Lecture 12 – Mathematical morphology II

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Agenda

- Morphological opening and closing
- Hit or Miss transform

MORPHOLOGICAL OPENING AND CLOSING

Morphological opening

- Remembering:
  - Erosion reduces/shrinks components in an image
  - Dilation enlarges/expands components in an image

- **Opening** softens the contour of an object, breaks up isthmuses, and eliminates thin protrusions

- The opening of a set A by a structuring element (SE) B is:
  - $A \circ B = (A \ominus B) \oplus B$
  - The **opening** of A by B is the erosion of A by B followed by a dilation by B

Morphological closing

- Closing also softens contours, however, differently from opening:
  - merges narrow discontinuities
  - eliminates small holes and
  - fills gaps (bays) in the contour

- The closing of a set A by a SE B is :
  - $A \cdot B = (A \oplus B) \ominus B$

- The **closing** of A by B is the dilation of A by B followed by erosion by B

Morphological opening and closing

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

Morphological opening and closing

Morphological opening

$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

Morphological opening and closing

Morphological opening
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Morphological opening and closing

Morphological opening
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Morphological opening and closing

Morphological opening
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Morphological opening and closing

Morphological opening
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Morphological closing
$(A \oplus B) \ominus B$

Morphological opening and closing

Morphological opening
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Morphological closing
$(A \oplus B) \ominus B$

$A \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	0	0	0
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1
4	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	0
8	0	1	1	1	0	0	1	0	0

Morphological opening and closing

Morphological opening
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Morphological closing
$(A \oplus B) \ominus B$

$A \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	0	0	0
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1
4	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	0
8	0	1	1	1	0	0	1	0	0

$(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	1	0	0	0	0
3	0	1	1	1	1	1	1	1	0
4	0	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

Morphological opening and closing

Morphological opening
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Morphological closing
$(A \oplus B) \ominus B$

$A \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	0	0	0
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1
4	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	0
8	0	1	1	1	0	0	1	0	0

$(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	1	0	0	0	0
3	0	1	1	1	1	1	1	1	0
4	0	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

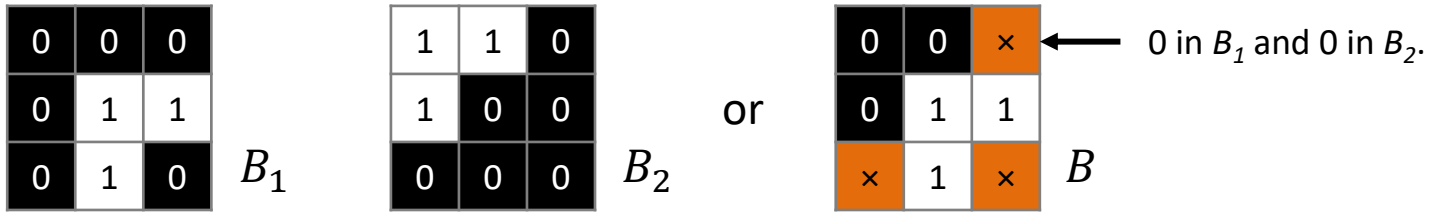
$(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	1	0	0	0	0
3	0	1	1	1	1	1	1	1	0
4	0	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

HIT OR MISS TRANSFORM

Hit or Miss transform

- The **hit-or-miss transform** is a basic tool for shape detection:
  - It uses two structuring elements to specify the pattern to be detected in the image.
    - B_1: checks (tests) object pixels (1's)
    - B_2: checks (tests) the background pixels (0's)
  - The hit-or-miss transform is defined as:
    - $A \circledast B = (A \ominus B_1) \cap (A^c \ominus (B_2))$



Hit or Miss transform

$A = C \cup D \cup E$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

D’

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

Hit or Miss transform

$A = C \cup D \cup E$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0
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0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

