

Aula 12 – Morfologia matemática II

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- Abertura e Fechamento morfológico
- Transformada Hit or Miss

ABERTURA E FECHAMENTO MORFOLÓGICO

Abertura morfológica

- Recordando:
 - A erosão reduz/diminui os componentes em uma imagem
 - A dilatação aumenta/expande os componentes em uma imagem
- A **abertura** suaviza o contorno de um objeto, rompe istmos e elimina saliências finas
- A abertura do conjunto A pelo EE B é:
 - $A \circ B = (A \ominus B) \oplus B$
 - A **abertura** de A por B é a erosão de A por B seguida de uma dilatação por B

Fechamento morfológico

- O fechamento também suaviza contornos, porém, diferente da abertura:
 - funde descontinuidades estreitas
 - elimina pequenos buracos e
 - preenche lacunas (baías) no contorno
- O fechamento do conjunto A pelo EE B é:
 - $A \cdot B = (A \oplus B) \ominus B$
- O fechamento de A por B é a dilatação de A por B seguida da erosão por B

Abertura e fechamento morfológico

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

Abertura e fechamento morfológico

Abertura morfológica

$$(A \ominus B) \oplus B$$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

Abertura e fechamento morfológico

Abertura morfológica
 $(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Abertura e fechamento morfológico

Abertura morfológica
 $(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Abertura e fechamento morfológico

Abertura morfológica
 $(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Abertura e fechamento morfológico

Abertura morfológica
 $(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

Fechamento morfológico
 $(A \oplus B) \ominus B$

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Abertura e fechamento morfológico

Abertura morfológica
 $(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

Fechamento morfológico
 $(A \oplus B) \ominus B$

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$A \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	0	0	0
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1
4	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	0
8	0	1	1	1	0	0	1	0	0

Abertura e fechamento morfológico

Abertura morfológica
 $(A \ominus B) \oplus B$

A

B

Fechamento morfológico
 $(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$A \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	0	0	0
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1
4	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	0
8	0	1	1	1	0	0	1	0	0

$(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	1	0	0	0	0
3	0	1	1	1	1	1	1	1	0
4	0	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

Abertura e fechamento morfológico

Abertura morfológica
 $(A \ominus B) \oplus B$

A

B

Fechamento morfológico
 $(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$A \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	0	0	0
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1
4	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	0
8	0	1	1	1	0	0	1	0	0

$(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	1	0	0	0	0
3	0	1	1	1	1	1	1	1	0
4	0	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

$(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	1	0	0	0	0
3	0	1	1	1	1	1	1	1	0
4	0	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

TRANSFORMADA HIT OR MISS

Transformada Hit or Miss

- A **transformada hit-or-miss** é uma ferramenta básica para a detecção de formas:
 - Utiliza dois elementos estruturantes para especificar o padrão a ser detectado na imagem.
 - B_1 : verifica (testa) os pixels de objetos (1's)
 - B_2 : verifica (testa) os pixels de fundo (0's)
 - A transformada hit-or-miss é definida como:
 - $A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$

0	0	0
0	1	1
0	1	0

 B_1

1	1	0
1	0	0
0	0	0

 B_2

ou

0	0	×
0	1	1
×	1	×

 B

← 0 em B_1 e 0 em B_2 .

Transformada Hit or Miss

$$A = C \cup D \cup E$$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	1	1	0	0
0	0	0	1	1	1	1	0	0	0	0	0	1	1	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

D'

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

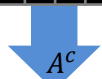
Transformada Hit or Miss

$$A = C \cup D \cup E$$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0



A^c

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

D'

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

Transformada Hit or Miss

$$A = C \cup D \cup E$$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

$$A \ominus D$$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A^c

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

D'

