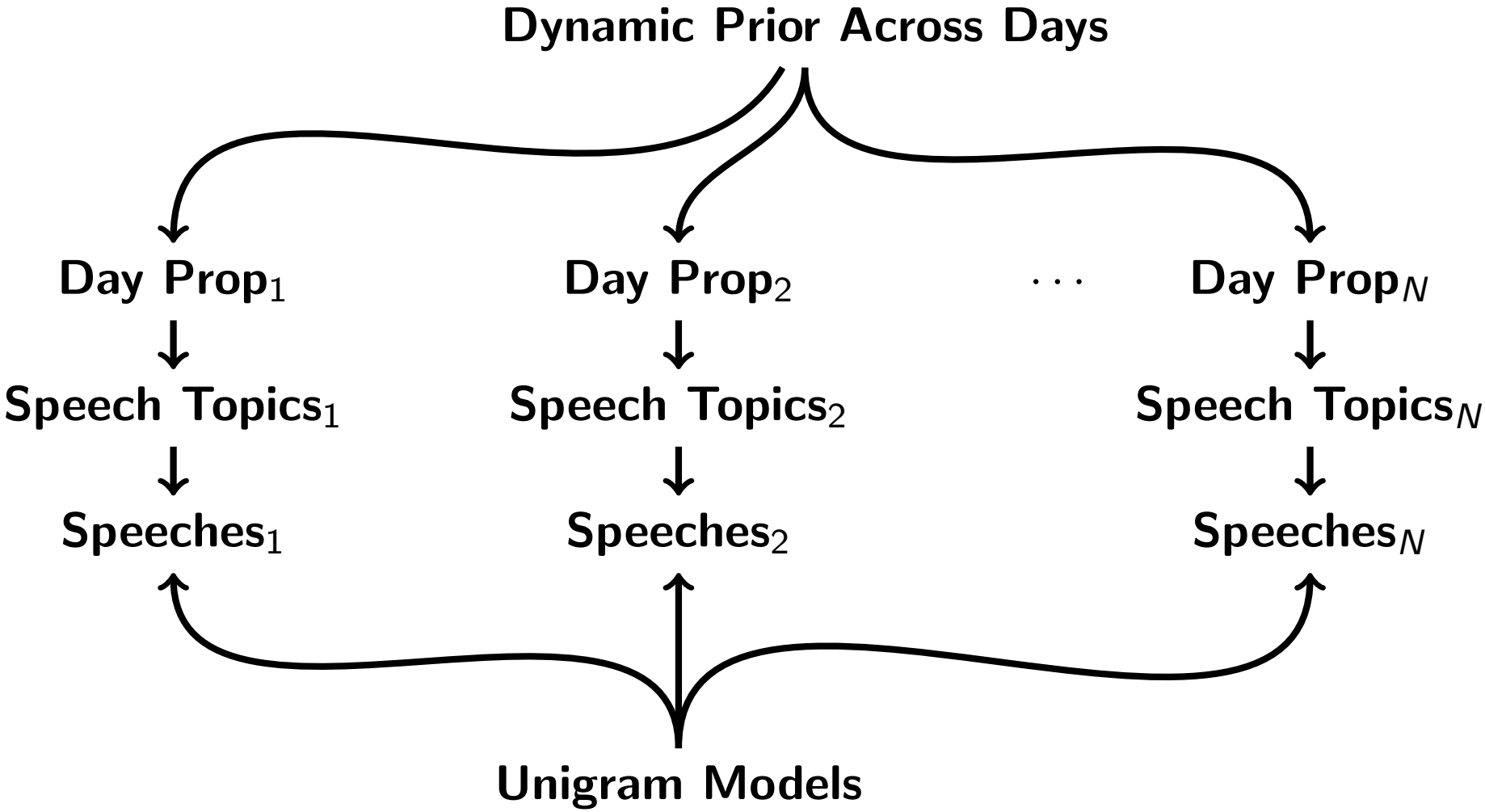
A General Hierarchical Structure

Dynamic Topic Model (Quinn et al 2010)



A General Hierarchical Structure

Dynamic Topic Model (Quinn et al 2010)



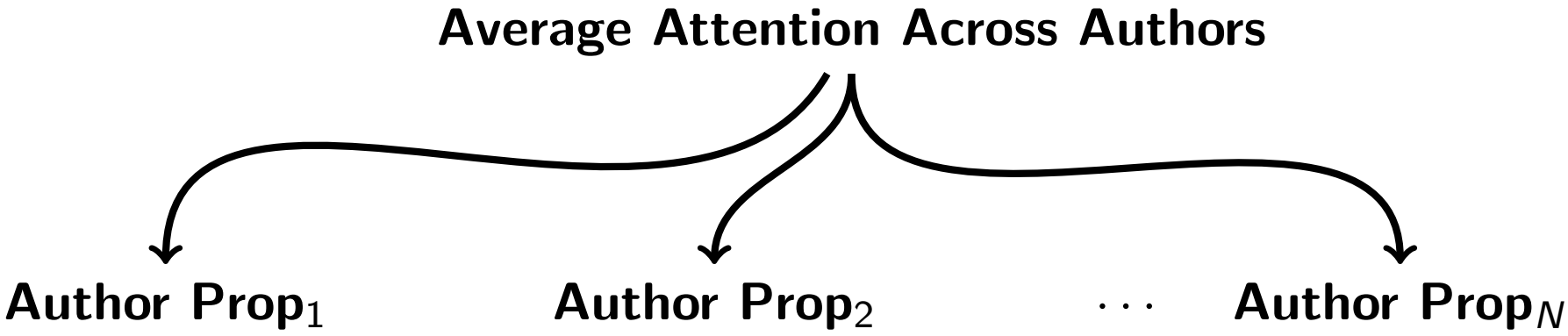
A General Hierarchical Structure

Expressed Agenda Model (Grimmer 2010)

Average Attention Across Authors

A General Hierarchical Structure

Expressed Agenda Model (Grimmer 2010)



A General Hierarchical Structure

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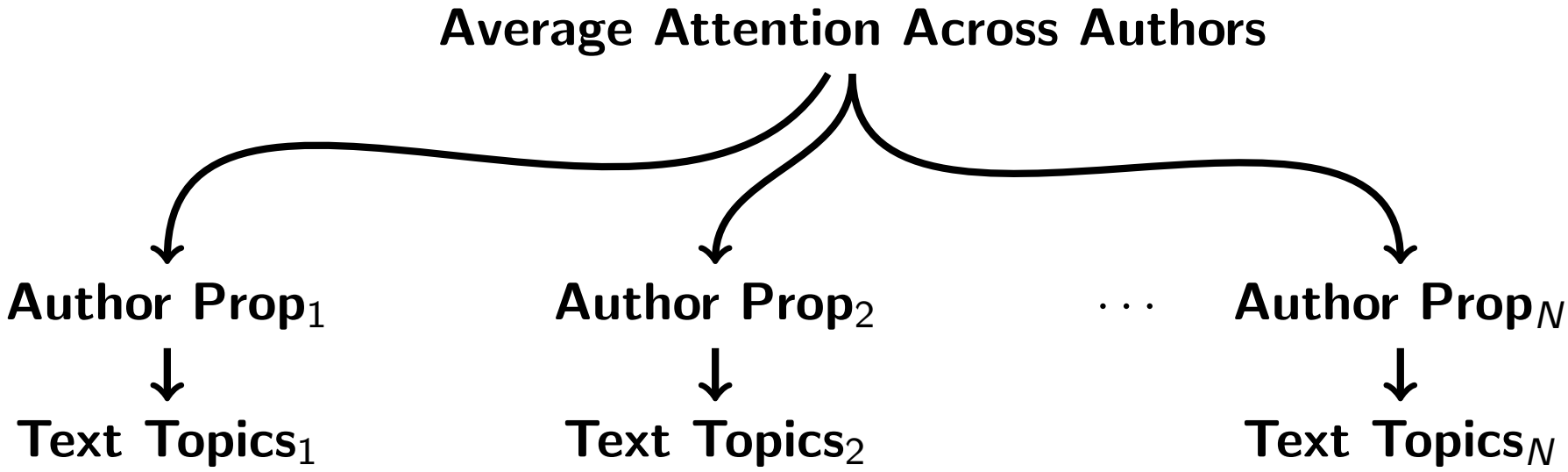

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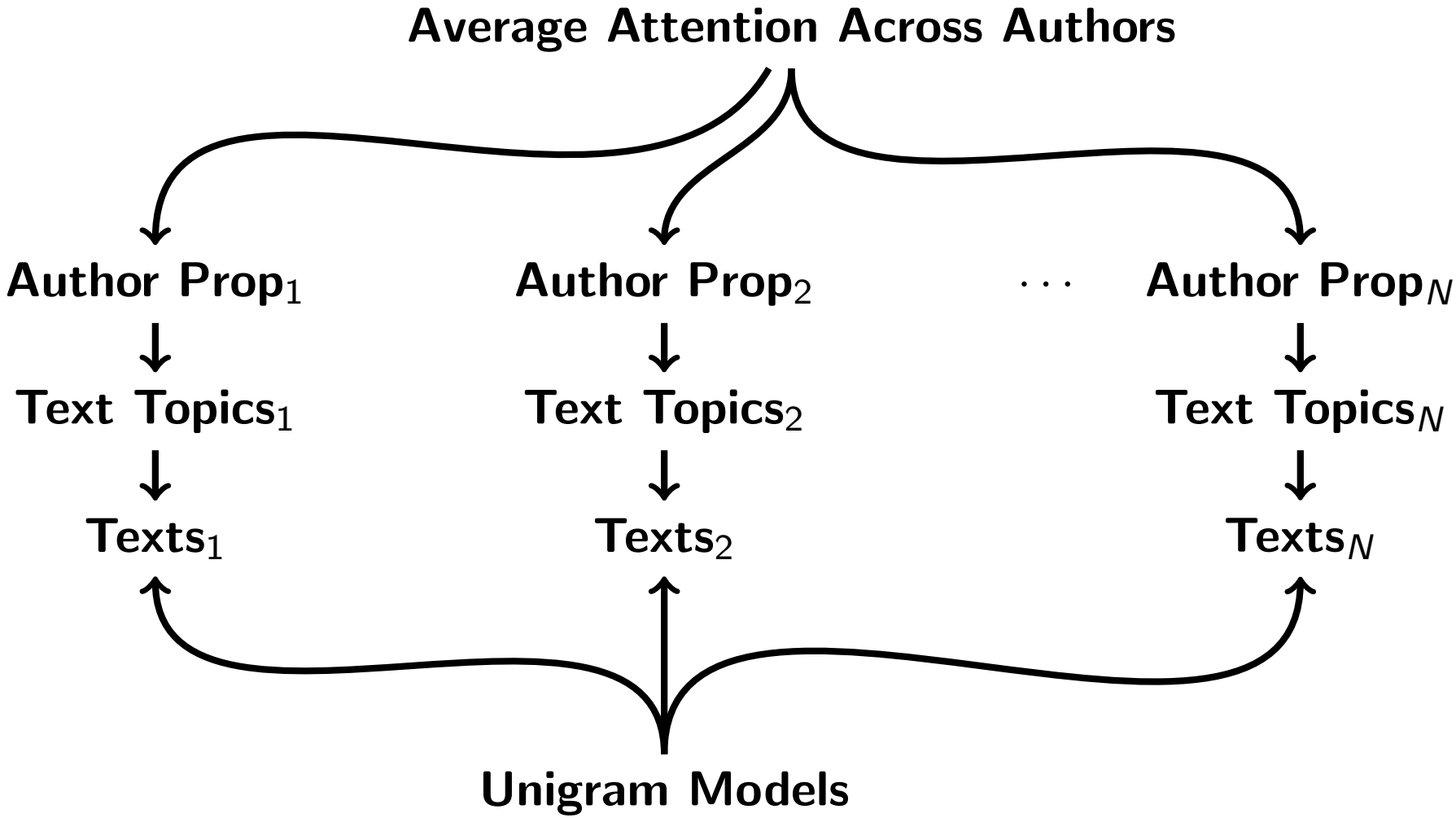

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$$g(\boldsymbol{W}, \gamma)$$

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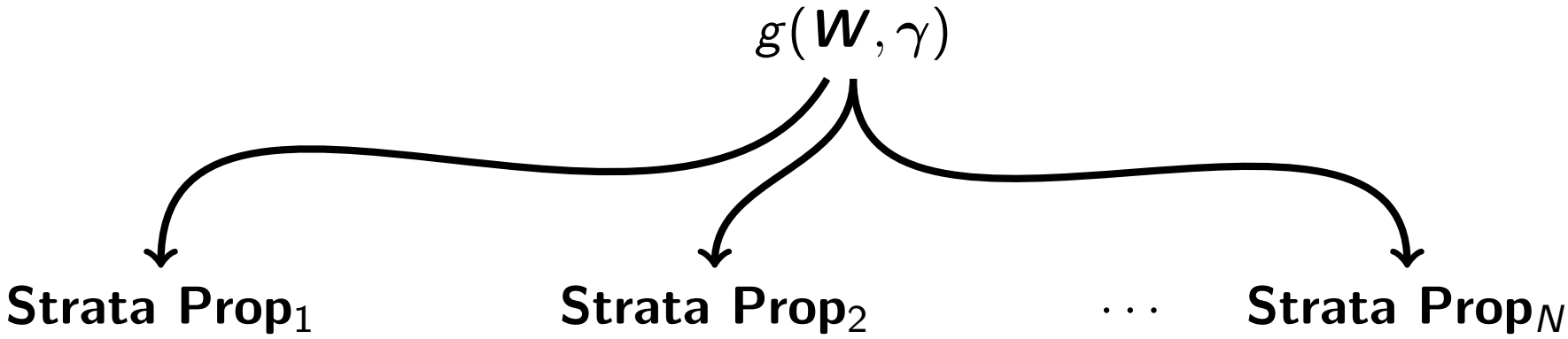

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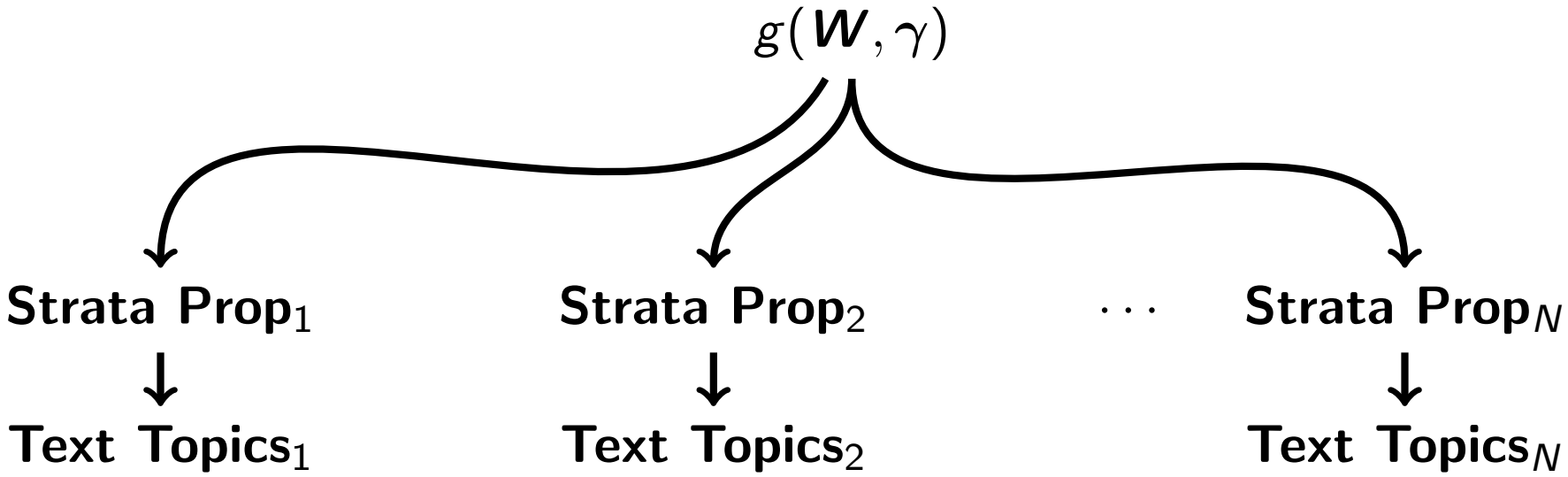

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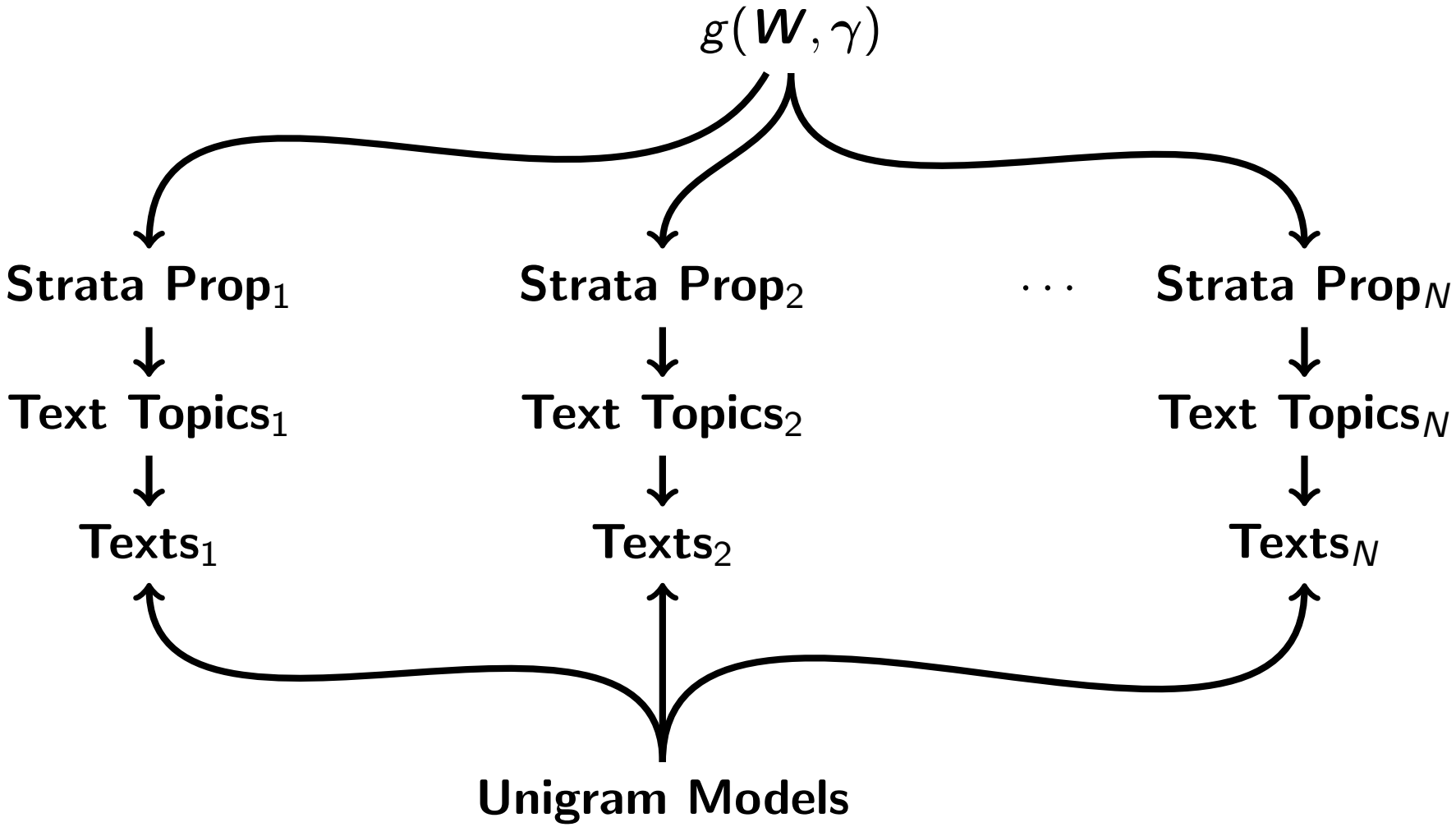

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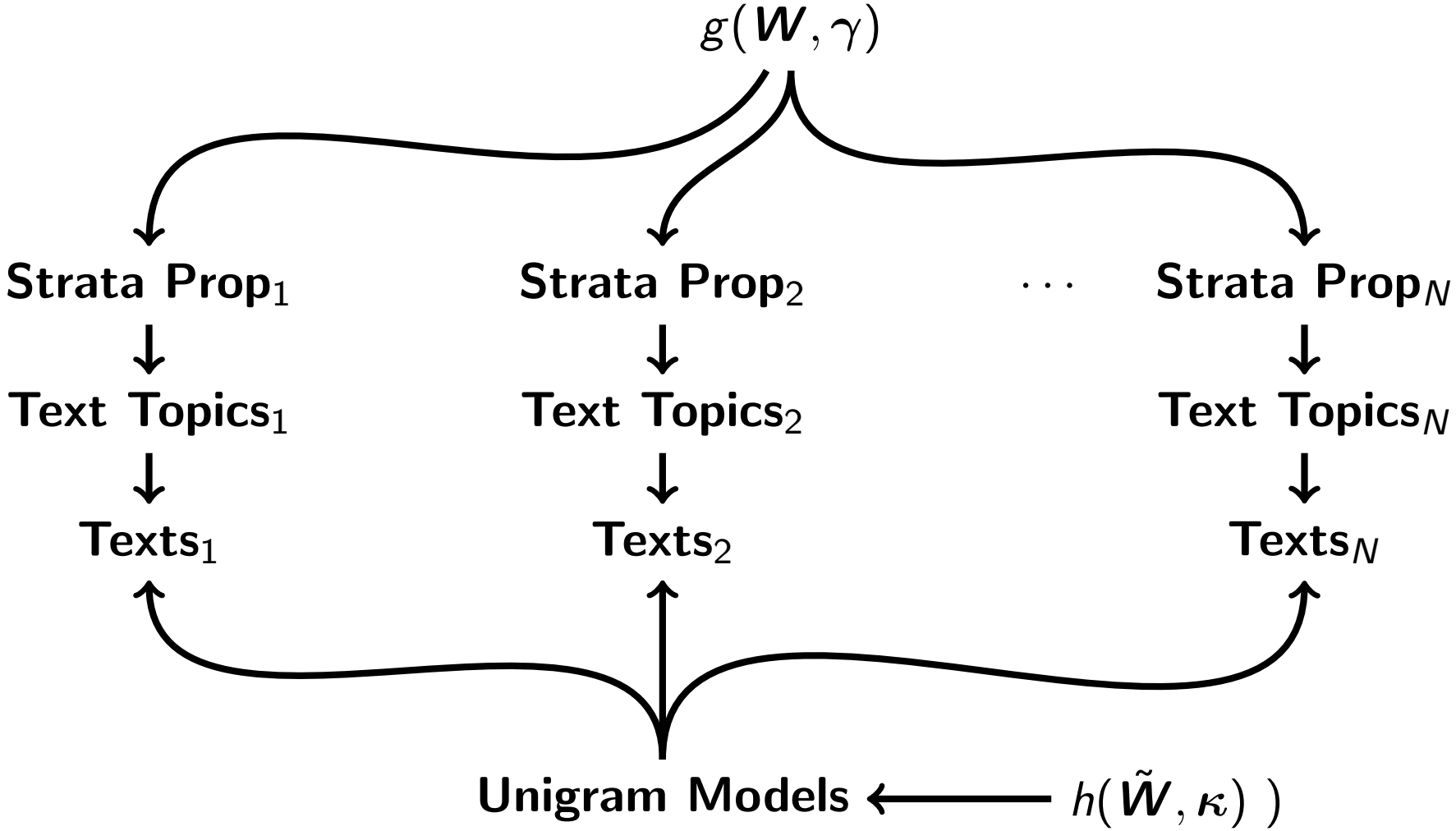

A General Hierarchical Structure

Conditioning on Unknown Covariates$\rightsquigarrow$ levels of mixtures at proportions (Grimmer 2013; Wallach 2008)

Mixture of Top. Attn. Models

A General Hierarchical Structure

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Strata Indic. **Topic Attention Models**

A General Hierarchical Structure

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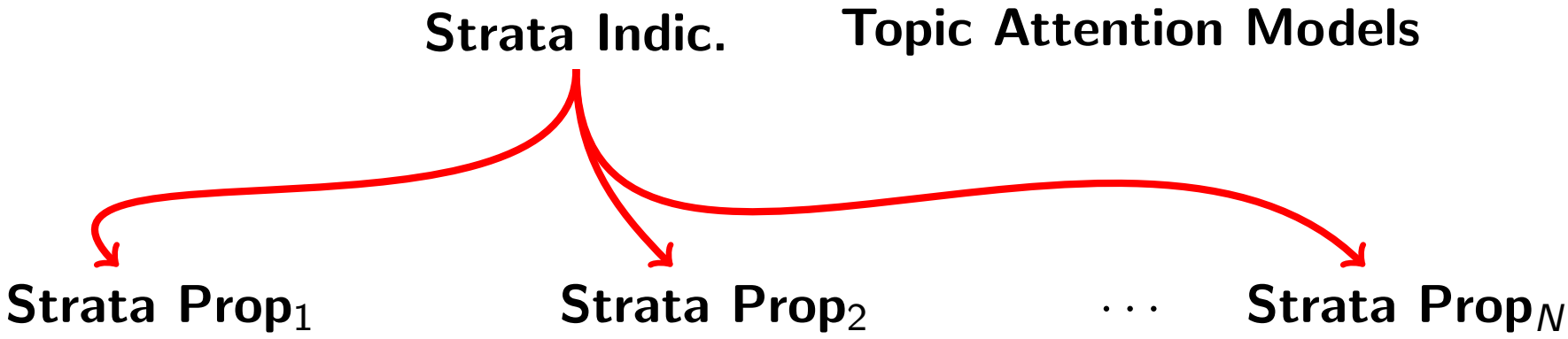

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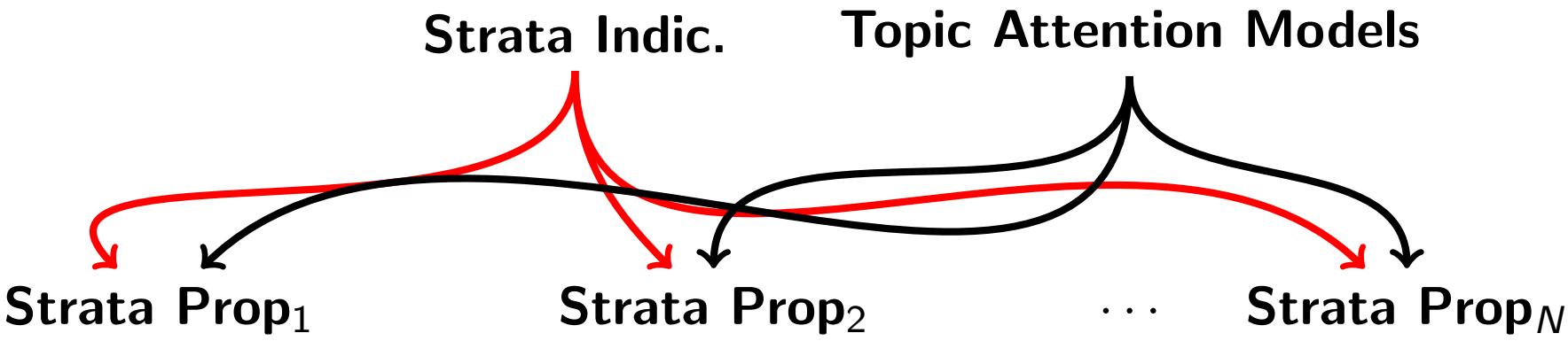

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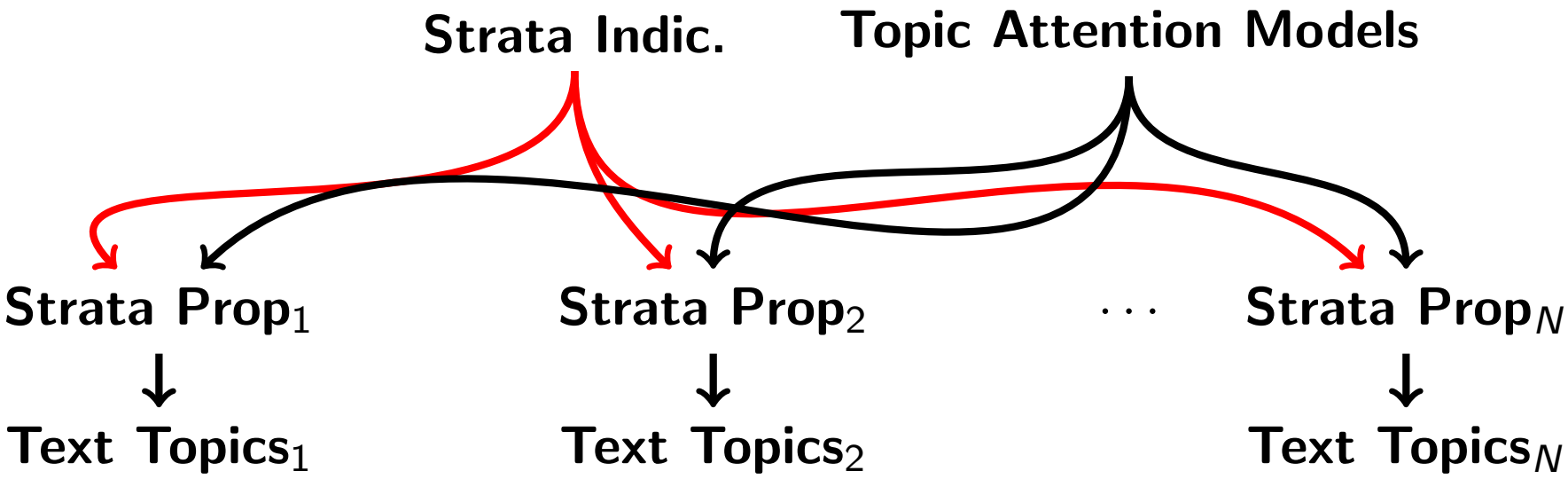

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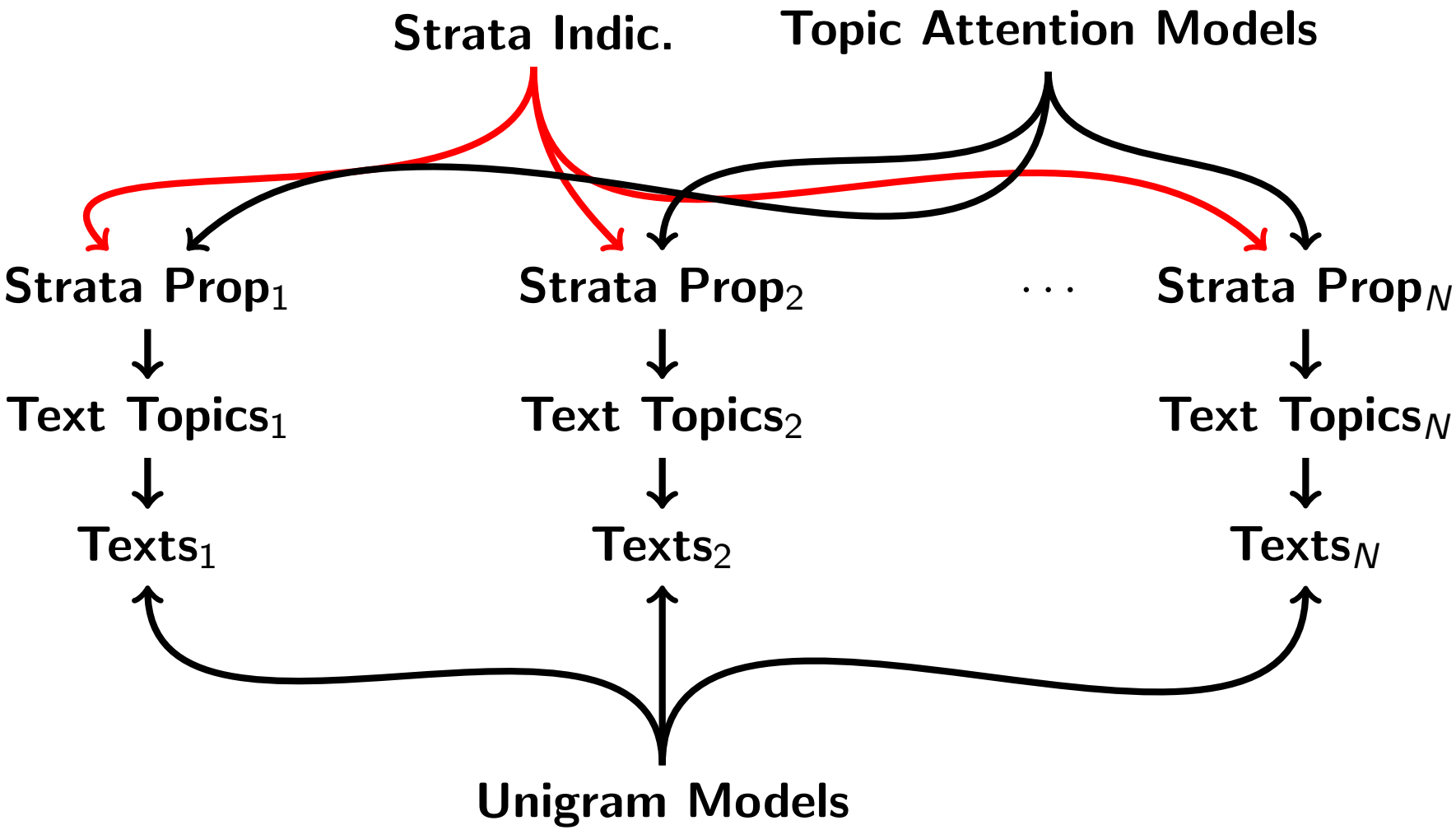

A General Hierarchical Structure

Conditioning on Unknown Covariates for Topics$\rightsquigarrow$ hierarchy of topics (Li and McCallum 2006; Blaydes, Grimmer, and McQueen 2017)

Population attention

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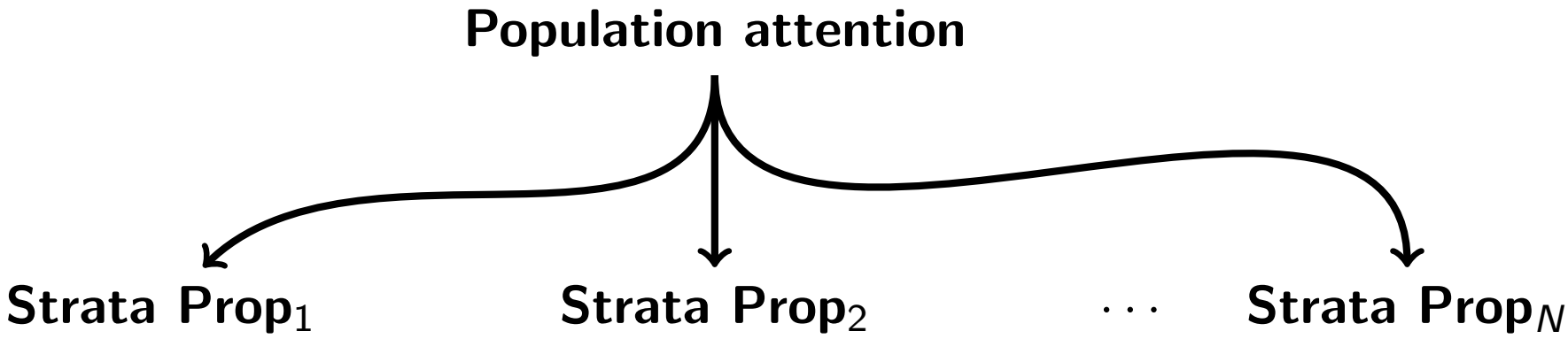

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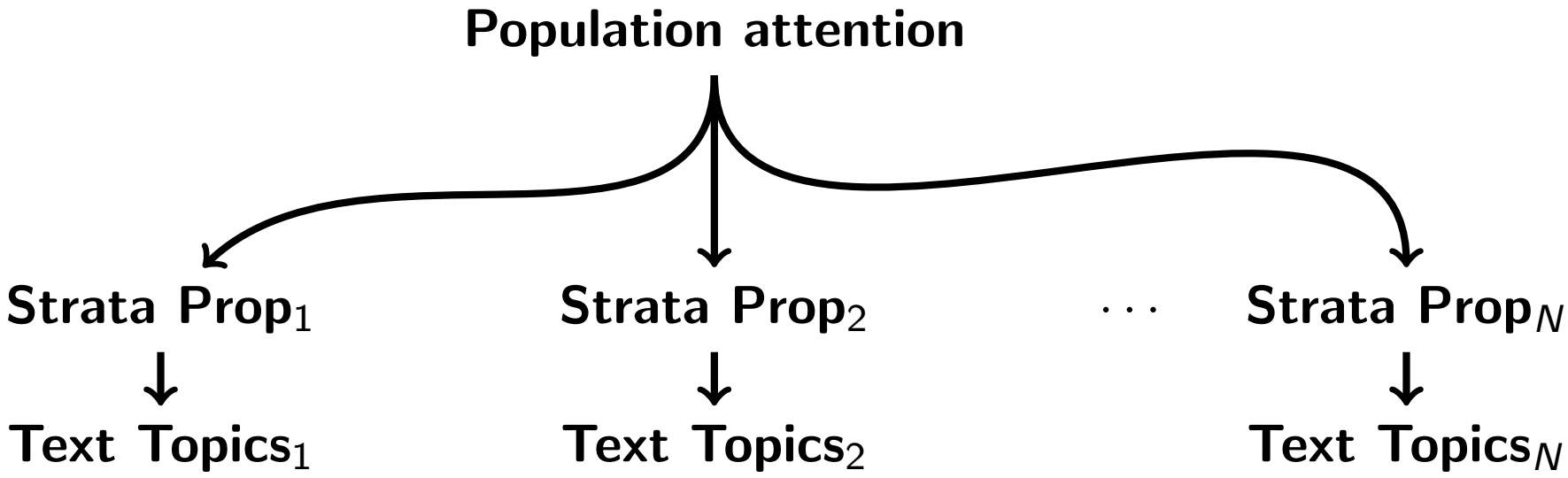

