Lecture 11 – Mathematical morphology I

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Agenda

- Mathematical morphology
- Basic operations with sets
- Erosion
- Dilation
- Duality
- Gray level mathematical morphology

MATHEMATICAL MORPHOLOGY

Mathematical morphology

- The language of mathematical morphology is set theory
  - Objects in an image are represented as sets
  - The set of all white (or black, depending on convention) pixels in a binary image is a complete representation of the image

- In binary images these sets are in Z^2
  - Each element of the set is a two-dimensional vector
  - Each dimension corresponds to the coordinates (x, y) of a white pixel in the image

- Gray level images can be represented as sets in Z^3
  - Two components of each element refer to the pixel coordinates
  - The third corresponds to its discrete intensity value

Representation of binary images as sets

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*
0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1	1
2	0	0	1	1	0	0	0	1
3	0	0	0	0	0	0	1	1
4	0	0	0	0	0	0	0	0
5	0	1	1	0	0	1	0	0
6	0	1	1	0	0	0	0	0
7	0	0	0	0	0	0	0	0

C_0 = { (1, 1), (1, 2), (1, 3), (2, 2), (2, 3) }
C_1 = { (1, 5), (1, 6), (1, 7), (2, 7), (3, 6), (3, 7) }
C_2 = { (5, 5) }
C_3 = { (5, 1), (5, 2), (6, 1), (6, 2) }

$$C_I = \bigcup_{i=0}^{N-1} C_i \,, \qquad for\ N\ objects$$

Intensity images as sets

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	1	0	5	7	5
2	0	0	2	3	0	0	0	6
3	0	0	0	0	0	0	4	7
4	0	0	0	0	0	0	0	0
5	0	1	2	0	0	3	0	0
6	0	1	3	0	0	0	0	0
7	0	0	0	0	0	0	0	0

C_0 = { (1, 1, 1), (1, 2, 2), (1, 3, 1), (2, 2, 2), (2, 3, 3) }
C_1 = { (1, 5, 5), (1, 6, 7), (1, 7, 5), (2, 7, 6), (3, 6, 4), (3, 7, 7) }
C_2 = { (5, 5, 3) }
C_3 = { (5, 1, 1), (5, 2, 2), (6, 1, 1), (6, 2, 3) }

$$C_I = \bigcup_{i=0}^{N-1} C_i \,, \qquad for\ N\ objects$$

BASIC OPERATIONS WITH SETS

Basic operations with sets

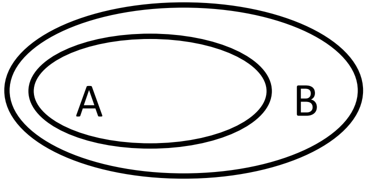
- Let A be a set of ordered pairs of real numbers
  - If a=(a_1, a_2) is an element of A, we have:
    - $a \in A$ (a is an element of A)
  - If a is not an element of A:
    - $a \notin A$ (a is not an element of A)
  - If a set contains no elements:
    - Empty set – ∅

- A set is defined by the contents of two keys
  - Ex.: $C = \{w|w = -d, d \in D\}$
  - C is the set of elements, w, such that w is formed by multiplying each of the elements of the set D by -1

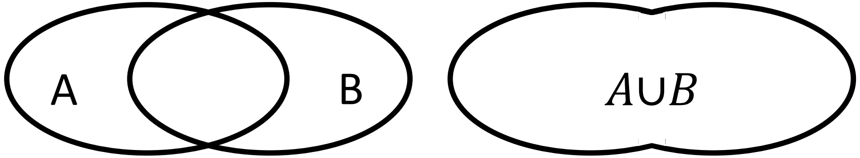
- One way to use sets in image processing is:
  - Consider the elements of the set as the coordinates of the pixels (ordered pairs of integers)
  - Each set represents regions (objects) in the image

Basic operations with sets

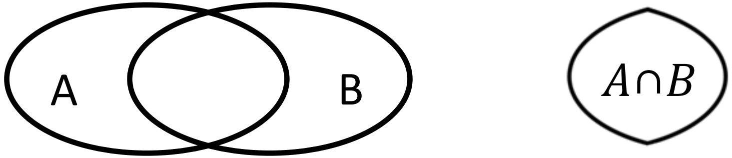
- If each element of set A is also an element of set B, then...
  - A is a subset of B
  - $A \subseteq B$