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

Transformada Hit or Miss

$$A = C \cup D \cup E$$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

$$A \ominus D$$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A^c

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

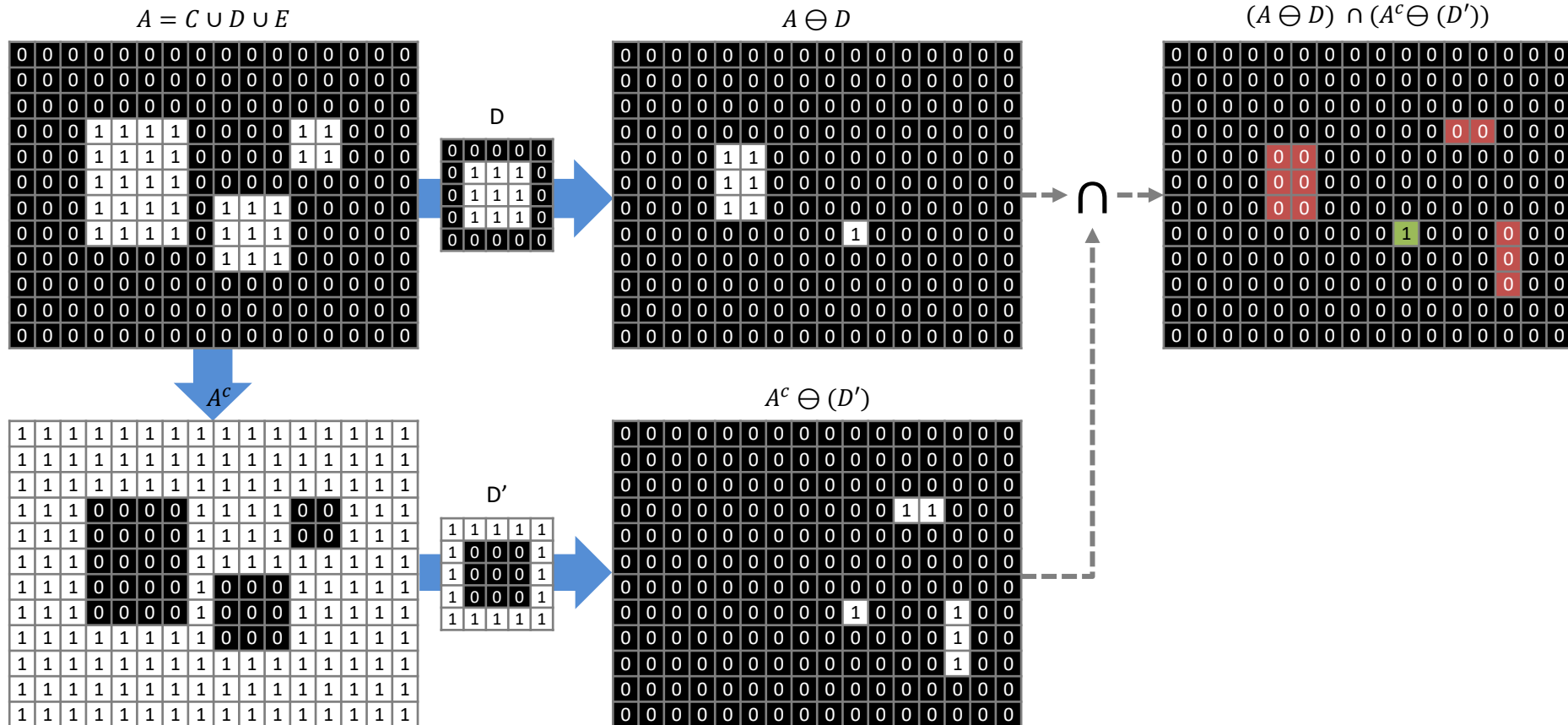
D'

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

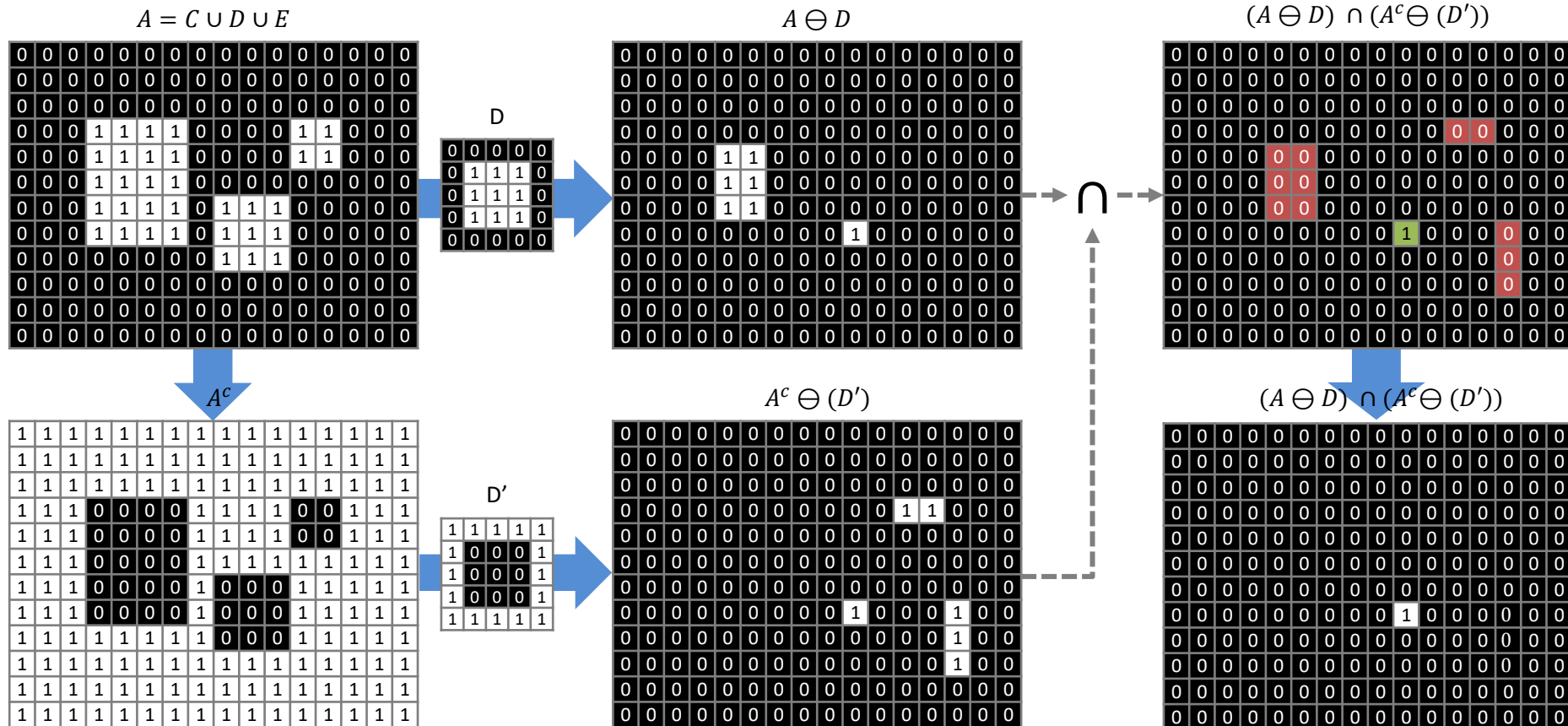
$$A^c \ominus (D')$$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Transformada Hit or Miss



