Parameter Choice for Microscopic Traffic Model to Reproduce Traffic Waves

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1 Synopsis

This report describes how the parameters of a combined follow-the-leader-optimal-velocity model (similar to the one given in [1]) are determined, so that the model reproduces the relevant characteristic features of traffic waves from an experiment [2]. The model and its parameters are implemented in the accompanying Matlab file `micro_model_human_driver_behavior_circular_road.m`. The microscopic model, which augmented an optimal velocity relaxation (as proposed by Bando et al. [4]) by a follow-the-leader term that prevents vehicles from colliding. The presence of both terms allows for parameter choices that render equilibrium flow states linearly unstable, yet vehicle trajectories remain well-defined for all times (exhibiting traffic waves). The ODE model reads as

$$\ddot{x}_j = b \cdot \frac{\dot{x}_{j+1} - \dot{x}_j}{(x_{j+1} - x_j)^n u} + a \cdot (V(x_{j+1} - x_j) - \dot{x}_j) \ ,$$

where

$$V(d) = V_{\mathrm{m}} \cdot \frac{\tanh(d/d_0 - 2) + \tanh(2)}{1 + \tanh(2)} \ .$$

This model is augmented by noise (described below in detail) that models human imperfections while driving. The following set of parameters has been determined to quantitatively reproduce the experimental trajectories well:

$$\nu = 2 \ , \ V_{\mathrm{m}} = 9.72 \text{ m/s} \ , \ d_0 = 2.23 \text{ m} \ , \ a = 0.5 \text{ /s} \ , \ b = 20 \text{ m}^\nu\text{/s} \ , \ \Delta t = 2 \text{ s} \ , \ \sigma = .25 \text{ m/s}^2 \ .$$

The vehicle trajectories of the experiment [2] are shown in Fig. 1. Vehicle trajectories produced by the model are shown in Fig. 2.

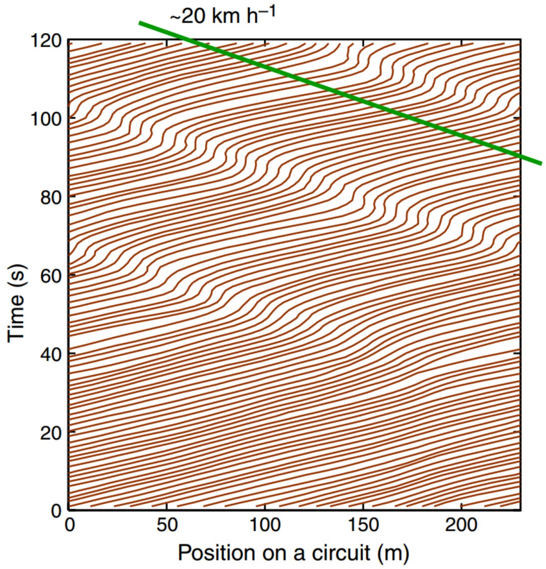

Figure 1: Trajectories of the experiment.

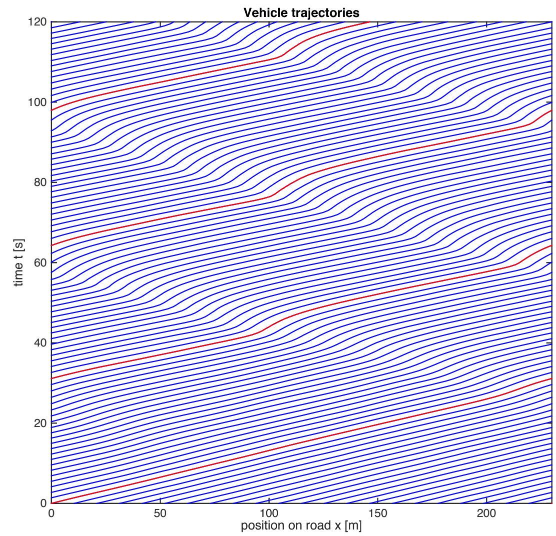

Figure 2: Trajectories of the model.

1.1 Variable Names

Table 1 shows the relevant variables, and their corresponding names in the Matlab file. The expressions $\sigma_{\Delta t}$ and $\dot{x}_j$ are discussed later in the text.

