Association Rule Learning and its exercises



A.I.

Big Data

Images

Videos

Audios

Texts

IoT

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關聯規則學習

Association Rule Learning

概念

- 在大型資料庫中發現項目間關聯的方法。
 - {牛奶, 麵包}→{可樂}:代表某人同時買了牛奶和 麵包,就可能會買可樂。
- 該方法常使用於電子商務上,通常可為促銷、 產品推薦等行銷活動的決策依據。



定義

- 商品的項目集合(itemset) · / = { /₁, /₂ ..., /_m} · #Item
- 交易資料庫(Database), $D = \{t_1, t_2, ..., t_n\}$ 。 #Transaction
- 關聯規則(Association Rule), $X \rightarrow Y$



案例

TID	網球拍	網 球	運動鞋	羽毛球
1	1	1	1	0
2	1	1	0	0
3	1	0	0	0
4	1	0	1	0
5	0	1	1	1
6	1	1	0	0

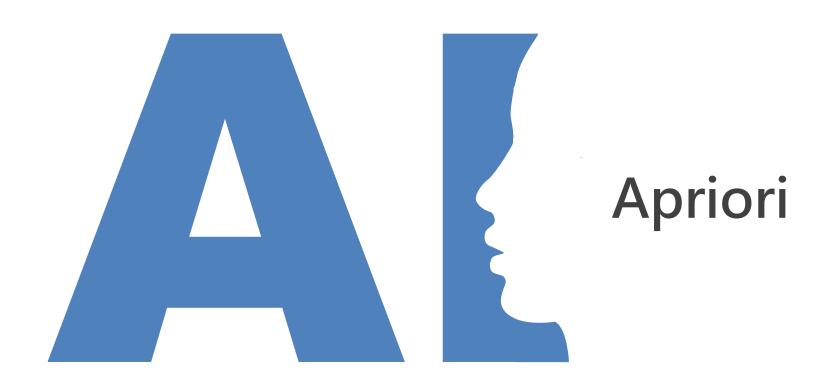
- 顧客購買記錄的資料庫 D,包含 6 個 Transactions
- 項目集 / = {網球拍, 網球, 運動鞋, 羽毛球}

觀察關聯規則,網球拍 → 網球。

- 1. Transaction 1, 2, 3, 4, 6 包含網球拍。
- 2. Transaction 1, 2, 6 同時包含網球拍和網球。
- 3. 支持度 = 3/6 = 0.5,信心度 = 3/5 = 0.6。
- 若最小支持度為 0.5,最小信心度為 0.6。
- 關聯規則"網球拍→網球" 是存在強關聯的。

- 1-itemset (4): {網球拍}, {網球}, {運動鞋}, {羽毛球}
- 2-itemset (6): {網球拍, 網球}, {網球拍, 運動鞋}, {網球拍, 羽毛球}, {網球,運動鞋},{網球,羽毛球},{運動鞋,羽毛球}
- 3-itemset (4): {網球拍, 網球, 運動鞋}, {網球拍, 網球, 羽毛球}, {網球拍, 運動鞋, 羽毛球} {網球,運動鞋,羽毛球}





概念

- 逐層搜索的迭代方法。
- *k*-itemset 用於探索(*k* + 1) itemset。
 - 1. 找出 frequent 1-itemset $L_1 \circ L_1$ 用來找 frequent 2-itemset $L_2 \circ L_2 \circ L_3$ 而 L_2 用來找到 L_3 。直到不能找到 k-itemset。
 - 2. 每找一個 L_k 需要掃描一次資料庫。為提高頻繁項集逐層產生的效率, Apriori 性質則可減少搜索。
- Apriori 性質: frequent itemset 的所有非空子集都必須是頻繁的。
 - 若某個 *k*-itemset 的 candidate 的 subsets 不在 (*k*-1)-itemset 時, 這個 candidate 就可以直接删除。