A^c

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1
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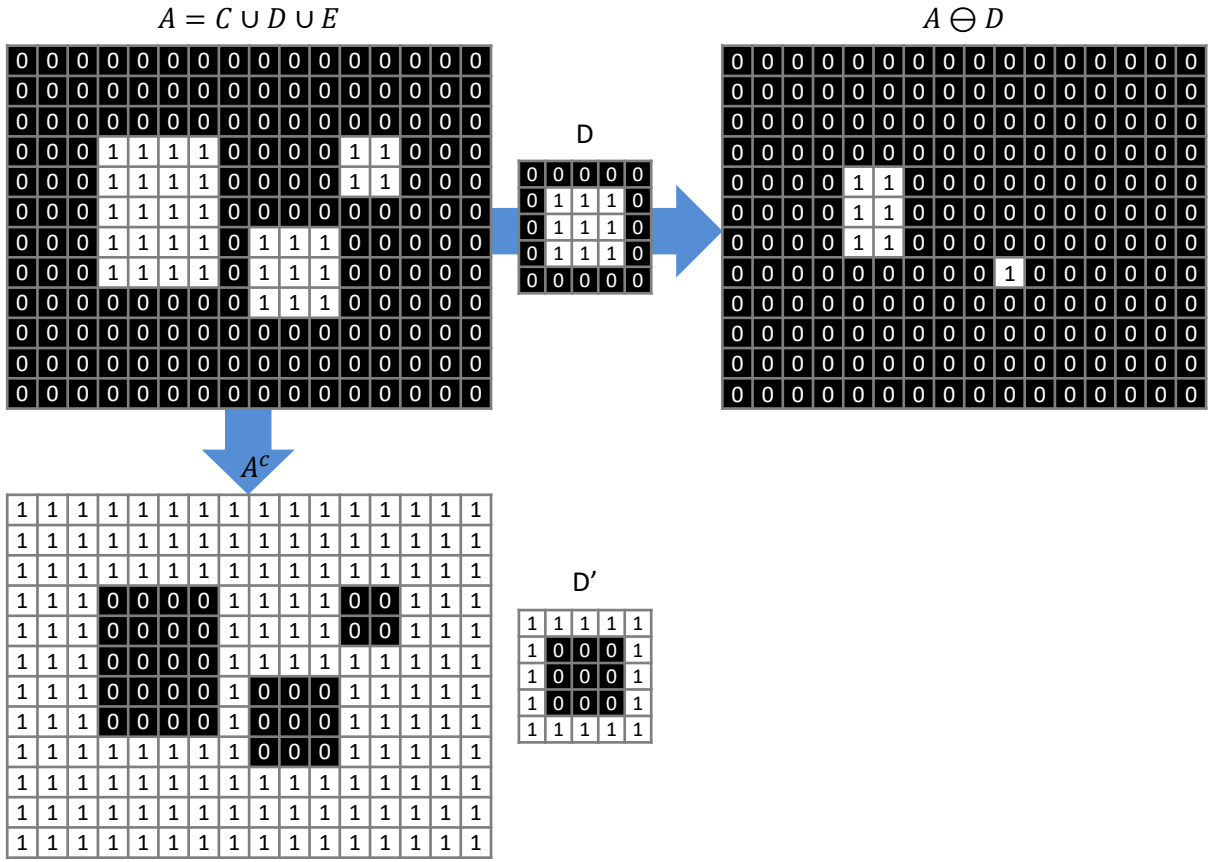
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

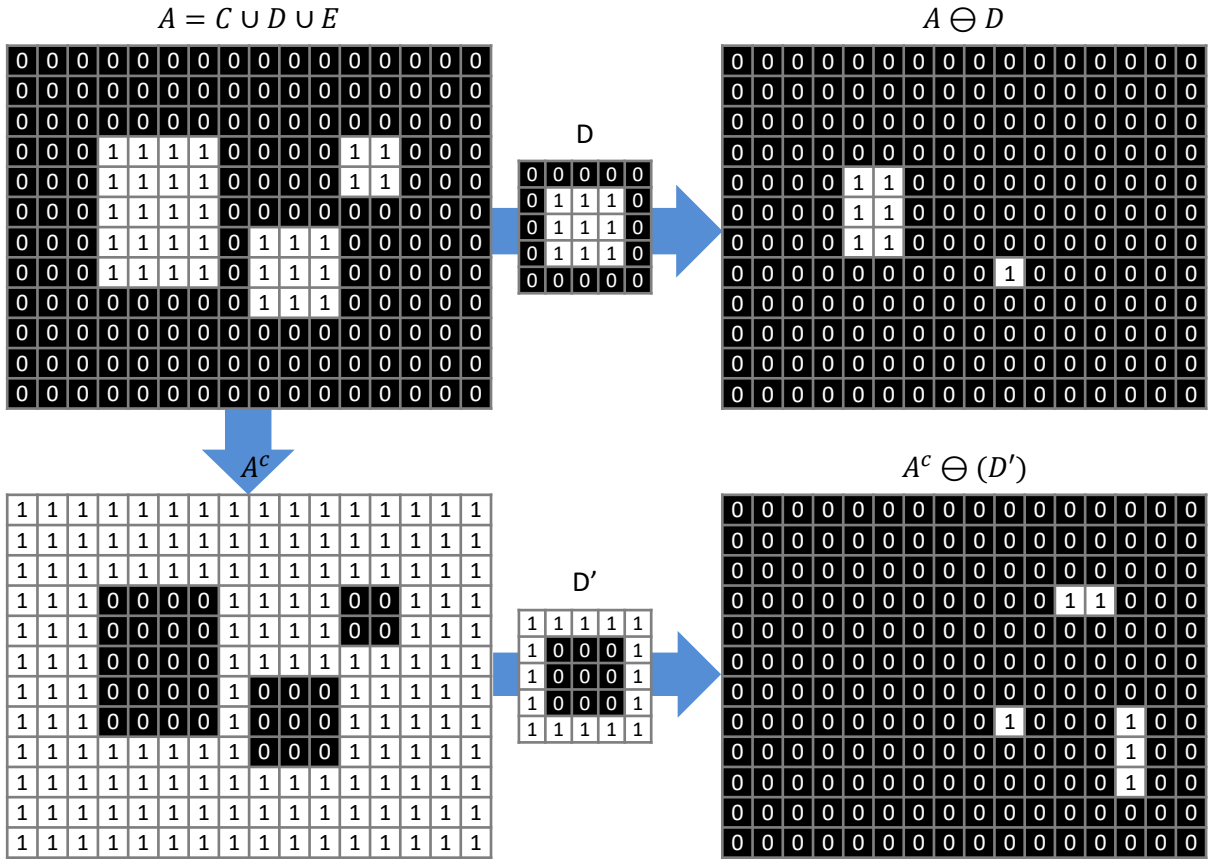
D’