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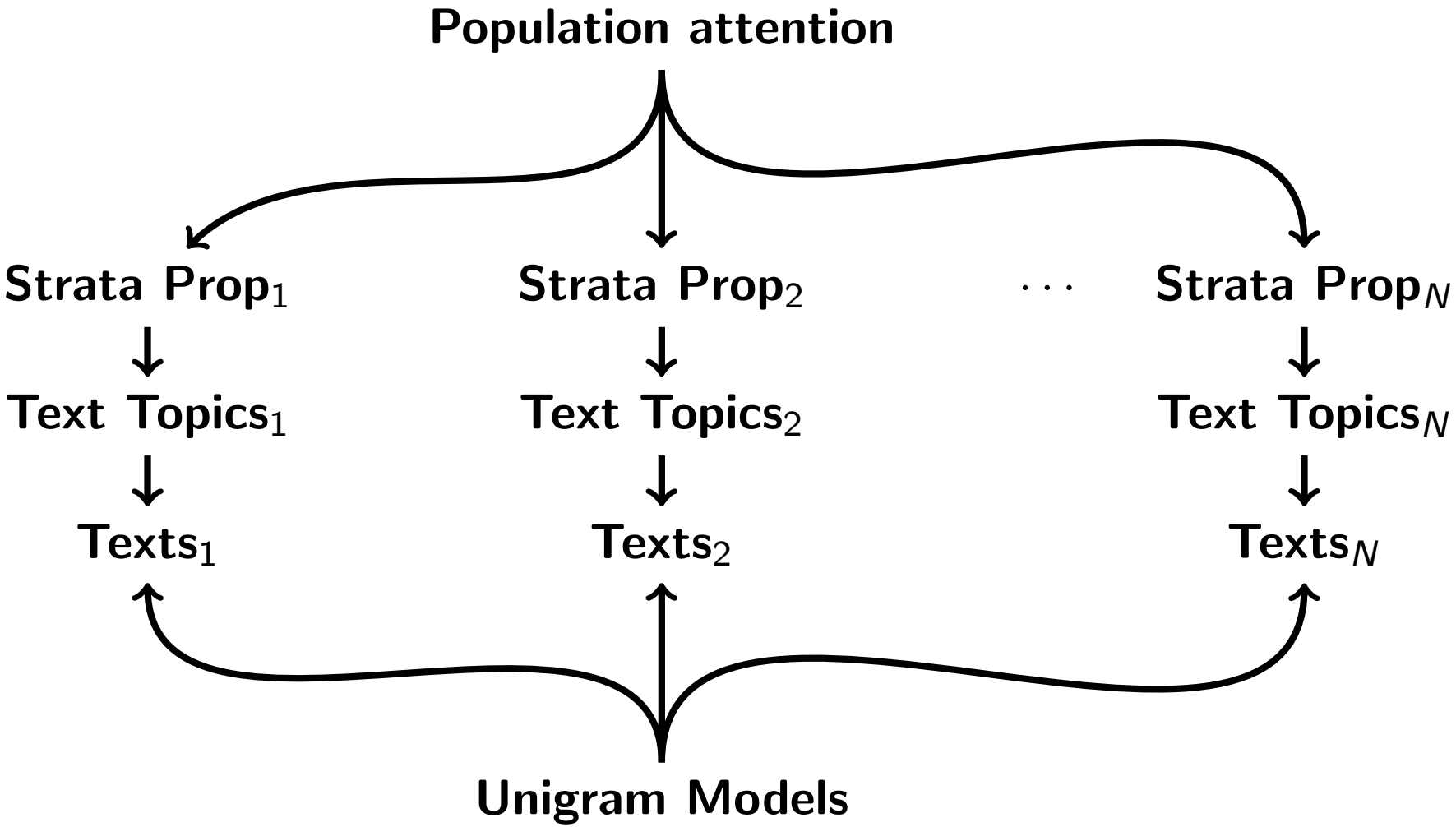

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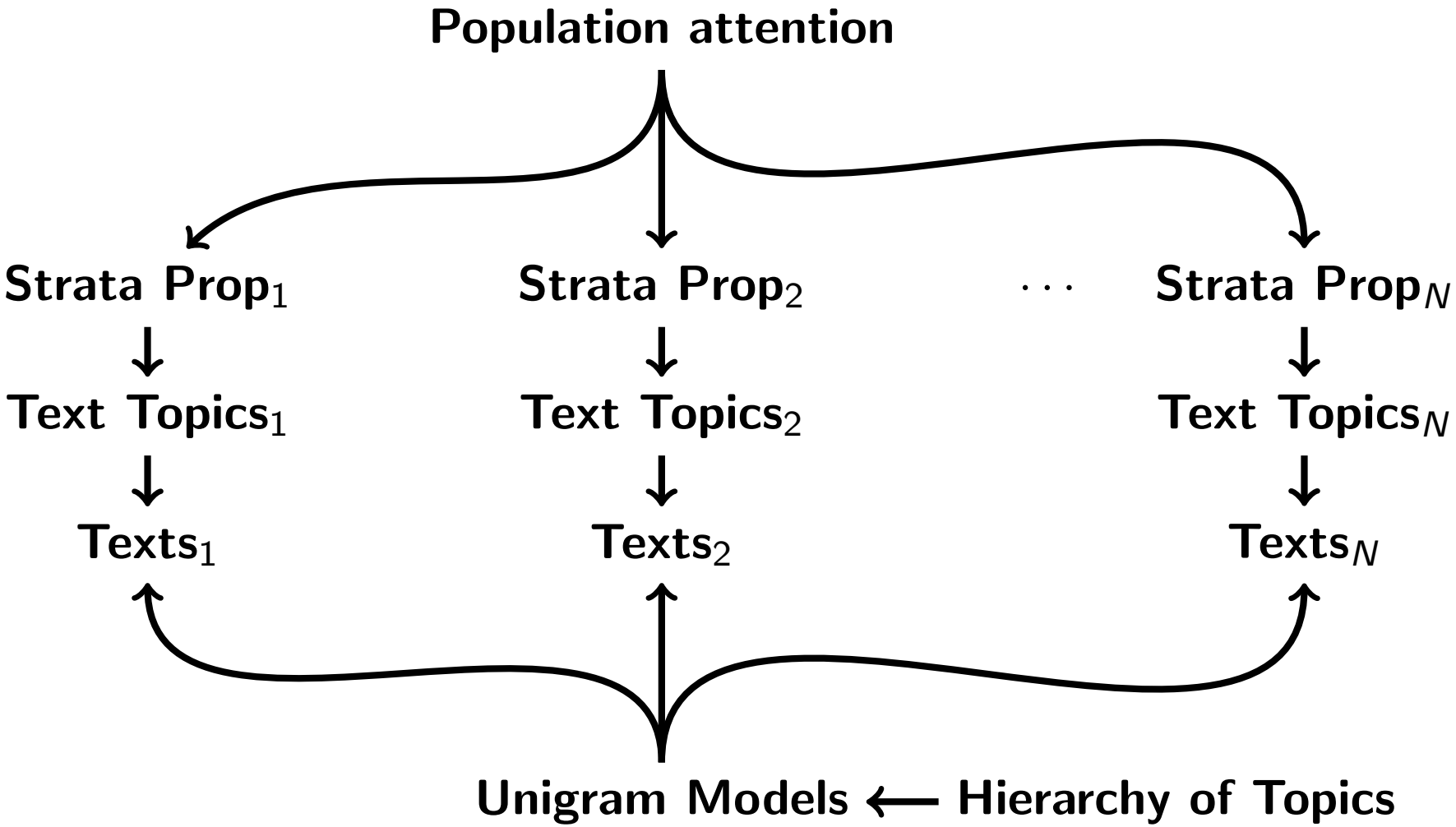

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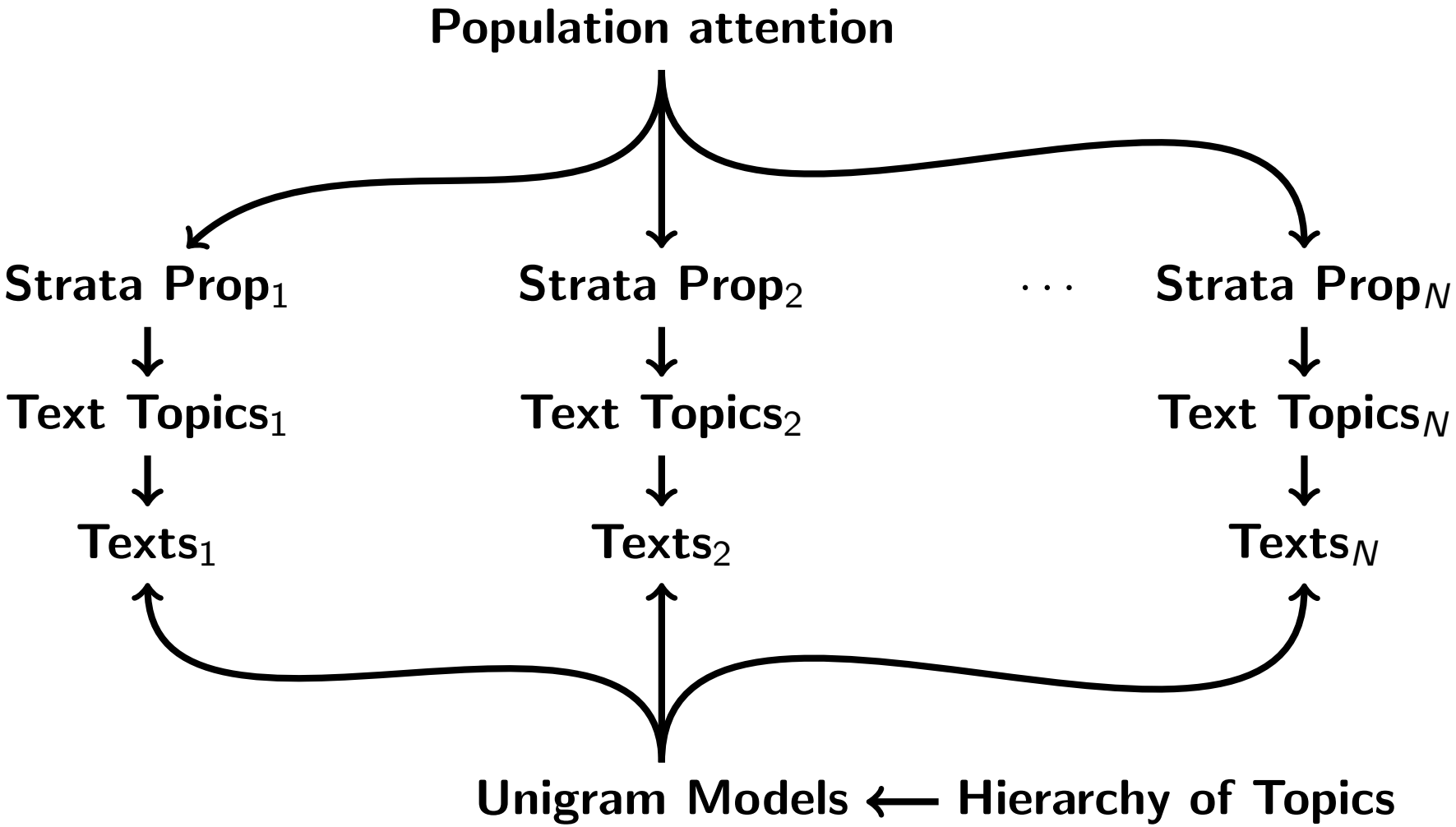

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  - Improved topics$\rightsquigarrow$ small word conditions, structure could help

Plan for the Class

1) Discuss model with unknown covariates for strata proportions$\rightsquigarrow$ presentational style
2) Discuss model with hierarchy of topics$\rightsquigarrow$ mirrors genre

Unknown Covariates for Issue Attention: Measuring Attention in Senate Press Releases

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- Characterize representation provided to constituents
- Divide attention over a set of topics
- Given attention to topics, write press releases

Presentational Styles$\rightsquigarrow$ Objective Function

- $\pi_{itk} \equiv$ Attention senator i allocates to issue k in year t
- $\pi_{itk} \equiv$ Probability press release is about issue k
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$$\boldsymbol{x}_{ijt} \sim \text{Multinomial}(n_{ijt}, \boldsymbol{\theta}_k).$$

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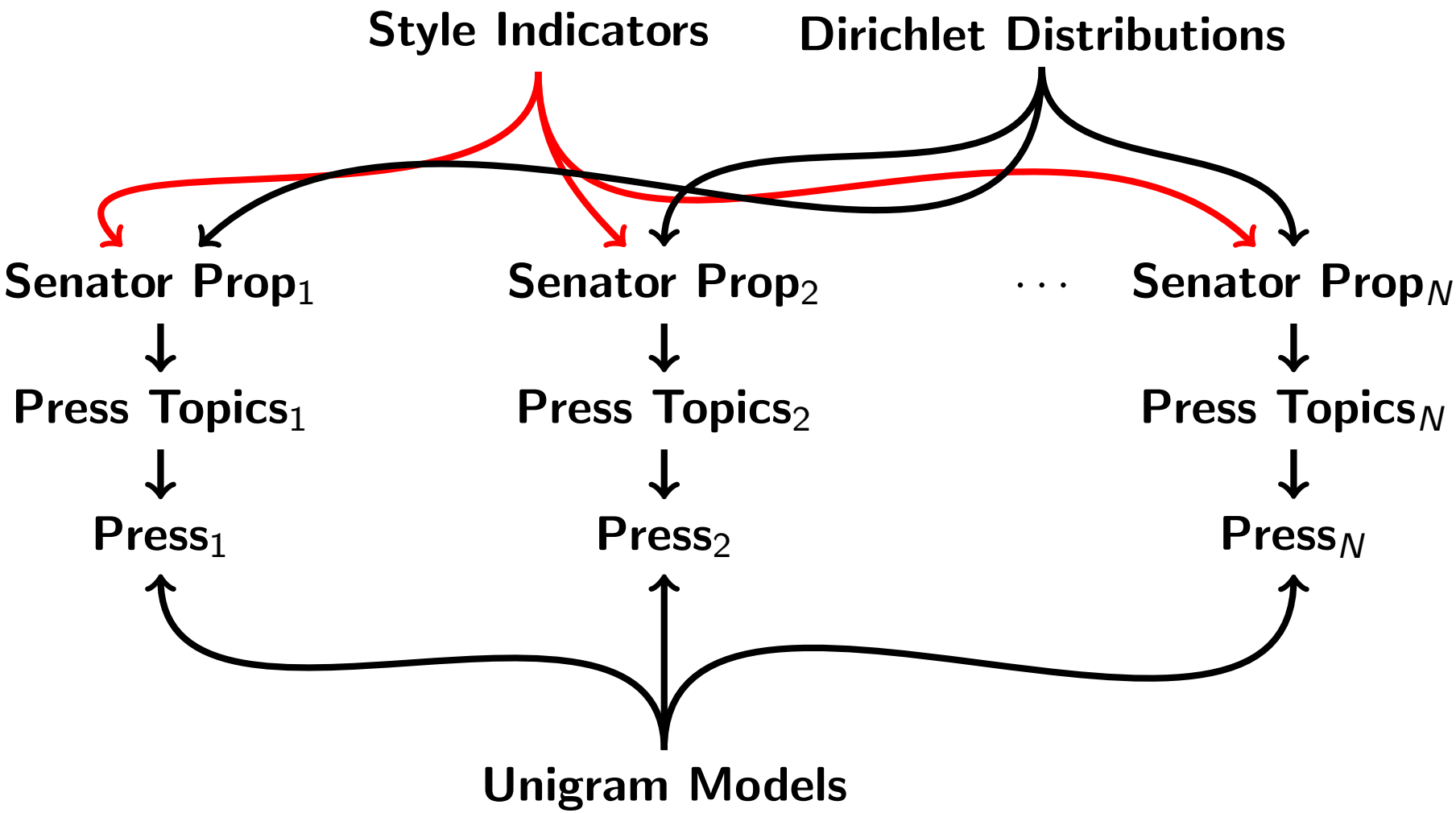
Mixture of Styles, Mixture of Topics



Posterior:

$$p(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau} | \boldsymbol{X}) \propto \prod_{k=1}^{K} \prod_{s=1}^{S} \frac{\exp(-\frac{\alpha_{ks}}{1/4})}{1/4} \times \frac{\Gamma(\sum_{w=1}^{W} \lambda_w)}{\prod_{w=1}^{W} \Gamma(\lambda_w)} \prod_{w=1}^{W} \theta_{k,w}^{\lambda_w - 1} \times$$

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1) Estimate with Variational Approximation
2) Determining number of clusters at top? (Grimmer, Shorey, Wallach, and Zlotnick, In Progress)

Posterior:

$$p(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau} | \boldsymbol{X}) \propto \prod_{k=1}^{K} \prod_{s=1}^{S} \frac{\exp(-\frac{\alpha_{ks}}{1/4})}{1/4} \times \frac{\Gamma(\sum_{w=1}^{W} \lambda_w)}{\prod_{w=1}^{W} \Gamma(\lambda_w)} \prod_{w=1}^{W} \theta_{k,w}^{\lambda_w - 1} \times$$
$$\prod_{i=1}^{N} \prod_{t=2005}^{2007} \prod_{s=1}^{S} \left[\beta_s \frac{\Gamma(\sum_{k=1}^{K} \alpha_{ks})}{\prod_{k=1}^{K} \Gamma(\alpha_{ks})} \prod_{k=1}^{K} \pi_{itk}^{\alpha_{ks} - 1} \prod_{j=1}^{D_{it}} \prod_{k=1}^{K} \left[\pi_{itk} \prod_{w=1}^{W} \theta_{kw}^{x_{ijtw}} \right]^{\tau_{ijtk}} \right]^{\sigma_{its}}$$

1) Estimate with Variational Approximation
2) Determining number of clusters at top? (Grimmer, Shorey, Wallach, and Zlotnick, In Progress)
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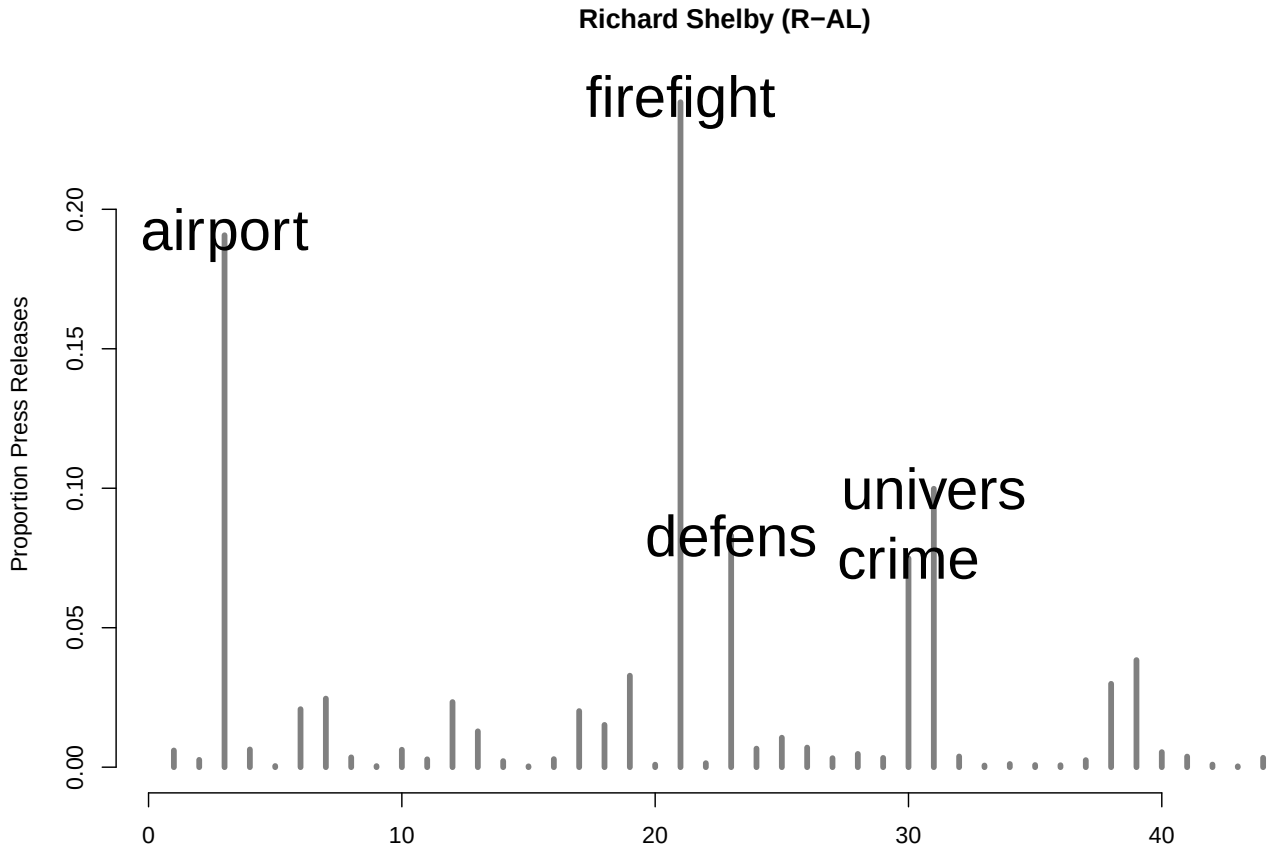
2) Determining number of clusters at top? (Grimmer, Shorey, Wallach, and Zlotnick, In Progress)
   - Non-parametric model$\rightsquigarrow$ statistical selection
   - Experiments/Coding Exercises to assess


Richard Shelby (R−AL)
firefight
airport
univers
defens
crime
Proportion Press Releases
0.00
0.05
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0.15
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0
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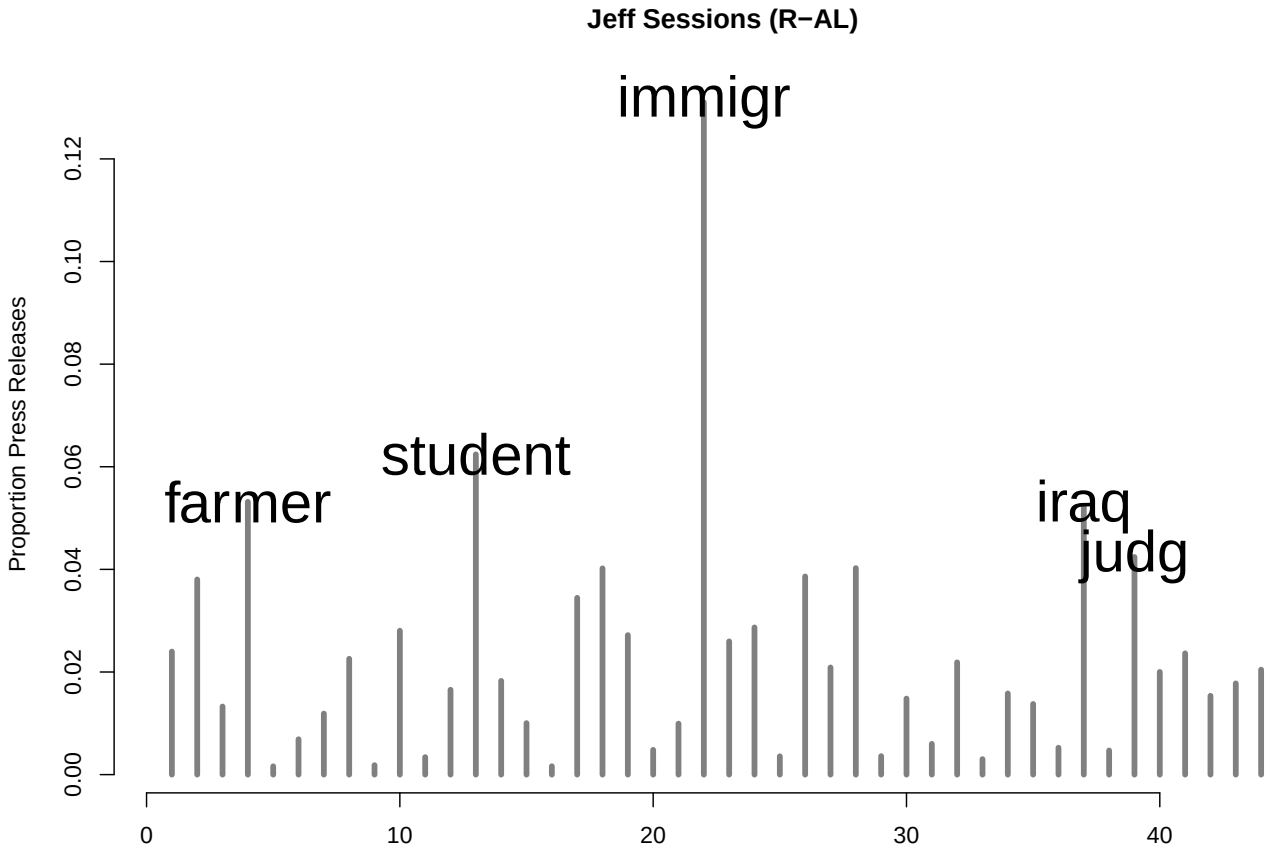
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0.12
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10
20
30
40

Notions of validity: From Quinn, Monroe, et al (2010)

- Semantic Validity: All categories are coherent and meaningful
- Convergent Construct Validity: Measures concur with existing measures in critical details.
- Discriminant Construct Validity: Measures differ from existing measures in productive ways.
- Predictive Measure: Measures from the model corresponds to external events in expected ways.
- Hypothesis Validity: Measures generated from the model can be used to test substantive hypotheses.

To establish utility of new measures, demonstrate variety of validations
None of these validations are performed using a canned statistic
All: require substantive knowledge on areas (and what we expect!) [

Home Style Measures, Semantic Validity

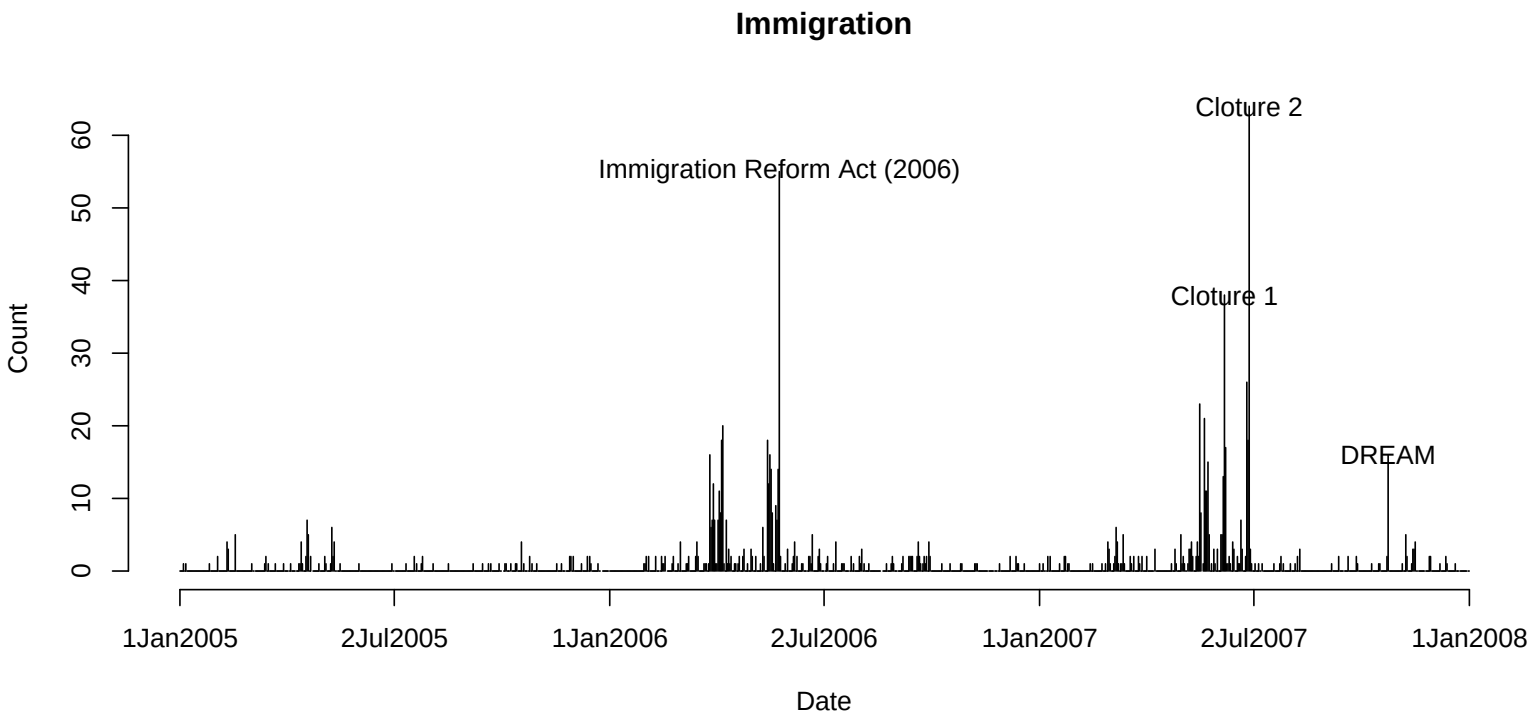
Must: Demonstrate to reader that topics are coherent and semantically meaningful

Description	Stems	%
Honorary	honor,prayer,rememb,fund,tribut	5.0
Transp. Grants	airport,transport,announc,urban,hud	4.8
Iraq	iraq,iraqi,troop,war,sectarian	4.7
DHS Policy	homeland,port,terrorist,dh,fema	4.1
Judicial Nom.	judg,court,suprem,nomin,nomine	3.8
Fire Dept. Grant	firefight,homeland,afgp,award,equip	3.7

How: examples in text are also useful.

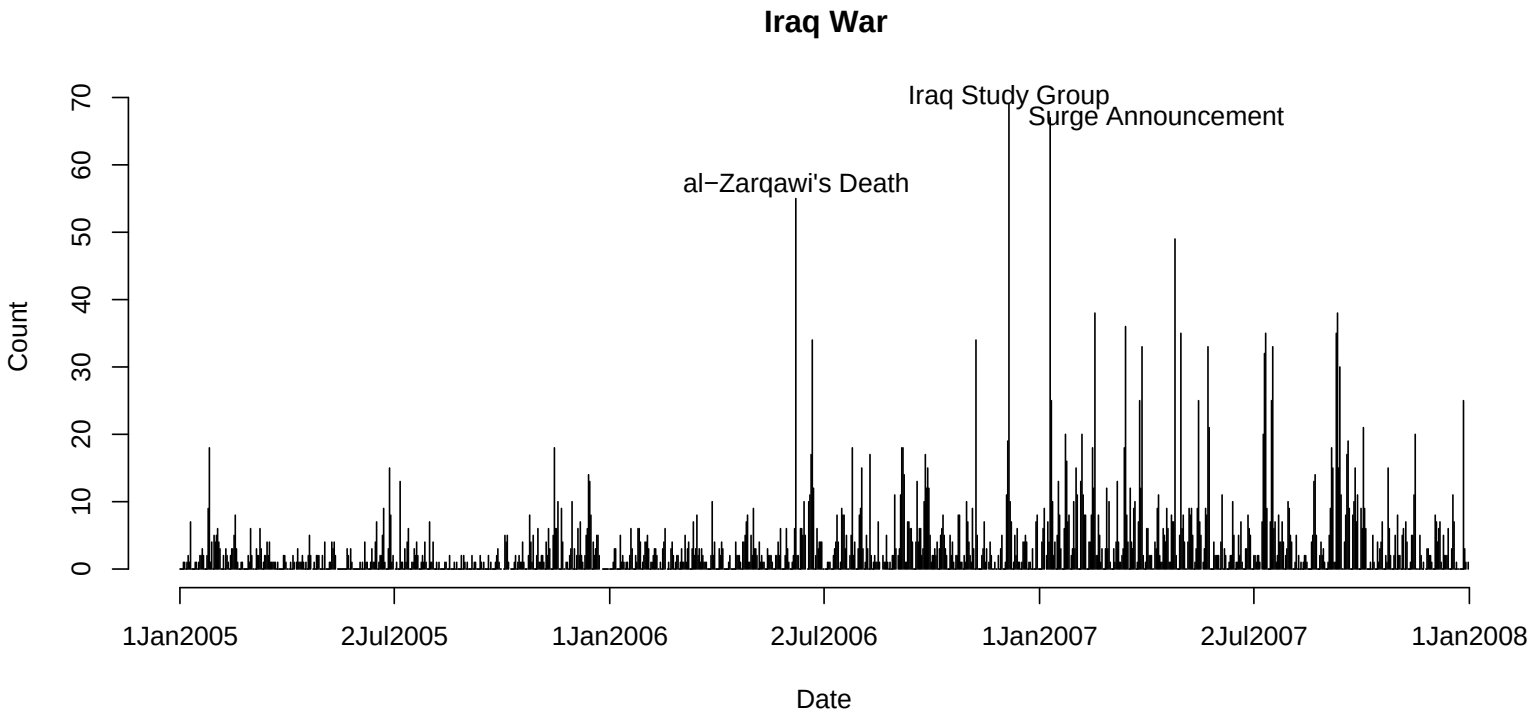
Home Style Measures, Convergent Validity