案例

TID	網球拍	網 球	運動鞋	羽毛球
1	1	1	1	0
2	1	1	0	0
3	1	0	0	0
4	1	0	1	0
5	0	1	1	1
6	1	1	0	0

- 顧客購買記錄的資料庫 D,包含 6 個 Transactions
- 項目集 /= {網球拍,網球,運動鞋,羽毛球}

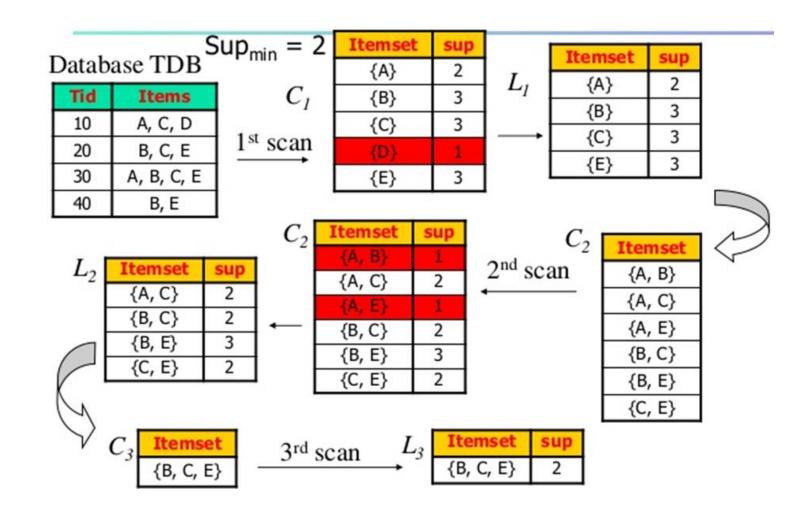
觀察關聯規則,網球拍 → 網球。

- 1. Transaction 1, 2, 3, 4, 6 包含網球拍。
- 2. Transaction 1, 2, 6 同時包含網球拍和網球。
- 3. 支持度 = 3/6 = 0.5,信心度 = 3/5 = 0.6。
- 若最小支持度為 0.5,最小信心度為 0.6。
- 關聯規則"網球拍→網球"是存在強關聯的。

- 1-itemset (4): {網球拍}, {網球}, {運動鞋}, {羽毛球}
- 2-itemset (7): {網球拍, 網球}, {網球拍, 運動鞋}, {網球拍, 羽毛球}, {網球,運動鞋}, {網球,羽毛球}, {運動鞋,羽毛球}
- 3-itemset (4): {網球拍, 網球, 運動鞋}, {網球拍, 網球, 羽毛球}, {網球拍, 運動鞋, 羽毛球} {網球,運動鞋,羽毛球}