- The union of sets A and B is:
  - The set of elements that belong to either set A, or B, or both
  - $C = A \cup B$



- The intersection of two sets A and B is:
  - The set of elements that belong to both sets
  - $D = A \cap B$

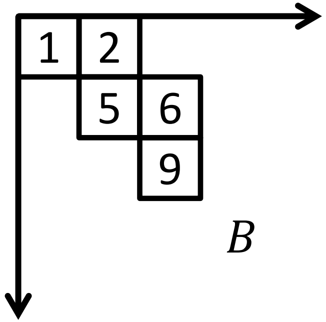


Basic operations with sets

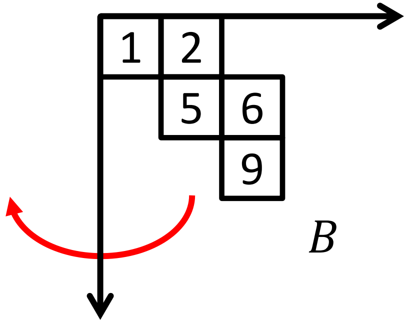
- The reflection of a set *B*, $\hat{B}$, is:
  - $\hat{B} = \{w | w = -b, for\ b \in B\}$
  - If B is the set of pixels that represent an object,
    - $\hat{B}$ is a set of pixels in *B* whose coordinates *(x, y)* were replaced by *(-x, -y)*.

- The translation of a set B at point (z_1, z_2), $(B)_z$, is:
  - $(B)_z = \{c | c = b + z, for\ b \in B\}$
  - If *B* is the set of pixels that represent an object,
    - $(B)_z$ is the set of pixels in *B* whose coordinates *(x, y)* have been replaced by $(x+z_1, y+z_2)$