Over time variation



Home Style Measures, Convergent Validity

Over time variation



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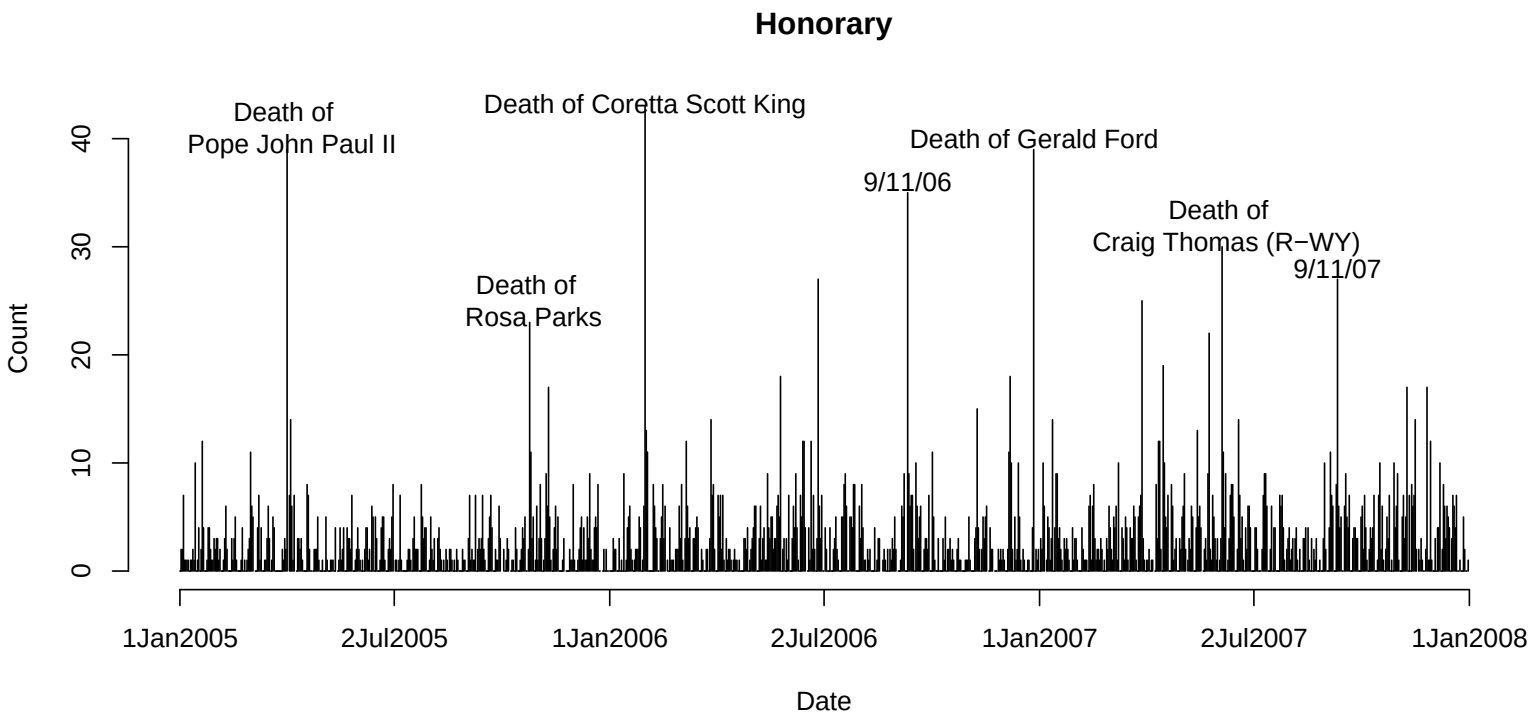
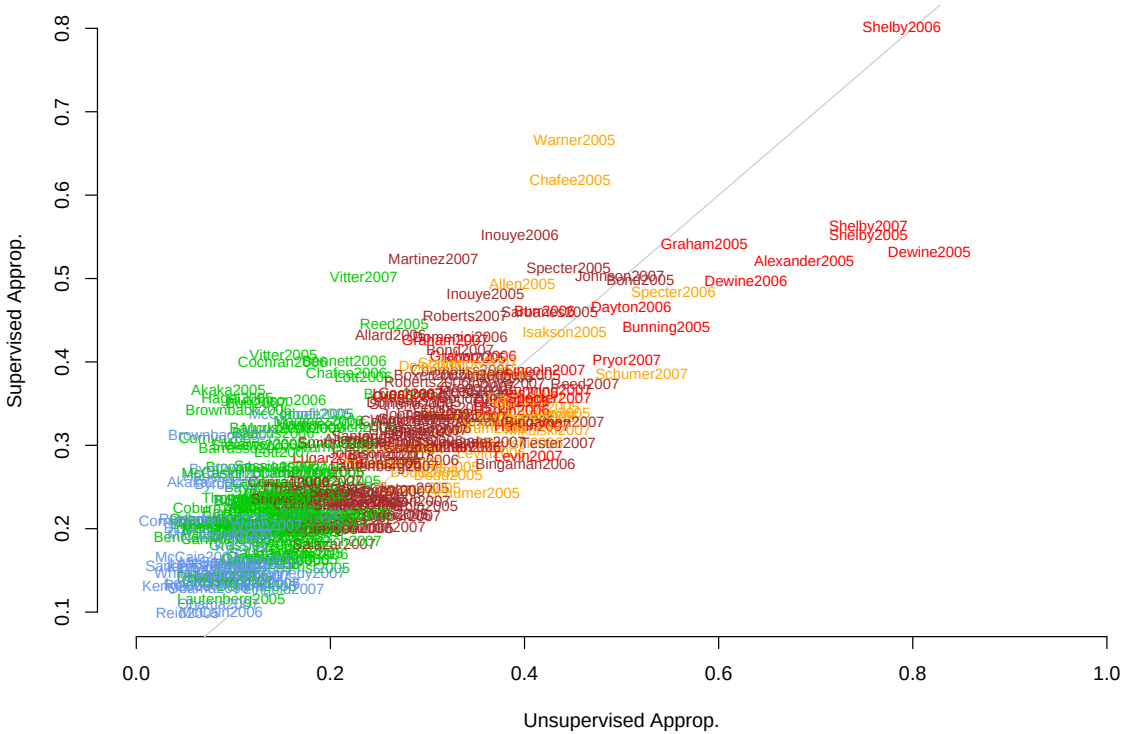
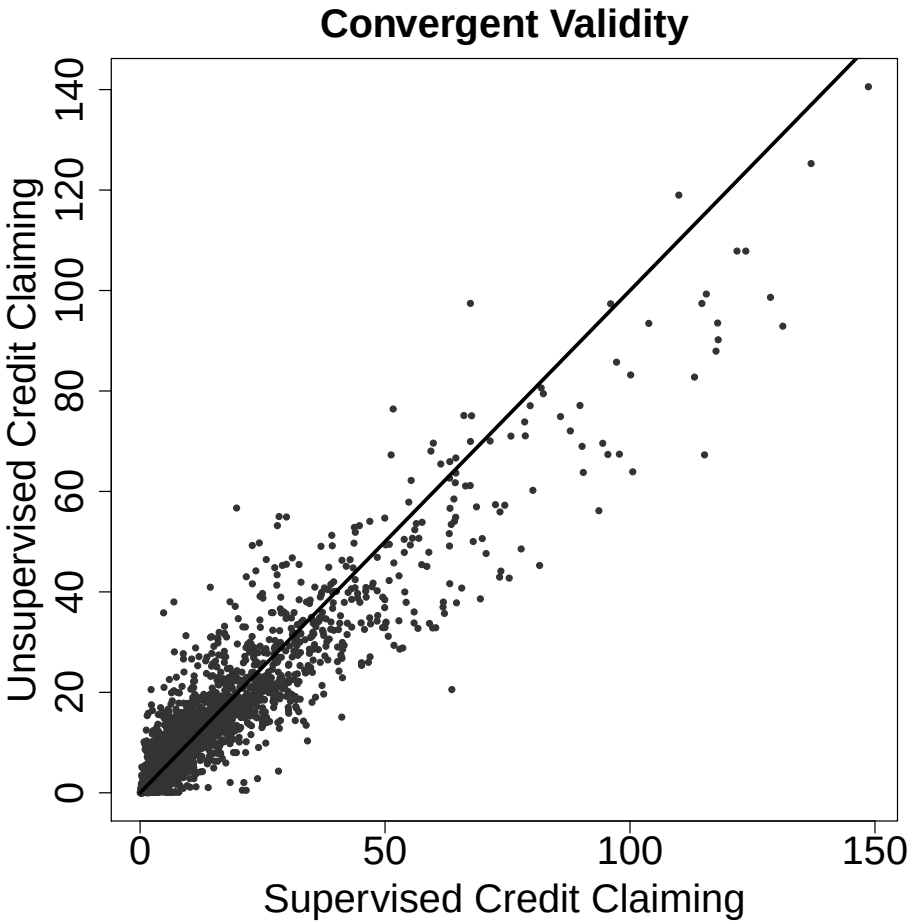


Home Style Measures, Convergent Validity

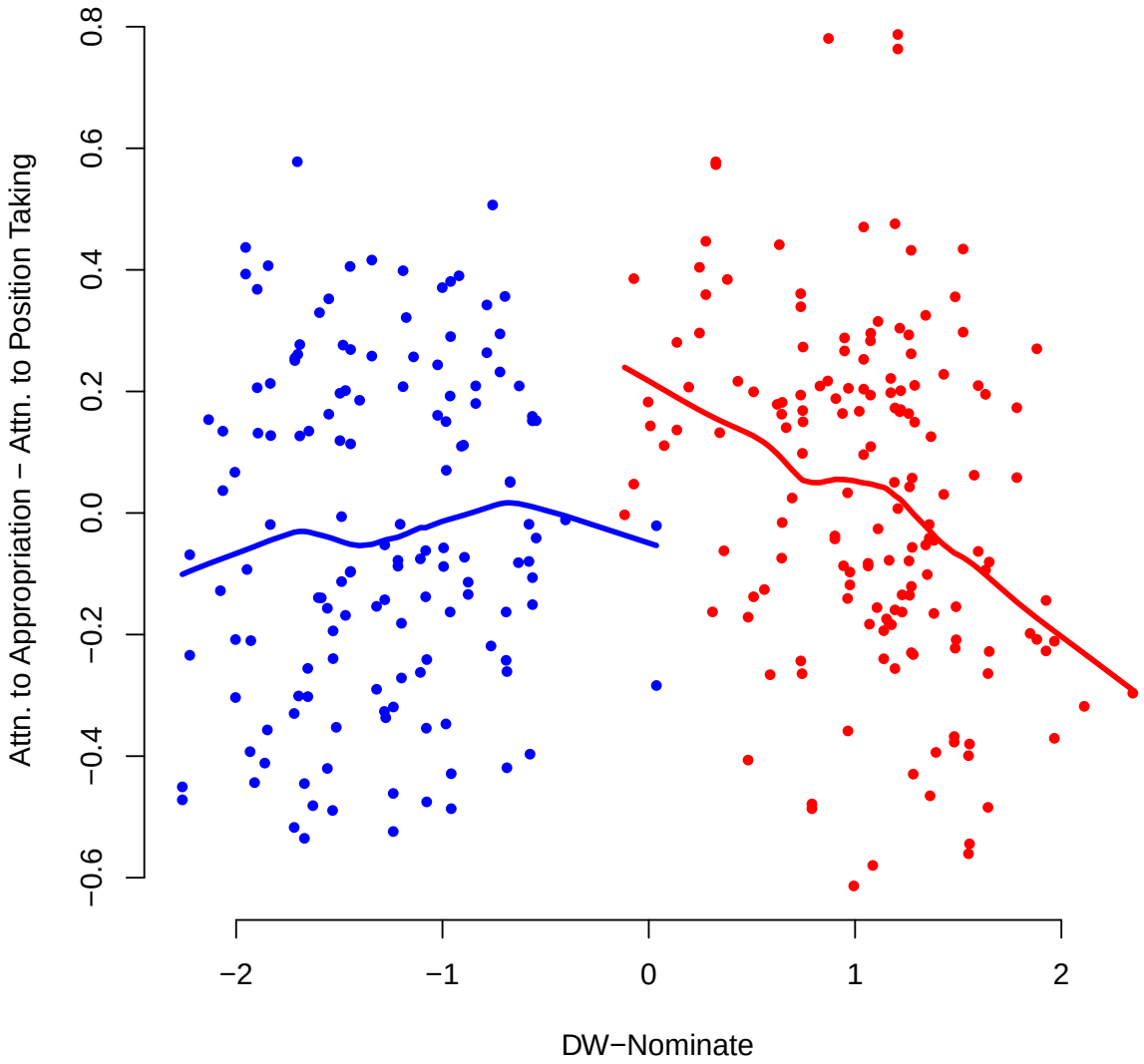
Supervised/Unsupervised Convergence



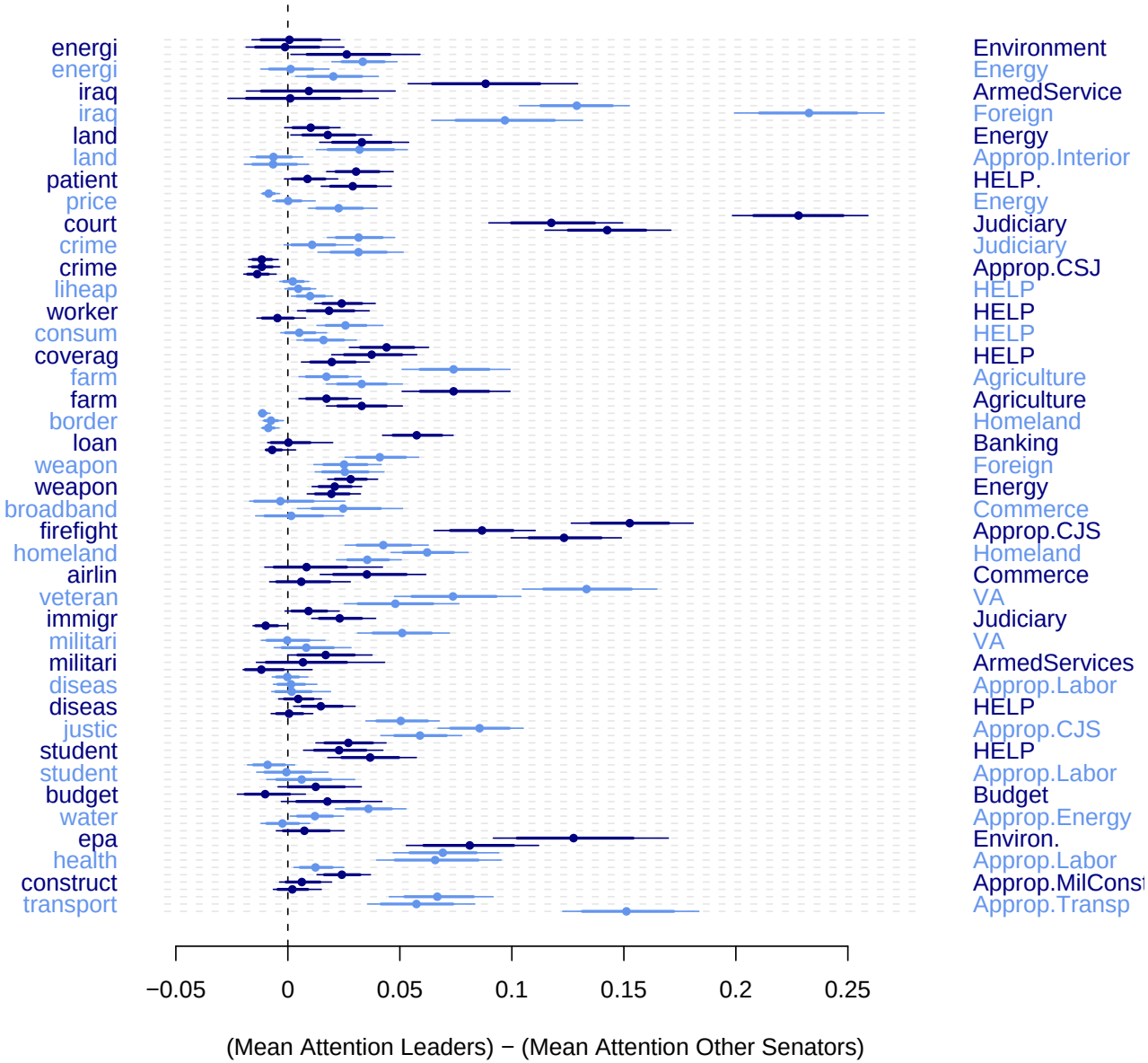
Home Style Measures, Convergent Validity



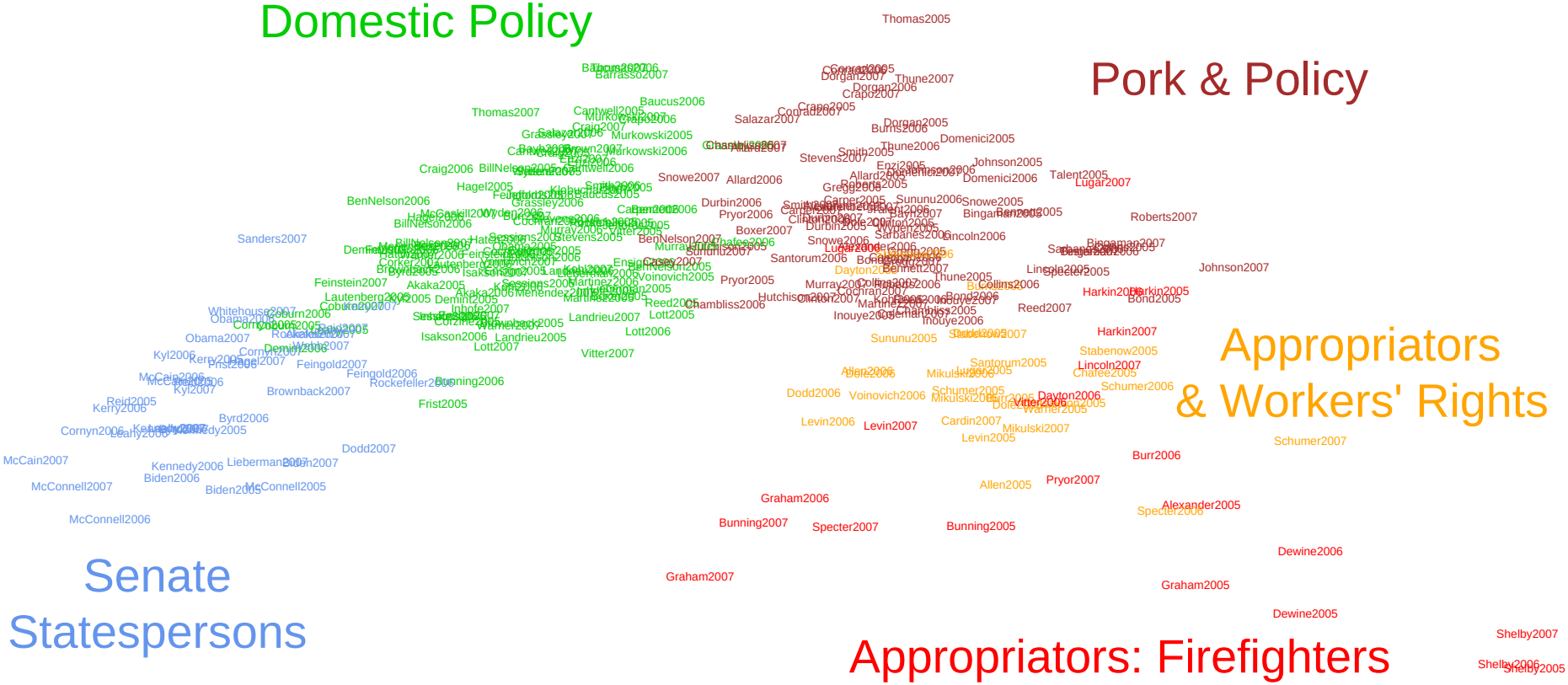
Discriminant Construct Validity



Predictive Validity



Hypothesis Validity



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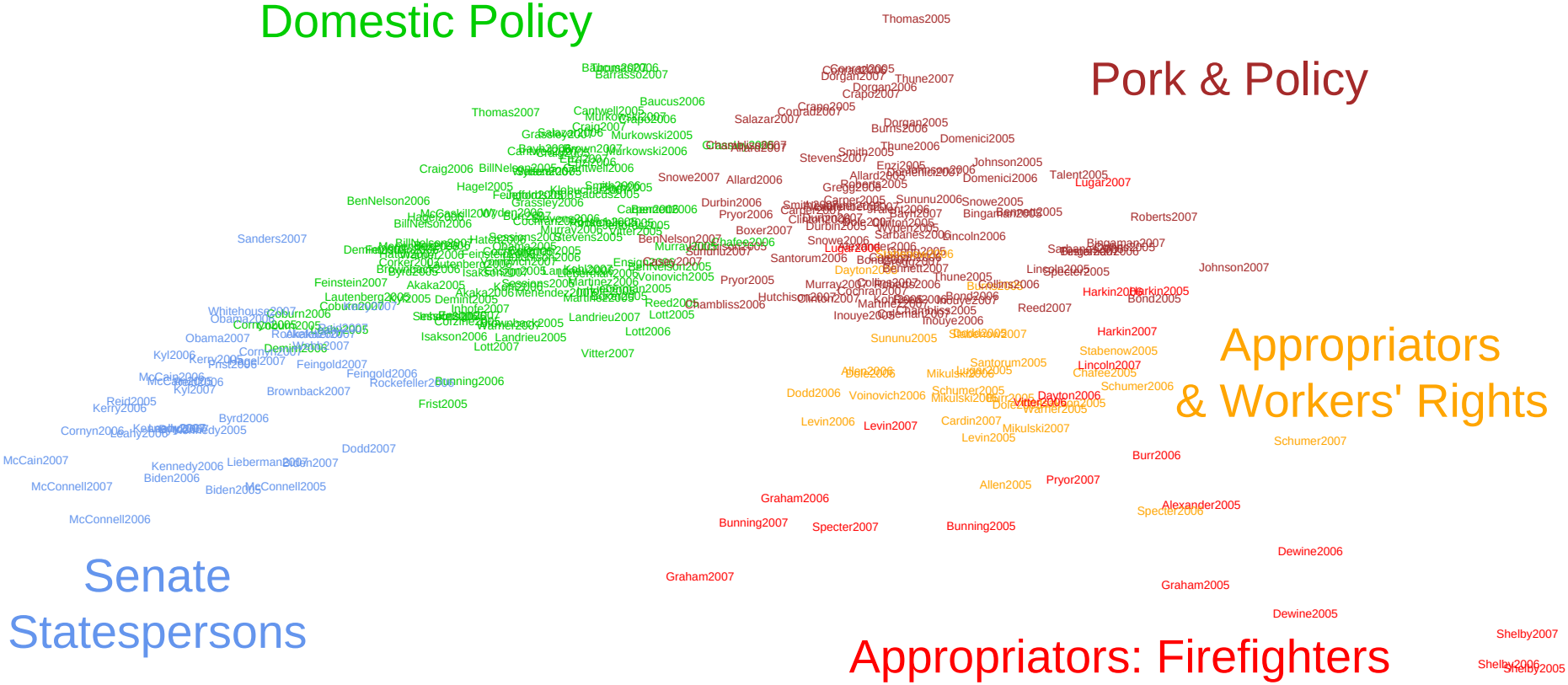
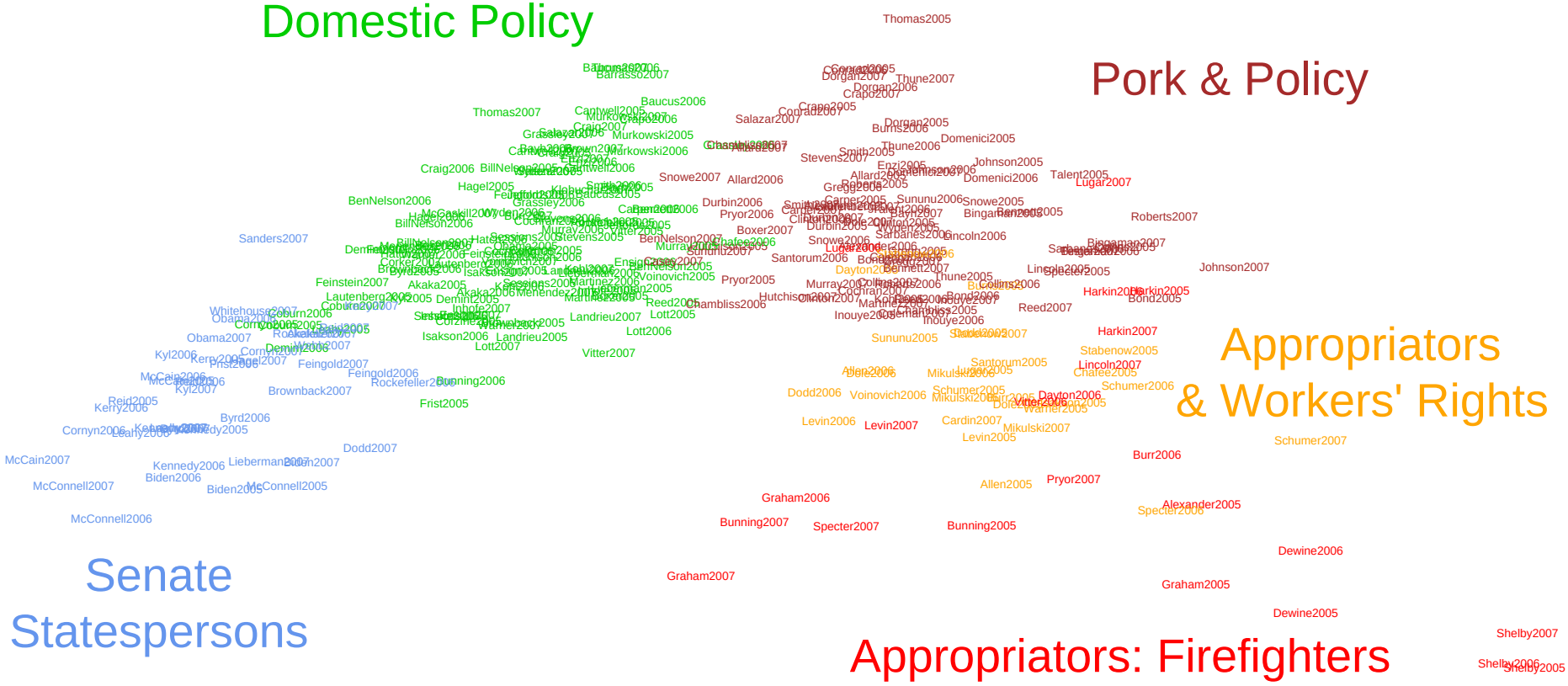


Senate Statesperson

- Iraq War
- Intelligence
- Intl. Relations

Hypothesis Validity



Senate Statesperson	Domestic Policy
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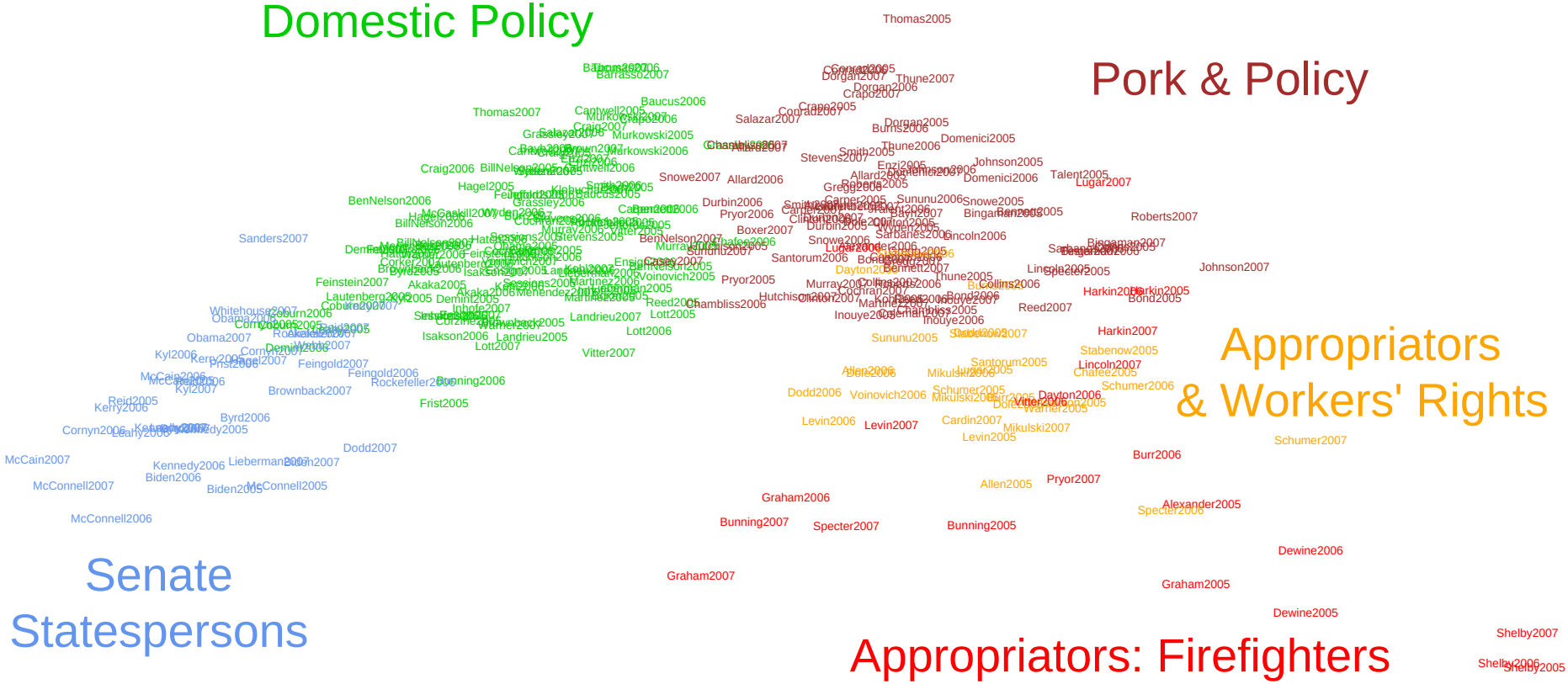
- Intelligence	- Gas prices
- Intl. Relations	- DHS
	- Consumer

Hypothesis Validity



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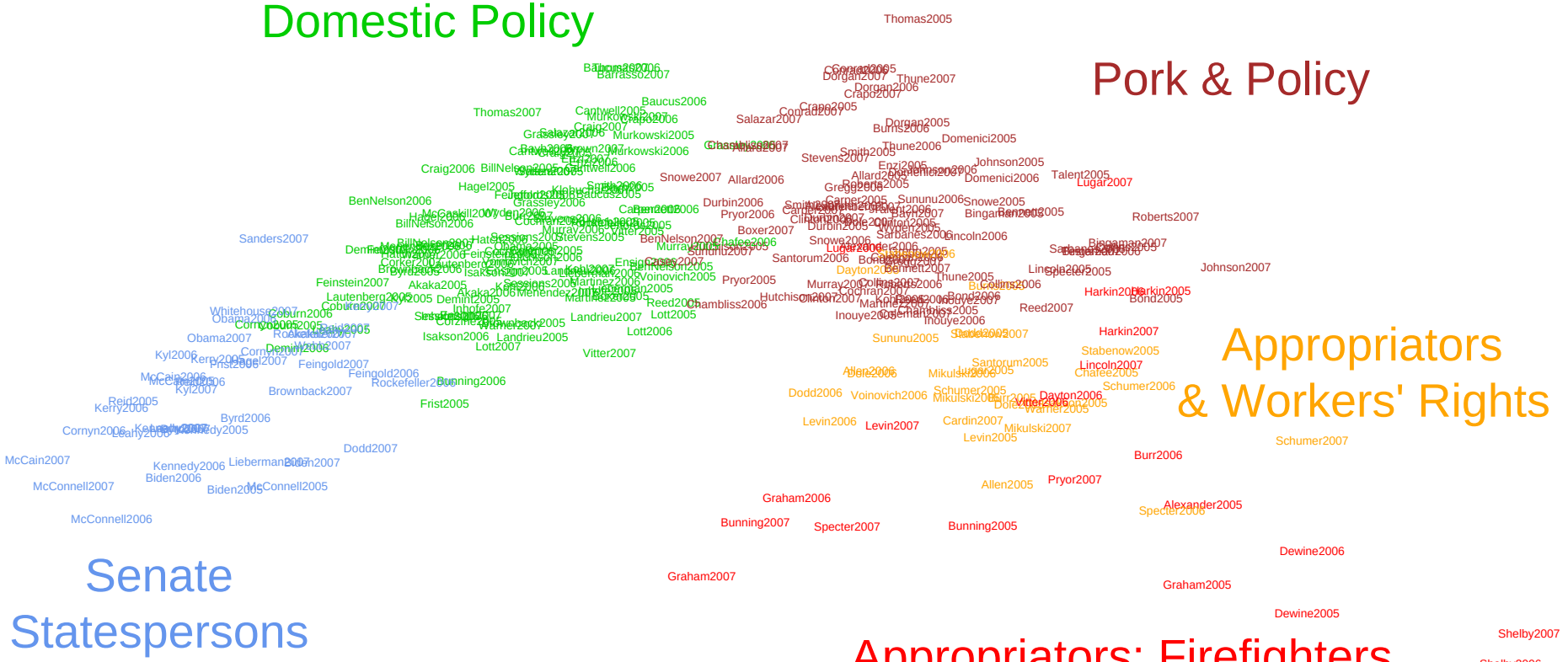
Pork & Policy

- WRDA grants
- Farming
- Health Care
- Education

Hypothesis Validity



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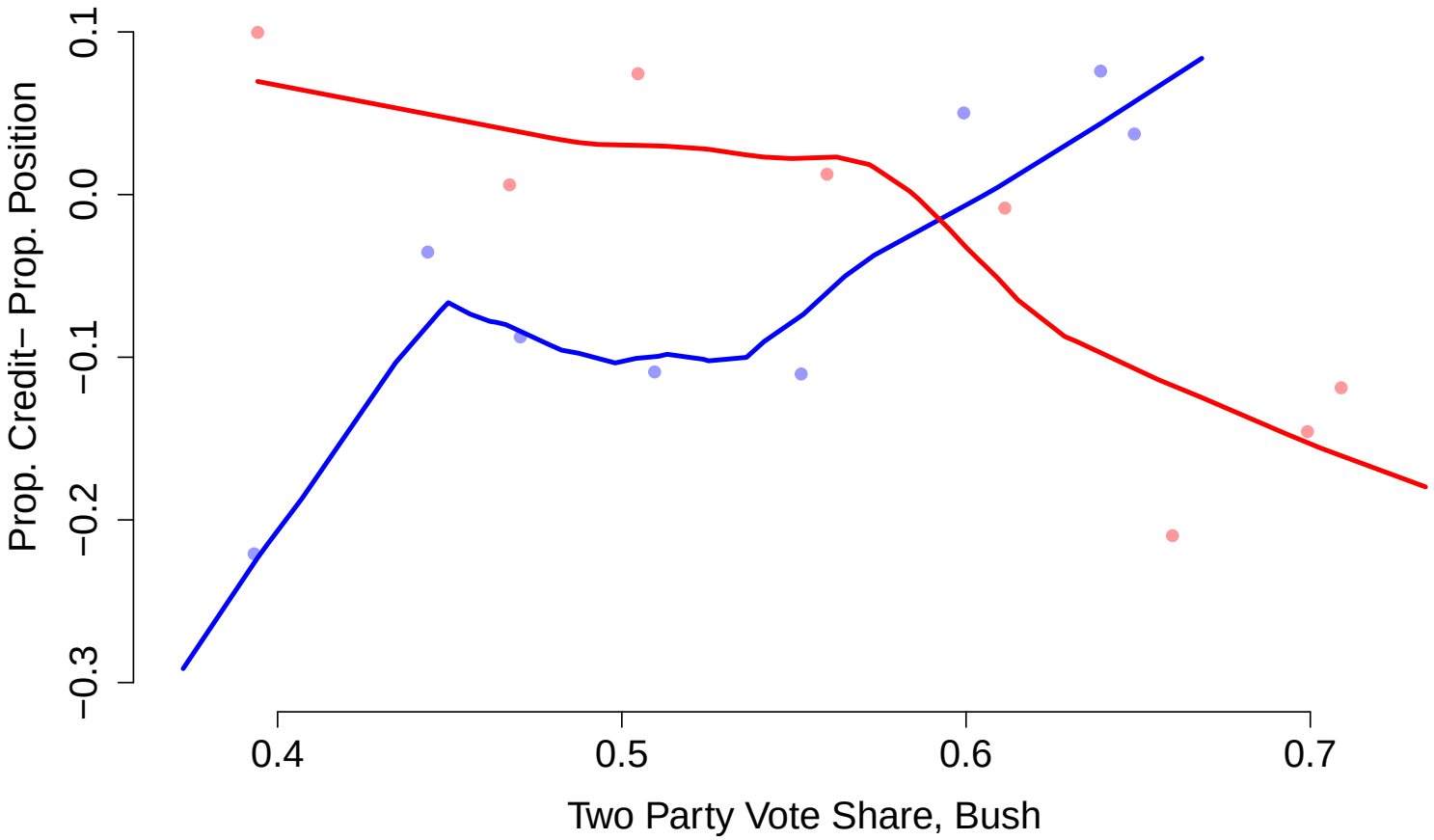
Appropriators

- Fire Grants
- Airport Grants
- University Money

Hypothesis Validity

Why do senators adopt different styles?