方法



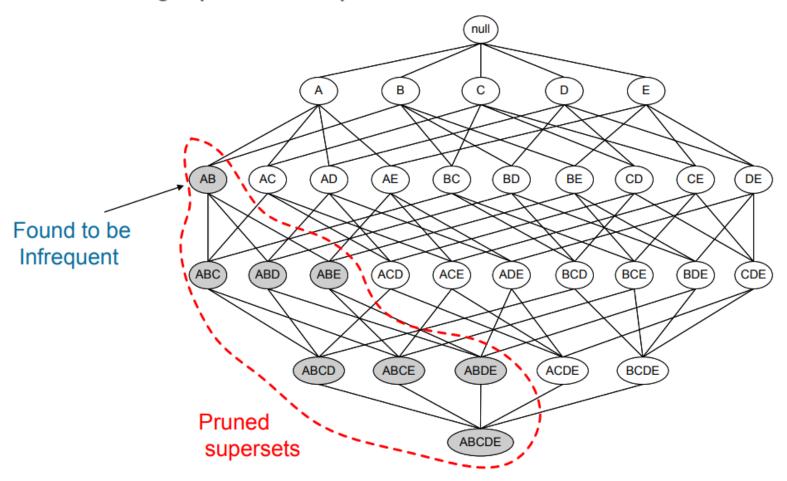
方法

- $1. C_3 = L_2$ 的組合
 - $L_2 = \{\{A, C\}, \{B, C\}, \{B, E\}, \{C, E\}\}\}$ {{A, C}, {B, C}, {B, E}, { C, E}} $= \{\{A, B, C\}, \{A, C, E\}, \{B, C, E\}\}$
- 2. 使用 Apriori 性質剪枝:某個 frequent itemset 的所有 subsets 必須是頻繁的, 對 candidate itemset C_3 ,我們可以刪除其非頻繁的 subsets :
 - {A, B, C} 的 2-itemset 是 {A, B}, {A, C}, {B, C},其中 {A, B} 不是 L₂的元素,所以刪除;
 - {A, C, E} 的 2-itemset 是 {A, C}, {A, E}, {C, E},其中 {A, E} 不是 ∠₂的元素,所以刪除;
 - {B, C, E} 的 2-itemset 是 {B, C}, {B, E}, {C, E}, 所有 2-itemset 都是 L₂的元素,因此保留。
- 3. 剪枝後得到 *C*₃ = {{B, C, E}}



剪枝

Illustrating Apriori Principle







Thinking Time

重點

- 1. 在每一步產生 candidate itemset 時產生的組合過 多,沒有排除不應該參與組合的元素。
- 2. 每 次 計 算 itemset 的 支 持 度 時 都 對 全 部 的 transactions 掃描一遍,造成龐大的I / O開銷。這 種代價是隨著資料的增加而產生幾何級數的增長。





FP-Growth

概念

- 不用產牛 candidate itemsets。
- 以樹(Tree)的結構儲存 frequent itemsets, 即 frequent pattern tree (FP-tree)。
- 只要遞迴地探勘這棵樹。



FP-tree 建造方法

TID	Items bought
100	{a, c, d, f, g, i, m, p}
200	{a, b, c, f, i, m, o}
300	{b, f, h, j, o}
400	{b, c, k, s, p}
500	{a, c, e, f, l, m, n, p}