The users of the Matlab file is able to specify all relevant parameters of the system such as the number of vehicles, the road length, and the run time. The current values match those used in the experiment [2]. The code then uses Matlab's `ode45` to compute the temporal evolution of the state vector, while periodically adding noise to the vehicle velocities. This noise represents human driver imperfections. The results are plotted, and the minimum distance, minimum velocity, and maximum velocity are outputted.

Variable	ν	$\mathrm{V_m}$	$\mathrm{d_0}$	Δt	σ	$\sigma_{\Delta t}$	$\dot{x}_j$
Matlab name	`nu`	Vm	`d0`	`dt_noise`	`sigma`	`sigma_dt`	`x0(2:2:end)`

Table 1: Variable names with their corresponding Matlab names.

2 Determining the Relevant Parameters

2.1 Experimental Parameters

From a combination of the text, video, and plots of the results of the 2008 Sugiyama et al. experiment [2], parameters of the experiment were derived and used as a basis for the realistic driver model. These values are as follows:

$v_{\min} = 0.33$ m/s (smallest vehicle velocity)
$v_{\max} = 10$ m/s (largest vehicle velocity)
$s = -6.4$ m/s (propagation speed of traffic waves)
$\ell = 4.5$ m (average vehicle length)
Thickness of wave = 5 s
Time for wave to develop ≈ 40 s
Only a single wave develops.

The values of $v_{\min}$ and $v_{\max}$ were found by measuring the slopes of the trajectories in the plot produced by the experiment and taking an average. The minimum value was measured as the slope over the braking period, and the maximum value was found measuring the slope of the trajectory immediately after it came out of the wave. These values are in keeping with those reported in the text of the experiment as well as the visual data from the video. The reported maximum velocity is about 11.11 m/s, and the minimum velocity was unreported but close to 0 m/s as seen in the video. Note that the best agreement of the model with the experiment required to choose V_{m} below the desired maximum velocity reported in the Sugiyama et al. paper. This is not unrealistic, as the largest velocity observed in our model could be larger than V_{m} due to the noise (see below).

The speed of the traffic wave was measured directly from the plot in Fig. 1 and found to be 6.4 m/s. Although the value reported in the paper is 5.56 m/s, this is a rough estimate only, as can be seen by the line drawn on the plot to approximate wave speed. Wave speed was instead measured by finding the time between the bottom of each wave and dividing 230 m, the road length, by this value.

The average vehicle length, ℓ, was determined from the video of the experiment [3]. The value was found by measuring the amount of open space at a few different times in the video in terms of vehicle length. The road length, 230 m, was divided by this value added to 22, the total number of vehicles, the find vehicle length.

The thickness of the wave was found from the plot in Fig. 1, as was the property of only one wave developing (this is also seen in the video). Finally, the time to develop is extracted from plot as well.

2.2 Changes to the Model

The first and most fundamental study of the model was the investigation of the influence of the power ν. It was founds that $\nu = 1$ leads to vehicle distances that are far too small, in particular in the presence of traffic waves and noise. Values for ν of 1, 2, 3, and 4 were tested. Higher values of ν render the follow-the-leader term less important for large vehicle distances, but significantly more important as vehicles get close. Thus, larger values of ν produce strong braking, thus preventing very small vehicle distances from arising. In the study, first stability plots for each power magnitude were generated. Plots for $\nu = 1$ and $\nu = 2$ can be found below:

Values of a and b were selected from similar areas on each plot, and the eigenvalues computed to ensure there were the same number of positive eigenvalues for each power