District Fit



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Blaydes, Grimmer, and McQueen 2017$\rightsquigarrow$ estimate nested topics to explore the Mirros for Princes

The Mirrors Genre (BGM 2017)

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Preprocessing Texts

47 books

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47 books $\rightsquigarrow$ Each divided into paragraphs

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Create feature space

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$$\mathbf{x}^*_{ij} = \frac{\mathbf{x}_{ij}}{\sqrt{\mathbf{x}'_{ij}\mathbf{x}_{ij}}}$$

Measuring Themes in the Mirrors

Model built around two hierarchies:

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2) Coarse topics $\rightsquigarrow$ granular topics (Li and McCallum 2006; Gopal and Yang 2014)

Measuring Themes in the Mirrors

Estimate four quantities of interest

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3) Each book i's $\mathbf{themes}_i$

$$\mathbf{themes}_i \quad = \quad (\text{theme}_{i1}, \text{theme}_{i2}, \ldots, \text{theme}_{i60})$$

Measuring Themes in the Mirrors

Estimate four quantities of interest

1) Granular topics (60)
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    - Each granular topic classified into one coarse topic
3) Each book i's $\textbf{themes}_i$
4) Each short segment's granular (and coarse) topic

A Hierarchy of Topics

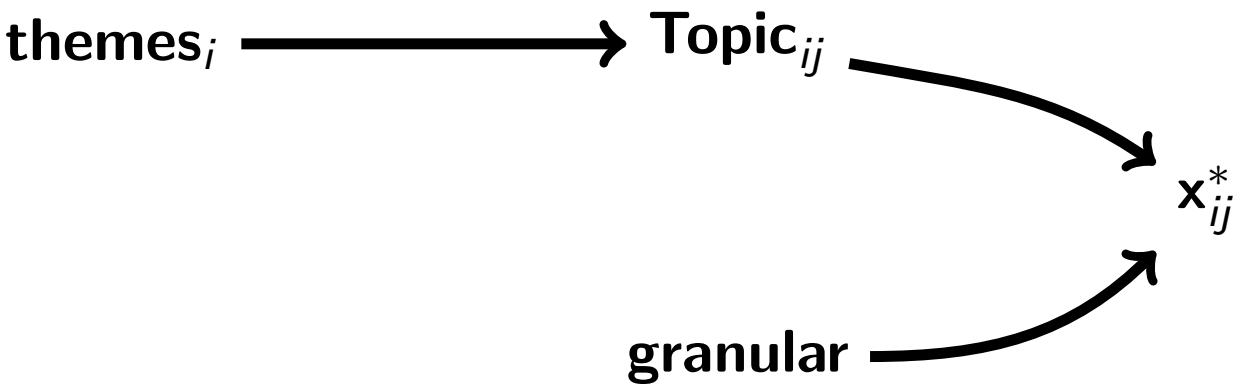
themes$_j$

A Hierarchy of Topics

$$\textbf{themes}_i \longrightarrow \textbf{Topic}_{ij}$$

$$\textbf{Topic}_{ij} \sim \text{Multinomial}(1, \textbf{themes}_i)$$

A Hierarchy of Topics



$$\begin{aligned} \textbf{Topic}_{ij} &\sim \text{Multinomial}(1, \textbf{themes}_i) \\ \mathbf{x}^*_{ij} | \text{Topic}_{ijk} = 1 &\sim \text{vMF}(\kappa, \textbf{granular}_k) \end{aligned}$$

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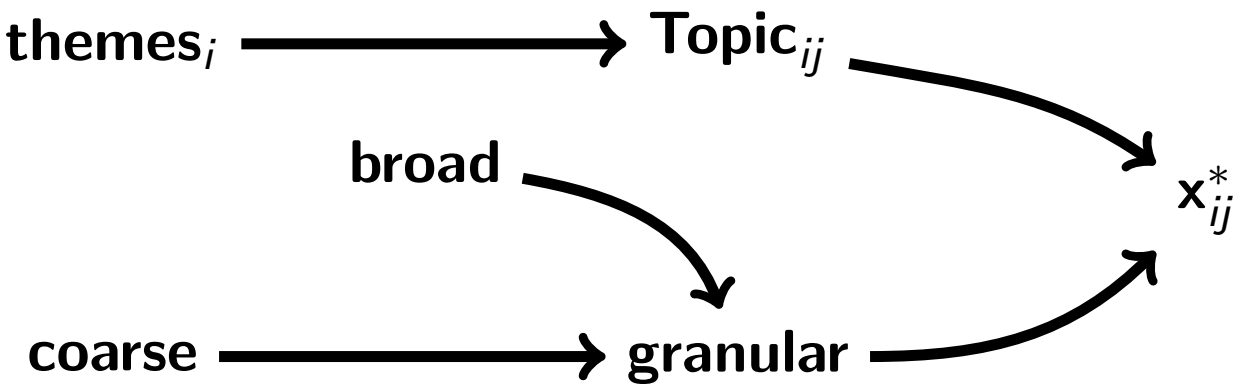

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\textbf{broad}_k &\sim \text{Multinomial}(1, \textbf{Broad Theme Prior}) \\
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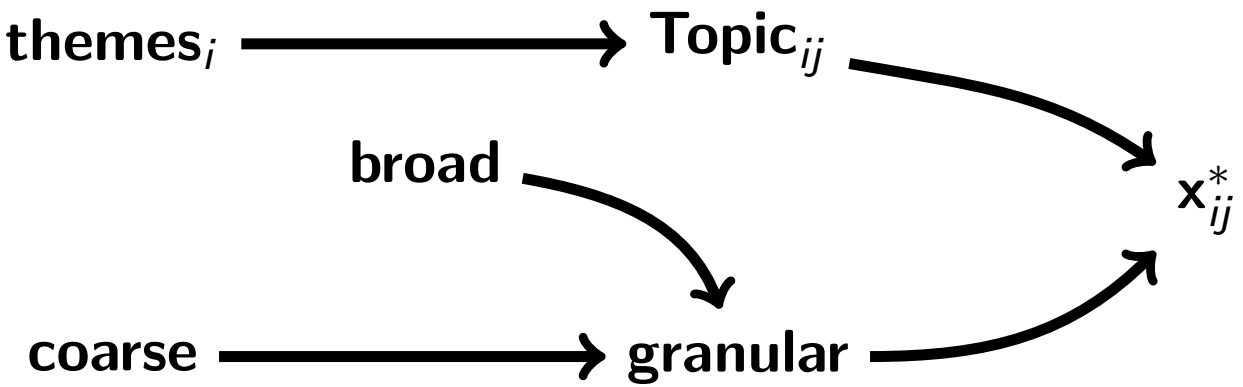

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Estimate model with Variational Approximation

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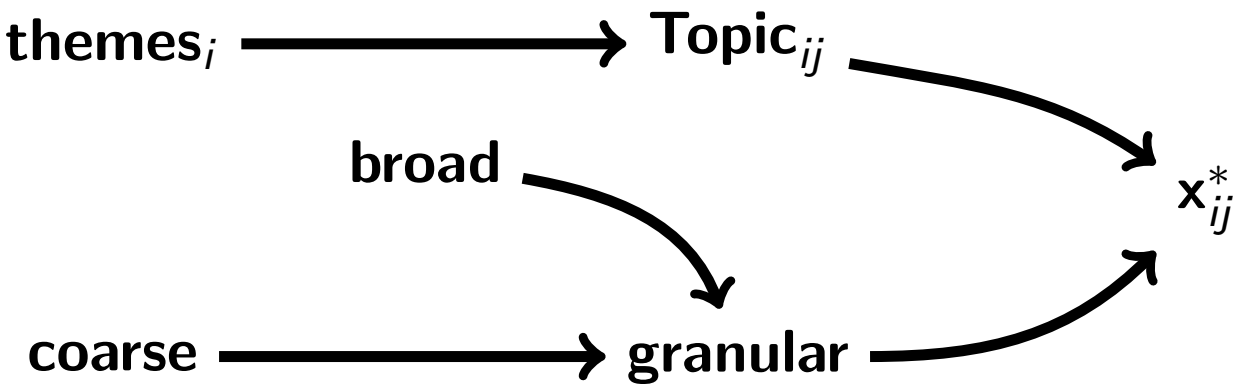

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Estimate model with Variational Approximation
Model selection: automatic model fit, qualitative evaluation

Interpreting Unsupervised Models

Two approaches to labeling output

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1) Computational: identify discriminating words

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Unsupervised models structure and guide our reading

Art of Rulership

Practices and ideals of political rule

Art of Rulership

Practices and ideals of political rule

king

Art of Rulership

Practices and ideals of political rule

king,princ

Art of Rulership

Practices and ideals of political rule

king,princ,citi

Art of Rulership

Practices and ideals of political rule

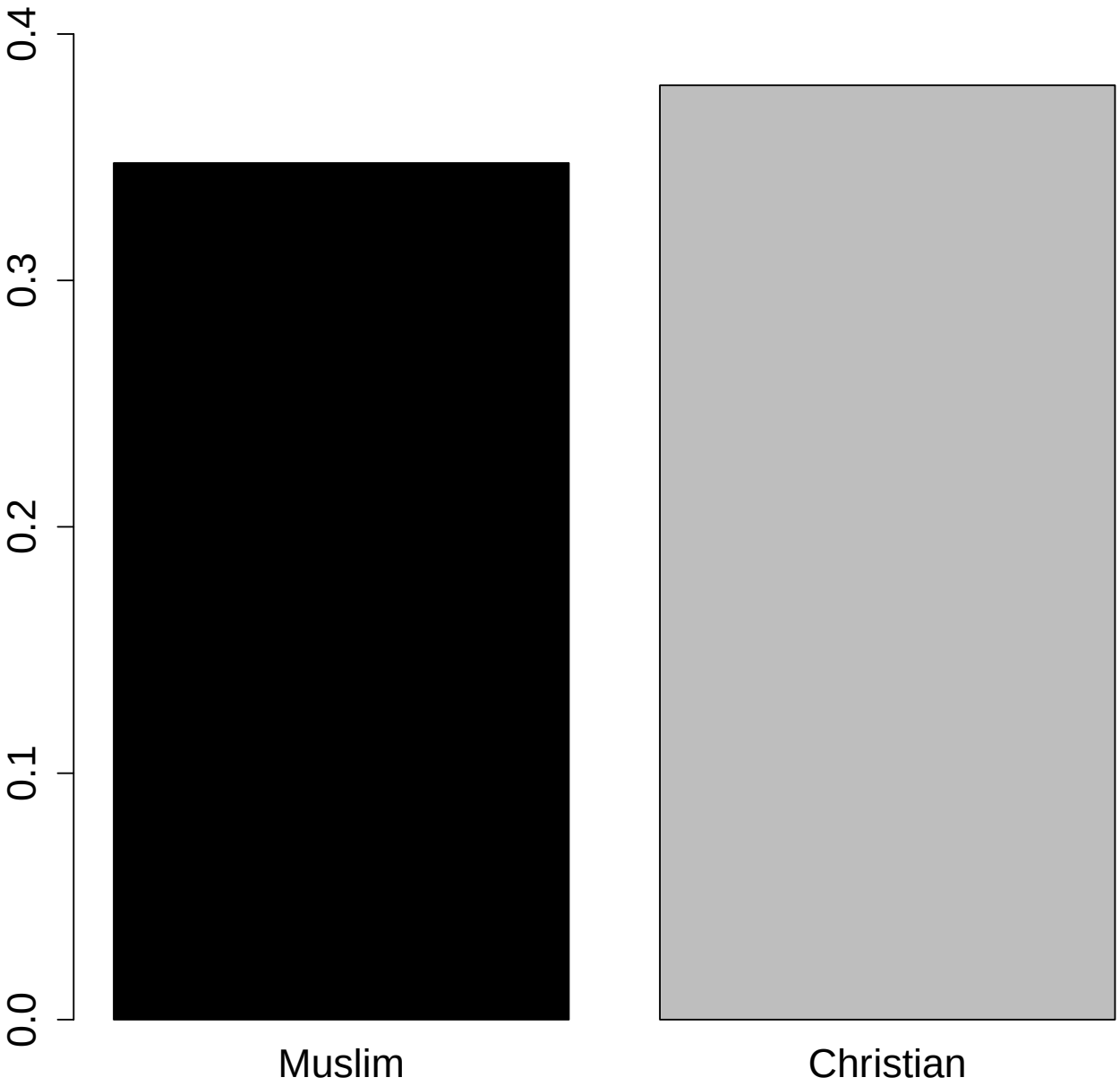
king,princ,citi,great,place,work,emperor,enemi,armi,letter

Art of Rulership

Practices and ideals of political rule

king,princ,citi,great,place,work,emperor,enemi,armi,letter

36.5% of paragraphs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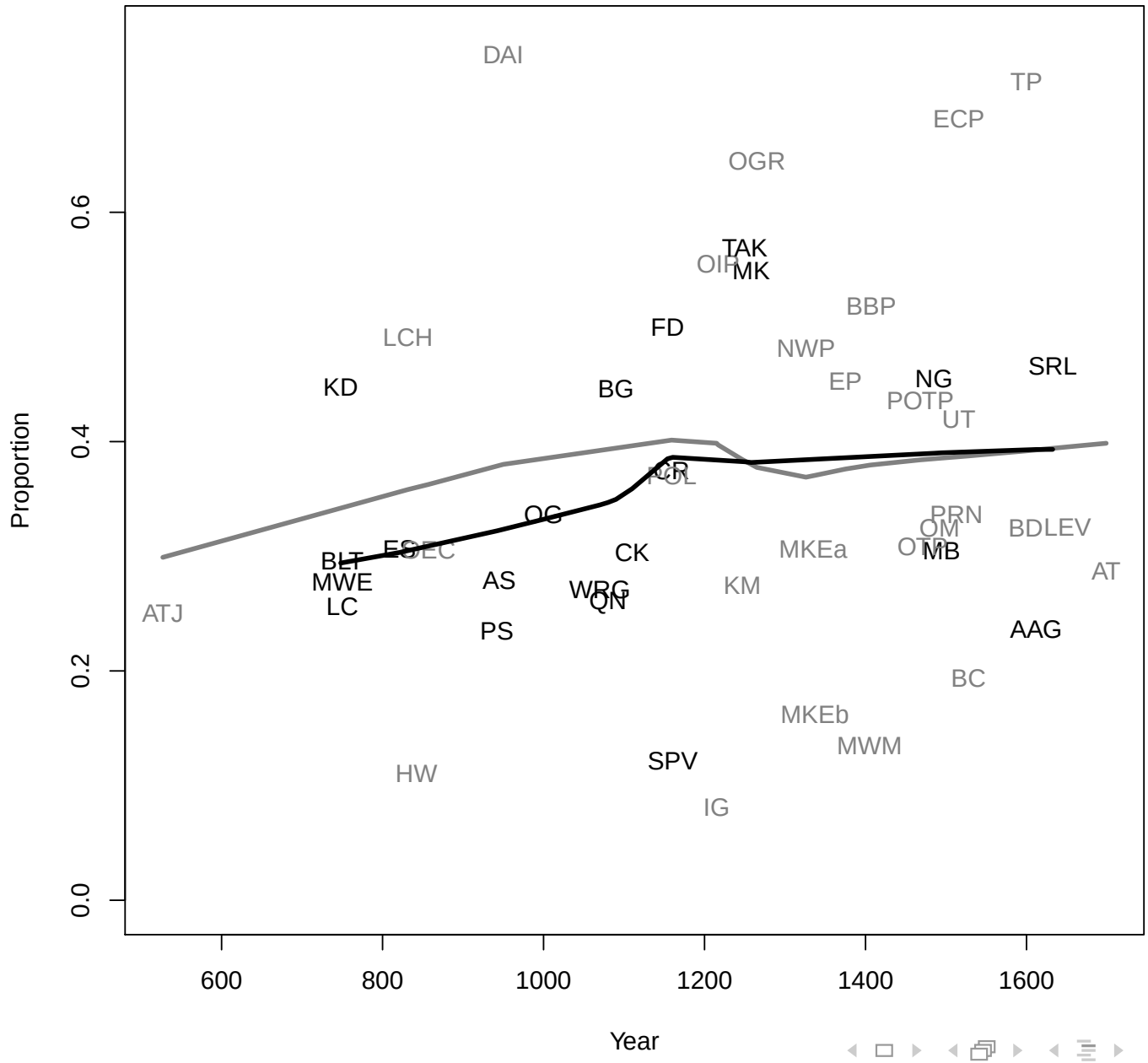
Muslim
Christian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Religion and Virtue

Connection between religion, virtue, justice and political rule

Religion and Virtue

Connection between religion, virtue, justice and political rule

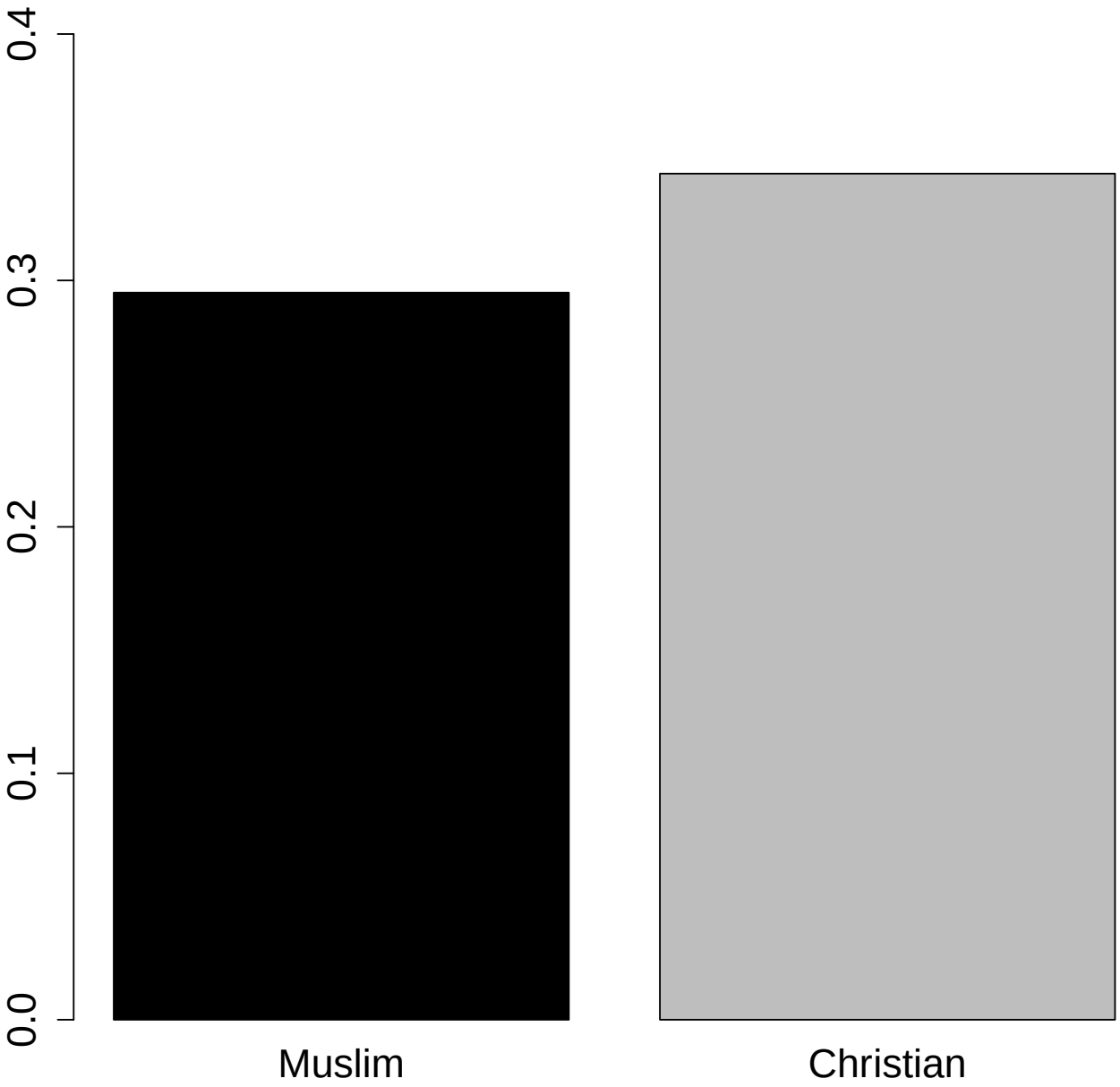
almighti,good,virtu,power,ruler,justic,prayer,rule,prophet,mena

Religion and Virtue

Connection between religion, virtue, justice and political rule

almighti,good,virtu,power,ruler,justic,prayer,rule,prophet,mena

32.2% of pargraphs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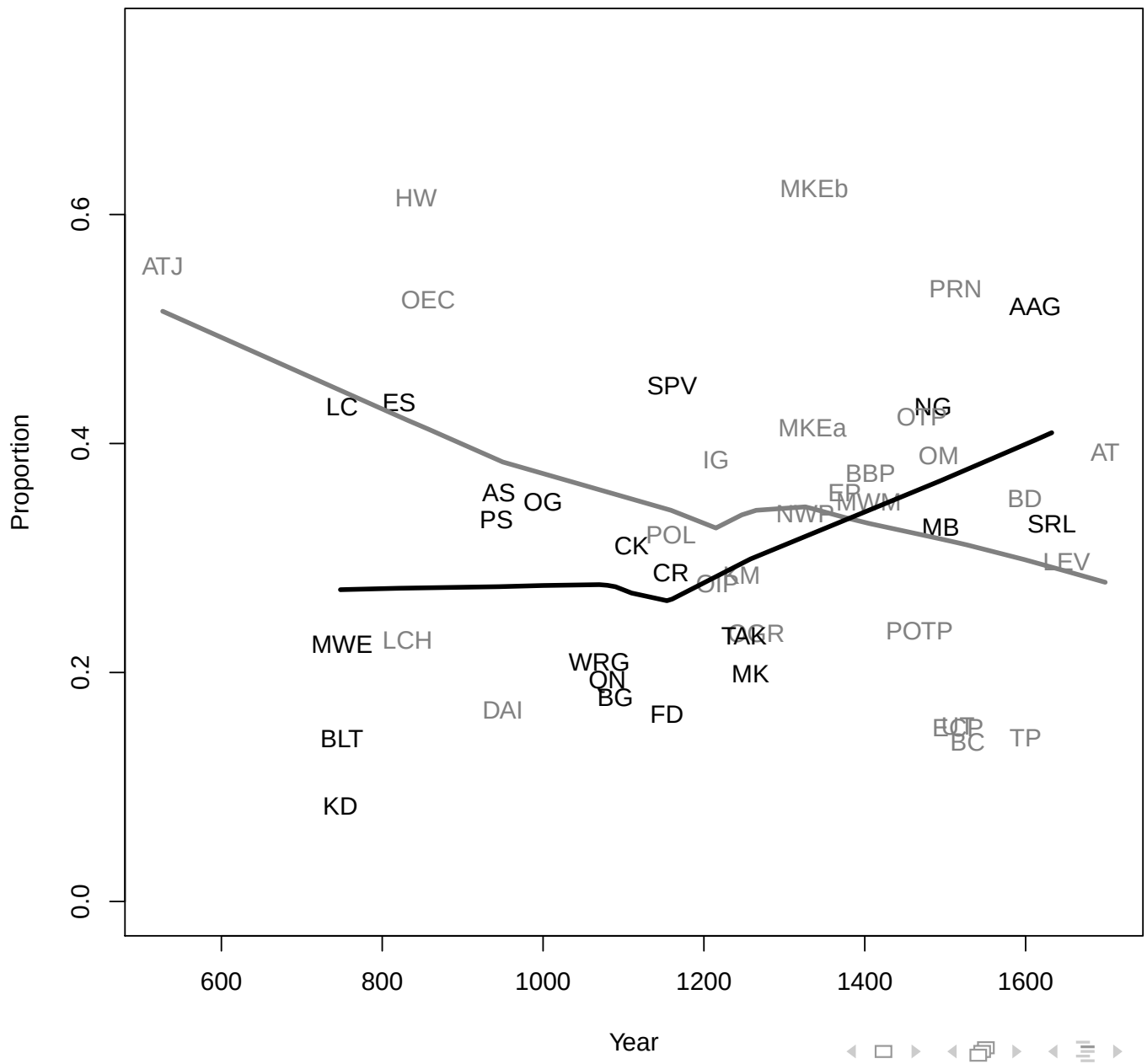
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Inner Life of the Ruler

Personal relationships, care for and practices of the self, and ultimate fate of the soul

Inner Life of the Ruler

Personal relationships, care for and practices of the self, and ultimate fate of the soul

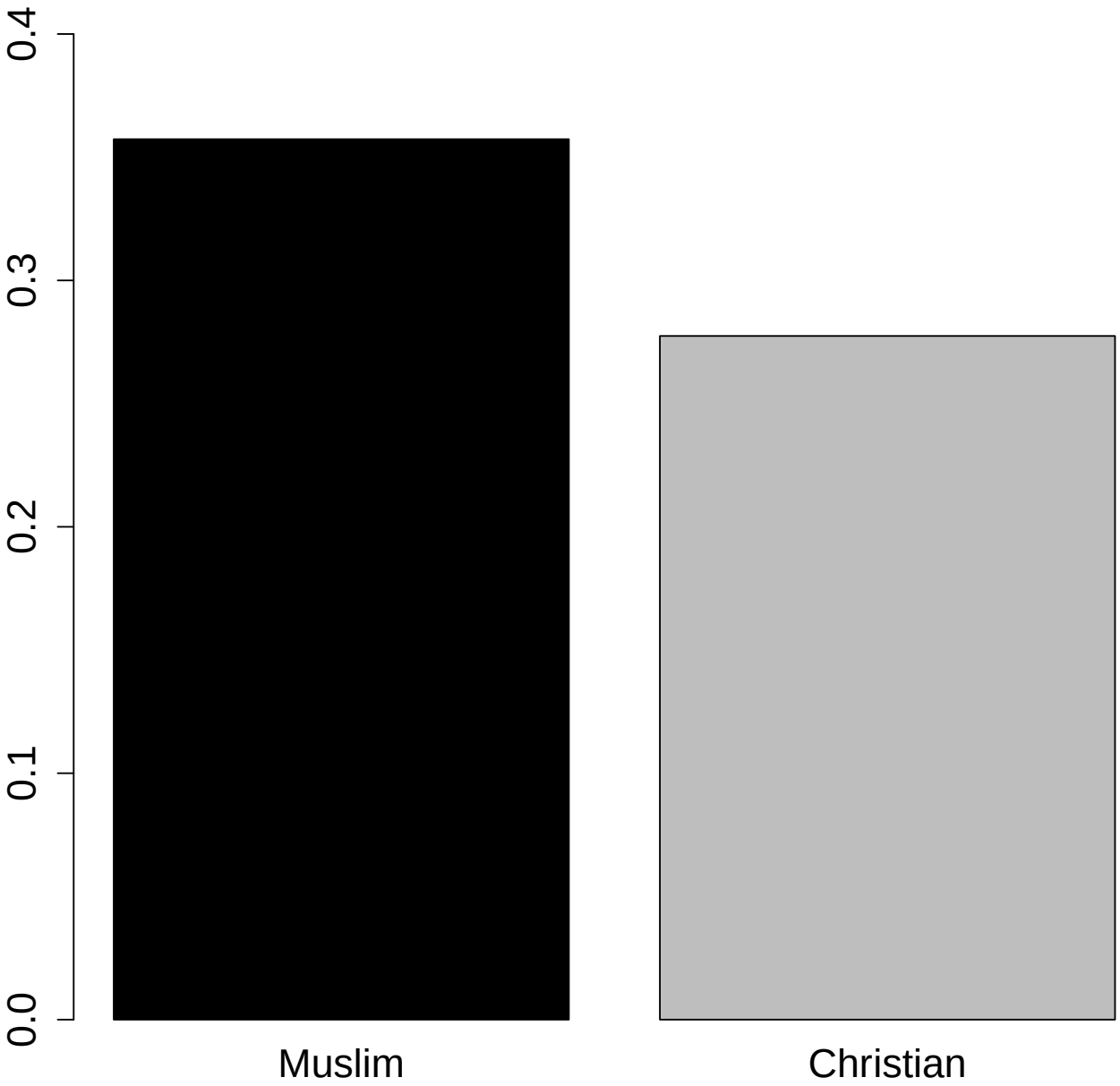
man,land,woman,know,bodi,eye,ladi,love,faculti,old

Inner Life of the Ruler

Personal relationships, care for and practices of the self, and ultimate fate of the soul

man,land,woman,know,bodi,eye,ladi,love,faculti,old

31.2% of paragraphs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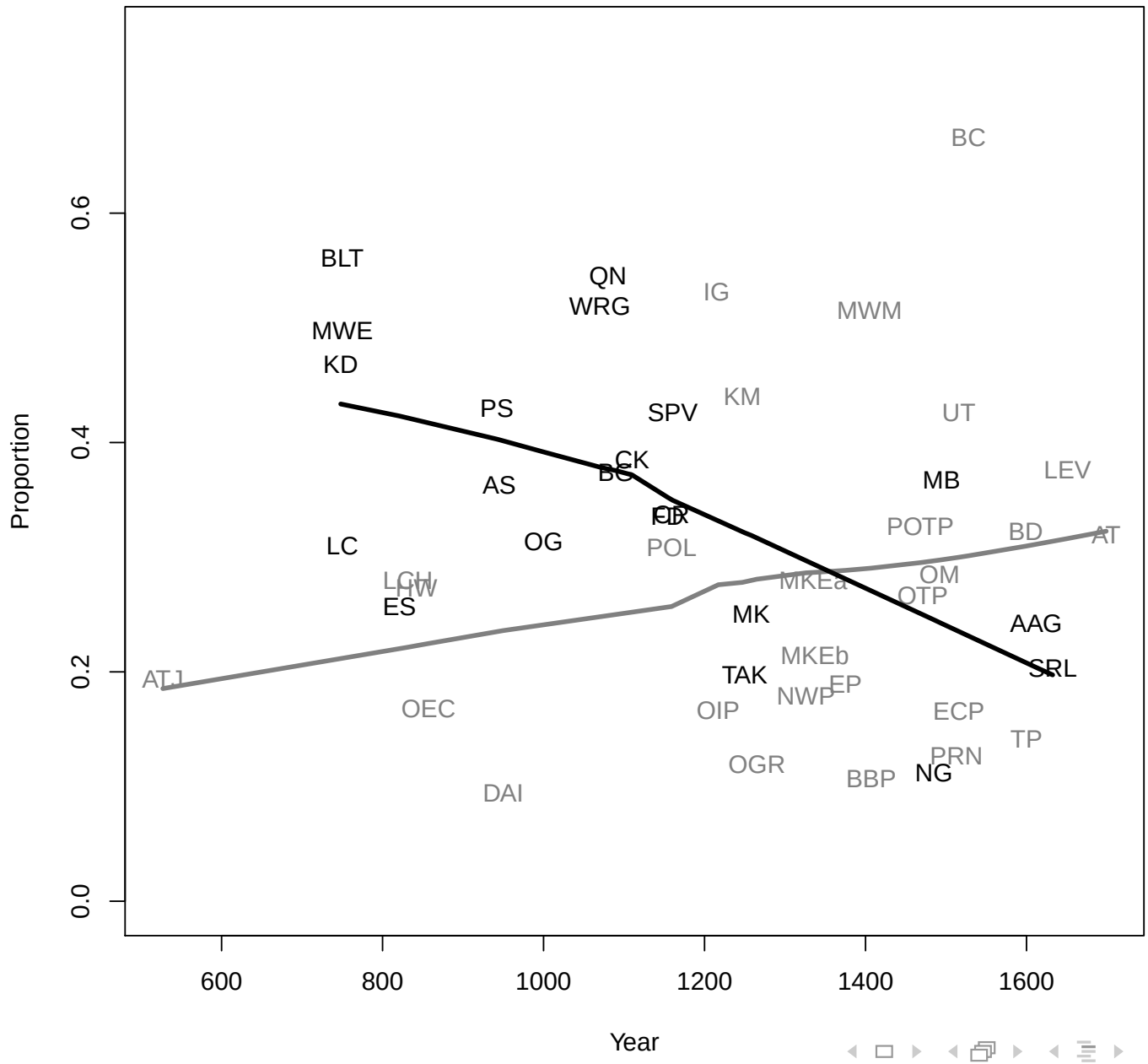
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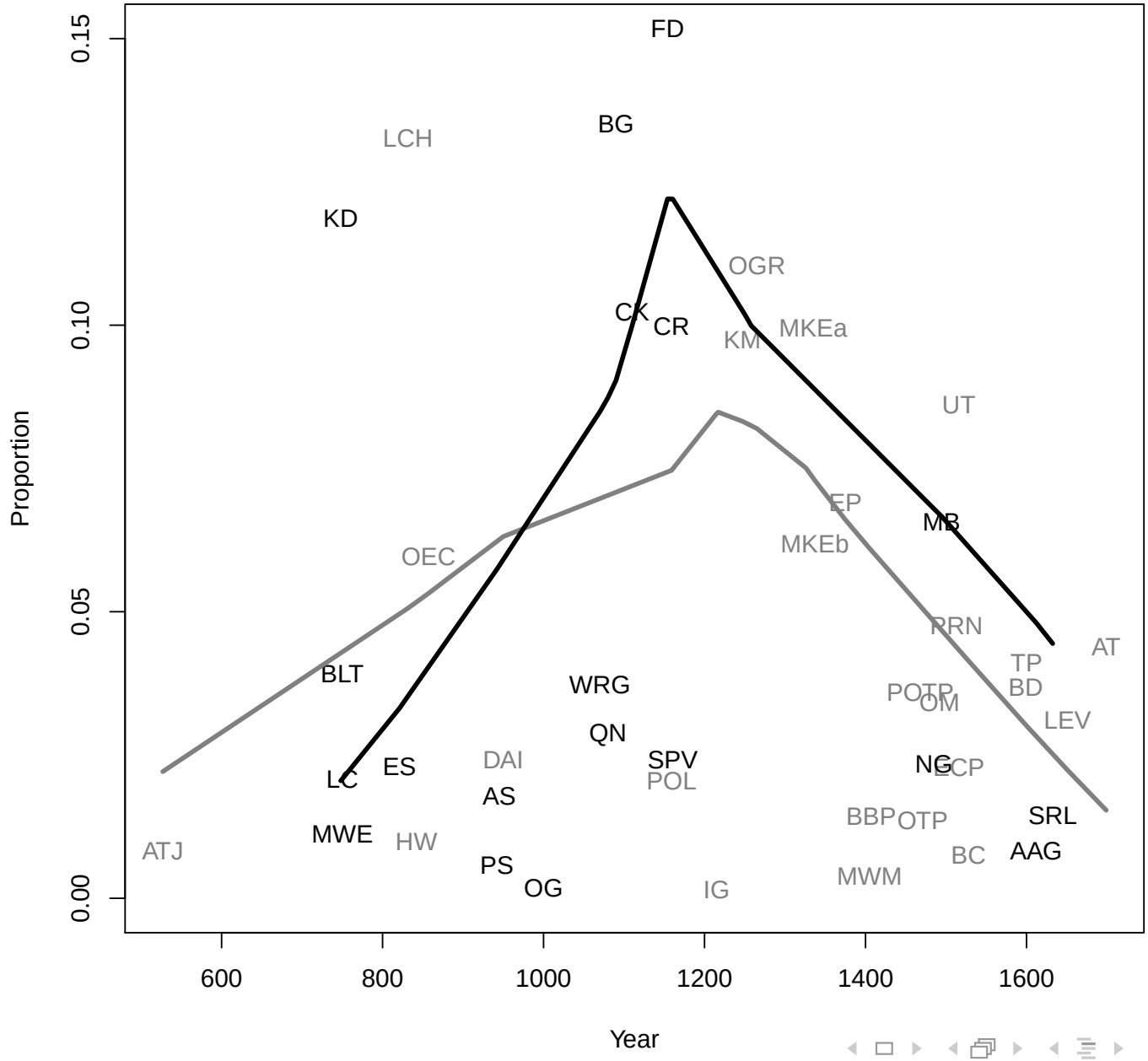
DAI

Granular: Best Practices for Ruling

king,princ,citi,great,place,work,emperor,enemi,armi,letter
king,kingdom,royal,minist,reign,father,court,majesti,presenc,war

6.2% of paragraphs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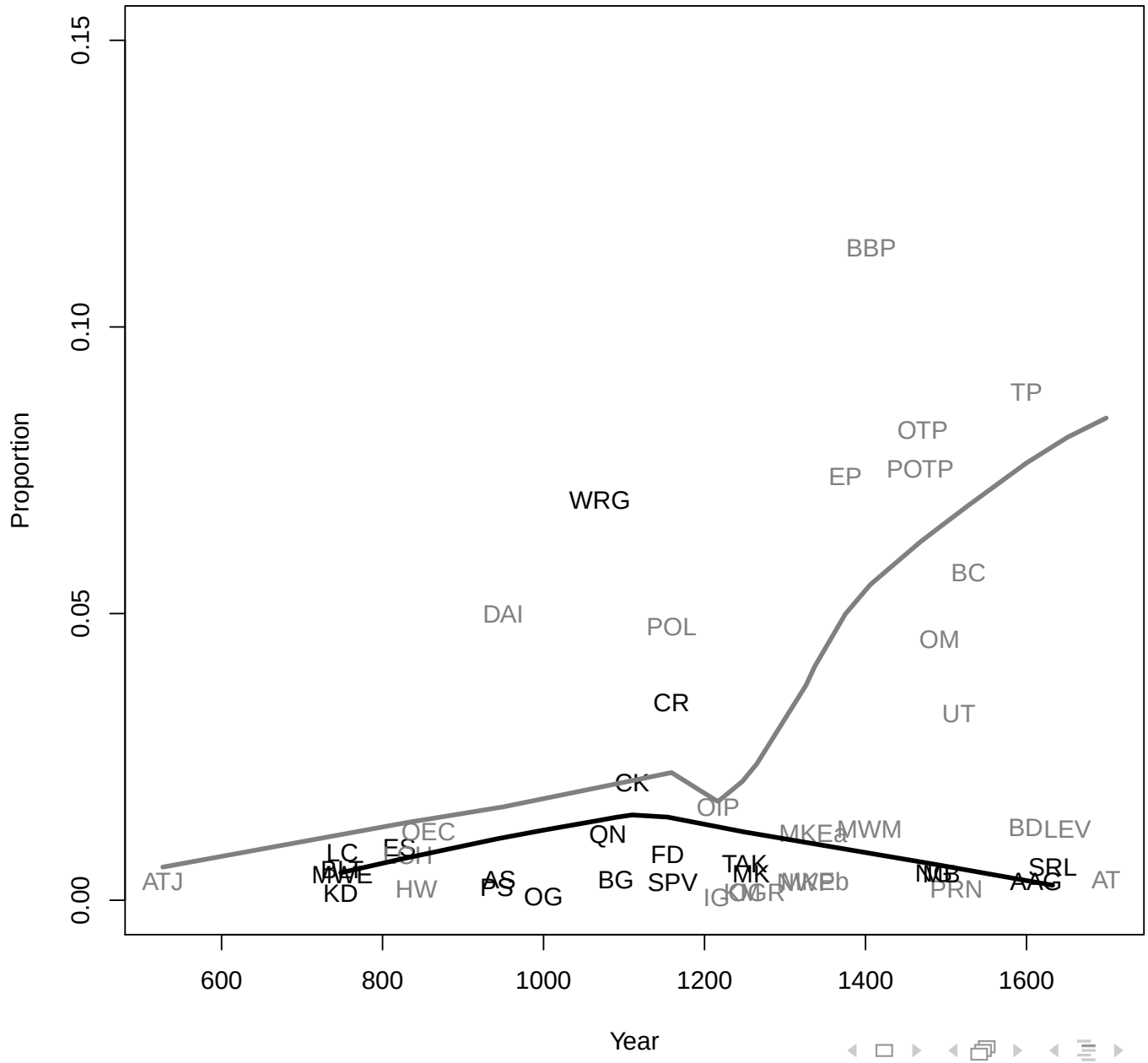
MWM

Granular: Characteristics that distinguish Just Ruler from Tyrant

king,princ,citi,great,place,work,emperor,enemi,armi,letter
king,kingdom,royal,minist,reign,father,court,majesti,presenc,war
princ,good,peopl,christian,tyranni,war,mind,ought,state,public