 $min_support = 3$

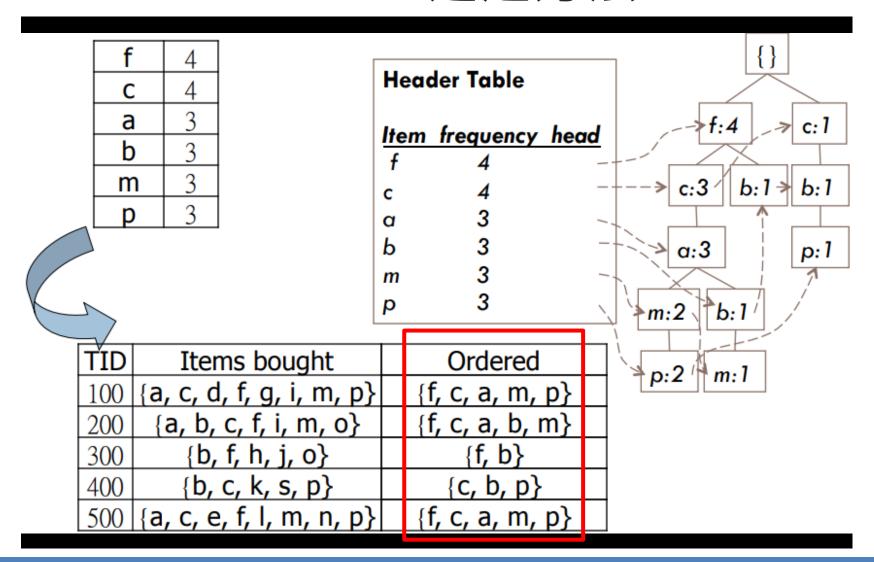


a	3
a b	3
С	4
f	4
m	3
р	3

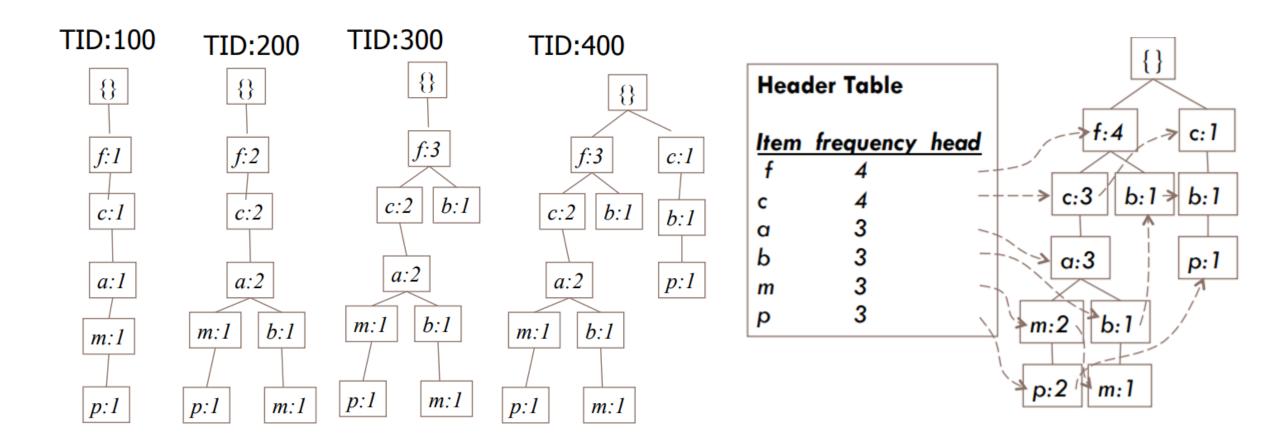


f	4
С	4
a	3
b	3
m	3
р	3

FP-tree 建造方法



FP-tree 建造方法



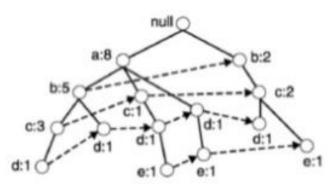


Mining Tree

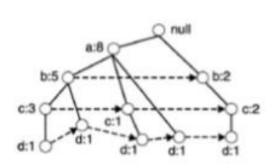
- Bottom-Up 探索,依序檢視每個項目。
- 遞迴建子樹,找到所有 k-itemsets。