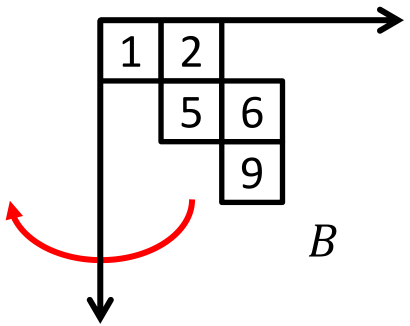
Reflection

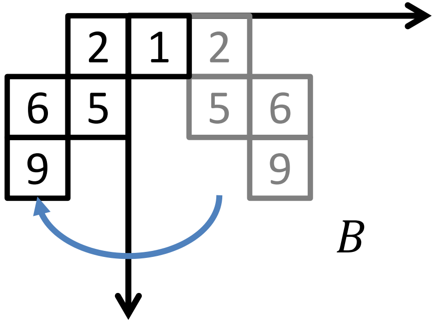


Reflection

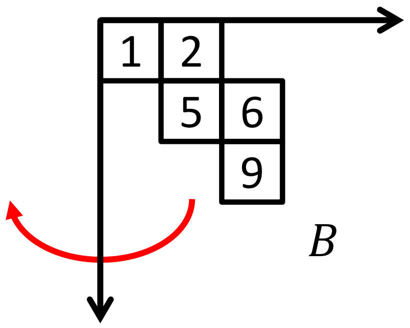


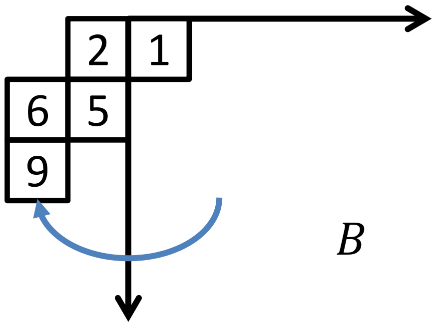
