Repository Notes

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This is done to understand better the codes on this repository.

1st Order Differential Equations

The first problem we are working on, is the mos simple differential equation we can have, in general is like follows

$$\frac{dx(t)}{dt} = f(x(t), t) \tag{1}$$

to solve this numerically, we have to approximate at some point, to do so, we consider

$$x(t) = x(t_0) + \left.\frac{dx}{dt}\right|_{t=t_0}(t - t_0) + \frac{1}{2} + \left.\frac{d^2x}{dt^2}\right|_{t=t_0}(t - t_0)^2 + \cdots \tag{2a}$$

so if we take the interval of integration to be small $(t - t_0 = \Delta t)$[3]

$$x(t) = x(t_0) + \Delta t f(x(t), t) \tag{2b}$$

[3] We are taking this consideration in order to get an application that can be used easily as a first example.

Nuclear Decay

There is a immediate application of this, and that is a nuclear decay model, this can me thought on two different ways, the first one, looking up the number of atoms that haven't decayed or looking at the probability of decay, the difference is a normalization constant. If we take N as the number of radioactive nuclei on a sample they follow the differential equation

$$\frac{dN}{dt} = -\frac{1}{\tau}N \tag{3}$$

where τ is called the *Mean lifetime* of the nuclei. The analytical solution for this is an exponentially decaying function

$$N(t) = N(t_0)\exp\left(-\frac{t}{\tau}\right) \tag{4}$$

Note: When working with codes, it is necessary to have some way to test it, in our case this model is the battle horse.

2nd Order Differential Equations

At some point it gets necessary to use higher order differential equations, specially on physics it is extremely important to solve second order ordinary differential equations (ODE)

$$\frac{d^2x(t)}{dt^2} = f(x(t), x'(t), t), \tag{5}$$

where $x'(t)$ denotes the first derivative of $x(t)$ respect to t.

To solve this numerically, we cannot use the same strategy used for the first order, because we have to know the first derivative, which not always happen. To solve this, we can use the fact that one second order differential equation can be decomposed on two first order coupled EDO,

$$\frac{dv(t)}{dt} = f(x(t), v(t), t), \tag{6a}$$

$$\frac{dx}{dt} = v \tag{6b}$$

So, now each can be solved as the before.

NOTE: This have to be done carefully, because as the two equations are coupled, they have to be solved simultaneously[4]

[4] This can be proofed easily, and it totally worth it as an exercise.

As the two of them have to be solved simultaneously, some questions arise, for instance, the computer only can perform one numeric operation at a time[5], which operations should we do first?, the order is important?, there is any physical consideration that can help us to solve those questions.

[5] This can be different when paralleling, I hope that we can discuss this part further on the notes.

- First Method: Euler. The method developed for the first order ODE is called the *Euler* method, is the simplest one we can use.

  In this case, we are evolving position and velocity each time step using the old values of the variables.

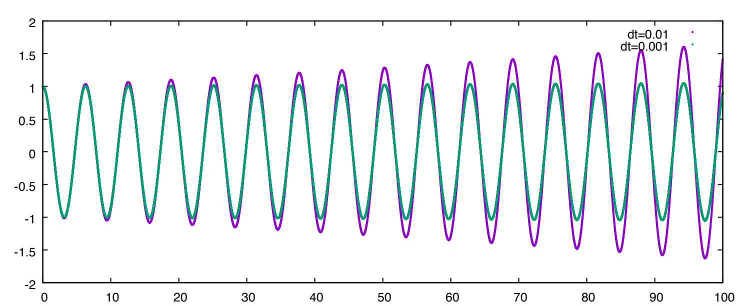

Figure 1: Comparison of the Euler method results, using different values of dt. It is clear that at the beginning it seems to be working, but as the time evolution passes, the numerical result gets farther from the analytical solution (a cosine function).

  So, a new method has to be proved in order to get better results.