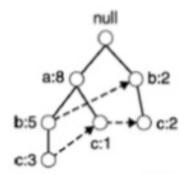
Mining Tree



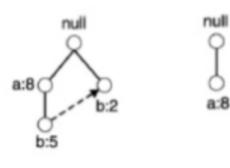
FP-tree



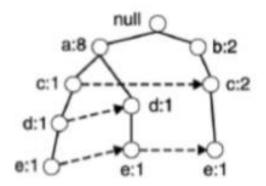
包含d節點 的子樹



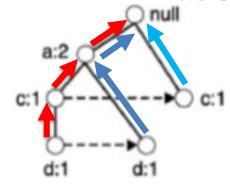
包含c節點 的子樹



包含b節點 包含a節點 的子樹 的子樹



包含e節點 的子樹



{e}的條件FP-tree If *Sup{b} < 2*





Thinking Time

重點

- · 避免多次掃描資料庫(for support), 節省了IO與運算成本。
- 不產生 candidate itemset。



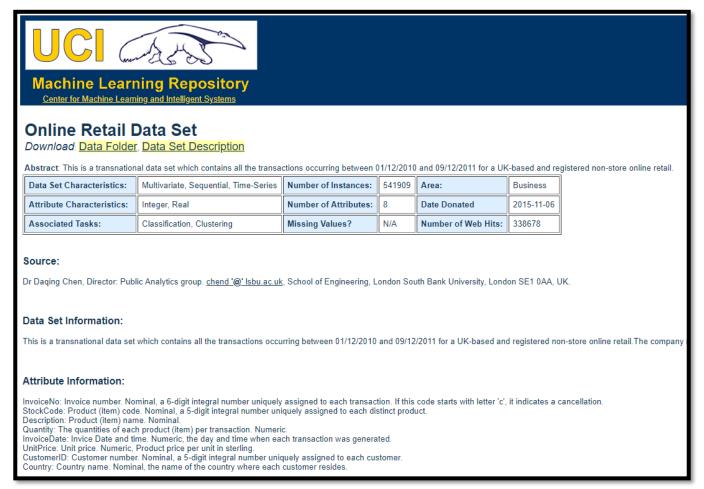
Reference

- https://zh.wikipedia.org/wiki/%E5%85%B3% E8%81%94%E8%A7%84%E5%88%99%E5%A D%A6%E4%B9%A0
- https://www.slideshare.net/waynechung944/ fp-growth-intro





Data Description



https://archive.ics.uci.edu/ml/datasets/Online%20Retail



Data Description

InvoiceNo	StockCode Description	Quantity Invo	iceDate UnitPri	ice CustomerID	Country
536365	85123A WHITE HANGING HEART T-LIGHT HOLDER	6	2010/12/1 08:26	2.55 17850	United Kingdom
536365	71053 WHITE METAL LANTERN	6	2010/12/1 08:26	3.39 17850	United Kingdom
536365	84406B CREAM CUPID HEARTS COAT HANGER	8	2010/12/1 08:26	2.75 17850	United Kingdom
536365	84029G KNITTED UNION FLAG HOT WATER BOTTLE	6	2010/12/1 08:26	3.39 17850	United Kingdom
536365	84029E RED WOOLLY HOTTIE WHITE HEART.	6	2010/12/1 08:26	3.39 17850	United Kingdom
536365	22752 SET 7 BABUSHKA NESTING BOXES	2	2010/12/1 08:26	7.65 17850	United Kingdom
536365	21730 GLASS STAR FROSTED T-LIGHT HOLDER	6	2010/12/1 08:26	4.25 17850	United Kingdom
536366	22633 HAND WARMER UNION JACK	6	2010/12/1 08:28	1.85 17850	United Kingdom
536366	22632 HAND WARMER RED POLKA DOT	6	2010/12/1 08:28	1.85 17850	United Kingdom
536367	84879 ASSORTED COLOUR BIRD ORNAMENT	32	2010/12/1 08:34	1.69 13047	United Kingdom
536367	22745 POPPY'S PLAYHOUSE BEDROOM	6	2010/12/1 08:34	2.1 13047	United Kingdom
536367	22748 POPPY'S PLAYHOUSE KITCHEN	6	2010/12/1 08:34	2.1 13047	United Kingdom
536367	22749 FELTCRAFT PRINCESS CHARLOTTE DOLL	8	2010/12/1 08:34	3.75 13047	United Kingdom
536367	22310 IVORY KNITTED MUG COSY	6	2010/12/1 08:34	1.65 13047	United Kingdom
536367	84969 BOX OF 6 ASSORTED COLOUR TEASPOONS	6	2010/12/1 08:34	4.25 13047	United Kingdom
536367	22623 BOX OF VINTAGE JIGSAW BLOCKS	3	2010/12/1 08:34	4.95 13047	United Kingdom
536367	22622 BOX OF VINTAGE ALPHABET BLOCKS	2	2010/12/1 08:34	9.95 13047	United Kingdom
536367	21754 HOME BUILDING BLOCK WORD	3	2010/12/1 08:34	5.95 13047	United Kingdom
536367	21755 LOVE BUILDING BLOCK WORD	3	2010/12/1 08:34	5.95 13047	United Kingdom
536367	21777 RECIPE BOX WITH METAL HEART	4	2010/12/1 08:34	7.95 13047	United Kingdom
536367	48187 DOORMAT NEW ENGLAND	4	2010/12/1 08:34	7.95 13047	United Kingdom
536368	22960 JAM MAKING SET WITH JARS	6	2010/12/1 08:34	4.25 13047	United Kingdom
536368	22913 RED COAT RACK PARIS FASHION	3	2010/12/1 08:34	4.95 13047	United Kingdom
536368	22912 YELLOW COAT RACK PARIS FASHION	3	2010/12/1 08:34	4.95 13047	United Kingdom
536368	22914 BLUE COAT RACK PARIS FASHION	3	2010/12/1 08:34	4.95 13047	United Kingdom
536369	21756 BATH BUILDING BLOCK WORD	3	2010/12/1 08:35	5.95 13047	United Kingdom
536370	22728 ALARM CLOCK BAKELIKE PINK	24	2010/12/1 08:45	3.75 12583	France
536370	22727 ALARM CLOCK BAKELIKE RED	24	2010/12/1 08:45	3.75 12583	France
536370	22726 ALARM CLOCK BAKELIKE GREEN	12	2010/12/1 08:45	3.75 12583	France
536370	21724 PANDA AND BUNNIES STICKER SHEET	12	2010/12/1 08:45	0.85 12583	France
536370	21883 STARS GIFT TAPE	24	2010/12/1 08:45	0.65 12583	France
536370	10002 INFLATABLE POLITICAL GLOBE	48	2010/12/1 08:45	0.85 12583	France
536370	21791 VINTAGE HEADS AND TAILS CARD GAME	24	2010/12/1 08:45	1.25 12583	France

