SCALING OF EMPIRICAL COMPARTMENTAL DISTRIBUTIONS

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Sociophysics is a physical approach to modeling social phenomena. Yet at times there is a lack of comparison between theoretical models and empirical (observed) data [1, 2]. To enable comparison some authors explicitly include space (geography) in their models [1, 3].

This research analyzes changes in empirical compartmental distributions and spin-lattice systems by rescaling the data. Here we consider empirical data from United Kingdom 2011 census. Further comparison is done against Ising and noisy voter models.

Firstly, a randomized spatial model is proposed to assess the nature of observed distributions of empirical compartments (distinct territorial units). Model takes empirical compartments from the finest scale and combines them randomly to replicate compartments of a coarser scale. Some diversity measures (e.g., standard deviation) are chosen for which relative changes in their values are calculated. It is shown that results produced by the proposed model follow the inverse square root law. However relative changes in the diversity measures for the empirical data differ from the theory thus suggesting existence of spatial structures.

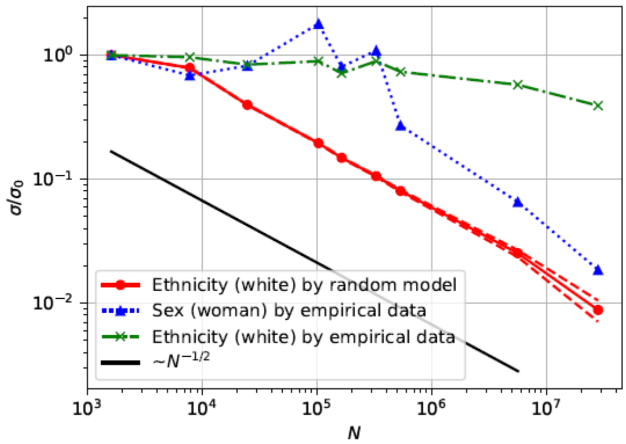

Figure 1. Relative changes of the standard deviation of chosen demographic indicators. Horizontal axis entails mean population in a compartment for the chosen scale.

Other considered diversity measures exhibit different power laws. For example, relative changes in Theil index (which is a case of generalized entropy) follow inverse linear law.

Building on the qualitative description of spatial structures a new measure is introduced for quantitative description. A numerical diversity index (I) is developed to describe certain configurations of spin-lattice models . These states are namely ferromagnetic, anti-ferromagnetic or random configuration. The latter is an instance of already proposed random model and is an intermediate between the other two.

Diversity index values are calculated for spin models at thermal equilibrium. Metropolis and Kawasaki dynamics are employed for simulation of Ising model as well as noisy voter model. Index values were evaluated for the empirical data to ascertain the demographics exhibiting recognizable spatial structures.

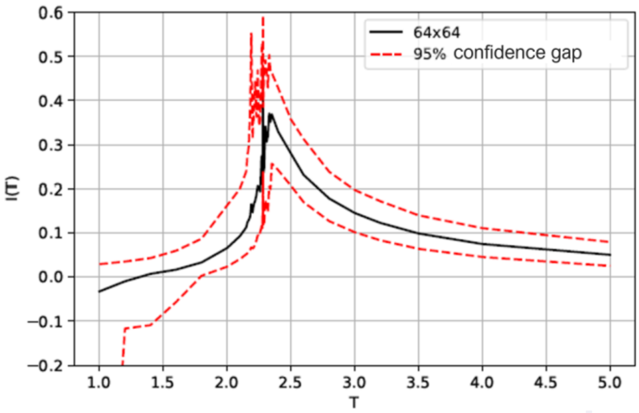

Figure 2. Diversity index values for Metropolis dynamics at equilibrium. Solid line shows index median value, dashed line shows 95% confidence gap. Critical temperature point is approximately at 2.269.

Proposed index I excelled at capturing spatial structure for Metropolis dynamics at critical temperature. However, it failed to capture polarization at low temperature values. Noisy voter model showed logarithmic dependence for values of index in terms of noise probability. For Kawasaki dynamics proposed index obtained biggest values at phase coexistence state. Results produced by Kawasaki dynamics were by far more comparable to the values for empirical data thus indicating better fit to model demographic phenomenon.

References

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