3.1% of paragraphs


Coarse Topic 1 Granular Topic 2
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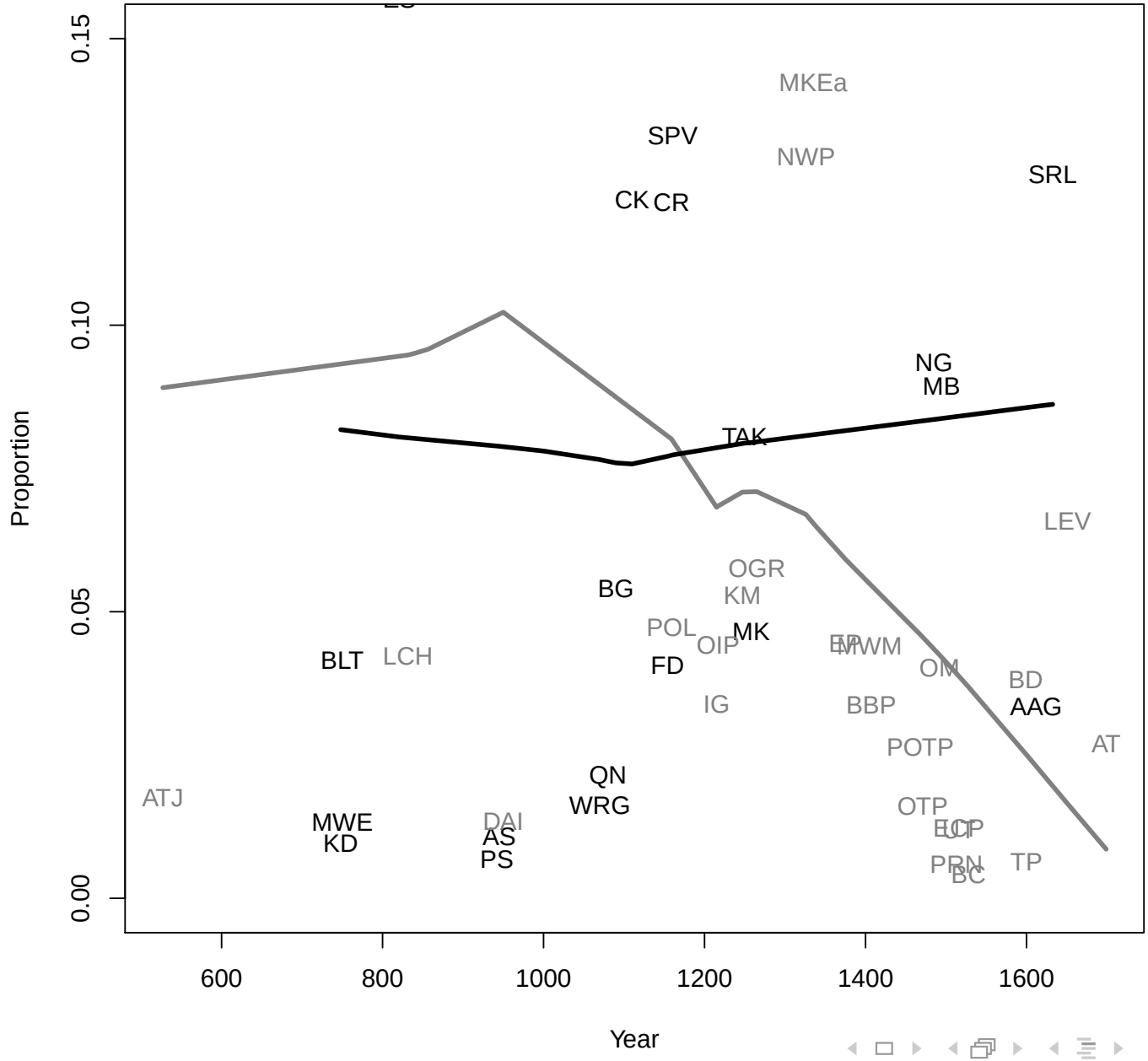
SRL

Granular: Religious Virtues and Political Ideals

almighti,good,virtu,power,ruler,justic,prayer,rule,prophet,mena
almighti,bless,grant,peac,messeng,prophet,merci,holi,command,grace

6.9% of paragraphs

Coarse Topic 2 Granular Topic 1

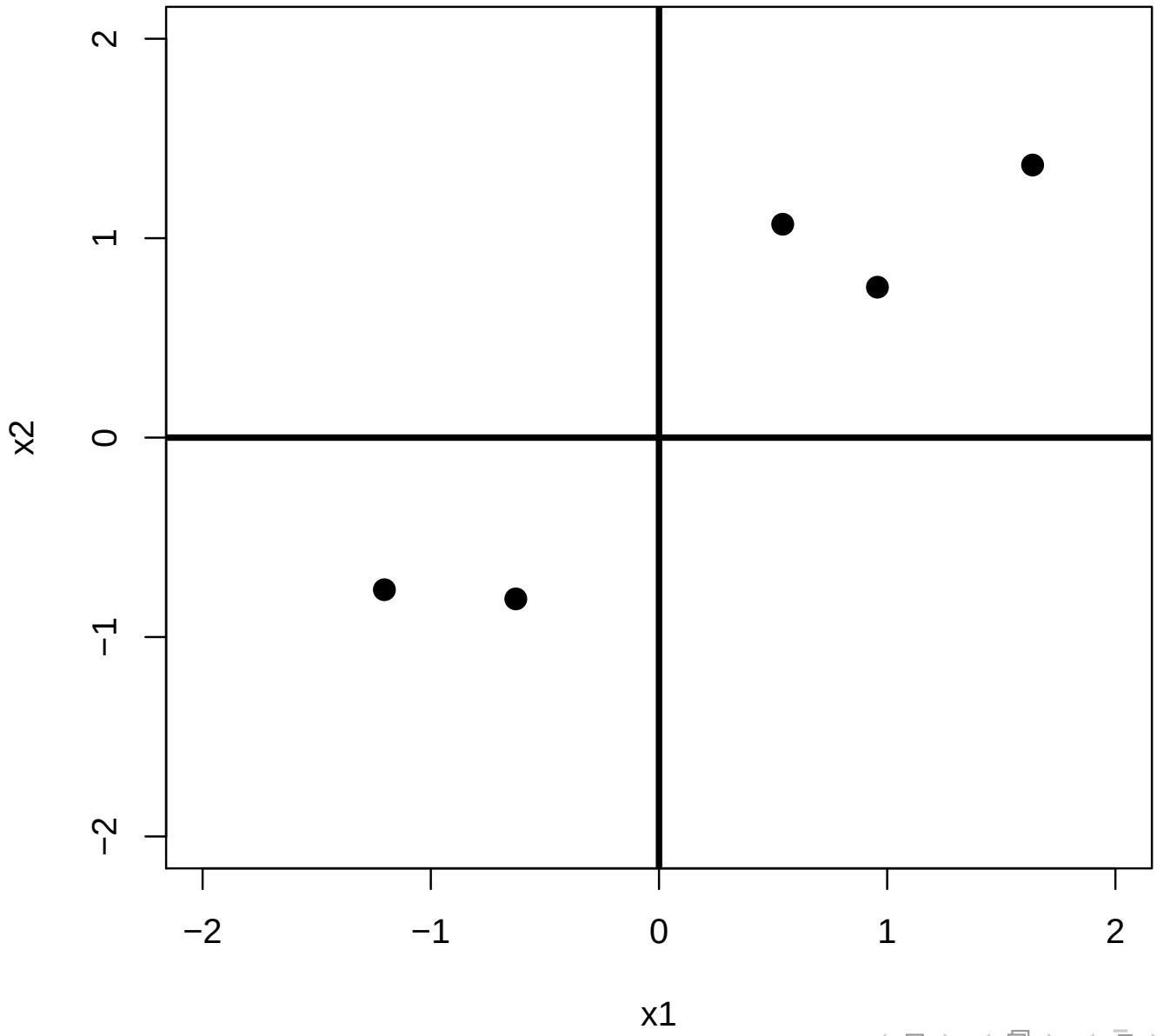


Principal Component Analysis

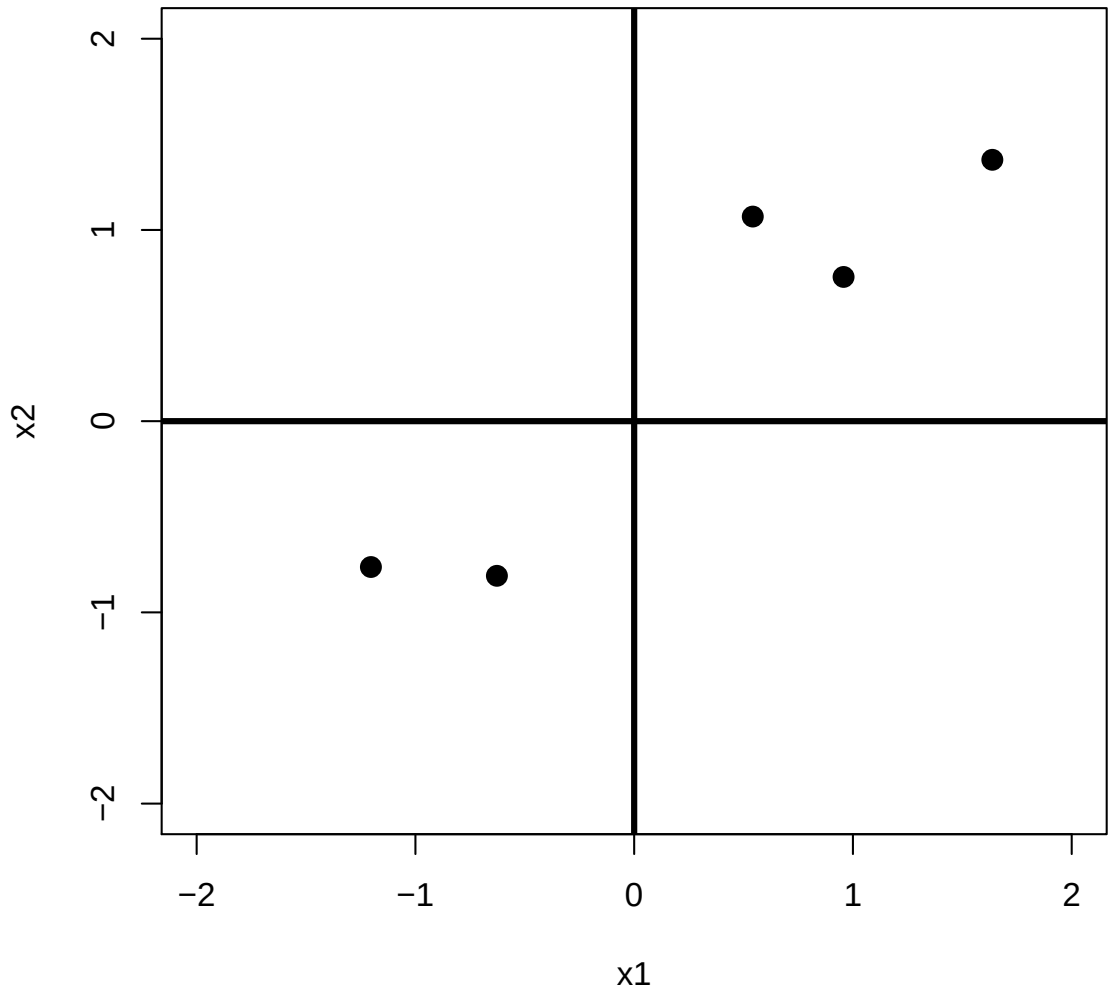
A Simple Two-Dimensional Example

Suppose we have the following observations:

$$\begin{aligned} x_1 &= (0.54, 1.07) \\ x_2 &= (-1.20, -0.76) \\ x_3 &= (-0.63, -0.81) \\ x_4 &= (0.96, 0.75) \\ x_5 &= (1.64, 1.37) \end{aligned}$$

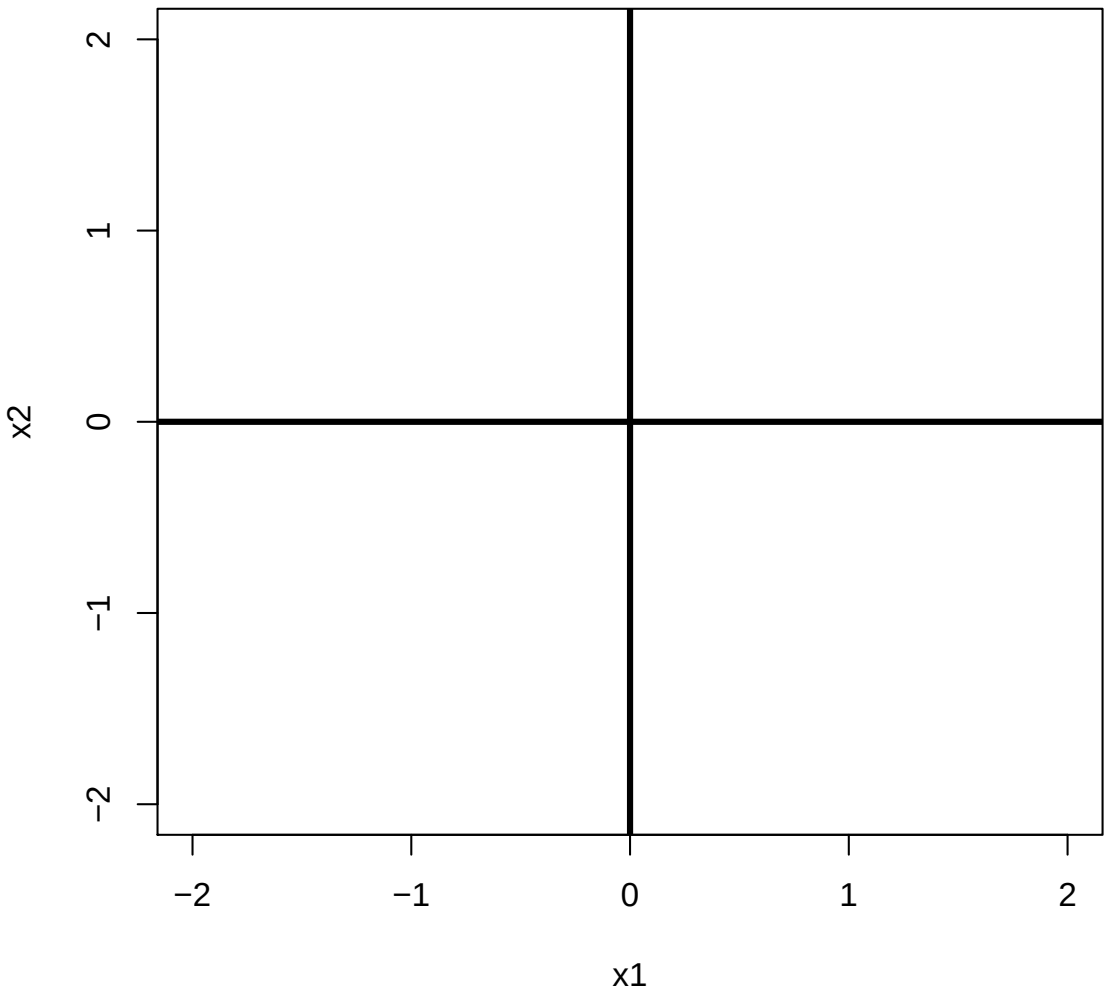

x2
2
1
0
−1
−2
−2
−1
0
1
2
x1

Goal: find line that summarizes bivariate information



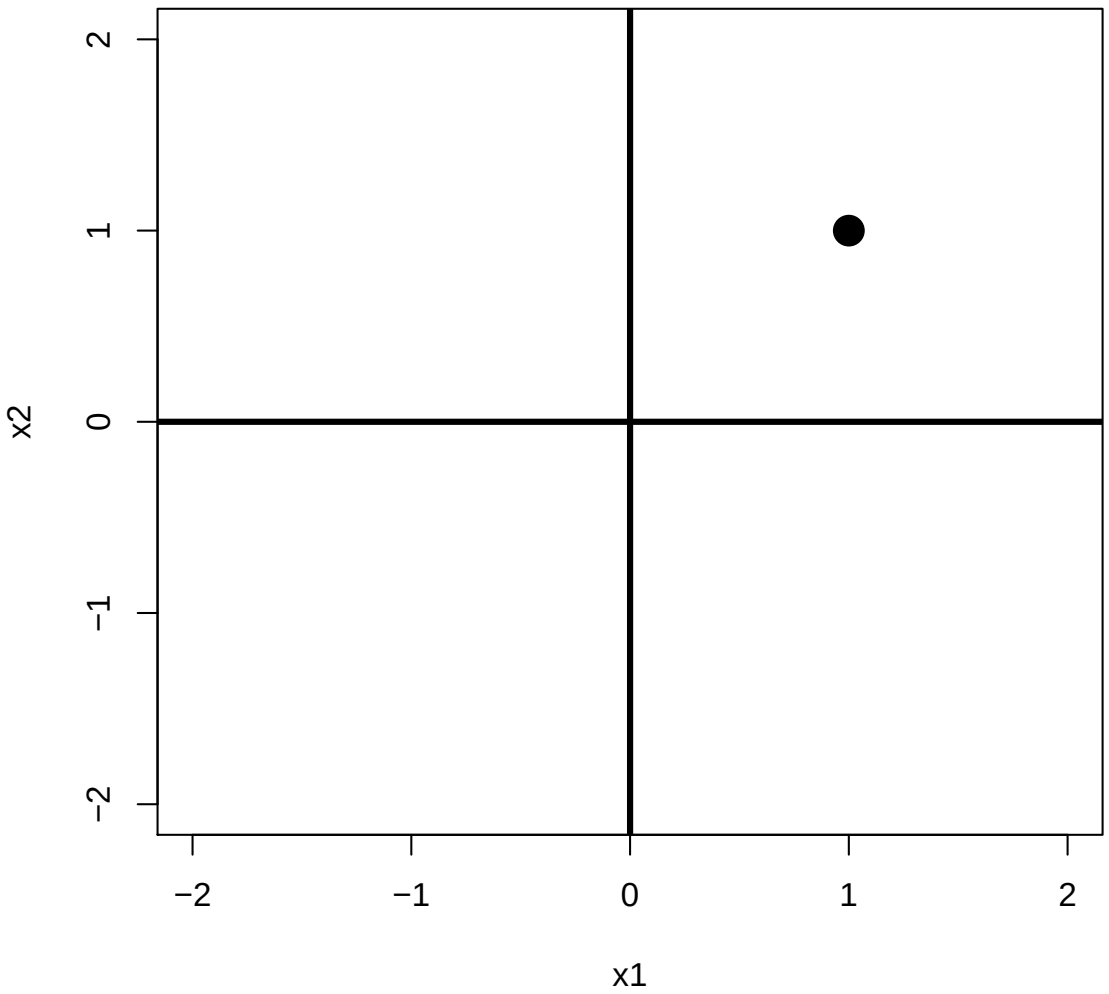
Vectors to Draw a Line

Suppose $\boldsymbol{w}_1 = (1, 1)$



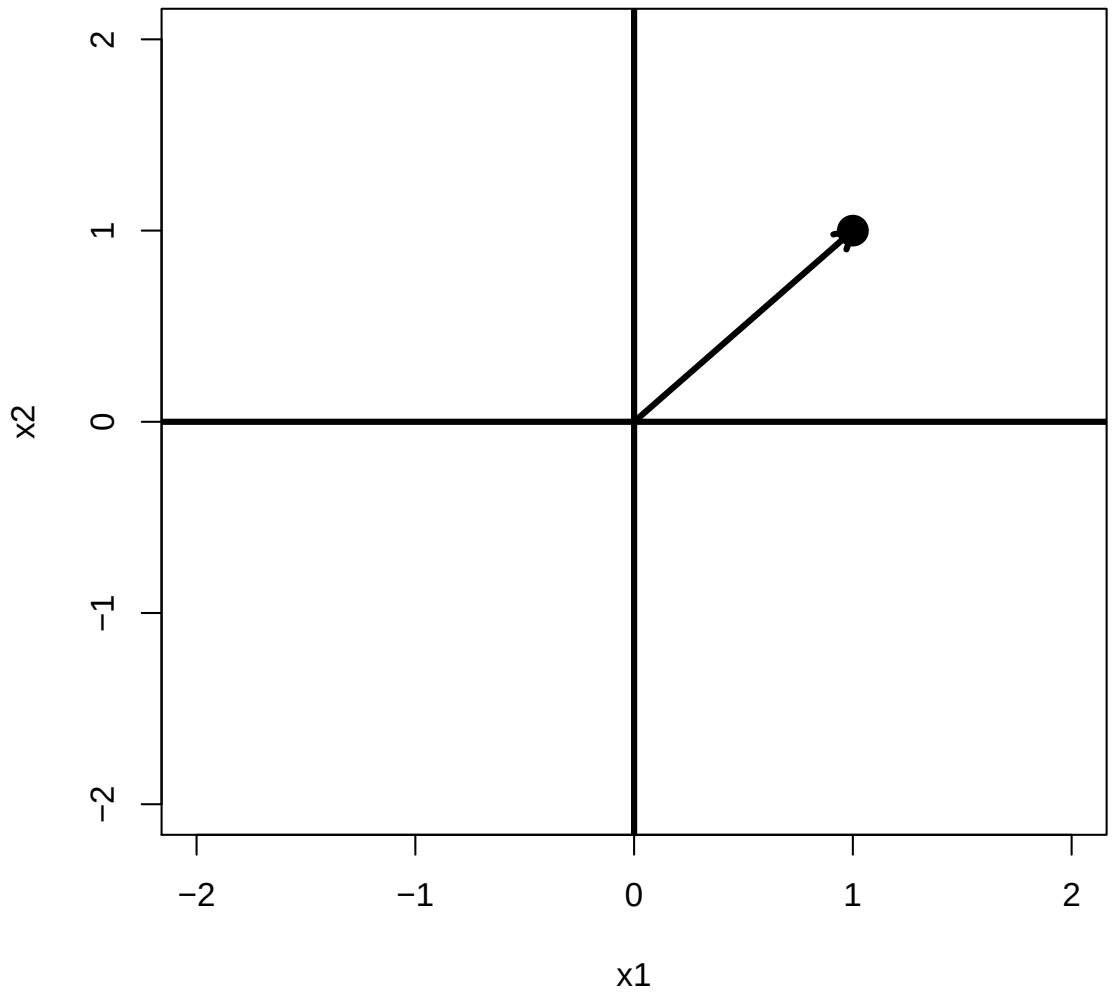
Vectors to Draw a Line

Suppose $\boldsymbol{w}_1 = (1, 1)$



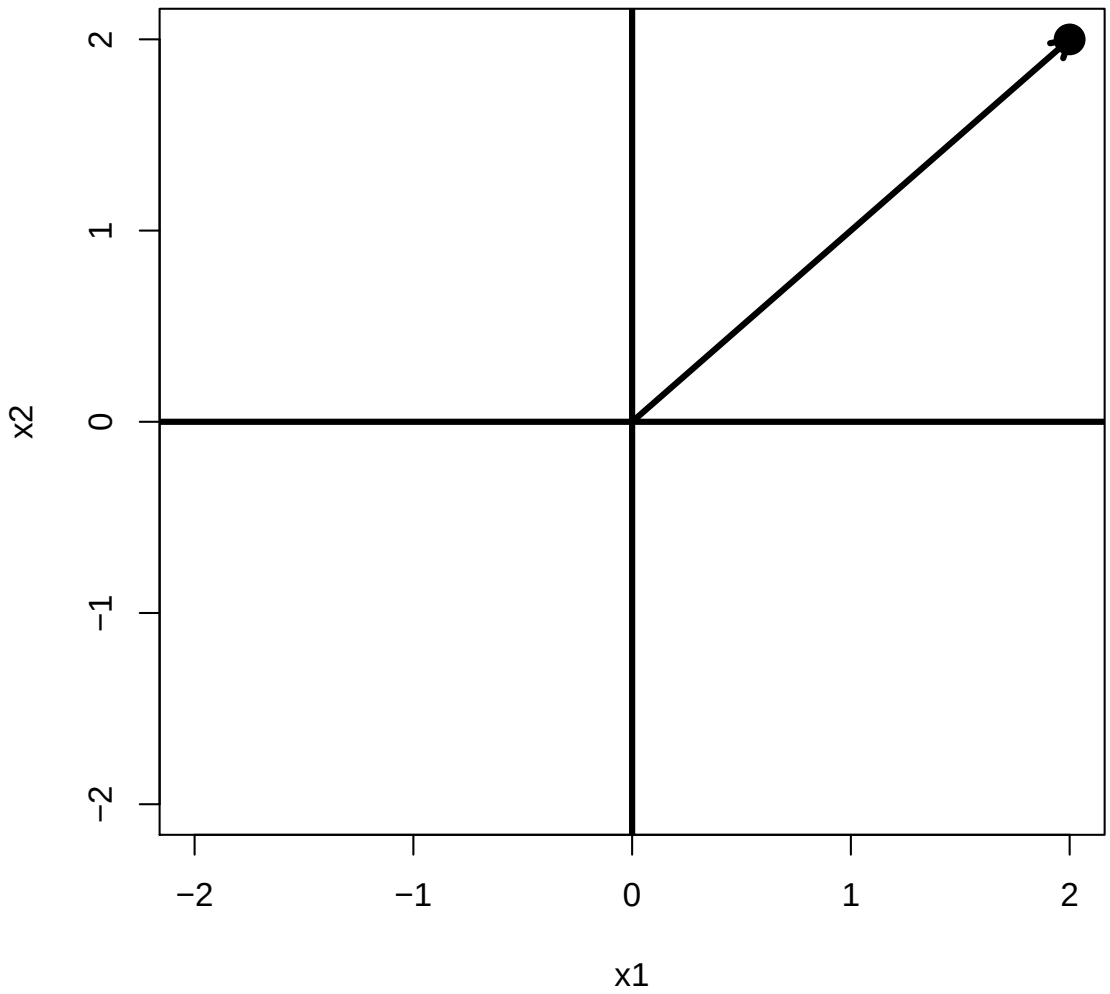
Vectors to Draw a Line

Suppose $\boldsymbol{w}_1 = (1, 1)$



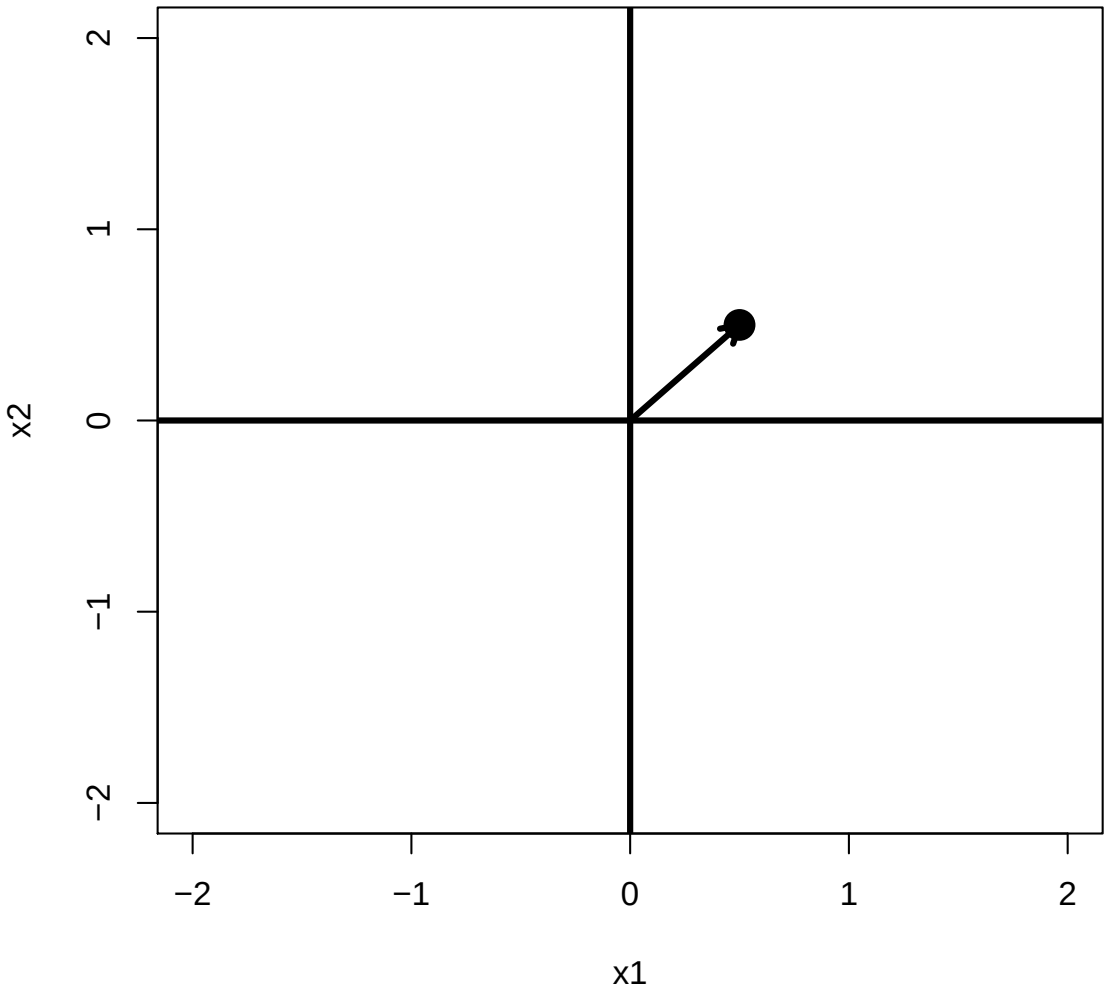
Vectors to Draw a Line

Suppose $\boldsymbol{w}_1 = (1, 1)$ $2\boldsymbol{w}_1 = (2, 2)$



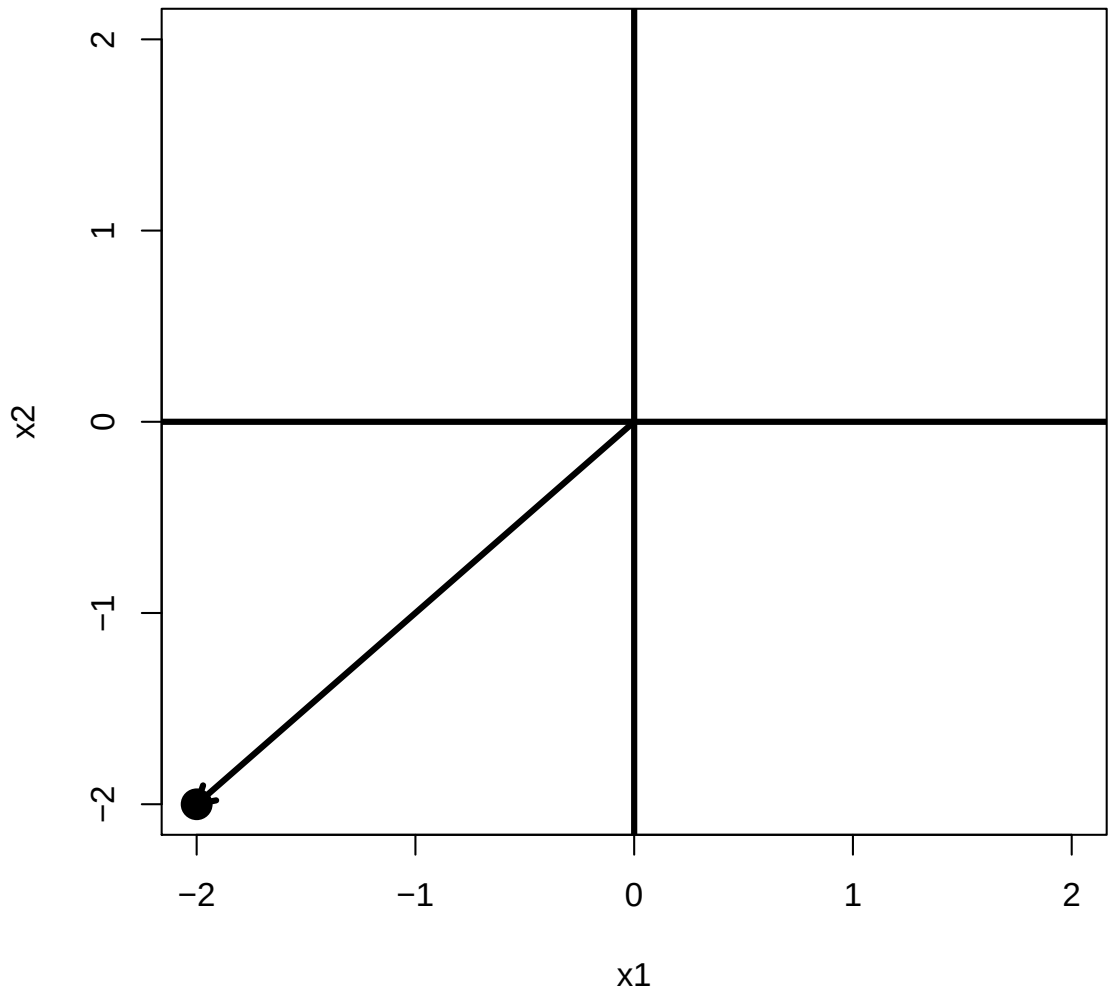
Vectors to Draw a Line

Suppose $\boldsymbol{w}_1 = (1, 1)$ $\frac{1}{2}\boldsymbol{w}_1 = (1/2, 1/2)$



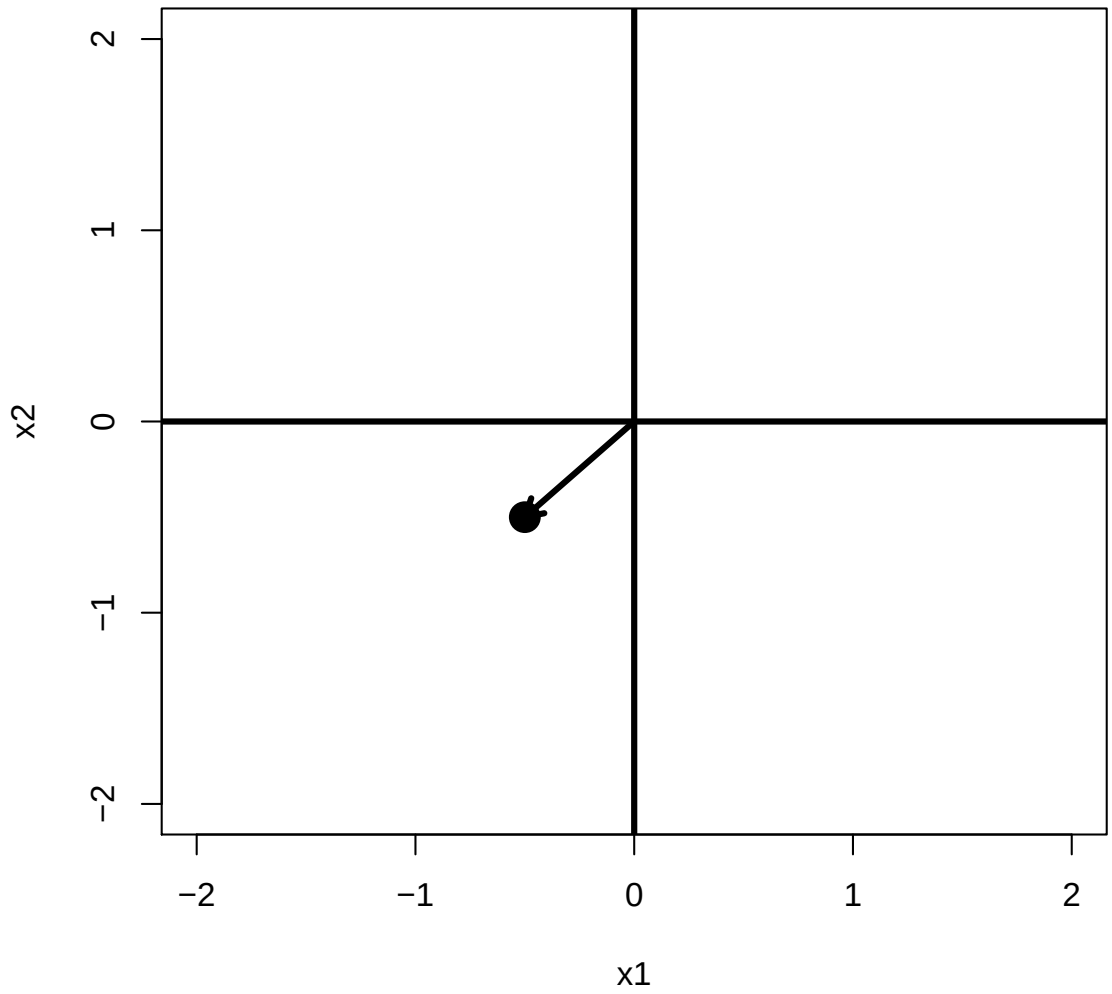
Vectors to Draw a Line

Suppose $\boldsymbol{w}_1 = (1, 1) \quad -2\boldsymbol{w}_1 = (-2, -2)$



Vectors to Draw a Line

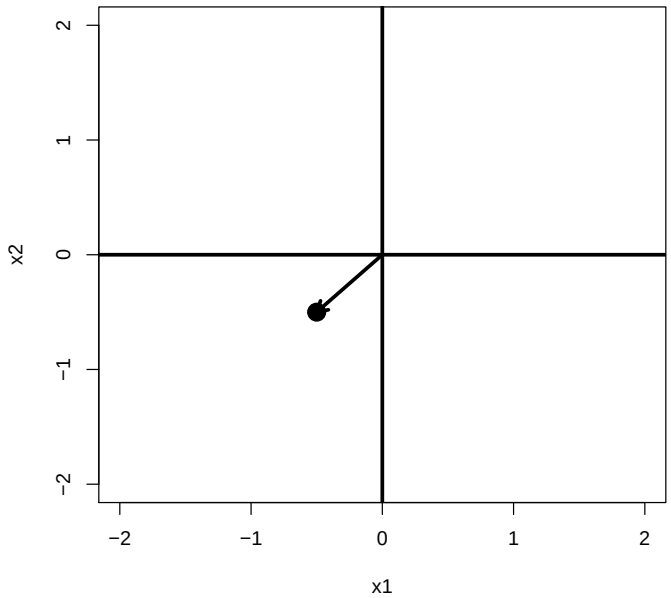
Suppose $\boldsymbol{w}_1 = (1, 1)$ $\quad -\frac{1}{2}\boldsymbol{w}_1 = (-1/2, -1/2)$