https://archive.ics.uci.edu/ml/datasets/Online%20Retail



Step 1 - Install mlxtend

pip install mlxtend xlrd

```
C:\Users\user>pip install mlxtend
Collecting mlxtend
 Downloading https://files.pythonhosted.org/packages/c0/ca/54fe0ae783ce81a467710d1c5fb41cfca0751
/mlxtend-0.16.0-py2.py3-none-any.whl (1.3MB)
Requirement already satisfied: scipy>=0.17 in c:\users\user\appdata\local\programs\python\python3
om mlxtend) (1.0.1)
Requirement already satisfied: setuptools in c:\users\user\appdata\local\programs\python\python36
m mlxtend) (39.0.1)
Requirement already satisfied: scikit-learn>=0.18 in c:\users\user\appdata\local\programs\python\
ges (from mlxtend) (0.19.1)
Requirement already satisfied: numpy>=1.10.4 in c:\users\user\appdata\local\programs\python\pytho
from mlxtend) (1.14.2)
Requirement already satisfied: matplotlib>=1.5.1 in c:\users\user\appdata\local\programs\python\p
es (from mlxtend) (2.2.2)
Requirement already satisfied: pandas>=0.17.1 in c:\users\user\appdata\local\programs\python\pyth
(from mlxtend) (0.22.0)
Requirement already satisfied: python-dateutil>=2.1 in c:\users\user\appdata\local\programs\pytho
kages (from matplotlib>=1.5.1->mlxtend) (2.6.1)
Requirement already satisfied: six>=1.10 in c:\users\user\appdata\local\programs\python\python36\
matplotlib>=1.5.1->mlxtend) (1.11.0)
Requirement already satisfied: cycler>=0.10 in c:\users\user\appdata\local\programs\python\python
rom matplotlib>=1.5.1->mlxtend) (0.10.0)
Requirement already satisfied: pyparsing!=2.0.4,!=2.1.2,!=2.1.6,>=2.0.1 in c:\users\user\appdata\
ython36\lib\site-packages (from matplotlib>=1.5.1->mlxtend) (2.2.0)
Requirement already satisfied: pytz in c:\users\user\appdata\local\programs\python\python36\lib\s
```



Step 2 – Import Libs & Data Preprocessing

```
import pandas as pd
    from mlxtend.frequent_patterns import apriori
    from mlxtend.frequent patterns import association rules
 4
    df=pd.read_excel('Online Retail.xlsx')
    df.head()
    df['Description'] = df['Description'].str.strip()
    df.dropna(axis=0, subset=['InvoiceNo'], inplace=True)
    df['InvoiceNo'] = df['InvoiceNo'].astype('str')
    df = df[~df['InvoiceNo'].str.contains('C')]
12
13
    basket = (df[df['Country'] =="France"]
        .groupby(['InvoiceNo', 'Description'])['Quantity']
14
        .sum().unstack().reset_index().fillna(0)
15
        .set index('InvoiceNo'))
16
```