Reflection



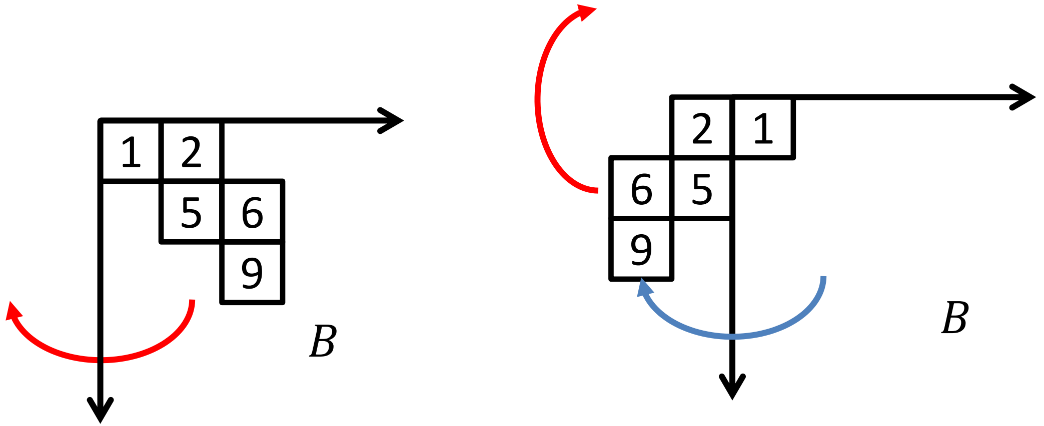


Reflection

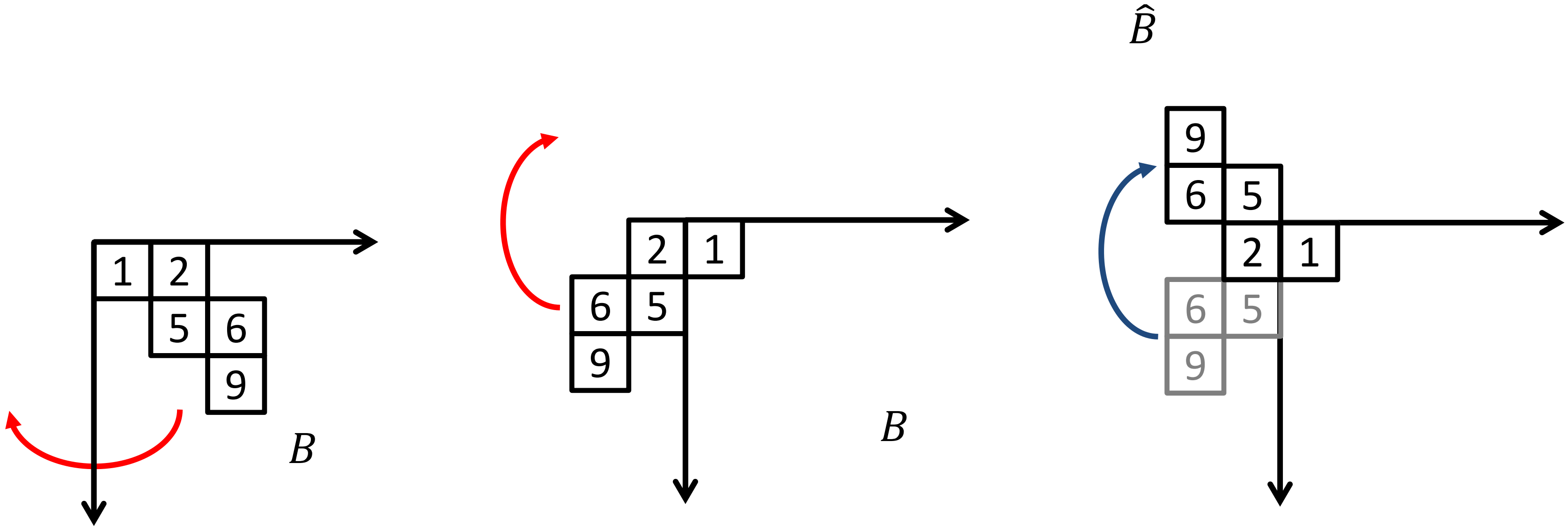




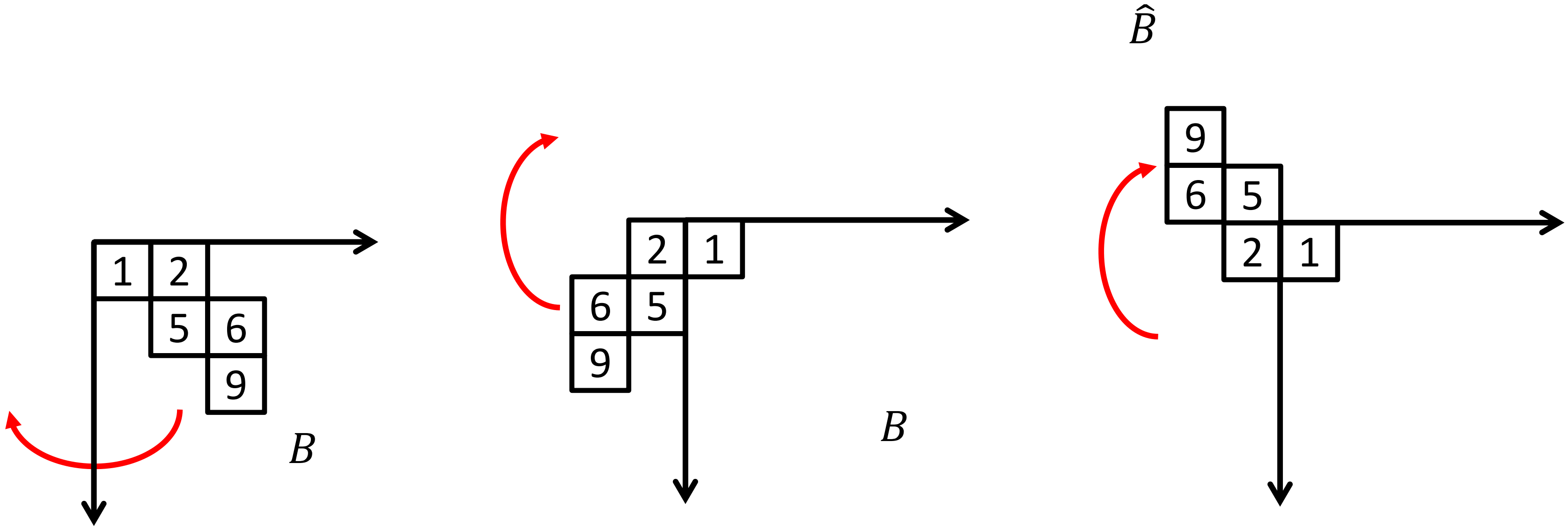
Reflection



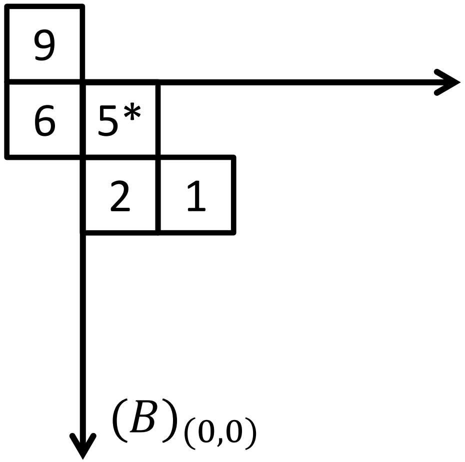