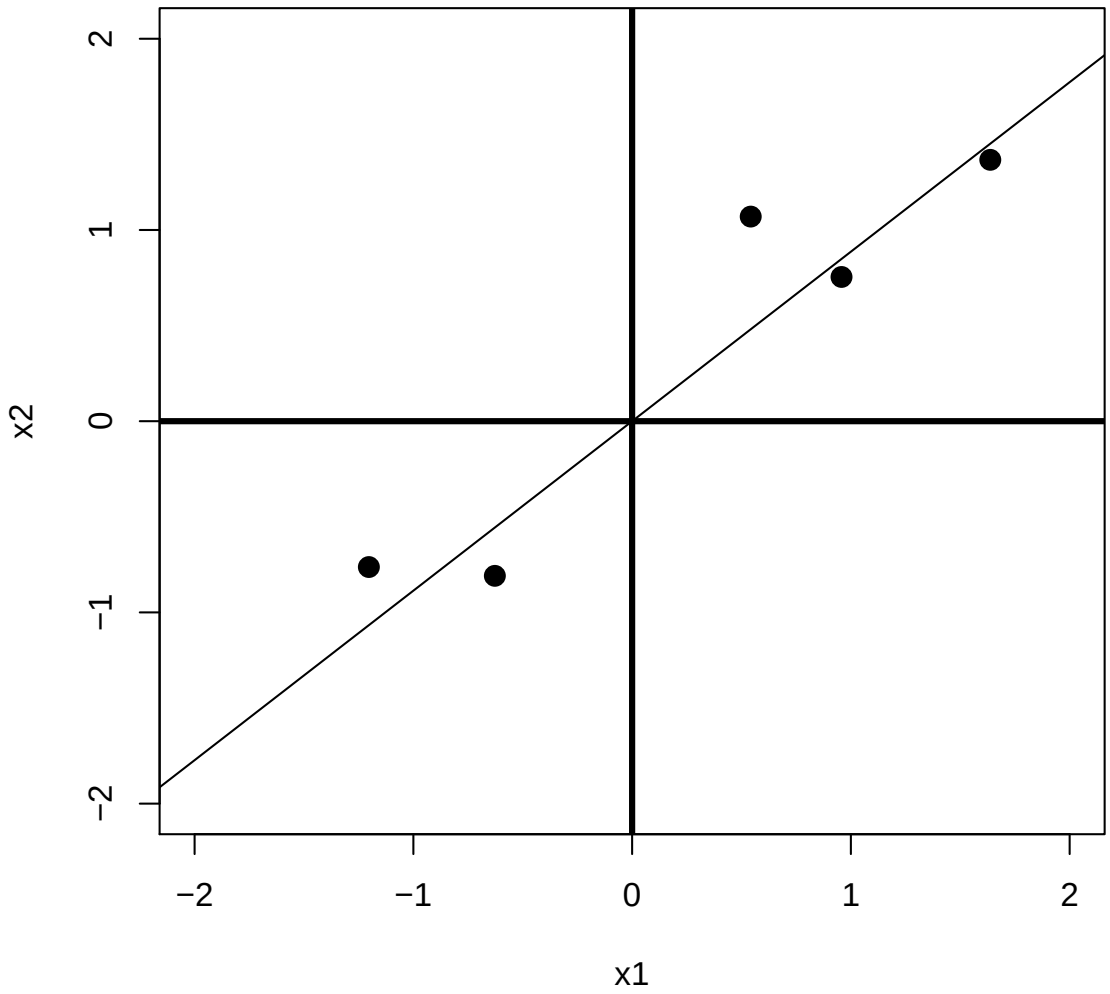
Vectors to Draw a Line

Suppose $\boldsymbol{w}_1 = (1, 1)$

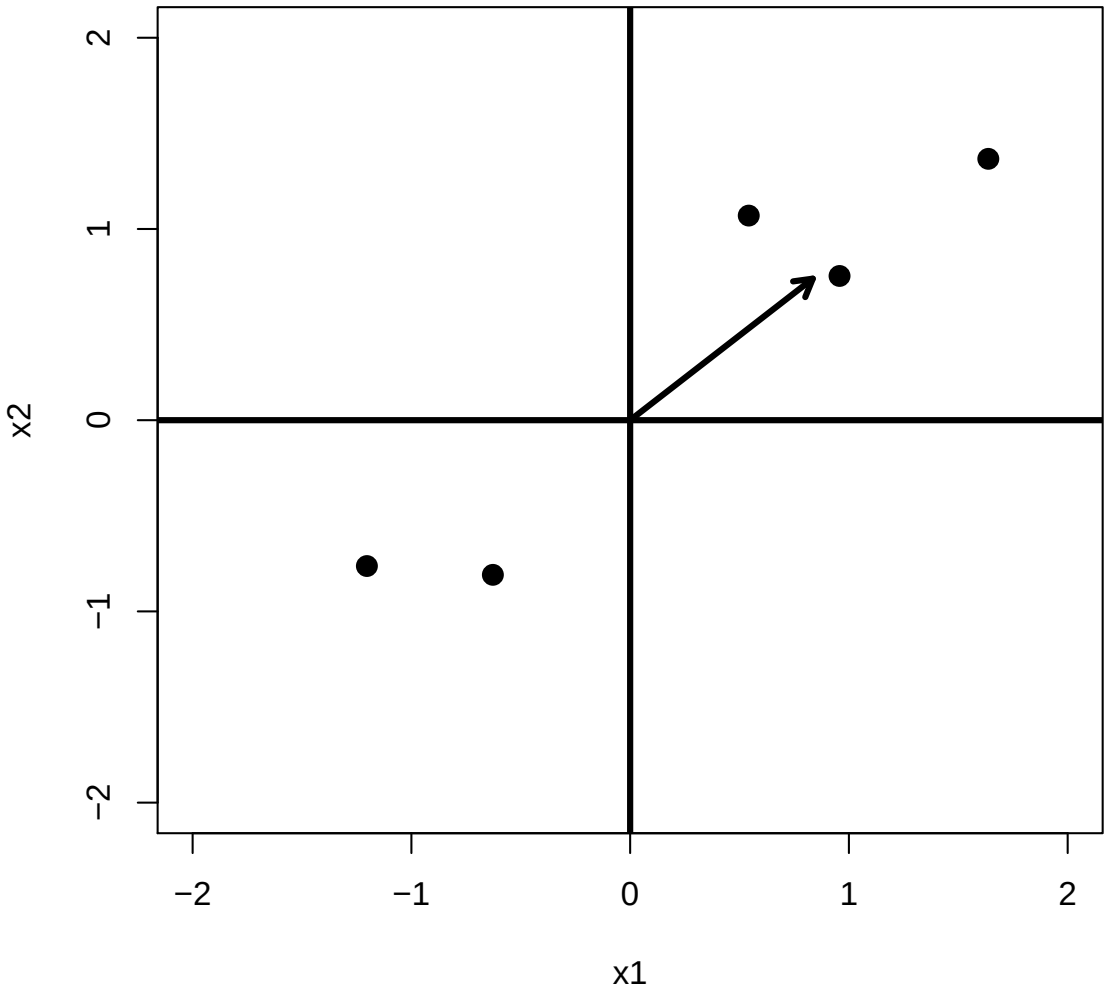
z_i = amount we shrink/flip $\boldsymbol{w}_1$ to approximate point i.



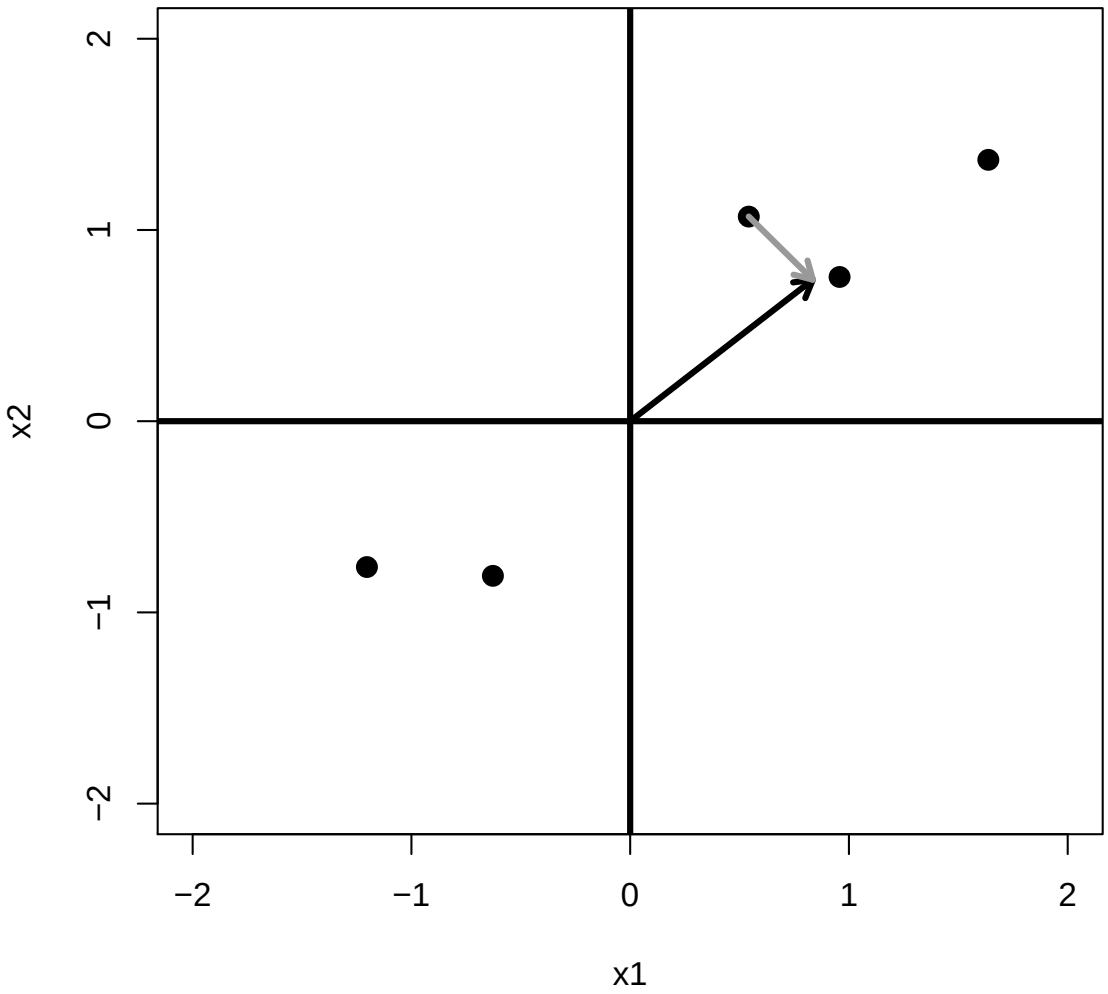
$\boldsymbol{w}_1 = (0.75, 0.66)$



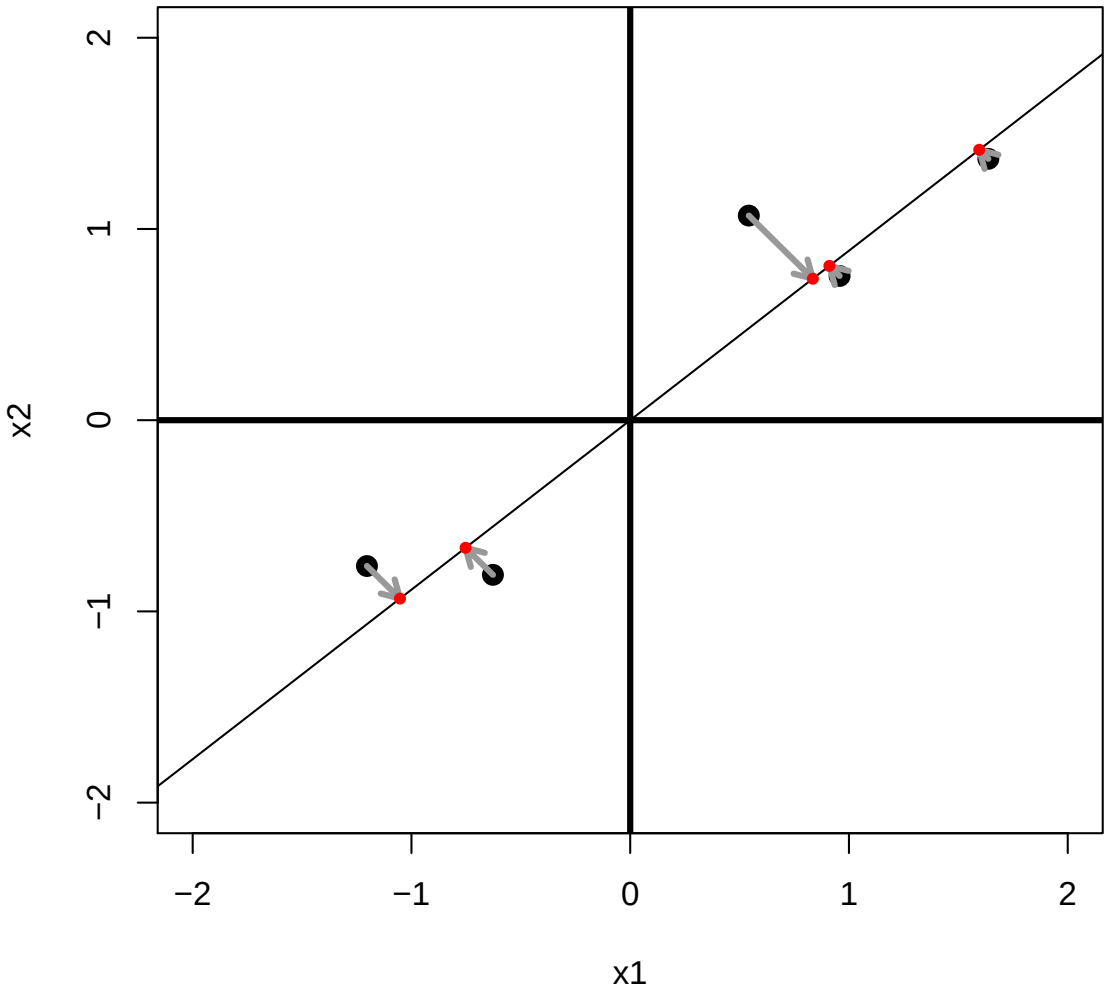
$\boldsymbol{w}_1 = (0.75, 0.66)$ $z_1 = 1.12$



$\boldsymbol{w}_1 = (0.75, 0.66)$ $z_1 = 1.12$



$\boldsymbol{w}_1 = (0.75, 0.66)$ $z_1 = 1.12$



Algebraic Representation

$$\boldsymbol{x}_i = z_i \boldsymbol{w}_1 + \boldsymbol{e}_i$$

Algebraic Representation

$$\begin{aligned} \boldsymbol{x}_i &= z_i \boldsymbol{w}_1 + \boldsymbol{e}_i \\ (x_{i1}, x_{i2}) &= (z_i w_{11} + e_{i1}, z_i w_{12} + e_{i2}) \end{aligned}$$

Algebraic Representation

$$\begin{aligned} \boldsymbol{x}_i &= z_i \boldsymbol{w}_1 + \boldsymbol{e}_i \\ (x_{i1}, x_{i2}) &= (z_i w_{11} + e_{i1}, z_i w_{12} + e_{i2}) \end{aligned}$$

Find $\boldsymbol{w}_1 = (w_{11}, w_{12})$ and z_i to minimize the error

Algebraic Representation

$$\begin{aligned} \boldsymbol{x}_i &= z_i \boldsymbol{w}_1 + \boldsymbol{e}_i \\ (x_{i1}, x_{i2}) &= (z_i w_{11} + e_{i1}, z_i w_{12} + e_{i2}) \end{aligned}$$

Find $\boldsymbol{w}_1 = (w_{11}, w_{12})$ and z_i to minimize the error

$$\text{error} = \frac{1}{N} \sum_{i=1}^{N} \big((x_{i1}, x_{i2}) - z_i(w_{11}, w_{12})\big)' \big((x_{i1}, x_{i2}) - z_i(w_{11}, w_{12})\big)$$

Algebraic Representation

$$\begin{aligned} \boldsymbol{x}_i &= z_i \boldsymbol{w}_1 + \boldsymbol{e}_i \\ (x_{i1}, x_{i2}) &= (z_i w_{11} + e_{i1}, z_i w_{12} + e_{i2}) \end{aligned}$$

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$$\begin{aligned} \text{error} &= \frac{1}{N} \sum_{i=1}^{N} \big((x_{i1}, x_{i2}) - z_i(w_{11}, w_{12})\big)' \big((x_{i1}, x_{i2}) - z_i(w_{11}, w_{12})\big) \\ &= \frac{1}{N} \sum_{i=1}^{N} (x_{i1} - z_i w_{11})^2 + (x_{i2} - z_i w_{12})^2 \end{aligned}$$

Three Dimensional Approximation

$$
\begin{aligned}
\boldsymbol{x}_1 &= (0.09, -1.02, -0.10) \\
\boldsymbol{x}_2 &= (0.09, 1.41, 0.67) \\
\boldsymbol{x}_3 &= (-0.81, -1.46, -0.54) \\
\boldsymbol{x}_4 &= (1.43, 0.26, 0.61) \\
\boldsymbol{x}_5 &= (1.23, 0.87, 1.33
\end{aligned}
$$

Find $\boldsymbol{w}_1 = (w_{11}, w_{12}, w_{13})$ and z_i to provide best one dimensional approximation.

Three-Dimensional Visualization

Three-Dimensional Visualization

$\boldsymbol{w}_1 = (0.48, 0.75, 0.46)$

$$\boldsymbol{x}_i = z_i \boldsymbol{w}_1 + \boldsymbol{e}_i$$

$$
\begin{aligned}
\boldsymbol{x}_i &= z_i \boldsymbol{w}_1 + \boldsymbol{e}_i \\
(x_{i1}, x_{i2}, x_{i3}) &= (z_i w_{11} + e_{i1}, z_i w_{12} + e_{i2}, z_i w_{13} + e_{i3})
\end{aligned}
$$

$$\begin{aligned} \boldsymbol{x}_i &= z_i \boldsymbol{w}_1 + \boldsymbol{e}_i \\ (x_{i1}, x_{i2}, x_{i3}) &= (z_i w_{11} + e_{i1}, z_i w_{12} + e_{i2}, z_i w_{13} + e_{i3}) \end{aligned}$$

Find $\boldsymbol{w}_1 = (w_{11}, w_{12}, w_{13})$ and z_i to minimize the error

$$
\begin{aligned}
\boldsymbol{x}_i &= z_i \boldsymbol{w}_1 + \boldsymbol{e}_i \\
(x_{i1}, x_{i2}, x_{i3}) &= (z_i w_{11} + e_{i1}, z_i w_{12} + e_{i2}, z_i w_{13} + e_{i3})
\end{aligned}
$$

Find $\boldsymbol{w}_1 = (w_{11}, w_{12}, w_{13})$ and z_i to minimize the error

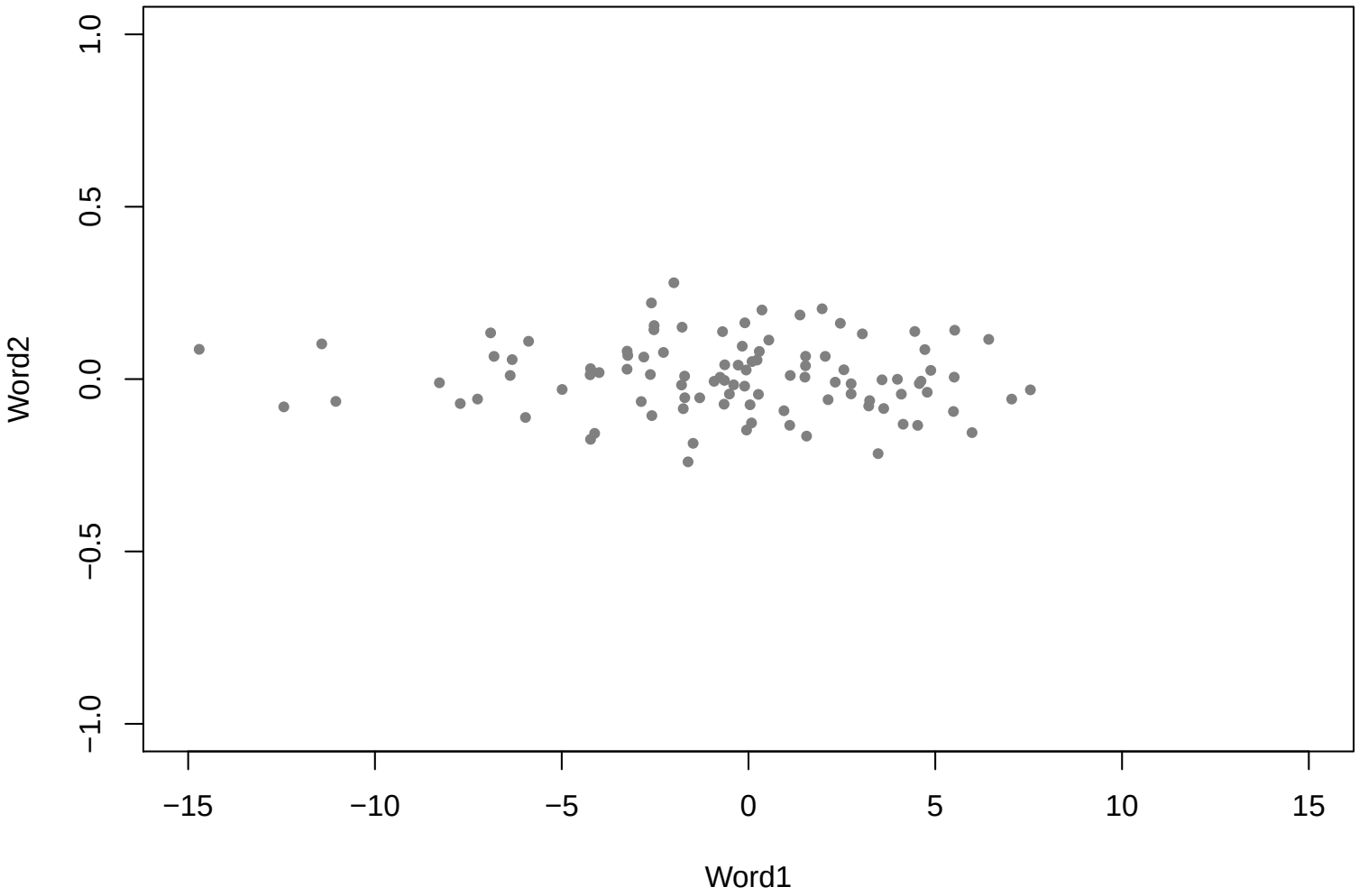
$$
\begin{aligned}
\text{error} &= \frac{1}{N} \sum_{i=1}^{N} \big((x_{i1}, x_{i2}, x_{13}) - z_i (w_{11}, w_{12}, w_{13})\big)' \\
&\quad \big((x_{i1}, x_{i2}, x_{i3}) - z_i (w_{11}, w_{12}, w_{13})\big)
\end{aligned}
$$

$$\begin{aligned} \boldsymbol{x}_i &= z_i \boldsymbol{w}_1 + \boldsymbol{e}_i \\ (x_{i1}, x_{i2}, x_{i3}) &= (z_i w_{11} + e_{i1}, z_i w_{12} + e_{i2}, z_i w_{13} + e_{i3}) \end{aligned}$$

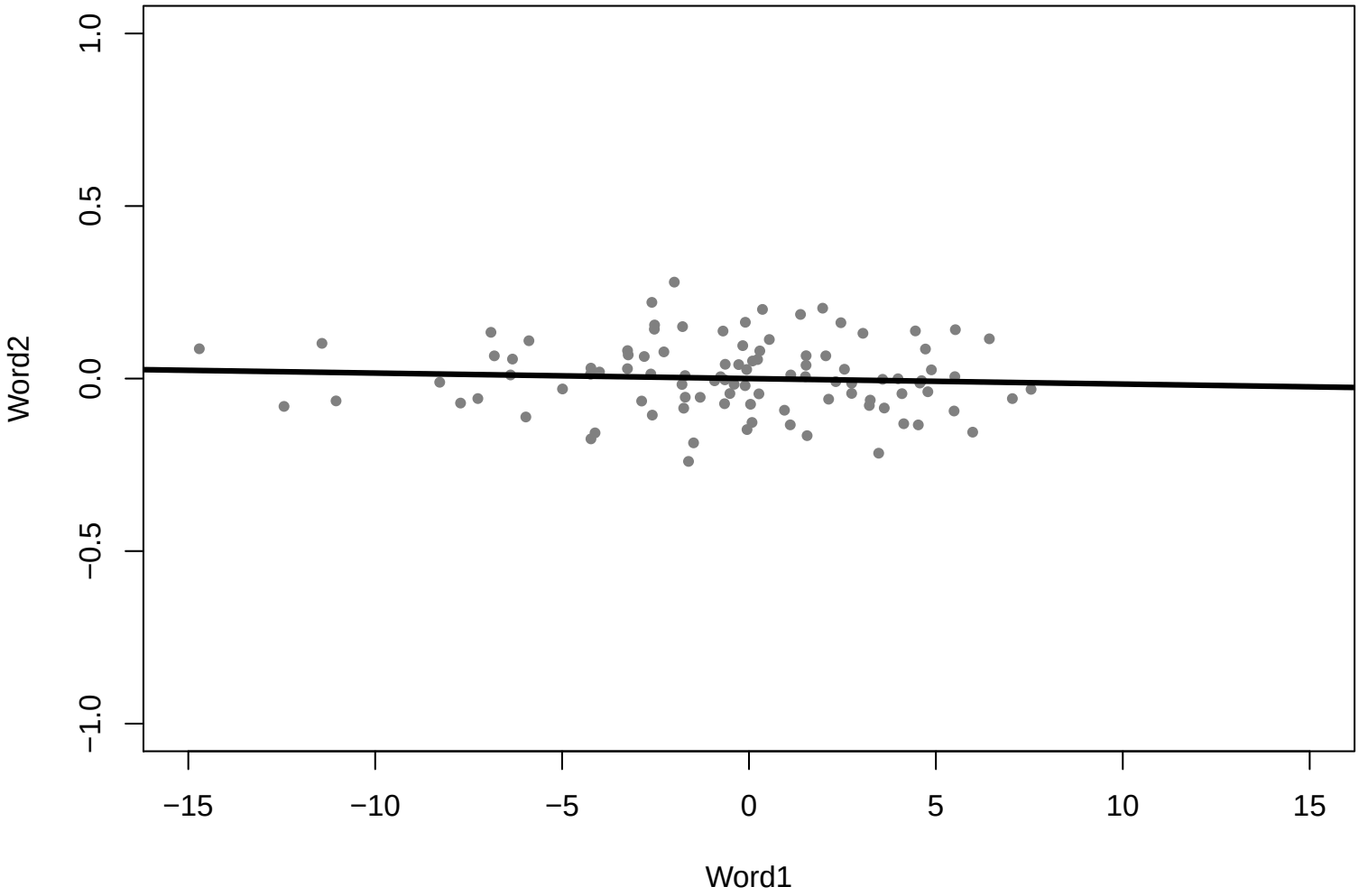
Find $\boldsymbol{w}_1 = (w_{11}, w_{12}, w_{13})$ and z_i to minimize the error

$$\begin{aligned} \text{error} &= \frac{1}{N} \sum_{i=1}^{N} \big((x_{i1}, x_{i2}, x_{13}) - z_i (w_{11}, w_{12}, w_{13})\big)' \\ &\quad \big((x_{i1}, x_{i2}, x_{i3}) - z_i (w_{11}, w_{12}, w_{13})\big) \\ &= \frac{1}{N} \sum_{i=1}^{N} (x_{i1} - z_i w_{11})^2 + (x_{i2} - z_i w_{12})^2 + (x_{i3} - z_i w_{13})^2 \end{aligned}$$

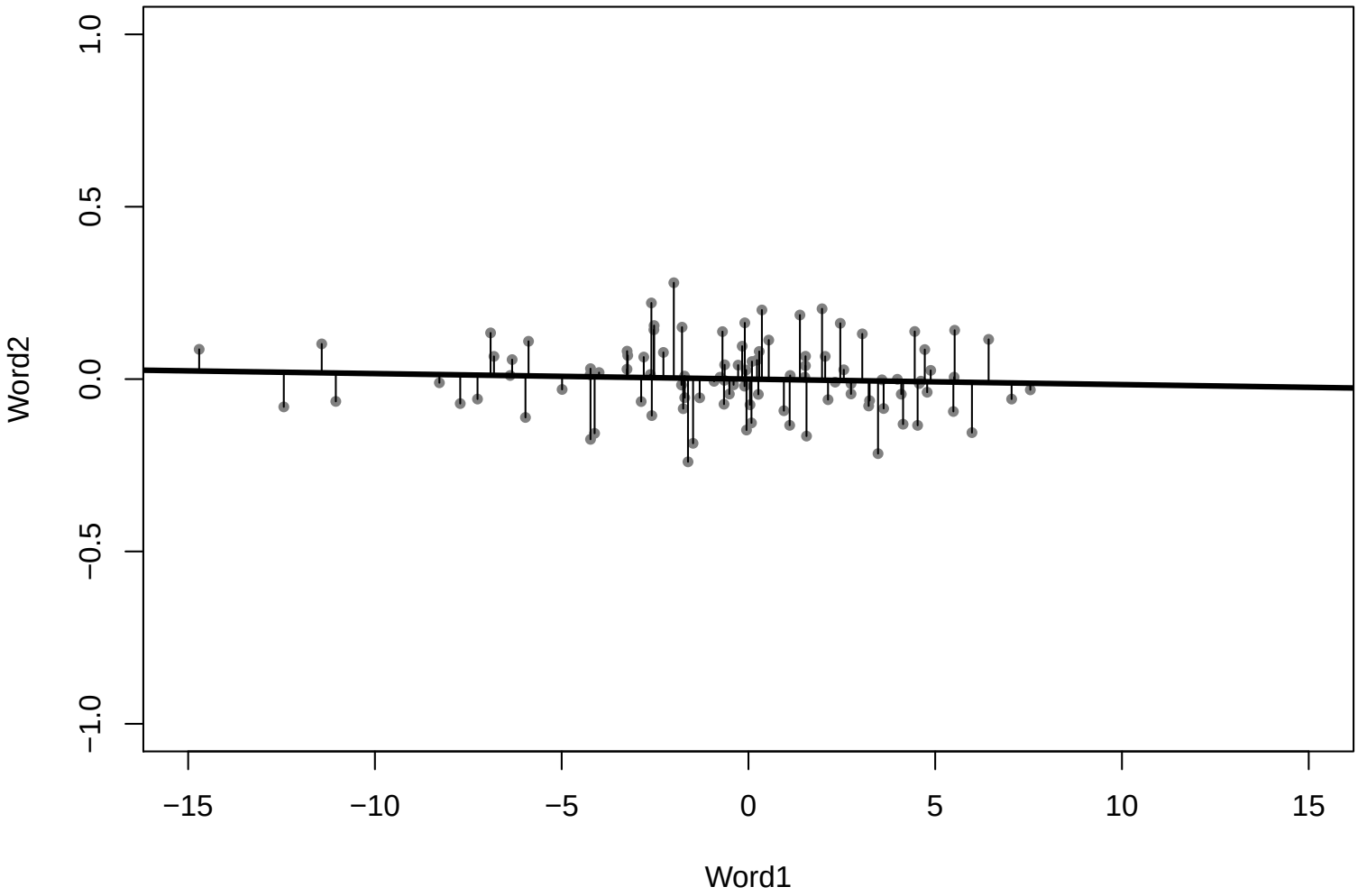
Principal Component Analysis



Principal Component Analysis



Principal Component Analysis



PCA Output

$$\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iJ})$$

PCA Output

$\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iJ})$
Principal Component Output:

PCA Output

$\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iJ})$
Principal Component Output:

1) K Principal Components $\boldsymbol{w}_k$

PCA Output

$\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iJ})$
Principal Component Output:

1) K Principal Components $\boldsymbol{w}_k$

$$\boldsymbol{w}_k = (w_{1k}, w_{2k}, \ldots, w_{Jk})$$

PCA Output

$\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iJ})$
Principal Component Output:

1) K Principal Components $\boldsymbol{w}_k$

$$\boldsymbol{w}_k = (w_{1k}, w_{2k}, \ldots, w_{Jk})$$

2) K component vector describing loadings on principal components for each document

PCA Output

$\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iJ})$
Principal Component Output:

1) K Principal Components $\boldsymbol{w}_k$

$$\boldsymbol{w}_k = (w_{1k}, w_{2k}, \ldots, w_{Jk})$$

2) K component vector describing loadings on principal components for each document

$$\boldsymbol{z}_i = (z_{1i}, z_{2i}, \ldots, z_{Ki})$$

An Introduction to Eigenvectors, Values, and Diagonalization

Definition

Suppose $\boldsymbol{A}$ *is an* $N \times N$ *matrix and* λ *is a scalar. If*

$$\boldsymbol{Ax} = \lambda \boldsymbol{x}$$

Then $\boldsymbol{x}$ *is an eigenvector and* λ *is the associated eigenvalue*

An Introduction to Eigenvectors, Values, and Diagonalization

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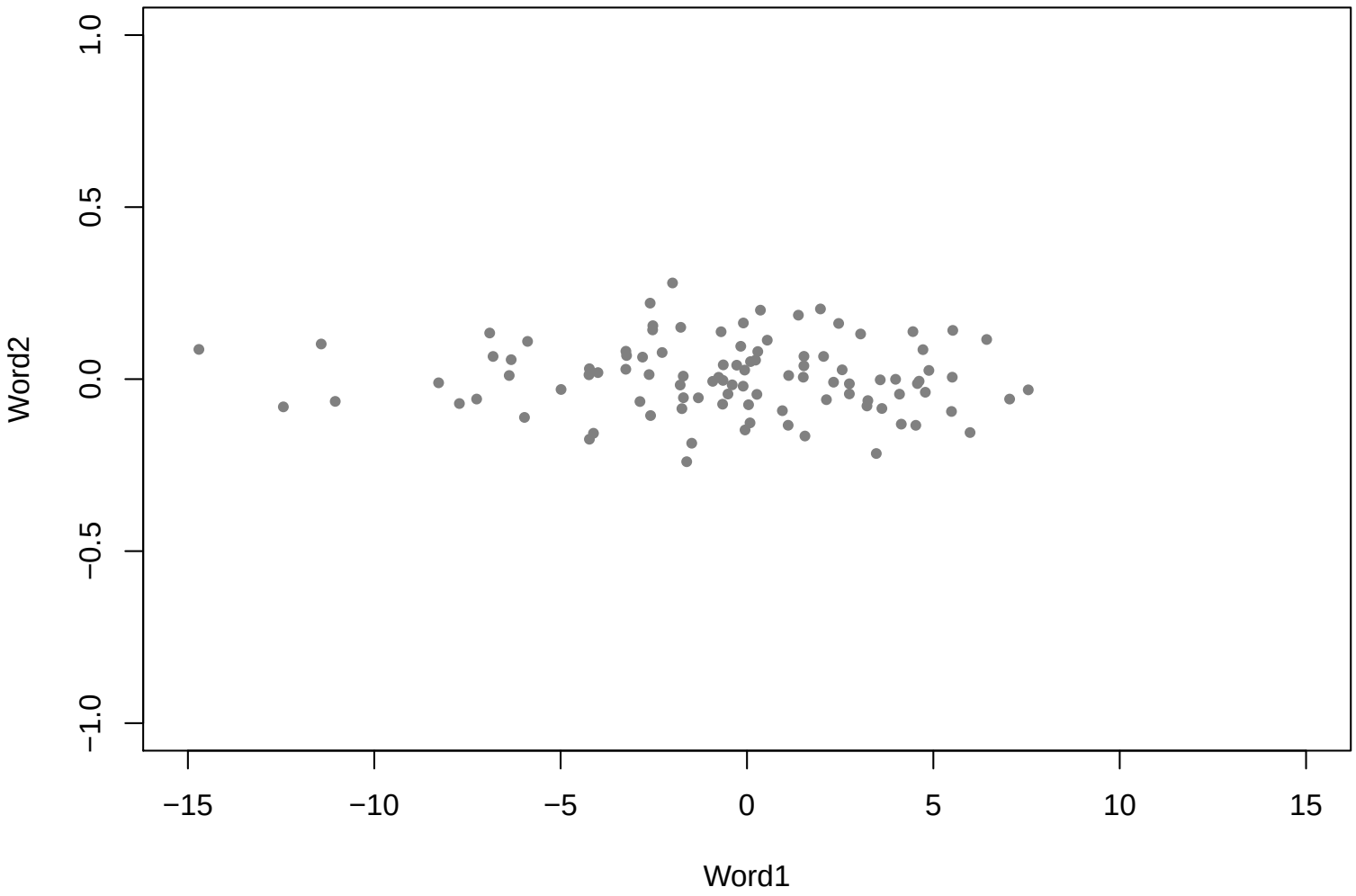
$$(\boldsymbol{A} - \lambda \boldsymbol{I}) = 0$$

Finding a Lower Dimensional Space (Manifold Learning)