Step 2 – Import Libs & Data Preprocessing

Description	10 COLOUR SPACEBOY PEN	12 COLOURED PARTY BALLOONS	12 EGG HOUSE PAINTED WOOD	12 MESSAGE CARDS WITH ENVELOPES	12 PENCIL SMALL TUBE WOODLAND	12 PENCILS SMALL TUBE RED RETROSPOT	12 PENCILS SMALL TUBE SKULL	12 PENCILS TALL TUBE POSY	12 PENCILS TALL TUBE RED RETROSPOT
InvoiceNo									
536370	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
536852	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
536974	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
537065	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
537463	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
537468	24.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
537693	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
537897	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
537967	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
538008	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
538093	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
538196	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
539050	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0
539113	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
539407	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
539435	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
539551	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
539607	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
539688	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
539727	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0



Step 3 – Data Preprocessing & **Association Rule Learning**

```
def encode_units(x):
        if x \le 0:
19
            return 0
20
       if x >= 1:
22
            return 1
23
    basket sets = basket.applymap(encode units)
24
    basket sets.drop('POSTAGE', inplace=True, axis=1)
26
27
    print(type(basket_sets))
28
29
    frequent_itemsets = apriori(basket_sets, min_support=0.07, use_colnames=True)
30
31
32
    rules = association_rules(frequent_itemsets, metric="lift", min_threshold=1)
33
    rules.head()
34
    print(rules[ (rules['lift'] >= 6) & (rules['confidence'] >= 0.8) ])
```

Step 3 – Results

	antecedents	ΔY	consequents	antecedent support	\consequent support	support	confidence	HIII	reverage	CONVICTION
2	(ALARM CLOCK BAKELIKE GREEN)	47	(ALARM CLOCK BAKELIKÉ RED)	0.096939	0.094388	0.079082	0.815789	8.642959	0.069932	4.916181
3	(ALARM CLOCK BAKELIKE RED)	47	(ALARM CLOCK BAKELIKE GREEN)			0.079082	0.837838	8.642959	0.069932	5.568878
16	(SET/6 RED SPOTTY PAPER PLATES)	47	(SET/20 ŘED RETROSPOT PAPER NAPKINS)	0.127551	0.132653	0.102041	0.800000	6.030769	0.085121	4.336735
18	(SET/6 RED SPOTTY PAPER PLATES)		(SET/6 RED SPOTTY PAPER CUPS)	0.127551	0.137755	0.122449	0.960000	6.968889	0.104878	21.556122
19	(SET/6 RED SPOTTY PAPER CUPS)	47	(SÉT/6 RED SPOTTY PAPER PLATES)	0.137755	0.127551	0.122449	0.888889	6.968889	0.104878	7.852041
20	(SET/6 RED SPOTTY PAPER PLATES, SET/6 RED SPOT		(SET/20 RED RETROSPOT PAPER NAPKINS)	0.122449	0.132653	0.099490	0.812500	6.125000	0.083247	4.625850
21	(SET/6 RED SPOTTY PAPER PLATES, SET/20 RED RET		(SET/6 RED SPOTTY PAPER CUPS)	0.102041	0.137755	0.099490	0.975000	7.077778	0.085433	34.489796
22	(SET/6 RED SPOTTY PAPER CUPS, SET/20 RED RETRO		(SÉT/6 RED SPOTTY PAPER PLATES)	0.102041	0.127551	0.099490	0.975000	7.644000	0.086474	34.897959
				·						