Reflection



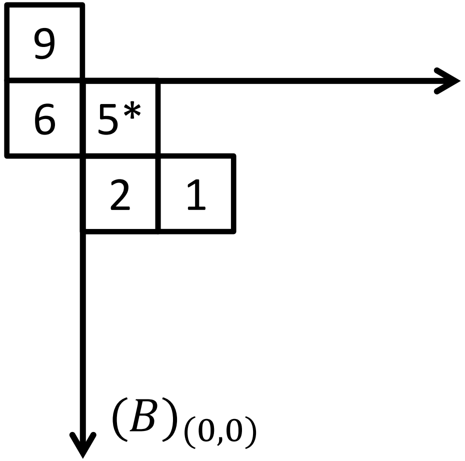
Reflection

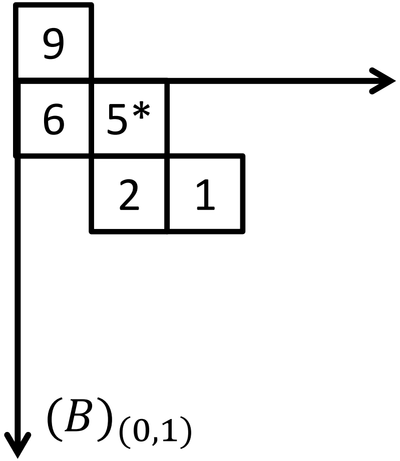


Translation

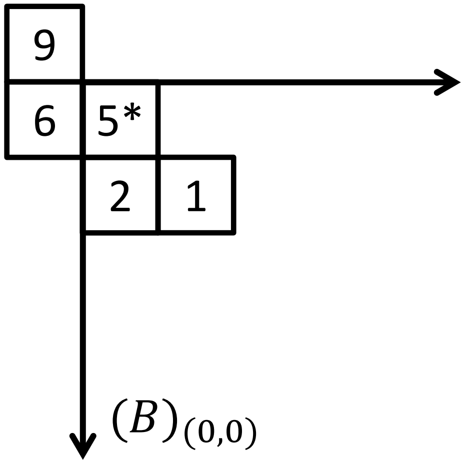


