## Contents

	Preface			
	Acknowledgements			
Pa	rt I	Fundamental machinery	1	
1	Intro	oduction	3	
	1.1	Differential equations	4	
	1.2	Partial differential equations	7	
	1.3	More examples	10	
	1.4	Importance of the forward model	17	
2			19	
	2.1	Background	19	
	2.2	Matrix and vector algebra	19	
	2.3	Simple statistics: regression	29	
	2.4	Least-squares	43	
	2.5	The singular vector expansion	69	
	2.6	Combined least-squares and adjoints	118	
	2.7	Minimum variance estimation and simultaneous equations	125	
	2.8	Improving recursively	136	
	2.9	Summary	143	
		Appendix 1. Maximum likelihood	145	
		Appendix 2. Differential operators and Green functions	146	
		Appendix 3. Recursive least-squares and Gauss-Markov solutions	148	
3	Extensions of methods		152	
	3.1	The general eigenvector/eigenvalue problem	152	
	3.2	Sampling	155	
	3.3	Inequality constraints: non-negative least-squares	164	
	3.4	Linear programming	166	
	3.5	Empirical orthogonal functions	169	
	3.6	Kriging and other variants of Gauss-Markov estimation	170	

viii Contents

	3.7	Non-linear problems	171
4	The time-dependent inverse problem: state estimation		
	4.1	Background	178
	4.2	Basic ideas and notation	180
	4.3	Estimation	192
	4.4	Control and estimation problems	214
	4.5	Duality and simplification: the steady-state filter and adjoint	229
	4.6	Controllability and observability	232
	4.7	Non-linear models	234
	4.8	Forward models	248
	4.9	A summary	250
		Appendix. Automatic differentiation and adjoints	250
5	Time-dependent methods – 2		
	5.1	Monte Carlo/ensemble methods	256
	5.2	Numerical engineering: the search for practicality	260
	5.3	Uncertainty in Lagrange multiplier method	269
	5.4	Non-normal systems	270
	5.5	Adaptive problems	273
		Appendix. Doubling	274
Pa	rt II	Applications	277
6	Applications to steady problems		
	6.1	Steady-state tracer distributions	280
	6.2	The steady ocean circulation inverse problem	282
	6.3	Property fluxes	309
	6.4	Application to real oceanographic problems	311
	6.5	Linear programming solutions	326
	6.6	The $\beta$ -spiral and variant methods	328
	6.7	Alleged failure of inverse methods	331
	6.8	Applications of empirical orthogonal functions (EOFs)	
		(singular vectors)	333
	6.9	Non-linear problems	335
7	Applications to time-dependent fluid problems		
	7.1	Time-dependent tracers	341
	7.2	Global ocean states by Lagrange multiplier methods	342
	7.3	Global ocean states by sequential methods	351
	7.4	Miscellaneous approximations and applications	354
	7.5	Meteorological applications	356
	References		357
	Index		367
	Colour plates between pp. 182 and 183.		