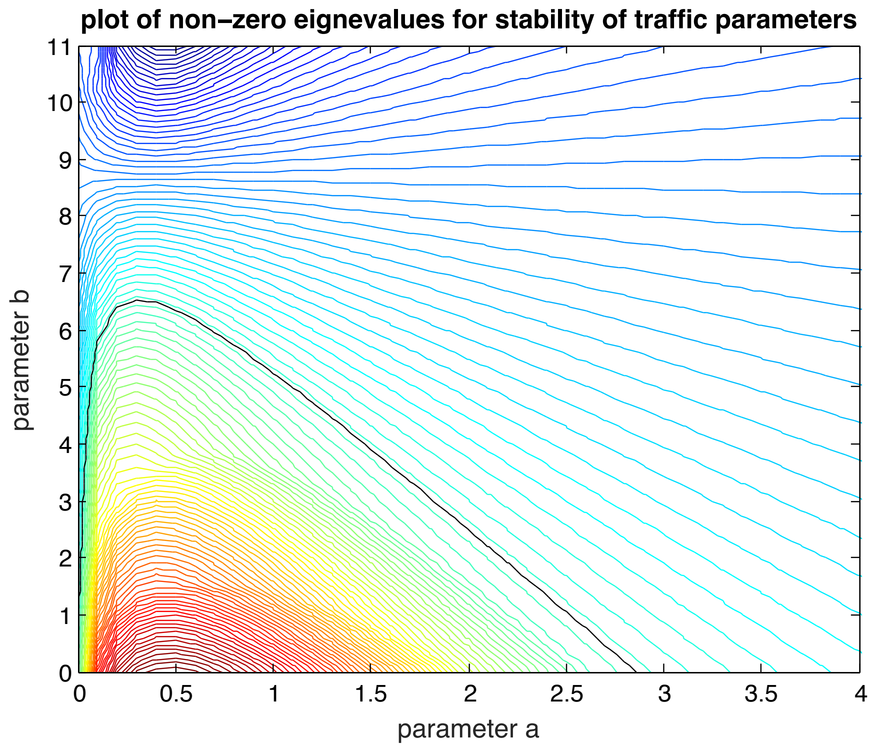

Figure 3: Stability plot for a and b with $\nu = 1$.

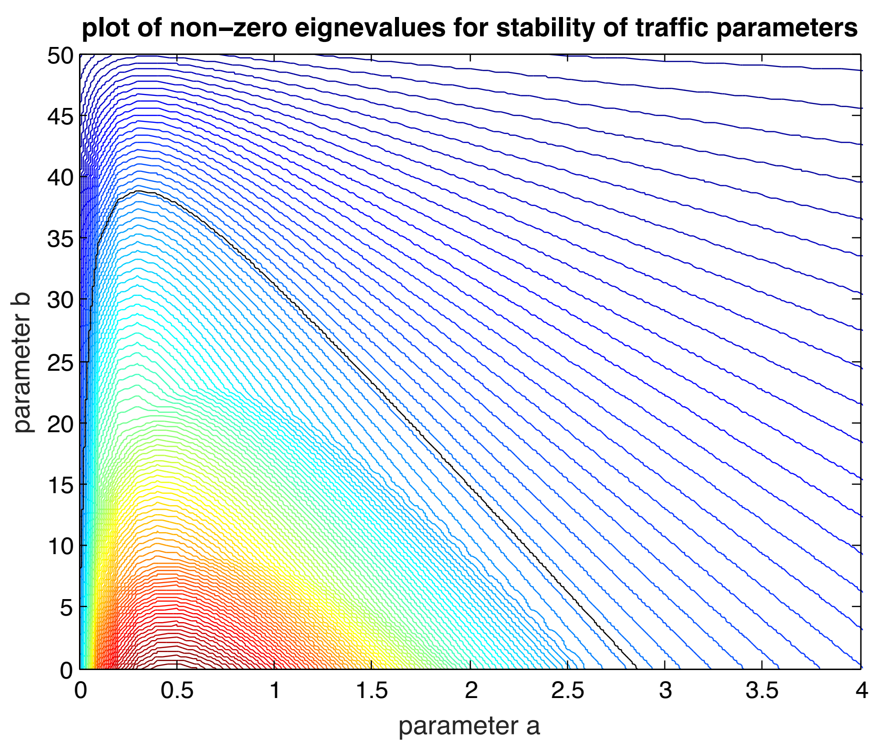

Figure 4: Stability plot for a and b with $\nu = 2$.

magnitude. A power of 2 significantly changed the minimum distance between vehicles, bringing it from less than 1 m to a realistic value of around 2 m. A minimum distance of 2 m is realistic for real stop-and-go traffic, especially since the vehicles are starting only 5.95 m apart. Increasing the power to 3 or 4 increased the minimum distance, but the differences were less significant. The value of $\nu = 2$ was chosen as it gave a sufficient minimum distance while still allowing vehicles to become close enough that they modeled stop-and-go traffic.

2.3 Maximum Velocity

The next model parameter is maximum velocity. The tanh equation used is based on the paper *Dynamical model of traffic congestion and numerical simulation* by Bando et al. [4]. The function is used as it was proposed in this paper.