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

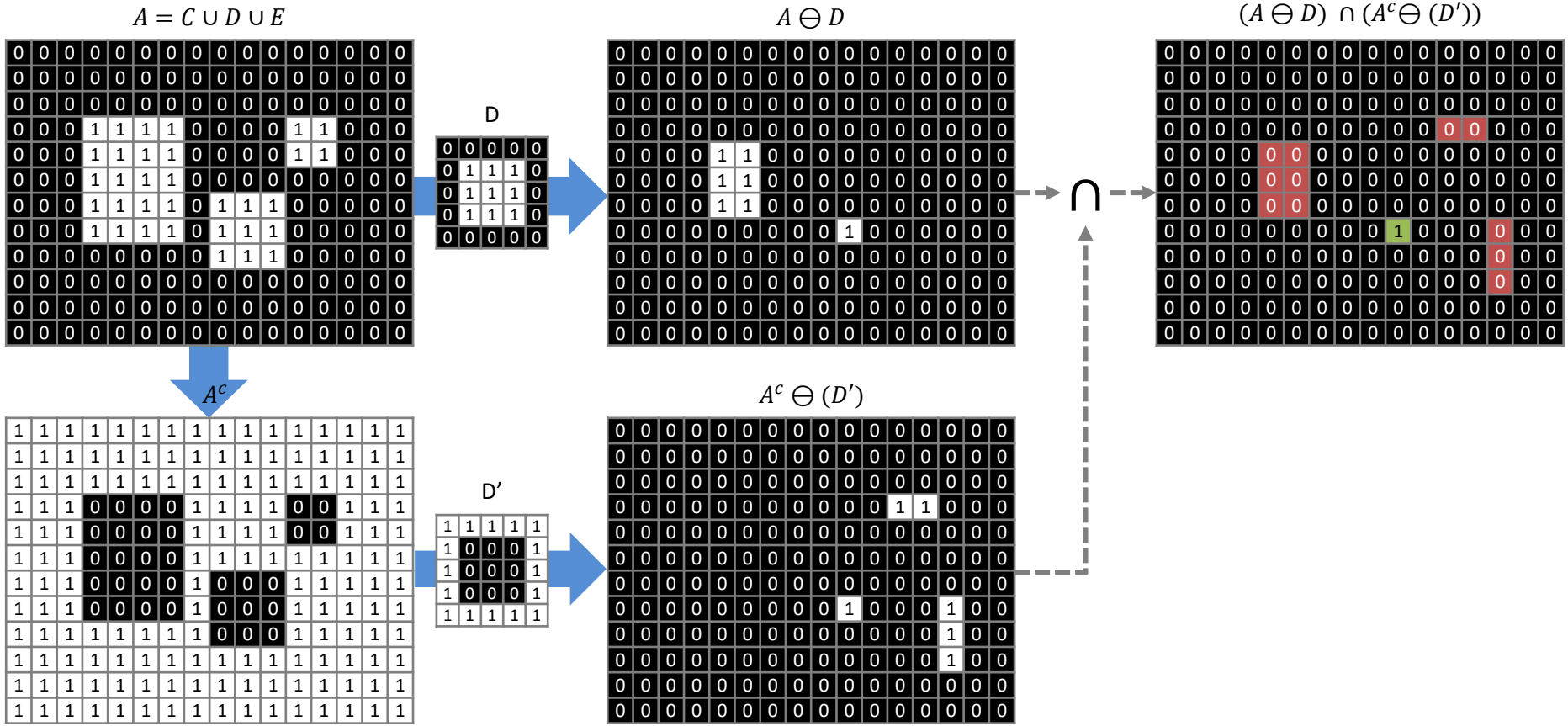
Hit or Miss transform



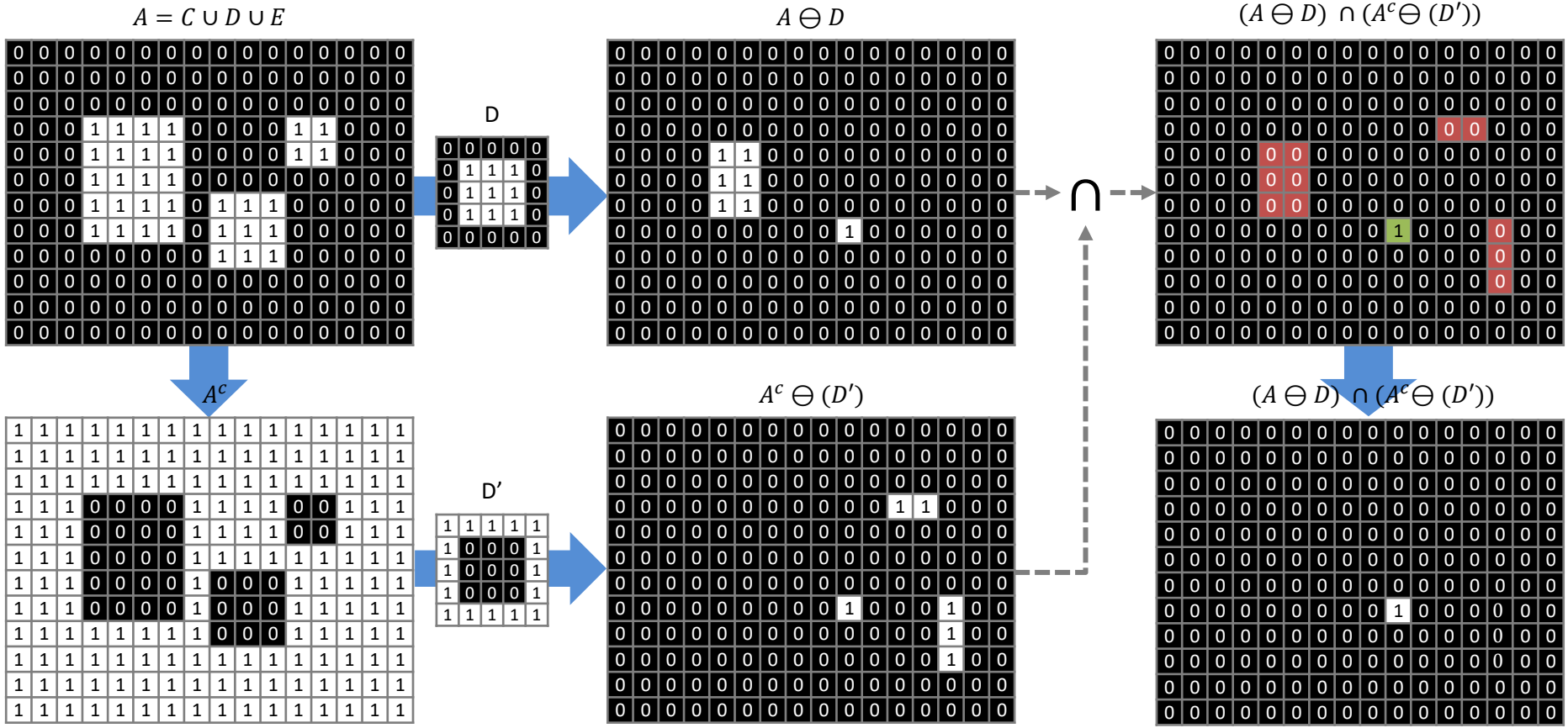
Hit or Miss transform



Hit or Miss transform



Hit or Miss transform

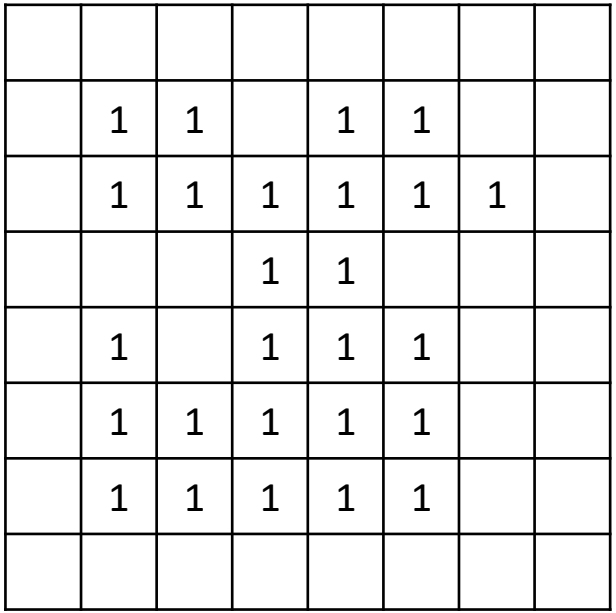


Corner detection using hit-or-miss transform

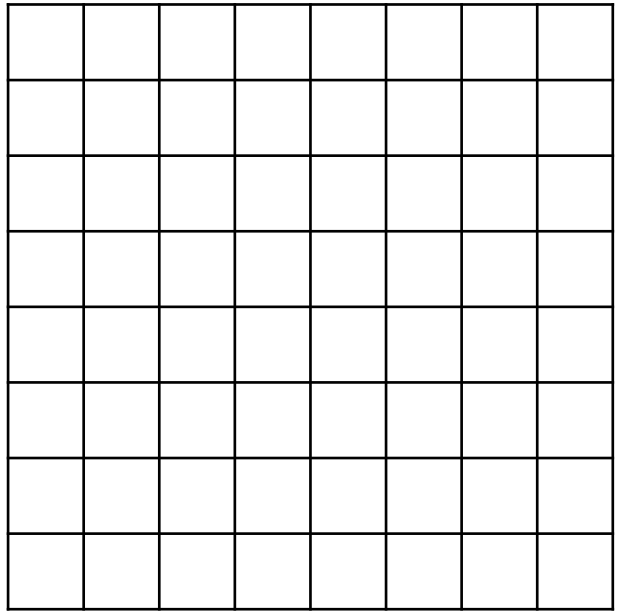
	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A

Corner detection using hit-or-miss transform



A



$$\bigcup_{k=1}^{4} \left(A \circledast B^k\right)$$

Obs. 1: 4-connectivity.
Obs. 2: × = “don’t care” if 0 or 1.

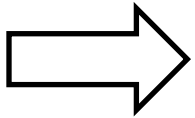
Corner detection using hit-or-miss transform

0	0	×
0	1	1
×	1	×

B^1

	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A



	1_{B^1}			1_{B^1}			

$$\bigcup_{k=1}^{4} (A \circledast B^k)$$

Obs. 1: 4-connectivity.
Obs. 2: × = “don’t care” if 0 or 1.

Corner detection using hit-or-miss transform

0	0	×
0	1	1
×	1	×

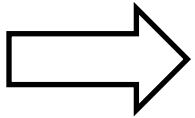
B^1

×	1	×
0	1	1
0	0	×

B^2

	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A



	1_{B^1}			1_{B^1}			
	1_{B^2}						
	1_{B^2}						

$$\bigcup_{k=1}^{4} (A \circledast B^k)$$

Obs. 1: 4-connectivity.
Obs. 2: × = “don’t care” if 0 or 1.