Transformada Hit or Miss



	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

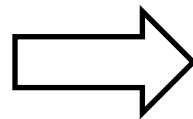
A

	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A

Obs. 1: 4-conectividade.

Obs. 2: × = não importa se 0 ou 1.



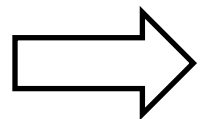
$$\bigcup_{k=1}^4 (A \circledast B^k)$$

0	0	×
0	1	1
×	1	×

B^1

	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

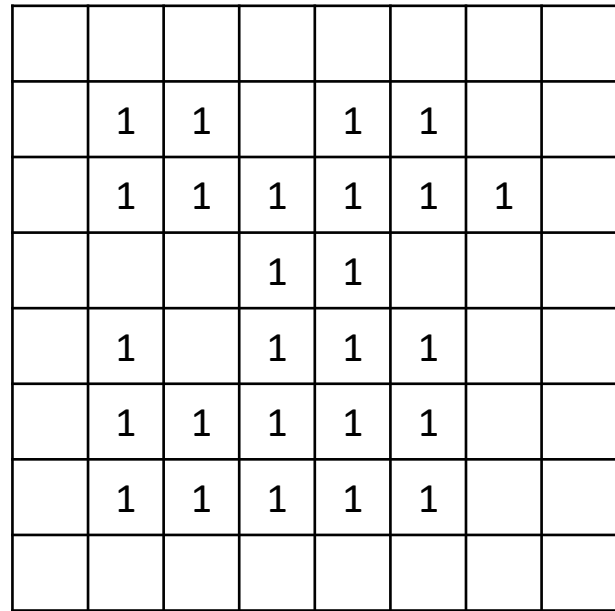
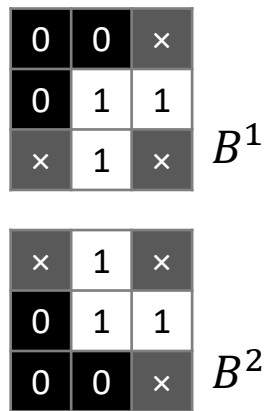
A



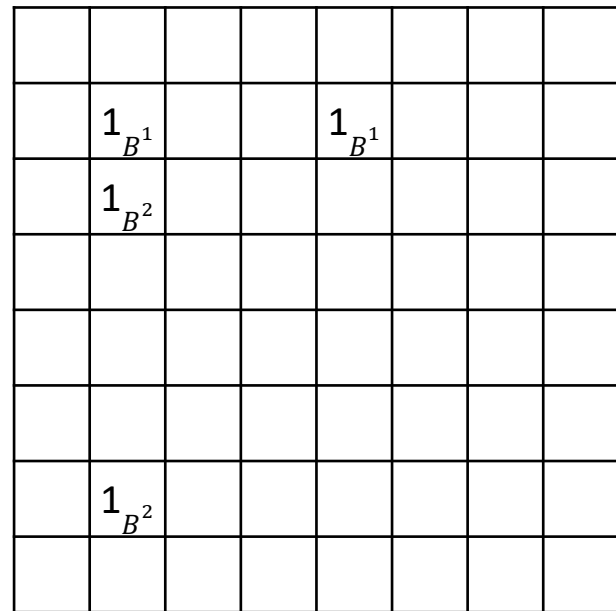
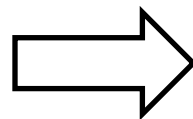
	1_{B^1}			1_{B^1}			

$$\bigcup_{k=1}^4 (A \circledast B^k)$$

- Obs. 1: 4-conectividade.
- Obs. 2: × = não importa se 0 ou 1.