Values between $V_{\mathrm{m}} = 8$ m/s and $V_{\mathrm{m}} = 16$ m/s were tested. For extremely small values the behavior was greatly varied and haphazard, often resulting in forward propagating waves. For larger values of V_{m}, it was difficult to find values of a and b that resulted in the correct wave speed. Decreasing a while increasing b often resulted in a double wave unless a was made sufficiently small, which then resulted in weak braking. If a was increased while decreasing b, a correct wave speed could not be attained unless b was made incredibly small, which still often resulted in a double wave, late development, and decreased braking time (i.e. braking lasted only for 2 seconds). All the values in the middle of the stability plot resulted in double waves. A value was chosen in between $V_{\mathrm{m}} = 9$ m/s and $V_{\mathrm{m}} = 10$ m/s as the $V_{\mathrm{m}} = 9$ m/s plot resulted in optimal braking speeds, but the waves developed too late and the trajectories were not curved enough, while the $V_{\mathrm{m}} = 10$ m/s plot had weak braking, but waves developed sooner and were more curved and thicker. The value of 35 km/hr (9.722 m/s) was chosen as it gives a somewhat round number in the unit km/hr.

Each value of V_{m} has a corresponding d_0 value based on the initial speed found from the 2008 Sugiyama et al. experiment [2]. The d_0 adjusts the velocity equation so vehicles that are 5.95 m apart (the experiment spacing) drive at 7.67 m/s, the initial speed of the trajectories in the experiment. To change V_{m}, see the corresponding table of d_0 values below:

$\mathrm{V_m}$ / $\mathrm{m/s^2}$	8	9	9.722	10	11	12	13	14	16
$\mathrm{d_0}$ / m	1.65	2.05	2.23	2.27	2.45	2.59	2.72	2.82	3.01

Table 2: V_{m} with corresponding d_0 values.

2.4 Follow-the-Leader and Optimal-Velocity Parameters

Three possible pairs of a and b values, the follow-the-leader and optimal velocity constants, were found to be suitable. Once a is selected, b is determined so that the speed of the waves is correct. The pair of $a = 0.5$ /s and $b = 20$ m^{ν}/s is the largest lower value of a that gives a reliably single wave (5% of the time a double wave occurs throughout). Another low pair, $a = 0.6$ /s and $b = 17$ m^{ν}/s (20% of the time a double wave occurs throughout and another 20% of the time a double wave occurs in the beginning), is less reliable but gives stronger braking (i.e., the wave is better pronounced). Smaller values of a yield weaker braking. For large a and small b, the choice a = 2.2 /s and b = 0.5 m$^\nu$/s yields the correct wave speed. This value yields much stronger braking but also much less reliably produces a single wave (50% double waves). Lower values of a in this region become less reliably a single wave while higher values of a push b to such small values such that the matching of the wave speed becomes extremely difficult or impossible. Of these three pairs, the choice a = 0.5 /s and b = 20 m$^\nu$/s is most preferable, while the other two can be used depending on the importance of keeping a single wave.

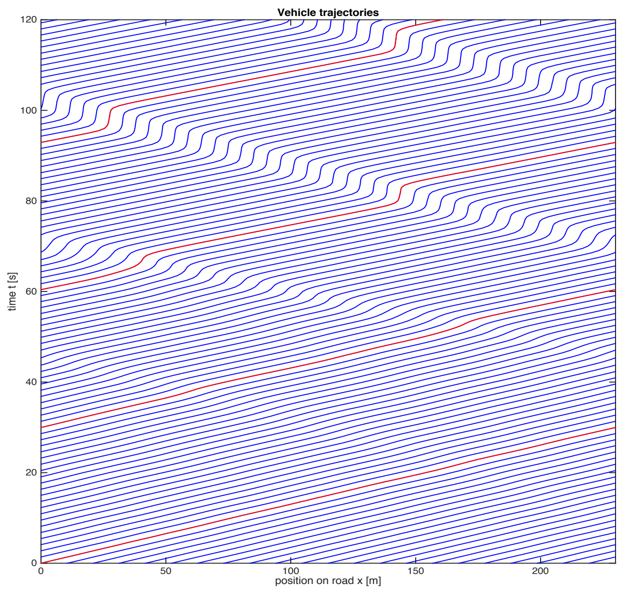

Figure 5: Trajectories for a = 2.2 /s and b = 0.5 m$^\nu$/s.

2.4.1 Double Waves

Sometimes a double wave may occur, as shown in Fig. 6. Users should be aware of this possibility even though it is a rare occurrence and will not affect average results if a large number of simulations are run.