Corner detection using hit-or-miss transform

0	0	×
0	1	1
×	1	×

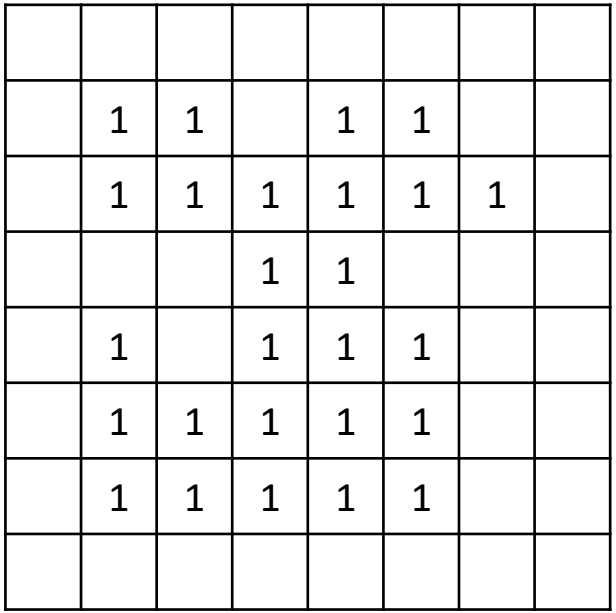
B^1

×	1	×
0	1	1
0	0	×

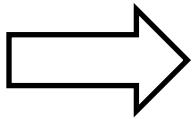
B^2

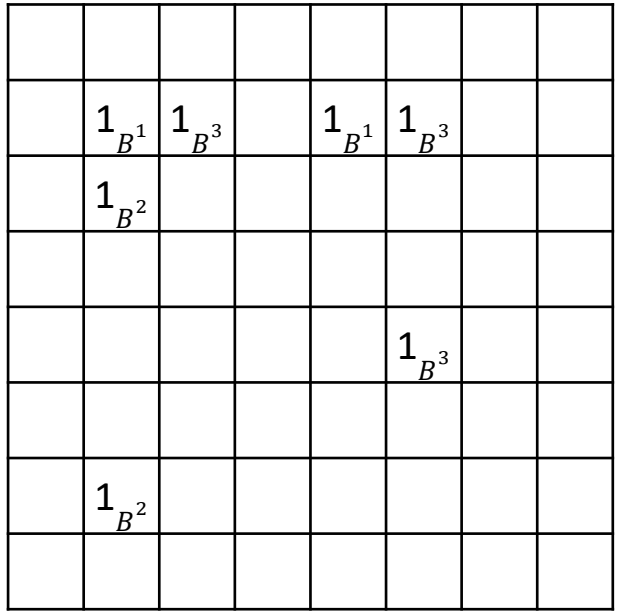
×	0	0
1	1	0
×	1	×

B^3



A





$$\bigcup_{k=1}^{4} (A \circledast B^k)$$

Obs. 1: 4-connectivity.
Obs. 2: × = “don’t care” if 0 or 1.

Corner detection using hit-or-miss transform

0	0	×
0	1	1
×	1	×

B^1

×	1	×
0	1	1
0	0	×

B^2

×	0	0
1	1	0
×	1	×

B^3

×	1	×
1	1	0
×	0	0

B^4

	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A

	1_{B^1}	1_{B^3}		1_{B^1}	1_{B^3}		
	1_{B^2}						
					1_{B^3}		
	1_{B^2}				1_{B^4}		

$$\bigcup_{k=1}^{4} (A \circledast B^k)$$

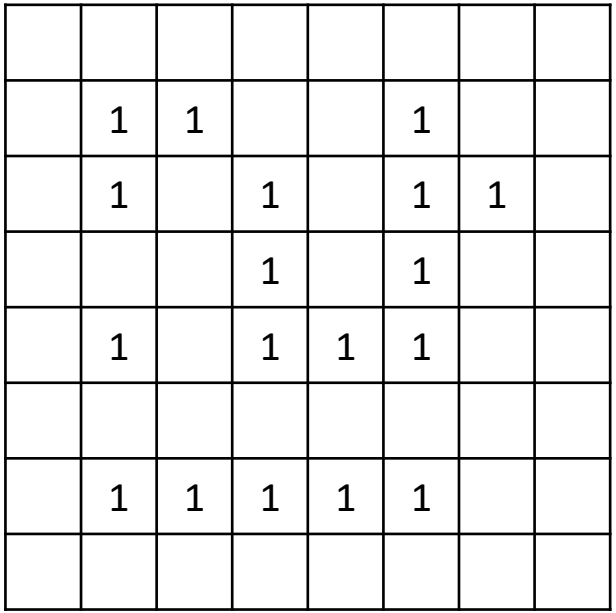