Generating Association Rules from Frequent **Itemsets**

```
import pandas as pd
from mlxtend.preprocessing import TransactionEncoder
from mlxtend.frequent patterns import apriori
dataset = [['Milk', 'Onion', 'Nutmeg', 'Kidney Beans', 'Eggs', 'Yogurt'],
           ['Dill', 'Onion', 'Nutmeg', 'Kidney Beans', 'Eggs', 'Yogurt'],
           ['Milk', 'Apple', 'Kidney Beans', 'Eggs'],
           ['Milk', 'Unicorn', 'Corn', 'Kidney Beans', 'Yogurt'],
           ['Corn', 'Onion', 'Onion', 'Kidney Beans', 'Ice cream', 'Eggs']]
te = TransactionEncoder()
te ary = te.fit(dataset).transform(dataset)
df = pd.DataFrame(te_ary, columns=te.columns_)
frequent itemsets = apriori(df, min support=0.6, use colnames=True)
frequent itemsets
```



Generating Association Rules from Frequent **Itemsets**

	support	itemsets
0	0.8	(Eggs)
1	1.0	(Kidney Beans)
2	0.6	(Milk)
3	0.6	(Onion)
4	0.6	(Yogurt)
5	0.8	(Kidney Beans, Eggs)
6	0.6	(Onion, Eggs)
7	0.6	(Milk, Kidney Beans)
8	0.6	(Onion, Kidney Beans)
9	0.6	(Kidney Beans, Yogurt)
10	0.6	(Onion, Kidney Beans, Eggs)



Generating Association Rules from Frequent **Itemsets**

from mlxtend.frequent_patterns import association_rules association_rules(frequent_itemsets, metric="confidence", min_threshold=0.7)

	antecedents	consequents	antecedent support	consequent support	support	confidence	lift	leverage	conviction
0	(Kidney Beans)	(Eggs)	1.0	0.8	0.8	0.80	1.00	0.00	1.000000
1	(Eggs)	(Kidney Beans)	0.8	1.0	0.8	1.00	1.00	0.00	inf
2	(Onion)	(Eggs)	0.6	0.8	0.6	1.00	1.25	0.12	inf
3	(Eggs)	(Onion)	0.8	0.6	0.6	0.75	1.25	0.12	1.600000
4	(Milk)	(Kidney Beans)	0.6	1.0	0.6	1.00	1.00	0.00	inf
5	(Onion)	(Kidney Beans)	0.6	1.0	0.6	1.00	1.00	0.00	inf
6	(Yogurt)	(Kidney Beans)	0.6	1.0	0.6	1.00	1.00	0.00	inf
7	(Onion, Kidney Beans)	(Eggs)	0.6	0.8	0.6	1.00	1.25	0.12	inf
8	(Onion, Eggs)	(Kidney Beans)	0.6	1.0	0.6	1.00	1.00	0.00	inf
9	(Kidney Beans, Eggs)	(Onion)	0.8	0.6	0.6	0.75	1.25	0.12	1.600000
10	(Onion)	(Kidney Beans, Eggs)	0.6	0.8	0.6	1.00	1.25	0.12	inf
11	(Eggs)	(Onion, Kidney Beans)	0.8	0.6	0.6	0.75	1.25	0.12	1.600000



Other Tools

PyFIM

http://www.borgelt.net/pyfim.html

```
import fim
# 啤酒和尿布数据
tracts = [\
    ['牛奶','面包'],\
    ['面包','尿布','啤酒','鸡蛋'],\
    ['牛奶','尿布','啤酒','可乐'],\
    ['面包','牛奶','尿布','啤酒'],\
    ['面包','牛奶','尿布','可乐'],\
# 关联分析,设置支持度至少 60%,自信度至少 80%
r = fim.fpgrowth(tracts, zmin=2, supp=60, conf=80, target='r')
print(r)
得到结果:
                        [('尿布', ('啤酒',), 3)]
```

Reference

- 1. http://rasbt.github.io/mlxtend/user_guide/ frequent_patterns/association_rules/
- 2. https://pbpython.com/market-basketanalysis.html
- 3. https://zhuanlan.zhihu.com/p/30600248