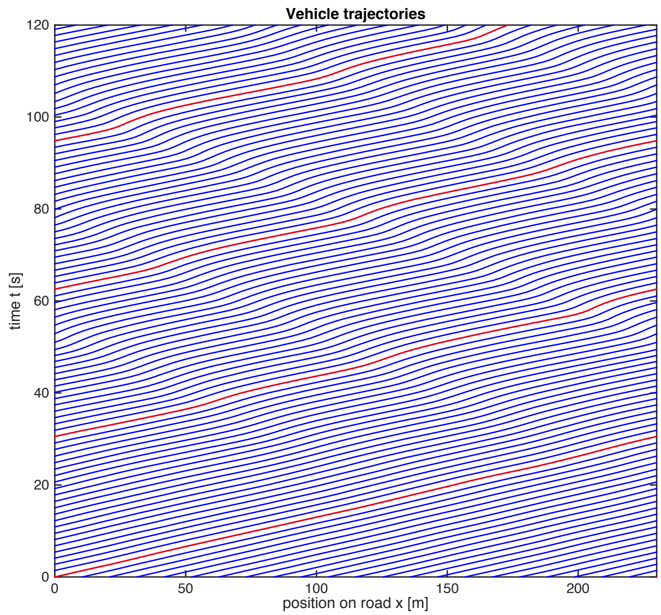

Figure 6: Trajectories for $a = 0.5$ /s and $b = 20$ m$^\nu$/s with a double wave.

2.5 Noise

Noise, which models imperfections in the driving behavior, is thought to act continuously (as a random walk) on the velocities of the vehicles. For computational purposes, noise is added to the vehicle velocities in (small) discrete time intervals. Every time increment Δt, a random value is drawn from a truncated Gaussian for each vehicle and added to the vehicle's velocity.

The relationship between the standard deviation of the noise random variable over the interval of length Δt, denoted $\sigma_{\Delta t}$, to the standard deviation incurred per second, σ, is given by the equation

$$\sigma_{\Delta t} = \sqrt{\frac{\Delta t}{1\ \mathrm{s}}} \cdot \sigma\ .$$

The reason for the square root is that the variances of the accumulating random events add. After each time increment Δt, a random number is sampled for each vehicle from a truncated normal distribution with standard deviation of about 1.[1] This value is then scaled with $\sigma_{\Delta t}$, and added to the vehicle velocity via equation

$$\dot{x}_j = \dot{x}_j + \mathrm{randtrunc}(1, n, 3) \cdot \sigma_{\Delta t}\ .$$

The reason for choosing a truncated normal distribution is that the long tails of a Gaussian may yield unphysically large accelerations. However, discarding samples further than 3 standard deviations away from the mean zero renders the truncated distribution very close to a true Gaussian.

[1] We choose a Gaussian of standard deviation 1, and redraw any sample larger in magnitude than 3. The resulting truncated Gaussian has a standard deviation of slightly less than 1.

Finally, as the addition of noise could produce negative vehicle velocities, we cap the velocities from below by zero via the equation

$$\dot{x}_j = \max(\dot{x}_j, 0)\ .$$

The value for Δt, the time between each addition of noise, was chosen next. Noise represents driver imperfections and speed fluctuations, so large intervals would be too spread out and too periodic to represent realistic random noise. For this reason values below 5 should be chosen. Increasing Δt had little effect on the model, resulting in only slightly lower braking speeds. Small values of Δt (less than or equal to 1 s) took a long time to compute and resulted in unusual behavior such as double waves, high braking speeds, and changes to the wave speed. A value of 1 s would work for this model, but 2 s was chosen since the difference to the model was insignificant but it computed more quickly.

Next, σ, the total amount of noise per second, was chosen. The larger σ, the earlier the instability kicks in, and thus the earlier the traffic waves become visible. However, to maintain mostly smooth waves and to avoid any noise that would disrupt a wave or result in zigzag trajectories, very high values of σ cannot be used. Among the values of σ that retain a smooth appearance of the vehicle trajectory (i.e., they do not look zig-zaggy), one of the largest possible was chosen, namely $\sigma = 0.25\ \mathrm{m/s}^2$. With the other model parameters, this choise yields a realistic time scale for the traffic waves to arise.

References

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[2] Y. Sugiyama *et al.* *Traffic jams without bottlenecks—experimental evidence for the physical mechanism of the formation of a jam*. New J. Phys., **10** 033001, 2008

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