Obs. 1: 4-connectivity.
Obs. 2: × = “don’t care” if 0 or 1.

End point detection using hit-or-miss

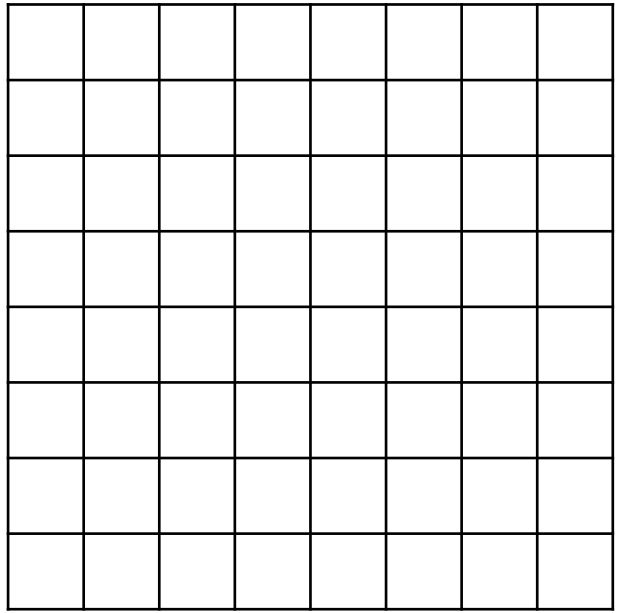
	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A

End point detection using hit-or-miss



A



$$\bigcup_{k=1}^{4} \left(A \circledast B^k\right)$$

Obs. 1: 4-connectivity.
Obs. 2: × = “don’t care” if 0 or 1.

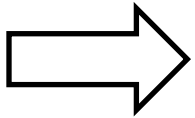
End point detection using hit-or-miss

×	0	×
0	1	0
×	×	×

B^1

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A



		1_{B^4}			1_{B^1}		
	1_{B^2}		1_{B^1}			1_{B^4}	
	1_{B^1}						
	1_{B^3}				1_{B^4}		

$$\bigcup_{k=1}^{4} (A \circledast B^k)$$

Obs. 1: 4-connectivity.
Obs. 2: × = “don’t care” if 0 or 1.