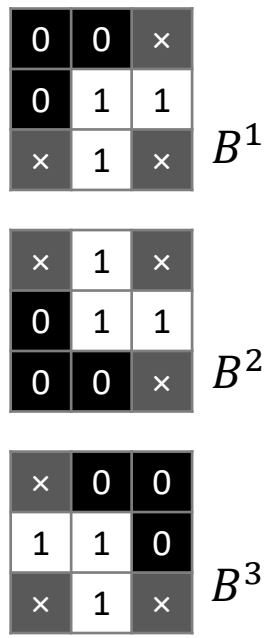


A



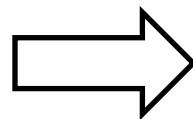
$$\bigcup_{k=1}^4 (A \circledast B^k)$$

- Obs. 1: 4-conectividade.
- Obs. 2: × = não importa se 0 ou 1.



	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A



	1_{B^1}	1_{B^3}		1_{B^1}	1_{B^3}		
	1_{B^2}						
					1_{B^3}		
	1_{B^2}						

$$\bigcup_{k=1}^4 (A \circledast B^k)$$

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

0	0	×
0	1	1
×	1	×

 B^1

×	1	×
0	1	1
0	0	×

 B^2

×	0	0
1	1	0
×	1	×

 B^3

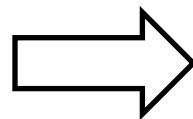
×	1	×
1	1	0
×	0	0

 B^4

	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.



	1_{B^1}	1_{B^3}		1_{B^1}	1_{B^3}		
	1_{B^2}						
						1_{B^3}	
	1_{B^2}					1_{B^4}	

$$\bigcup_{k=1}^4 (A \circledast B^k)$$

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

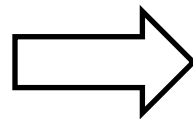
A

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A

Obs. 1: 4-conectividade.

Obs. 2: × = não importa se 0 ou 1.



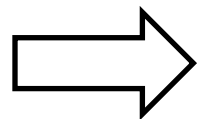
$$\bigcup_{k=1}^4 (A \circledast B^k)$$

×	0	×
0	1	0
×	×	×

B^1

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A



		1_{B^4}			1_{B^1}		
	1_{B^2}		1_{B^1}			1_{B^4}	
	1_{B^1}						
	1_{B^3}				1_{B^4}		

$$\bigcup_{k=1}^4 (A \circledast B^k)$$

- Obs. 1: 4-conectividade.
- Obs. 2: × = não importa se 0 ou 1.

×	0	×
0	1	0
×	×	×

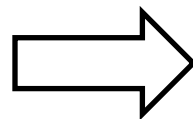
B^1

×	×	×
0	1	0
×	0	×

B^2

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A



		1_{B^4}			1_{B^1}		
	1_{B^2}		1_{B^1}			1_{B^4}	
	$1_{B^{1,2}}$						
	1_{B^3}				1_{B^4}		

$$\bigcup_{k=1}^4 (A \circledast B^k)$$

- Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

×	0	×
0	1	0
×	×	×

B^1

×	×	×
0	1	0
×	0	×

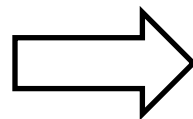
B^2

×	0	×
0	1	×
×	0	×

B^3

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A



		1_{B^4}			1_{B^1}		
	1_{B^2}		1_{B^1}			1_{B^4}	
	$1_{B^{1,2,3}}$						
	1_{B^3}				1_{B^4}		

$$\bigcup_{k=1}^4 (A \circledast B^k)$$

- Obs. 1: 4-conectividade.
- Obs. 2: × = não importa se 0 ou 1.

×	0	×
0	1	0
×	×	×

 B^1

×	×	×
0	1	0
×	0	×

 B^2

×	0	×
0	1	×
×	0	×

 B^3

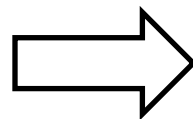
×	0	×
×	1	0
×	0	×

 B^4

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.



		1_{B^4}			1_{B^1}		
	1_{B^2}		1_{B^1}			1_{B^4}	
	$1_{B^{1,2,3,4}}$						
	1_{B^3}				1_{B^4}		

$$\bigcup_{k=1}^4 (A \circledast B^k)$$

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  howpublished = {\url{https://github.com/joaofmari/SIN392_Introduction-to-digital-image-processing_2023}}
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FIM