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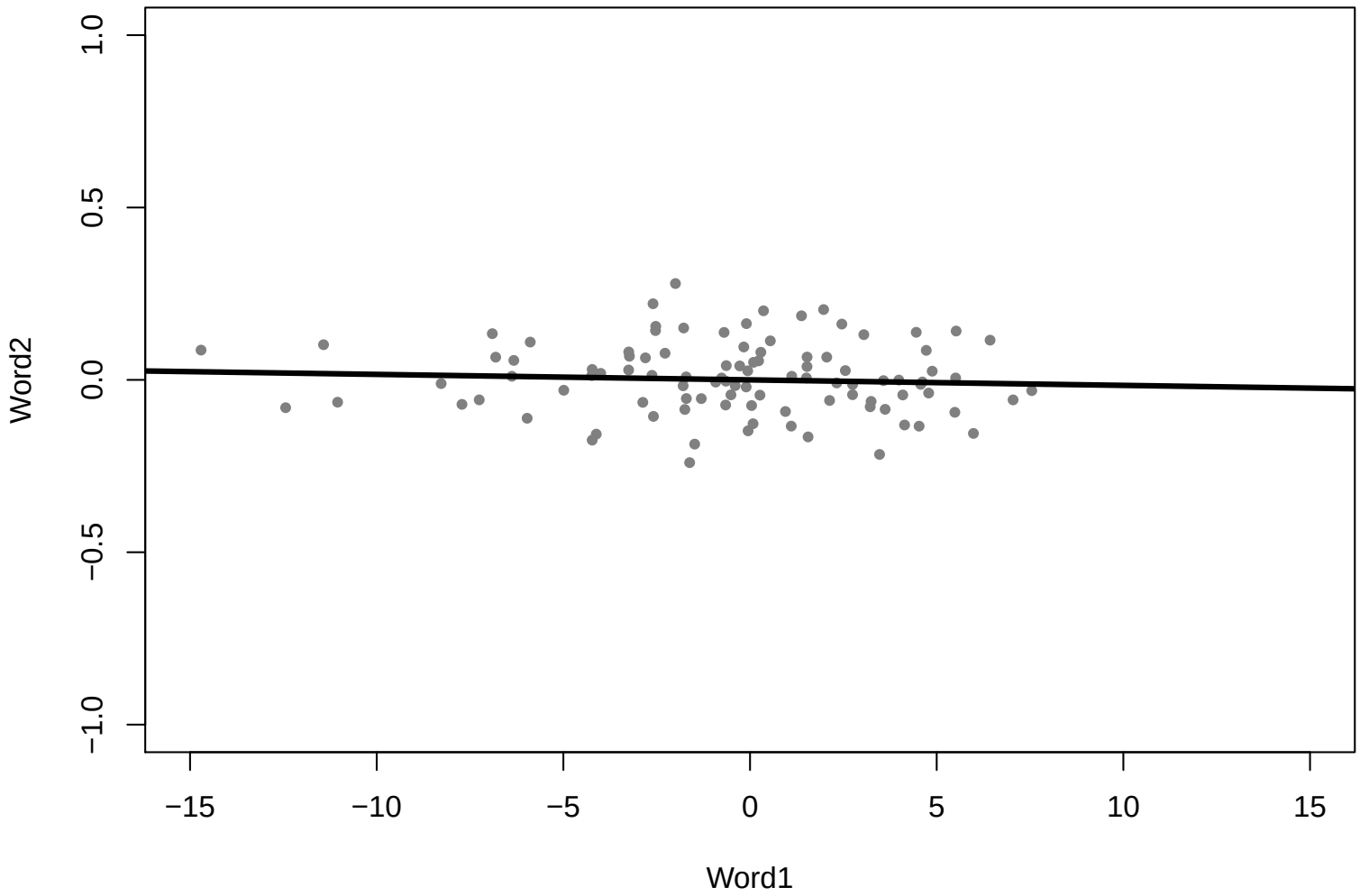

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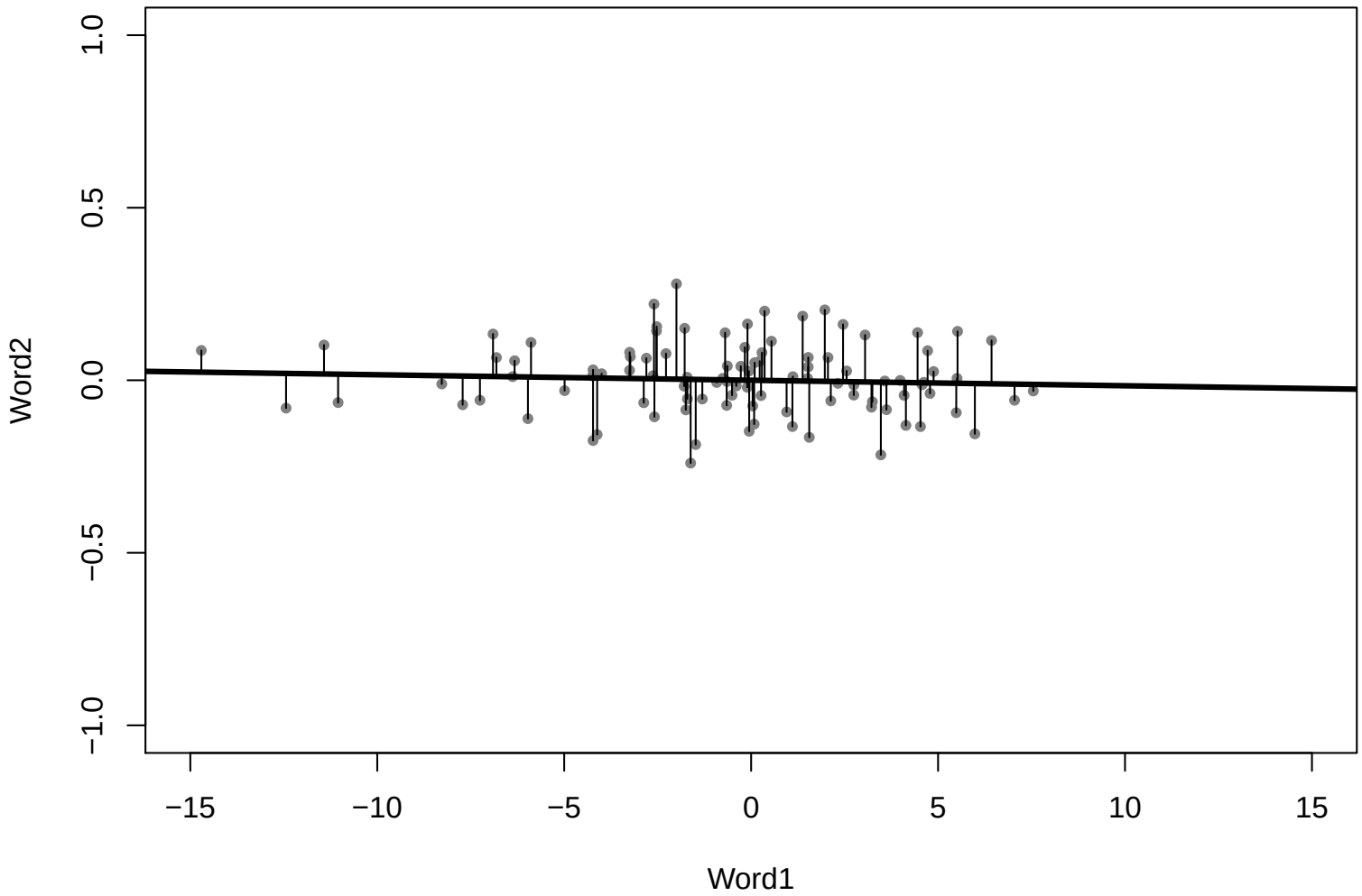

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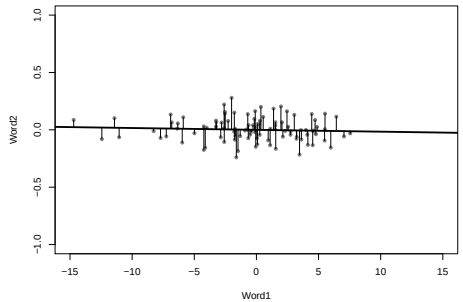
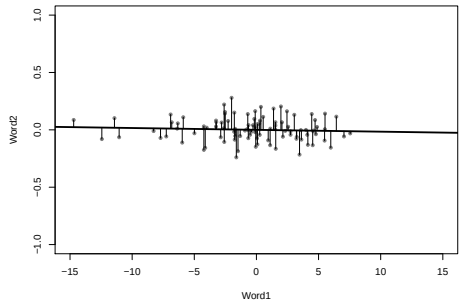


Original data:

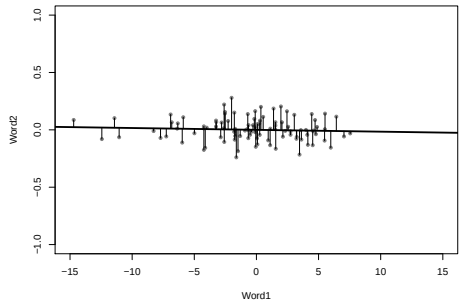
Finding a Lower Dimensional Space (Manifold Learning)



Original data:

$$\boldsymbol{x}_i = (x_{i1}, x_{i2})$$

Finding a Lower Dimensional Space (Manifold Learning)



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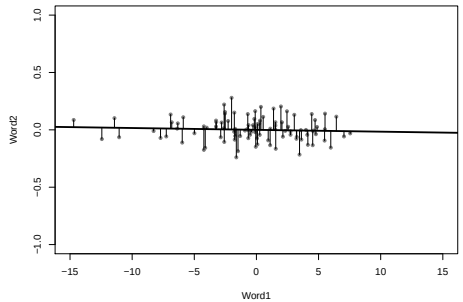
Finding a Lower Dimensional Space (Manifold Learning)



Original data:

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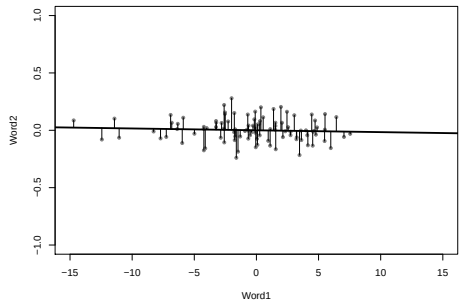
Which we approximate with

$$\begin{aligned} \tilde{\boldsymbol{x}}_i &= z_i \boldsymbol{w}_1 \\ &= z_i (w_{11}, w_{12}) \end{aligned}$$

Finding a Lower Dimensional Space (Manifold Learning)



Original data $\boldsymbol{x}_i \in \Re^J$

$$\boldsymbol{x}_i \quad = \quad (x_{i1}, x_{i2}, \ldots, x_{iJ})$$

Which we approximate with $L \leq J$ weights z_{il} and vectors $\boldsymbol{w}_l \in \Re^J$

$$\tilde{\boldsymbol{x}}_i \quad = \quad z_{i1}\boldsymbol{w}_1 + z_{i2}\boldsymbol{w}_2 + \ldots + z_{iL}\boldsymbol{w}_L$$

Define $\boldsymbol{\theta} = (\underbrace{\boldsymbol{Z}}_{N \times L}, \underbrace{\boldsymbol{W}_L}_{L \times J})$

Principal Component Analysis$\rightsquigarrow$ Objective function

Consider 1-dimensional case ($L = 1$), centered data, and $||\boldsymbol{w}_1|| = 1$.

Principal Component Analysis$\rightsquigarrow$ Objective function

Consider 1-dimensional case ($L = 1$), centered data, and $||\boldsymbol{w}_1|| = 1$.

$$f(\boldsymbol{\theta}, \boldsymbol{X}) = \frac{1}{N} \sum_{i=1}^{N} ||\boldsymbol{x}_i - z_{i1} \boldsymbol{w}_1||^2$$

Principal Component Analysis$\rightsquigarrow$ Objective function

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$\boldsymbol{w}_1'\boldsymbol{w}_1 = 1$

Principal Component Analysis$\rightsquigarrow$ Optimization

Optimization:

Principal Component Analysis$\rightsquigarrow$ Optimization

Optimization:

$$\frac{\partial f(\boldsymbol{\theta}, \boldsymbol{X})}{\partial z_{i1}} = -\frac{2\boldsymbol{w}_1'\boldsymbol{x}_i + 2z_{i1}}{N}$$

Principal Component Analysis$\rightsquigarrow$ Optimization

Optimization:

$$\begin{aligned} \frac{\partial f(\boldsymbol{\theta}, \boldsymbol{X})}{\partial z_{i1}} &= -\frac{2\boldsymbol{w}_1' \boldsymbol{x}_i + 2z_{i1}}{N} \\ 0 &= -\frac{2\boldsymbol{w}_1' \boldsymbol{x}_i + 2z_{i1}^*}{N} \end{aligned}$$

Principal Component Analysis$\rightsquigarrow$ Optimization

Optimization:

$$\begin{aligned}
\frac{\partial f(\boldsymbol{\theta}, \boldsymbol{X})}{\partial z_{i1}} &= -\frac{2\boldsymbol{w}_1^{'}\boldsymbol{x}_i + 2z_{i1}}{N} \\
0 &= -\frac{2\boldsymbol{w}_1^{'}\boldsymbol{x}_i + 2z_{i1}^*}{N} \\
z_{i1}^* &= \boldsymbol{w}_1^{'}\boldsymbol{x}_i
\end{aligned}$$

Principal Component Analysis$\rightsquigarrow$ Optimization

Substituting in z_{i1}^*

Principal Component Analysis$\rightsquigarrow$ Optimization

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$$= \frac{1}{N}\sum_{i=1}^{N}(\boldsymbol{x}_i - z_{i1}^*\boldsymbol{w}_1)'(\boldsymbol{x}_i - z_{i1}^*\boldsymbol{w}_1)$$

Principal Component Analysis$\rightsquigarrow$ Optimization

Substituting in z_{i1}^*

$$
\begin{aligned}
&= \frac{1}{N}\sum_{i=1}^{N}(\boldsymbol{x}_i - z_{i1}^*\boldsymbol{w}_1)'(\boldsymbol{x}_i - z_{i1}^*\boldsymbol{w}_1) \\
&= \frac{1}{N}\sum_{i=1}^{N}(\underbrace{\boldsymbol{x}_i'\boldsymbol{x}_i}_{\text{Constant}} - 2z_{i1}^* \underbrace{\boldsymbol{w}_1'\boldsymbol{x}_i}_{z_{i1}^*} + (z_{i1}^*)^2 \underbrace{\boldsymbol{w}_1'\boldsymbol{w}_1}_{1})
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&= -\frac{1}{N}\sum_{i=1}^{N}(z_{i1}^*)^2 + c \\
&= -\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{w}_1'\boldsymbol{x}_i\boldsymbol{x}_i'\boldsymbol{w}_1
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&= -\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{w}_1'\boldsymbol{x}_i\boldsymbol{x}_i'\boldsymbol{w}_1 \\
&= -\boldsymbol{w}_1'\boldsymbol{\Sigma}\boldsymbol{w}_1
\end{aligned}
$$

Principal Component Analysis$\rightsquigarrow$ Optimization

$$= \quad -\boldsymbol{w}_1^{'}\boldsymbol{\Sigma}\boldsymbol{w}_1$$

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&= \frac{1}{N}\sum_{i=1}^{N} z_{i1}^2 - 0 \\
&= \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{w}_1^{'}\boldsymbol{x}_i\boldsymbol{x}_i^{'}\boldsymbol{w}_1 = \boldsymbol{w}_1^{'}\boldsymbol{\Sigma}\boldsymbol{w}_1
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Minimize reconstruction error

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Minimize reconstruction error $\rightsquigarrow$ maximize variance of projected data

Principal Component Analysis$\rightsquigarrow$ Optimization

Maximize variance, subject to constraints

Principal Component Analysis$\rightsquigarrow$ Optimization

Maximize variance, subject to constraints

$$g(\boldsymbol{z}^*, \boldsymbol{w}_1, \boldsymbol{X}) \quad = \quad \boldsymbol{w}_1^{'} \boldsymbol{\Sigma} \boldsymbol{w}_1 - \lambda_1 (\boldsymbol{w}_1^{'} \boldsymbol{w}_1 - 1)$$

Principal Component Analysis$\rightsquigarrow$ Optimization

Maximize variance, subject to constraints

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$\boldsymbol{w}_1^* =$ Eigenvector of $\boldsymbol{\Sigma}$

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$\boldsymbol{w}_1^*$ = Eigenvector of $\boldsymbol{\Sigma}$ (!!!!!!)

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$\boldsymbol{w}_1^*$ = Eigenvector of $\boldsymbol{\Sigma}$ (!!!!!!)
We want $\boldsymbol{w}_1$ to maximize variance and

$$\boldsymbol{w}_1^{'}\boldsymbol{\Sigma}\boldsymbol{w}_1 = \lambda_1$$

So $\boldsymbol{w}_1$ is eigenvector associated with the largest eigenvalue λ_1

An Introduction to Eigenvectors, Values, and Diagonalization

Theorem

Suppose $\boldsymbol{A}$ *is an invertible* $N \times N$ *matrix with* N *linearly independent eigenvectors. Then we can write* $\boldsymbol{A}$ *as,*

$$\boldsymbol{A} = \boldsymbol{W}' \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_N \end{pmatrix} \boldsymbol{W}$$

where $\boldsymbol{W} = (\boldsymbol{w}_1, \boldsymbol{w}_2, \dots, \boldsymbol{w}_N)$ *is an* $N \times N$ *matrix with the* N *eigenvectors as column vectors.*

An Introduction to Eigenvectors, Values, and Diagonalization

Definition

Suppose A is a covariance matrix. Then, we can write A as

$$\boldsymbol{A} = \boldsymbol{W}' \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_N \end{pmatrix} \boldsymbol{W}$$

Where $\lambda_1 > \lambda_2 > \ldots > \lambda_N \geq 0$.
We will call $\boldsymbol{w}_1$ *the first eigenvector,* $\boldsymbol{w}_2$ *the second eigenvector, ...,* $\boldsymbol{w}_j$ *the* j^{th} *eigenvector.*

Back to Principal Components

Theorem

Suppose we want to approximate N observations $\boldsymbol{x}_i \in \Re^J$ with $L < J$ orthogonal-unit length vectors $\boldsymbol{w}_l \in \Re^J$ with associated scores z_{il} to minimize reconstruction error:

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Application of Principal Components in R

Consider press releases from 2005 US Senators

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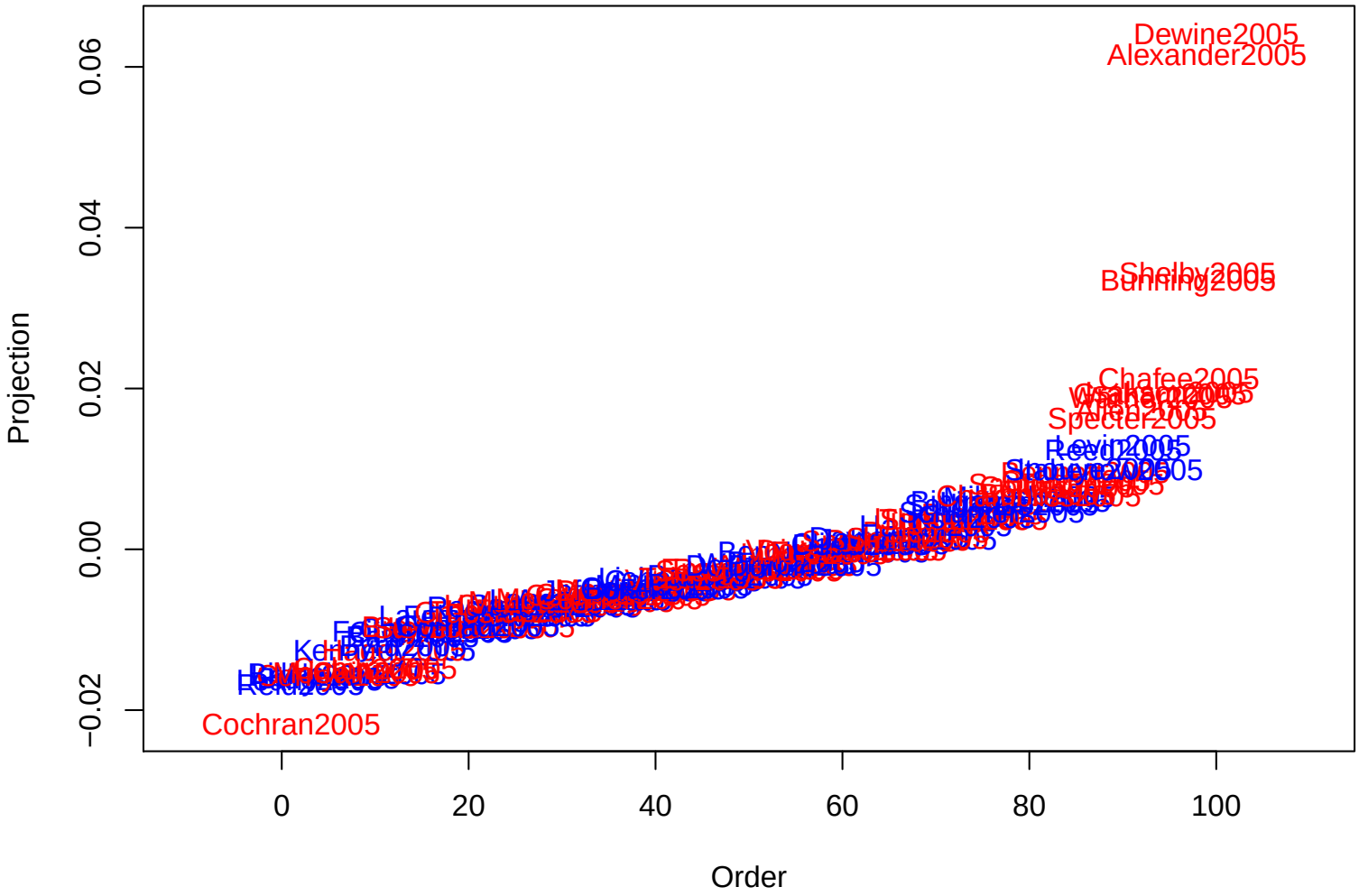
`dtm`: 100×2796 matrix containing word rates for senators
`prcomp(dtm)` applies principal components

```
load("SenateTDM.RData")
dtm<- t(tdm)
for(z in 1:100){
dtm[z,]<- dtm[z,]/sum(dtm[z,])
}

store<- prcomp(dtm, scale = F)
scores<- store$x[,1]
```

Application of Principal Components in R



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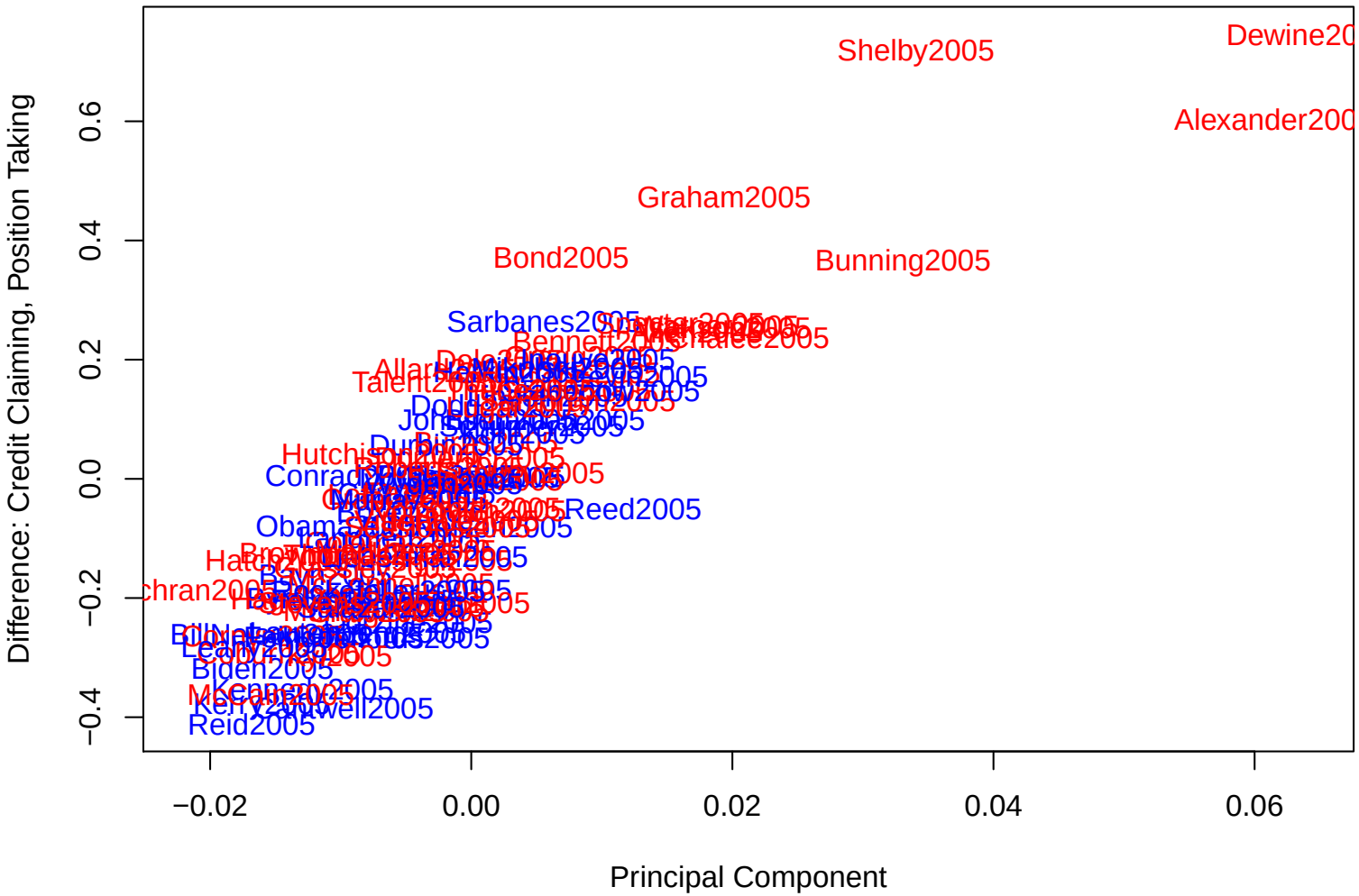

Probabilistic Principal Components (Tipping and Bishop 1999)

$$
\begin{aligned}
\boldsymbol{x}|\boldsymbol{w} &\sim \text{Multivariate Normal}(\boldsymbol{Z}\boldsymbol{W} + \boldsymbol{\mu}, \sigma^2\boldsymbol{I}) \\
\boldsymbol{w} &\sim \text{Multivariate Normal}(\boldsymbol{0}, \boldsymbol{I}) \\
\boldsymbol{x} &\sim \text{Multivariate Normal}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \\
\boldsymbol{\Sigma} &= \boldsymbol{W}\boldsymbol{W}^{'} + \sigma^2\boldsymbol{I}
\end{aligned}
$$

1) Log-likelihood $\rightsquigarrow$ straightforward
2) Optimization via EM-Algorithm
3) Corresponds to traditional PCA is $\lim_{\sigma^2} \to 0$
4) Closely related to Factor analysis.

How do we select the number of dimensions L?$\rightsquigarrow$ Model

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Recommendation$\rightsquigarrow$ look for Elbow

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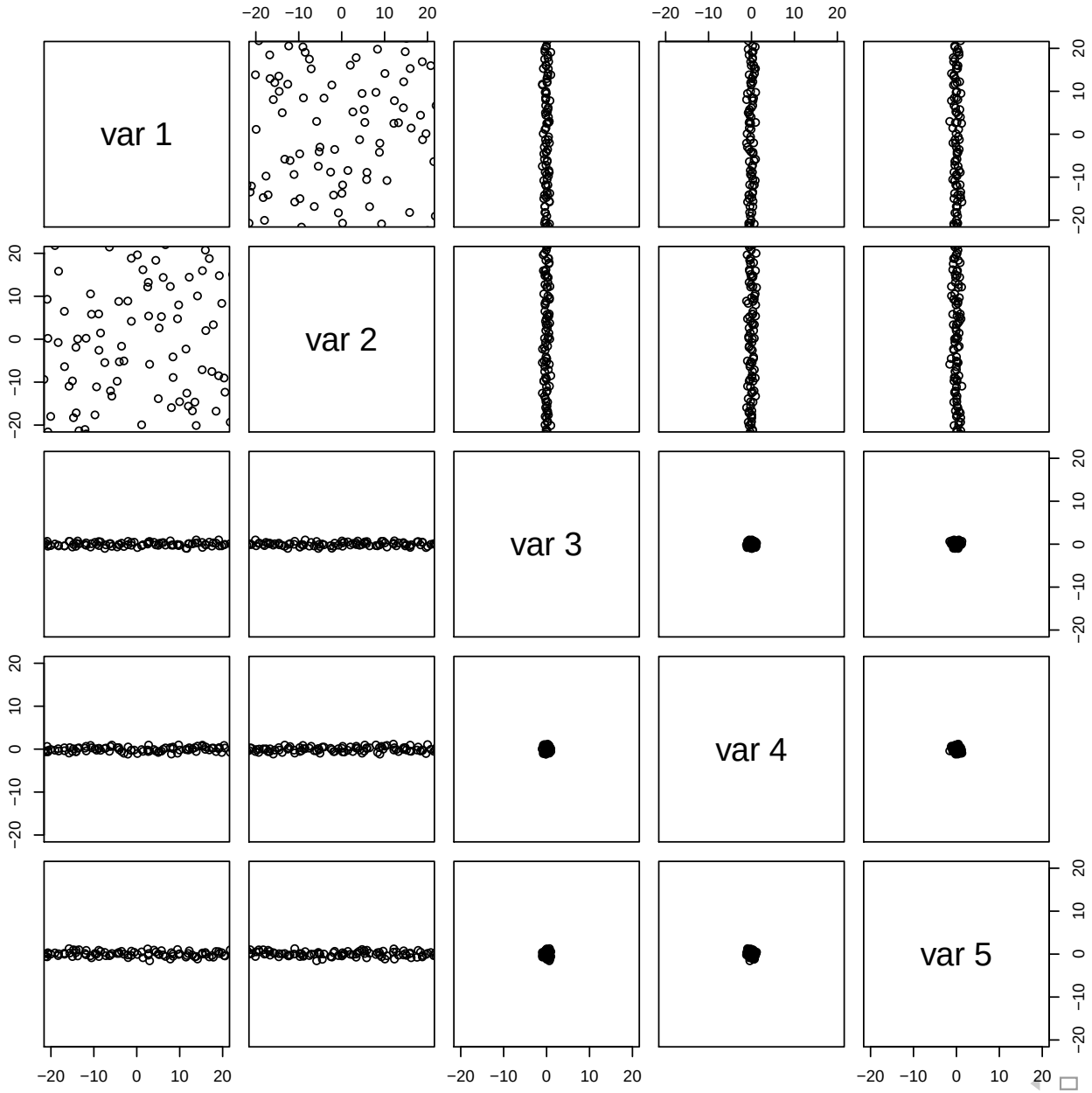

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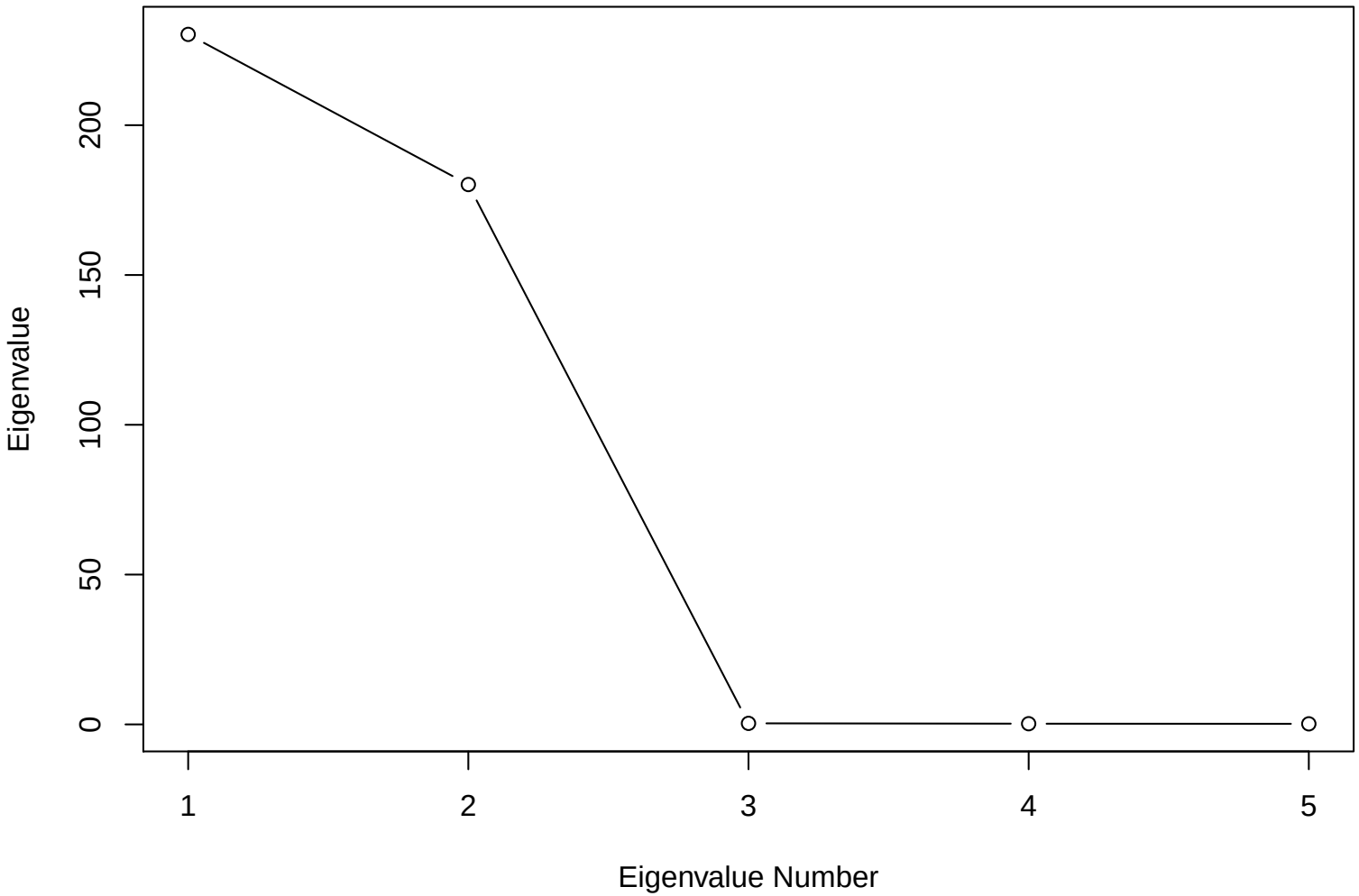

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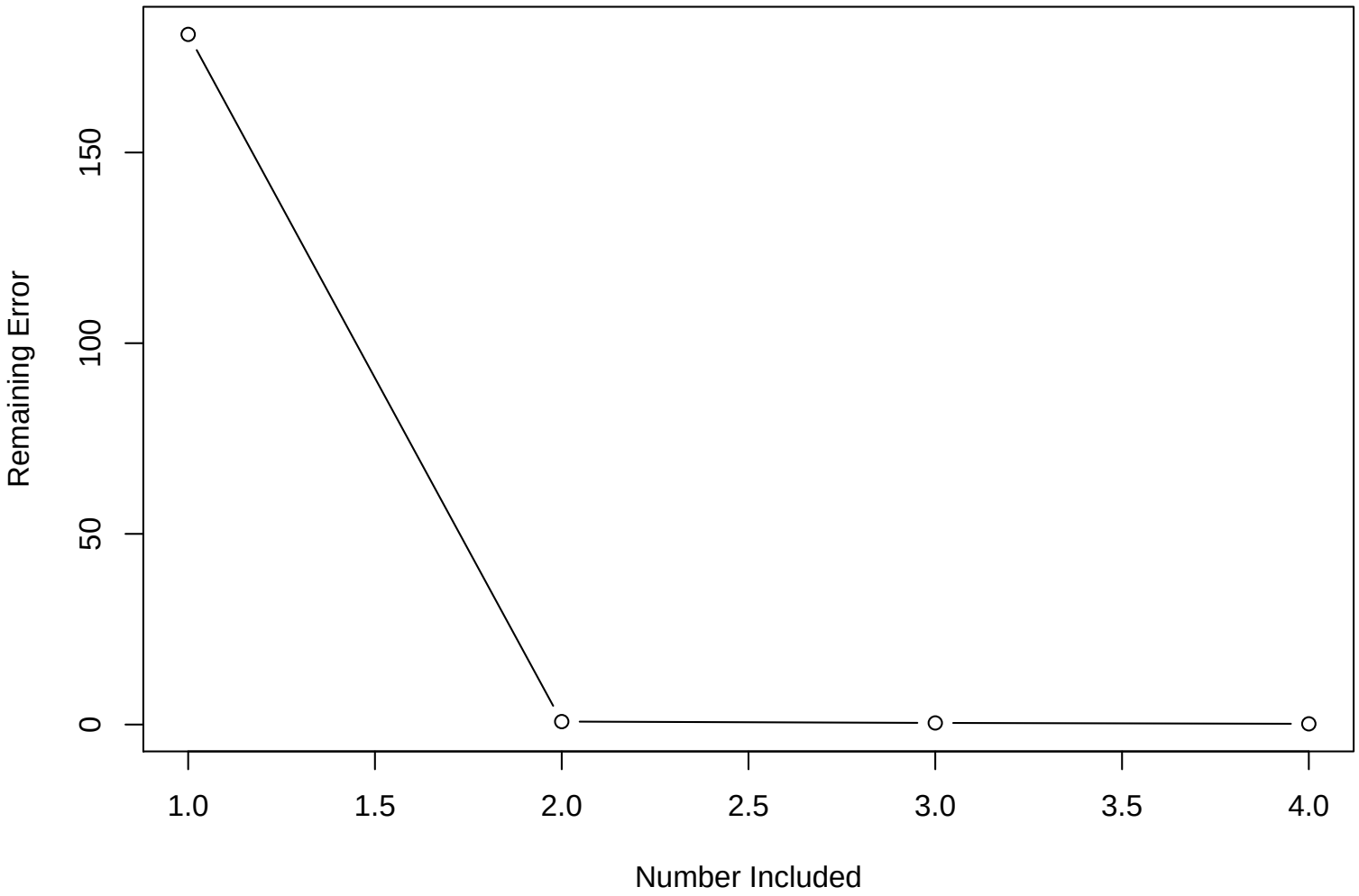

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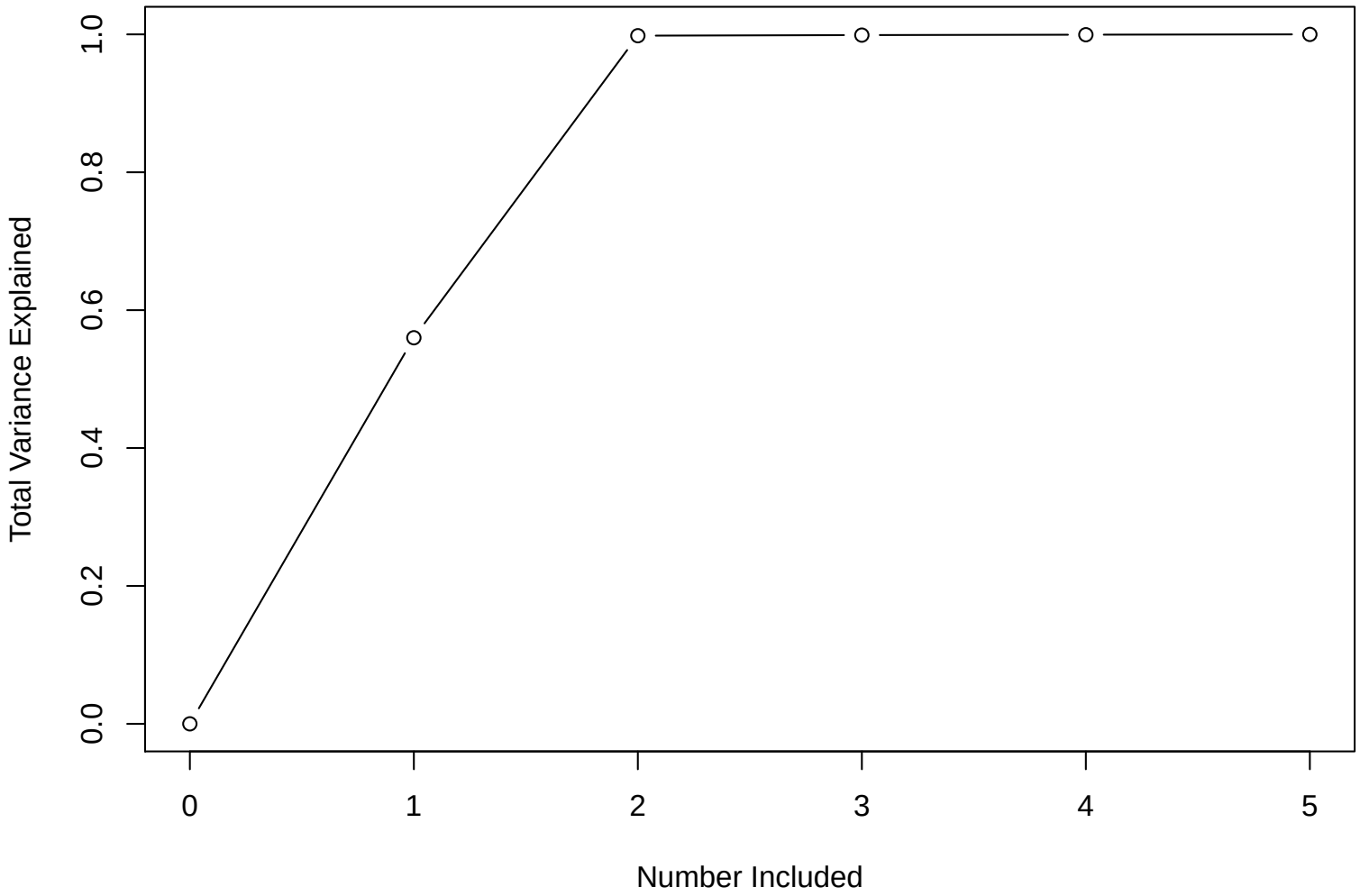

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Mathematical model$\rightsquigarrow$ insufficient to make modeling decision

Appendix

Kernel Principal Component Analysis

Define a Kernel $(N \times N)$ matrix as:

$$\boldsymbol{K} = \begin{pmatrix} k(\boldsymbol{x}_1, \boldsymbol{x}_1) & k(\boldsymbol{x}_1, \boldsymbol{x}_2) & \dots & k(\boldsymbol{x}_1, x_N) \\ k(\boldsymbol{x}_2, \boldsymbol{x}_1) & k(\boldsymbol{x}_2, \boldsymbol{x}_2) & \dots & k(\boldsymbol{x}_2, \boldsymbol{x}_N) \\ \vdots & \vdots & \ddots & \vdots \\ k(\boldsymbol{x}_N, \boldsymbol{x}_1) & k(\boldsymbol{x}_N, \boldsymbol{x}_2) & \dots & k(\boldsymbol{x}_N, \boldsymbol{x}_N) \end{pmatrix}$$

where $k(\cdot, \cdot)$ is a function that behaves like a similarity function.