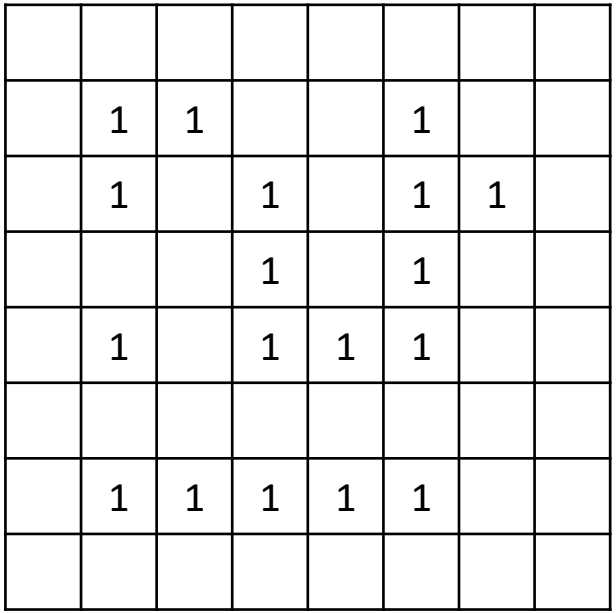
End point detection using hit-or-miss

×	0	×
0	1	0
×	×	×

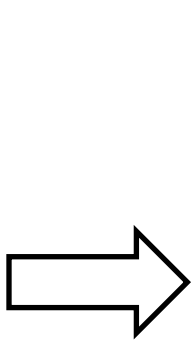
B^1

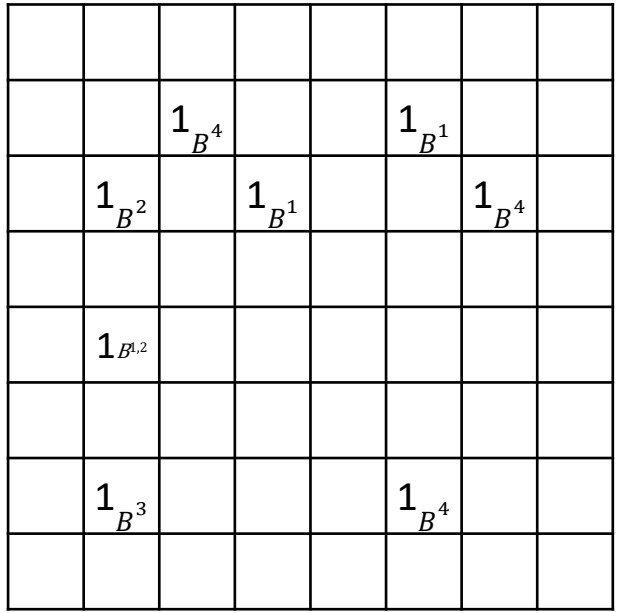
×	×	×
0	1	0
×	0	×

B^2



A

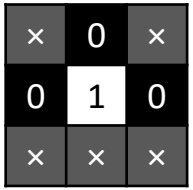




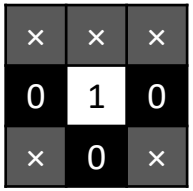
$$\bigcup_{k=1}^{4} (A \circledast B^k)$$

Obs. 1: 4-connectivity.
Obs. 2: × = “don’t care” if 0 or 1.

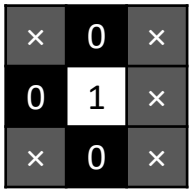
End point detection using hit-or-miss



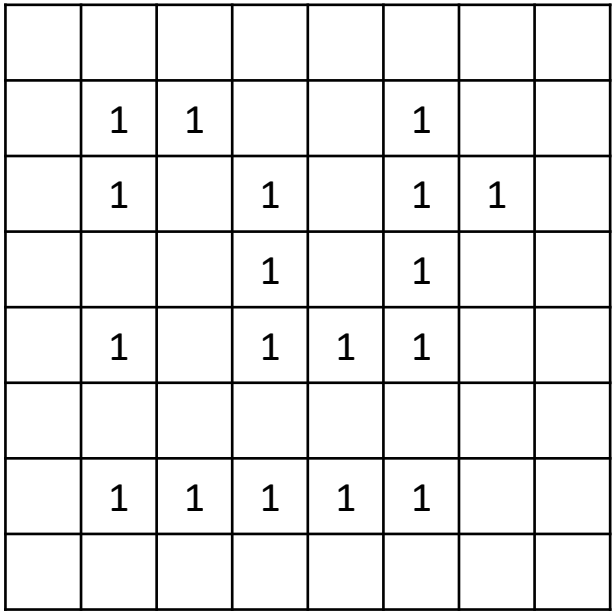
B^1



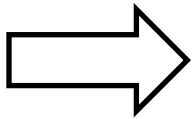
B^2

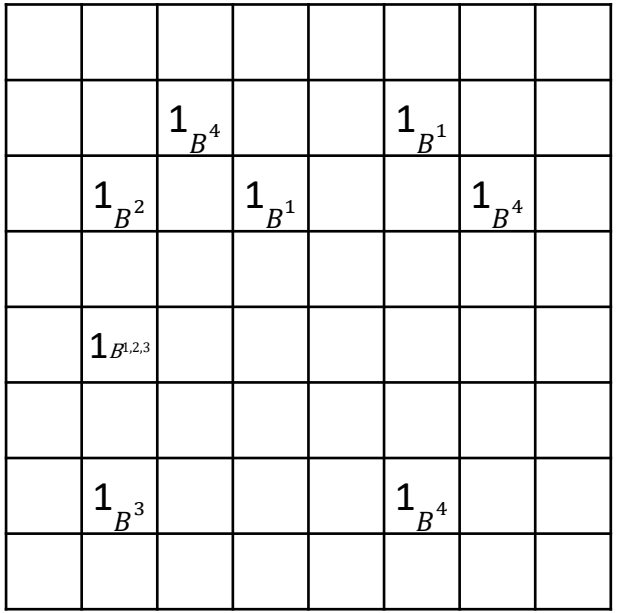


B^3



A

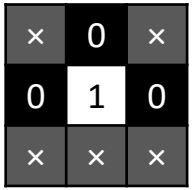




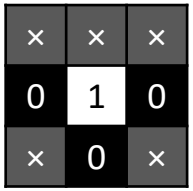
$$\bigcup_{k=1}^{4} (A \circledast B^k)$$

Obs. 1: 4-connectivity.
Obs. 2: × = “don’t care” if 0 or 1.

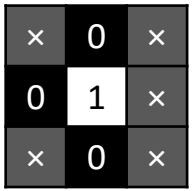
End point detection using hit-or-miss



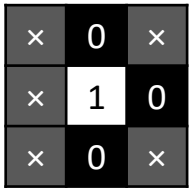
B^1



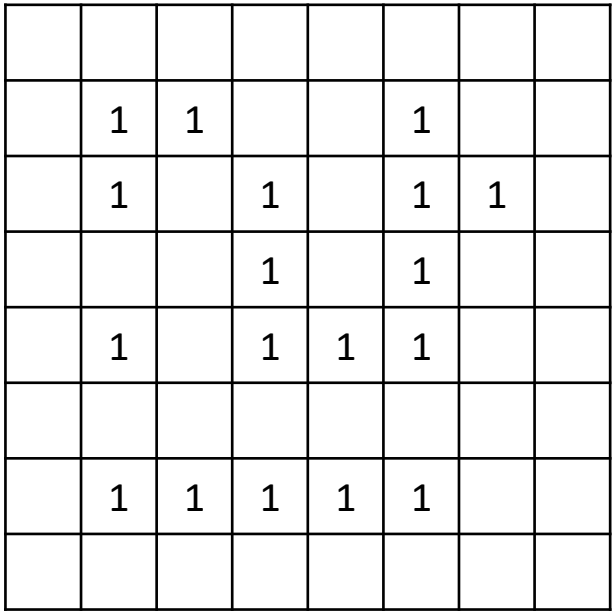
B^2



B^3

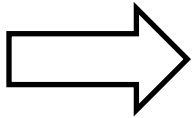


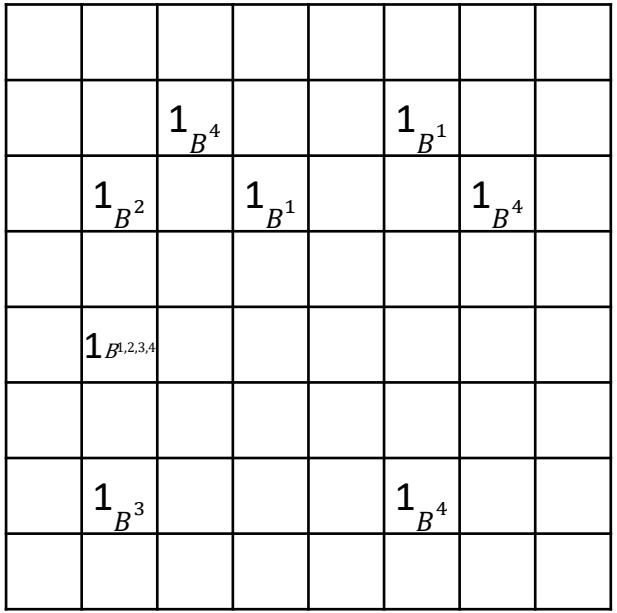
B^4



A

Obs. 1: 4-connectivity.
Obs. 2: × = “don’t care” if 0 or 1.





$$\bigcup_{k=1}^{4} (A \circledast B^k)$$

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THE END