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$$\boldsymbol{K} = \begin{pmatrix} k(\boldsymbol{x}_1, \boldsymbol{x}_1) & k(\boldsymbol{x}_1, \boldsymbol{x}_2) & \dots & k(\boldsymbol{x}_1, x_N) \\ k(\boldsymbol{x}_2, \boldsymbol{x}_1) & k(\boldsymbol{x}_2, \boldsymbol{x}_2) & \dots & k(\boldsymbol{x}_2, \boldsymbol{x}_N) \\ \vdots & \vdots & \ddots & \vdots \\ k(\boldsymbol{x}_N, \boldsymbol{x}_1) & k(\boldsymbol{x}_N, \boldsymbol{x}_2) & \dots & k(\boldsymbol{x}_N, \boldsymbol{x}_N) \end{pmatrix}$$

where $k(\cdot, \cdot)$ is a function that behaves like a similarity function. Where we suppose this matrix emerges from applying $\phi : \Re^J \to \Re^M$ to the data and then taking the inner product:

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Compute PCA of $\boldsymbol{\Phi}$ from $\boldsymbol{\Phi}\boldsymbol{\Phi}'$

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PCA of $\boldsymbol{X}$

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PCA of $\boldsymbol{X}$ Eigenvectors of $\boldsymbol{X}'\boldsymbol{X}$ ($\frac{1}{N}$ doesn't affect eigenvectors)

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Kernel PCA

Center $\boldsymbol{K}$?
Use centering matrix $\boldsymbol{H}$

$$\begin{aligned} \boldsymbol{H} &= \boldsymbol{I}_N - \frac{(\mathbf{1}_N \mathbf{1}_N')}{N} \\ \boldsymbol{K}_{\text{center}} &= \boldsymbol{H}\boldsymbol{K}\boldsymbol{H} \end{aligned}$$

Spirling and Indian Treaties

Spirling (2013): model Treaties between US and Native Americans

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Spirling and Indian Treaties

How do we preserve word order and semantic language?

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After stemming, stopping, bag of wording:

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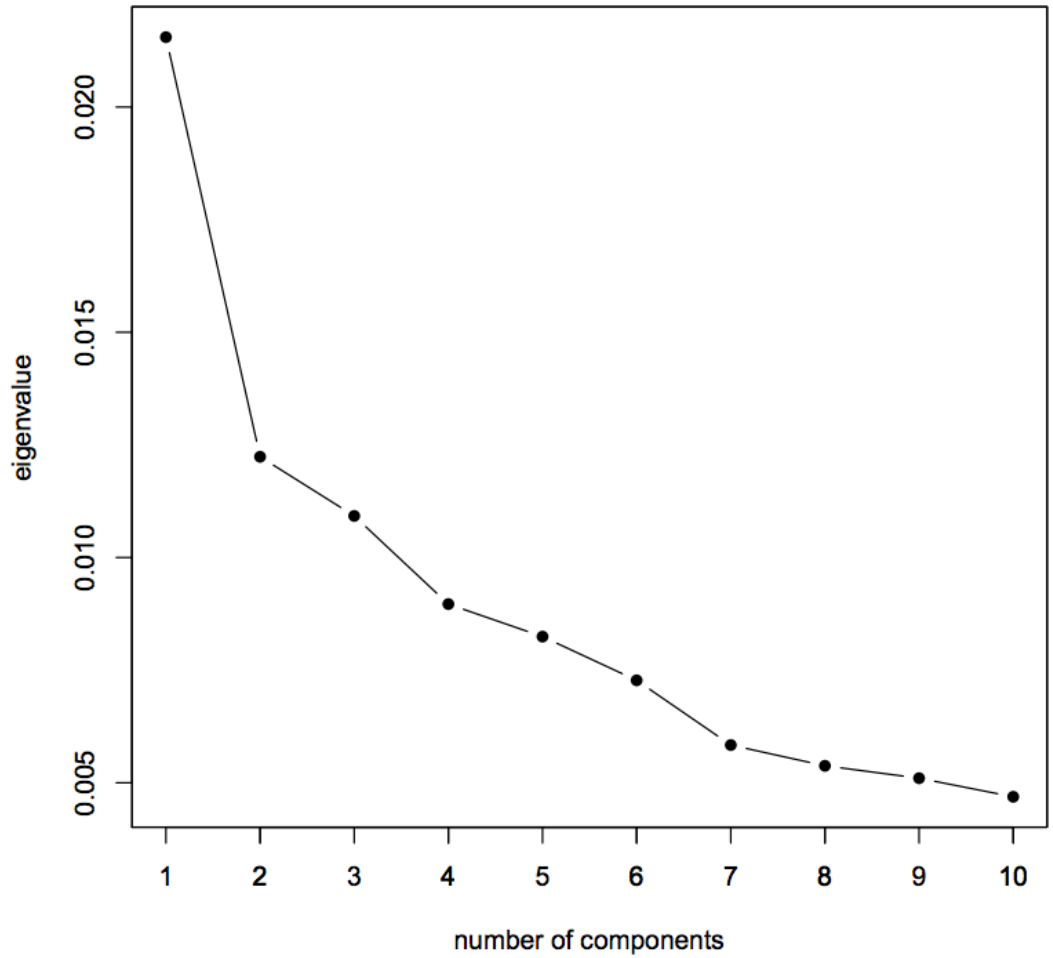
$\boldsymbol{\phi}(\boldsymbol{x}_i) \approx \binom{32}{5}$ element long count vector

Spirling and Indian Treaties



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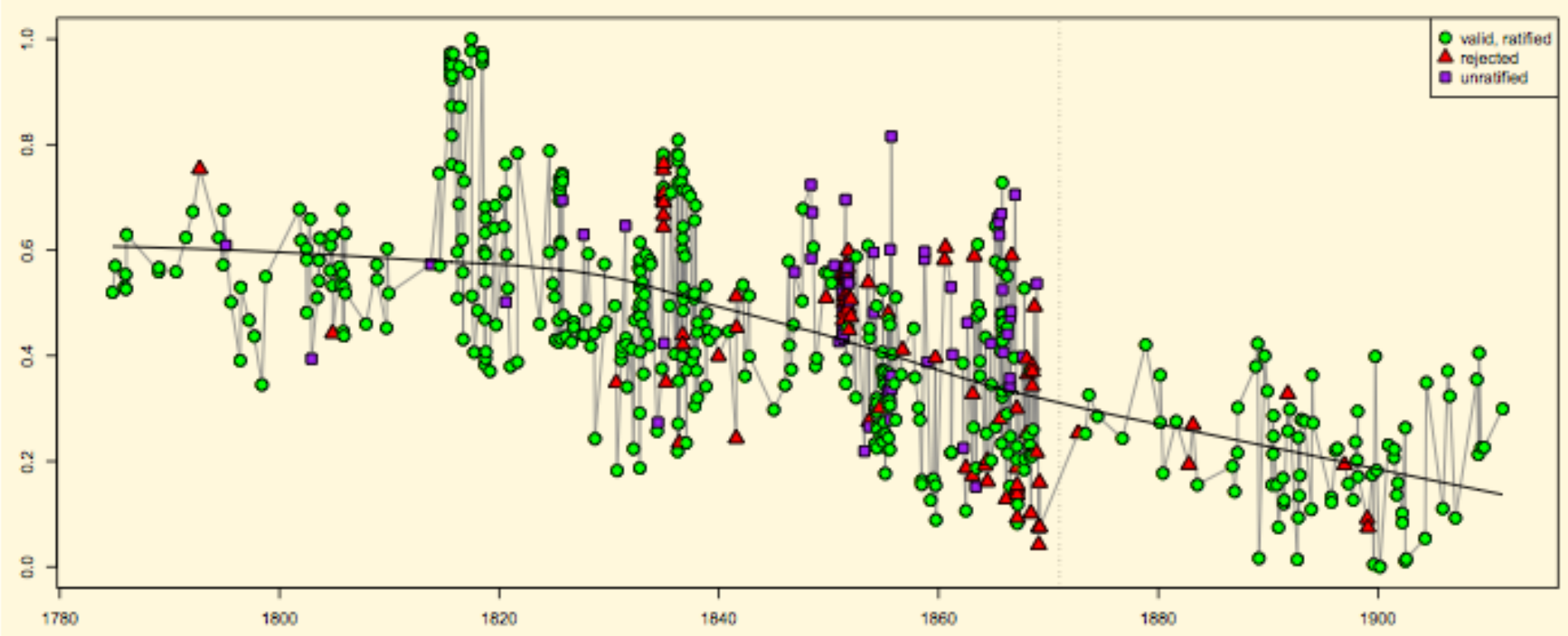

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Scaling$\rightsquigarrow$ placing actors in low-dimensional space (like principal components!)

Estimating Ideal Points: Roll Call Data and the US Congress

US Congress and Roll Call

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Healthy skepticism!

Our plan

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1) Wordscores$\rightsquigarrow$ "supervised" scaling

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2) Wordfish$\rightsquigarrow$ single dimension

Wordscores$\rightsquigarrow$ Big in Europe

Wordscores, Like the Hoff$\rightsquigarrow$ Big in Europe

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$\boldsymbol{x}_i \rightsquigarrow$ aggregation across documents.

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Wordscores: Optimization

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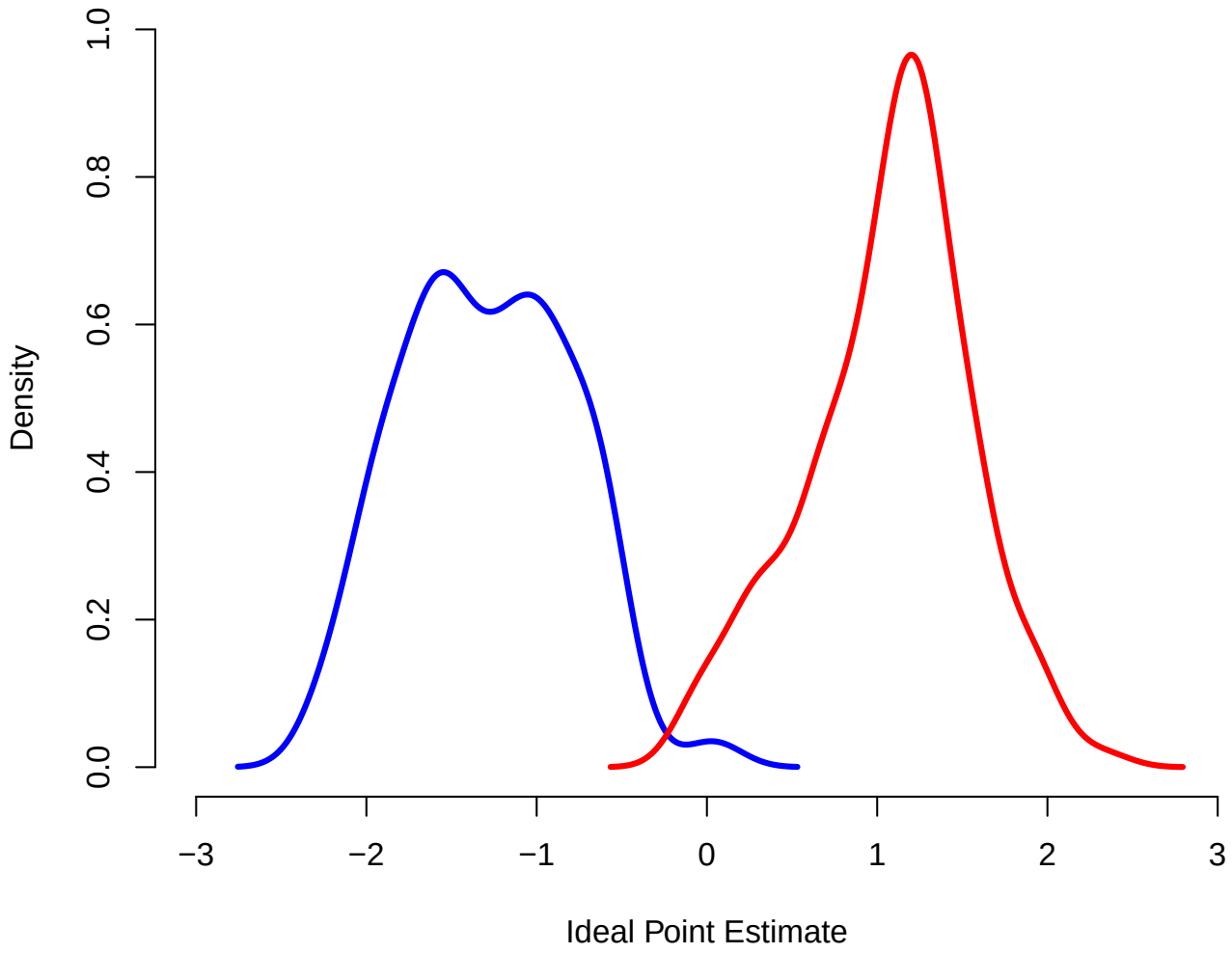
Applied to the Senate Press Releases

$L =$ Ted Kennedy
$C =$ Tom Coburn
Apply to other senators.

Applying to Senate Press Releases⇝ Gold Standard Scaling from NOMINATE



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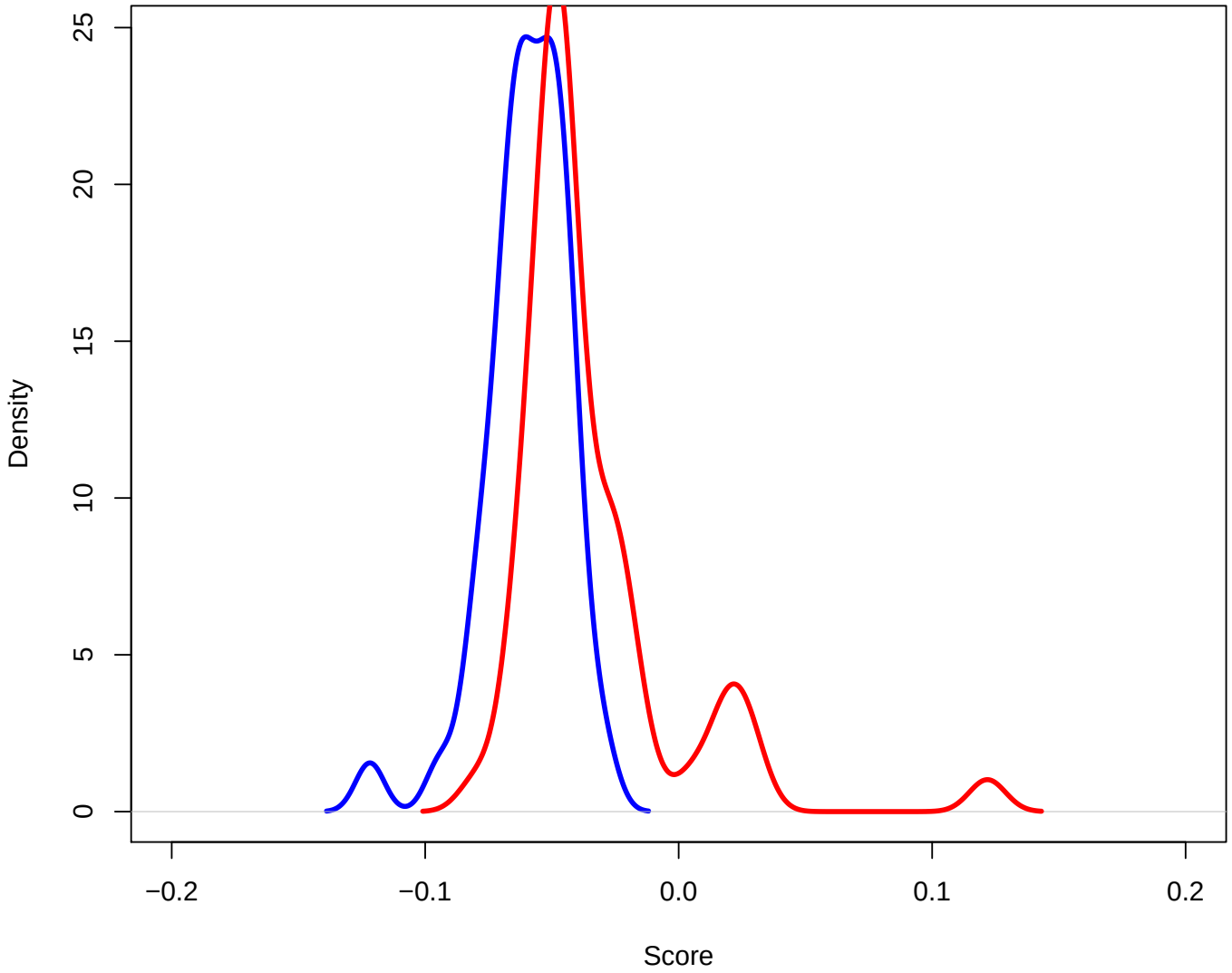

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- False prediction problem$\rightsquigarrow$ speech accomplishes many goals, only some of which are ideological

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To be fair: fast, nonparametric, and novel [trailblazing] method for scoring documents (starts conversation)

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Simplest model: Principal Components

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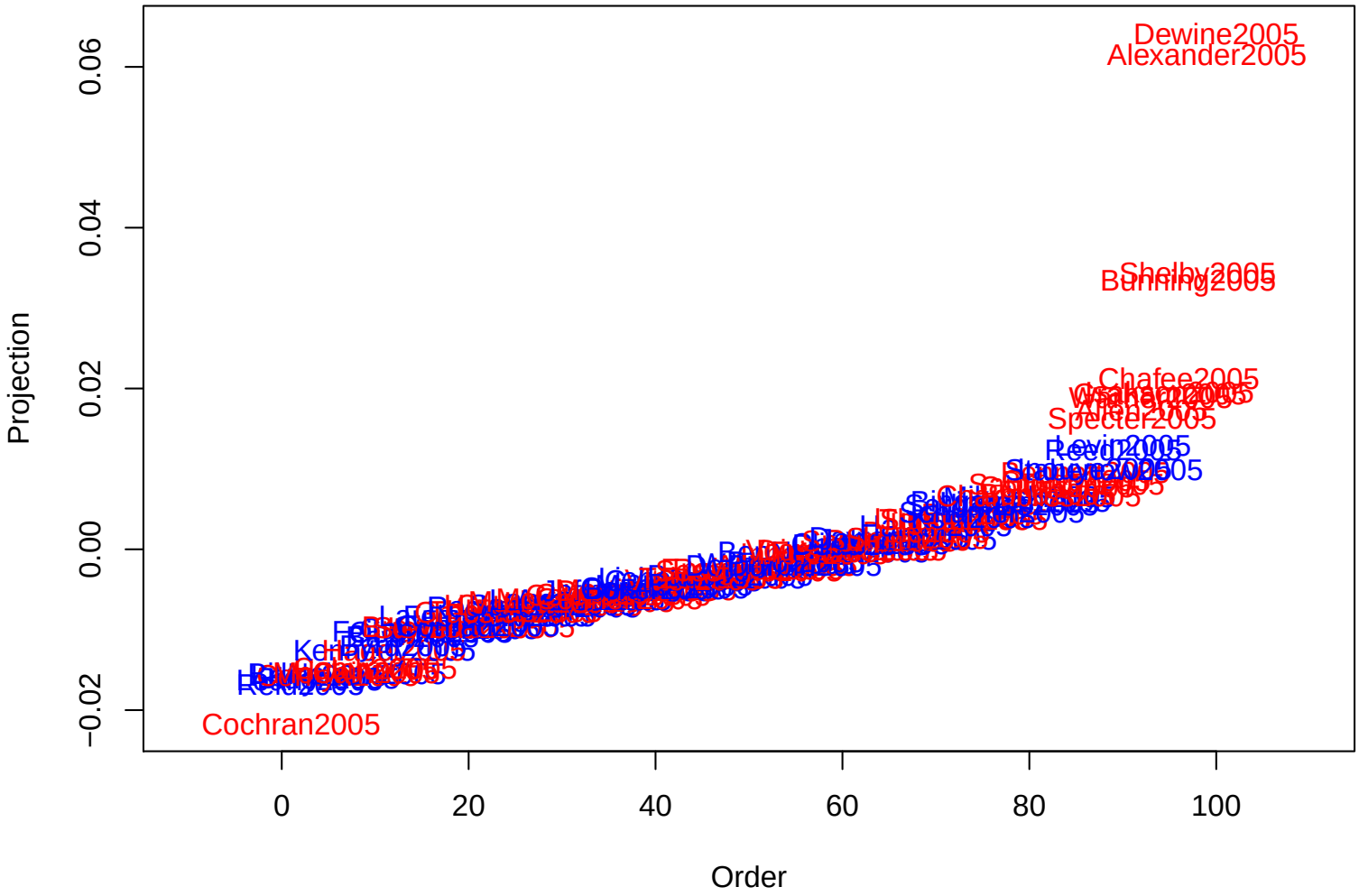
`prcomp(dtm)` applies principal components

Application of Principal Components in R



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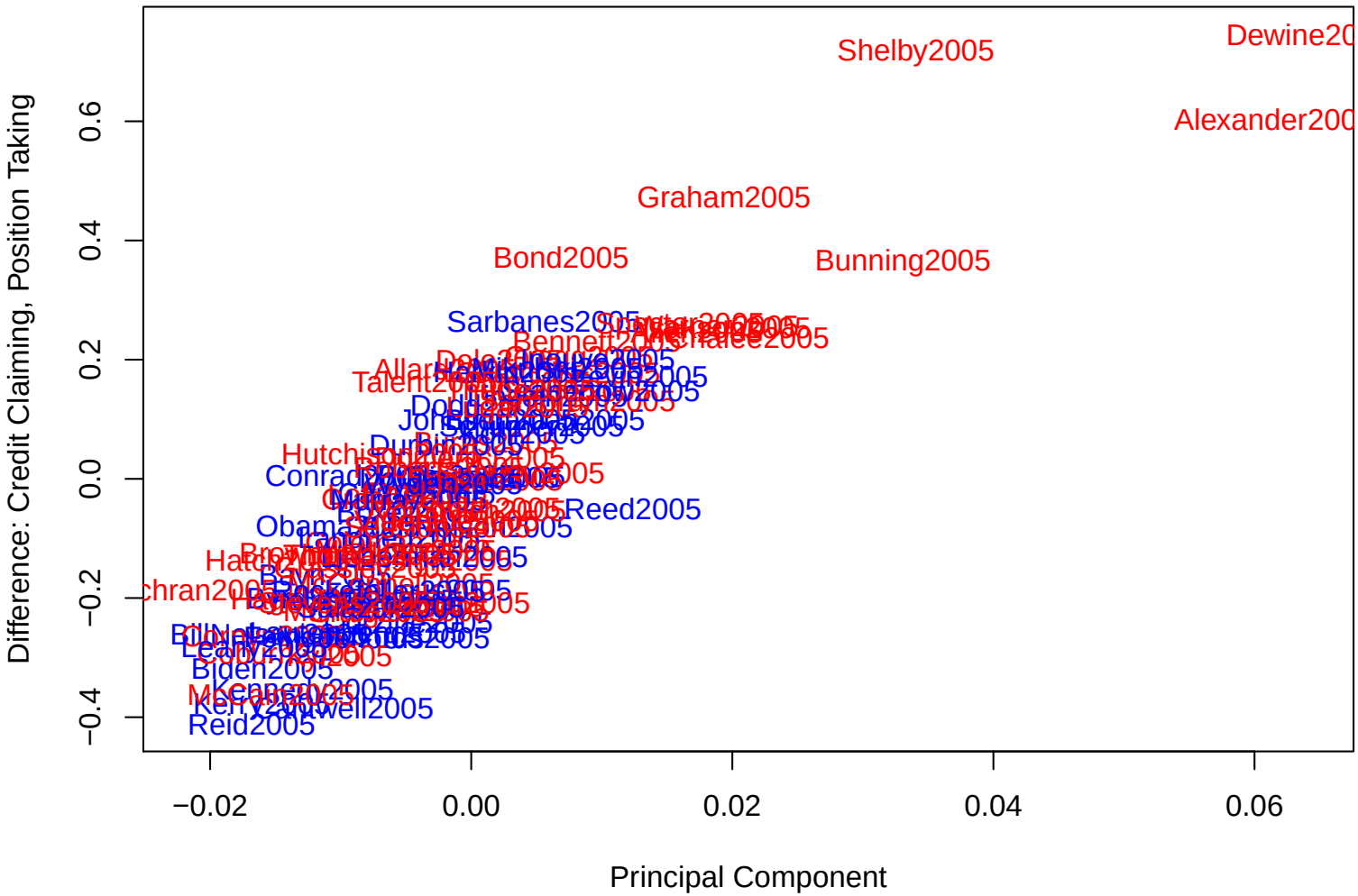

Probabilistic Unsupervised Embeddings

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Principal components is powerful$\rightsquigarrow$ statistical model for unsupervised scaling

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Item Response Theory (IRT)

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Monroe and Maeda (2005) and Slopkin and Proksch (2008) develop similar algorithms

WordFish Objective Function

Suppose we have legislator i.

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"Regression" of x_{ij} on ideal points θ_i, where we have to learn θ_i

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Implies the following posterior distribution:

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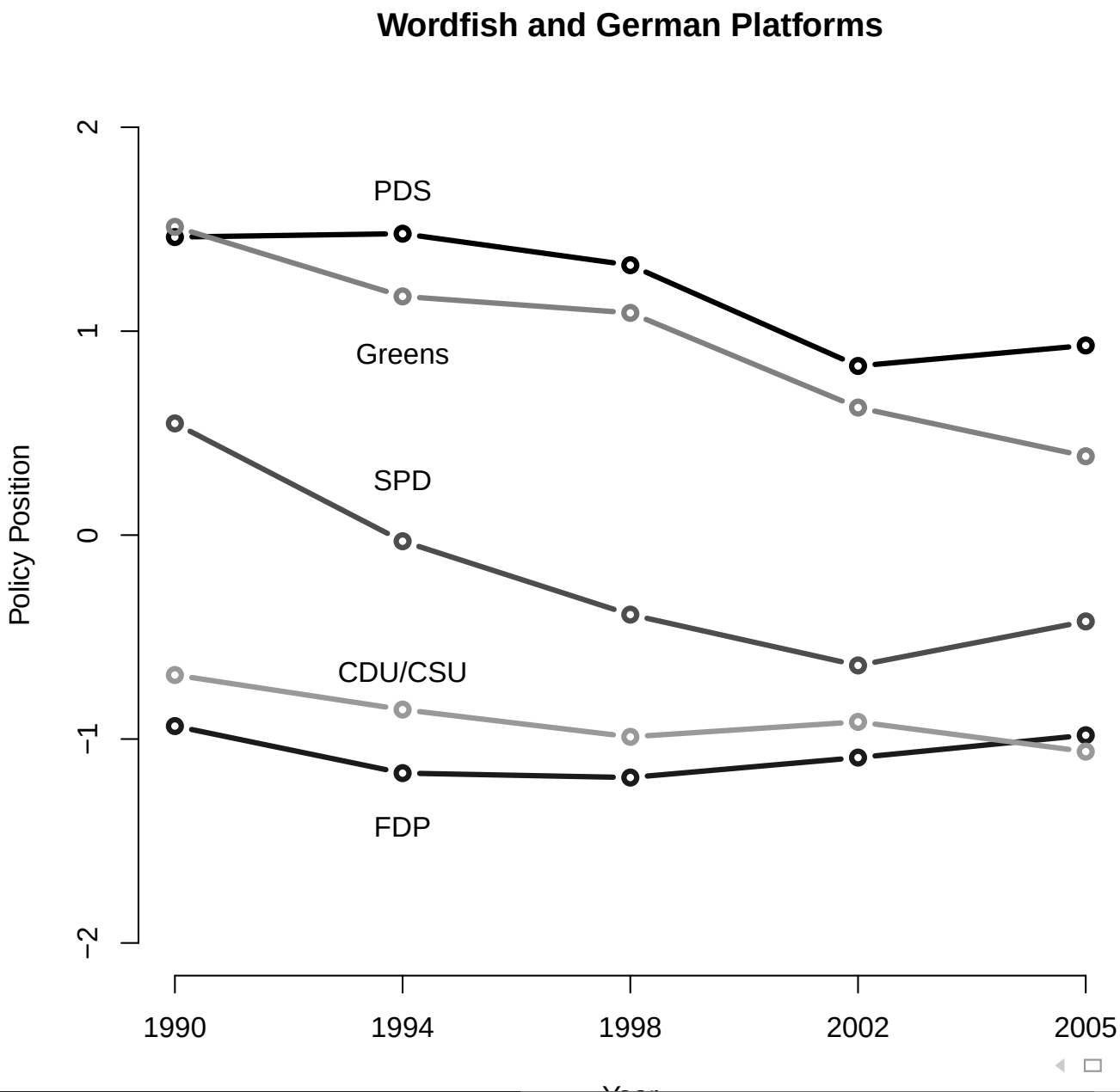
`Wordfish` package in `R`

Applications: German Party Manifestos



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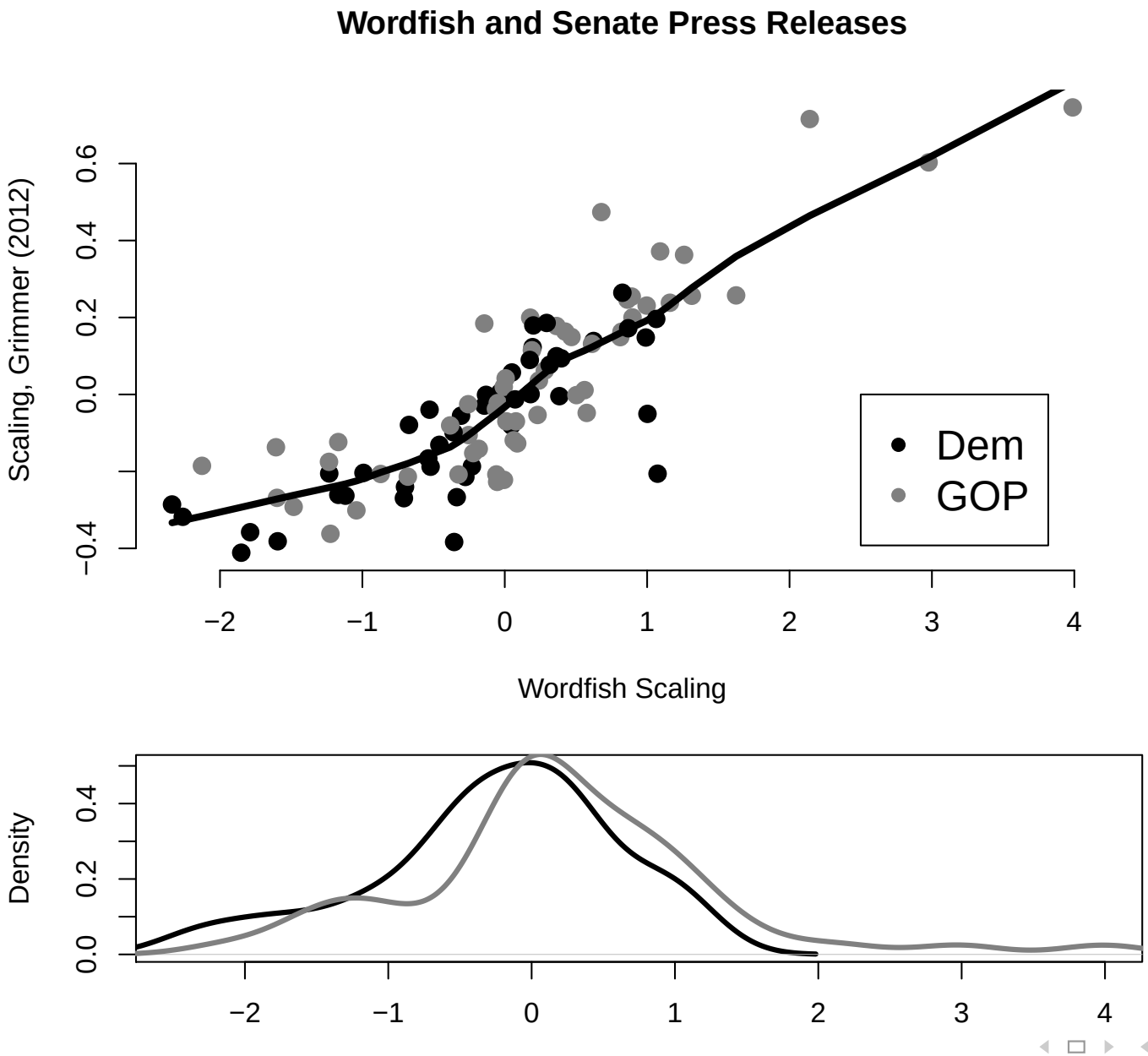

The Problem with Text-Based Scaling

What does validation mean?

1) Replicate NOMINATE, DIME, or other gold standards?
2) Agreement with experts
3) Prediction of other behavior

Must answer this to make progress on pure text scaling