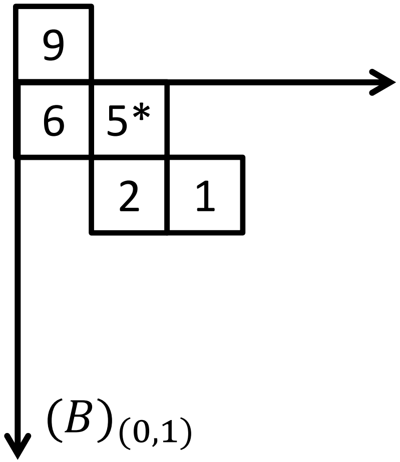
Translation

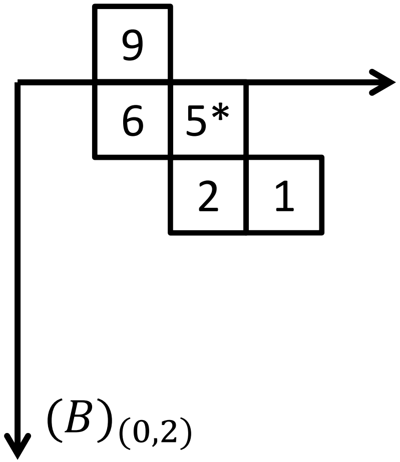




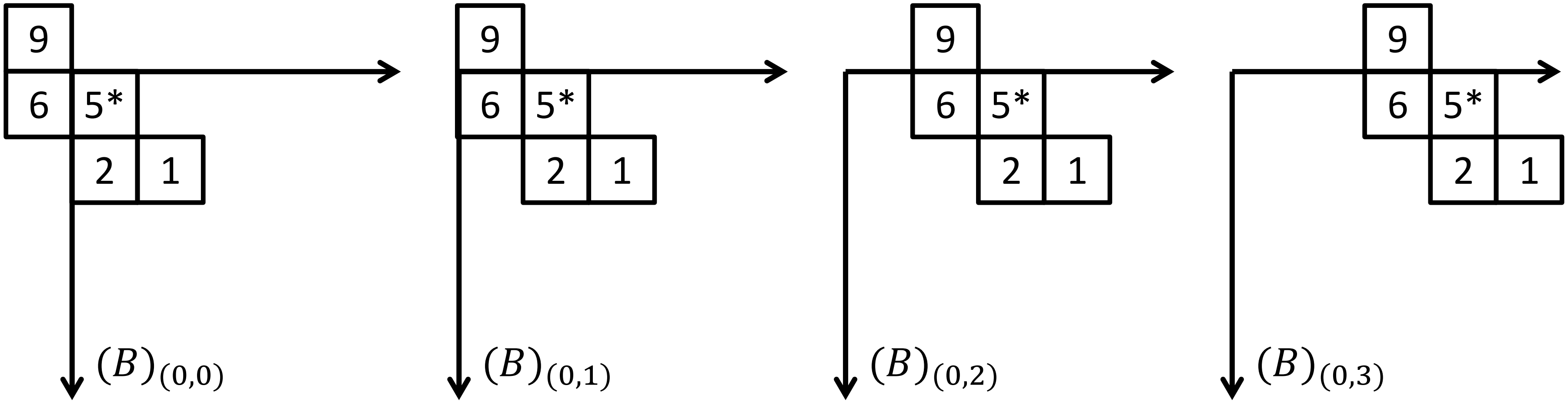
Translation



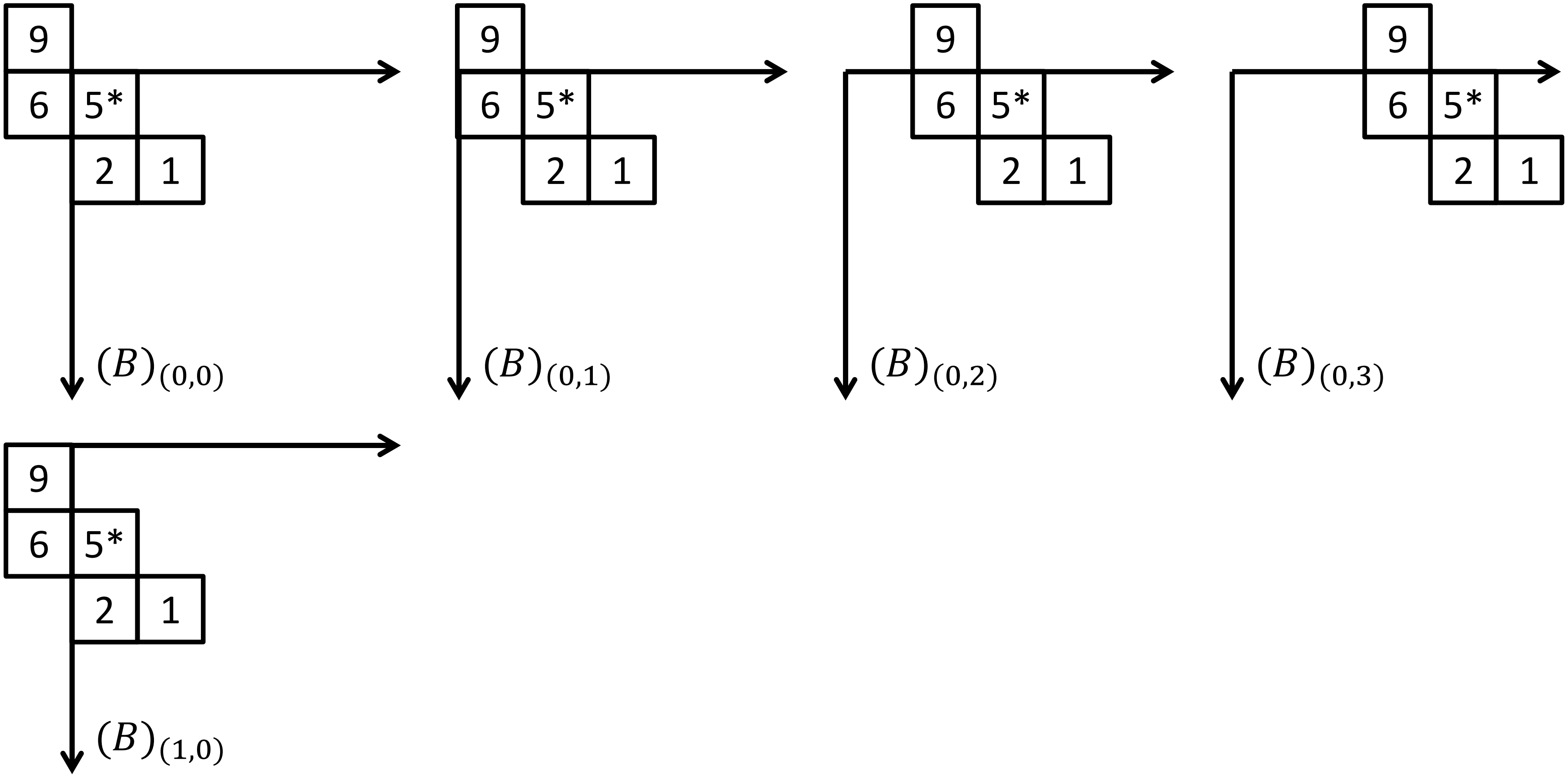




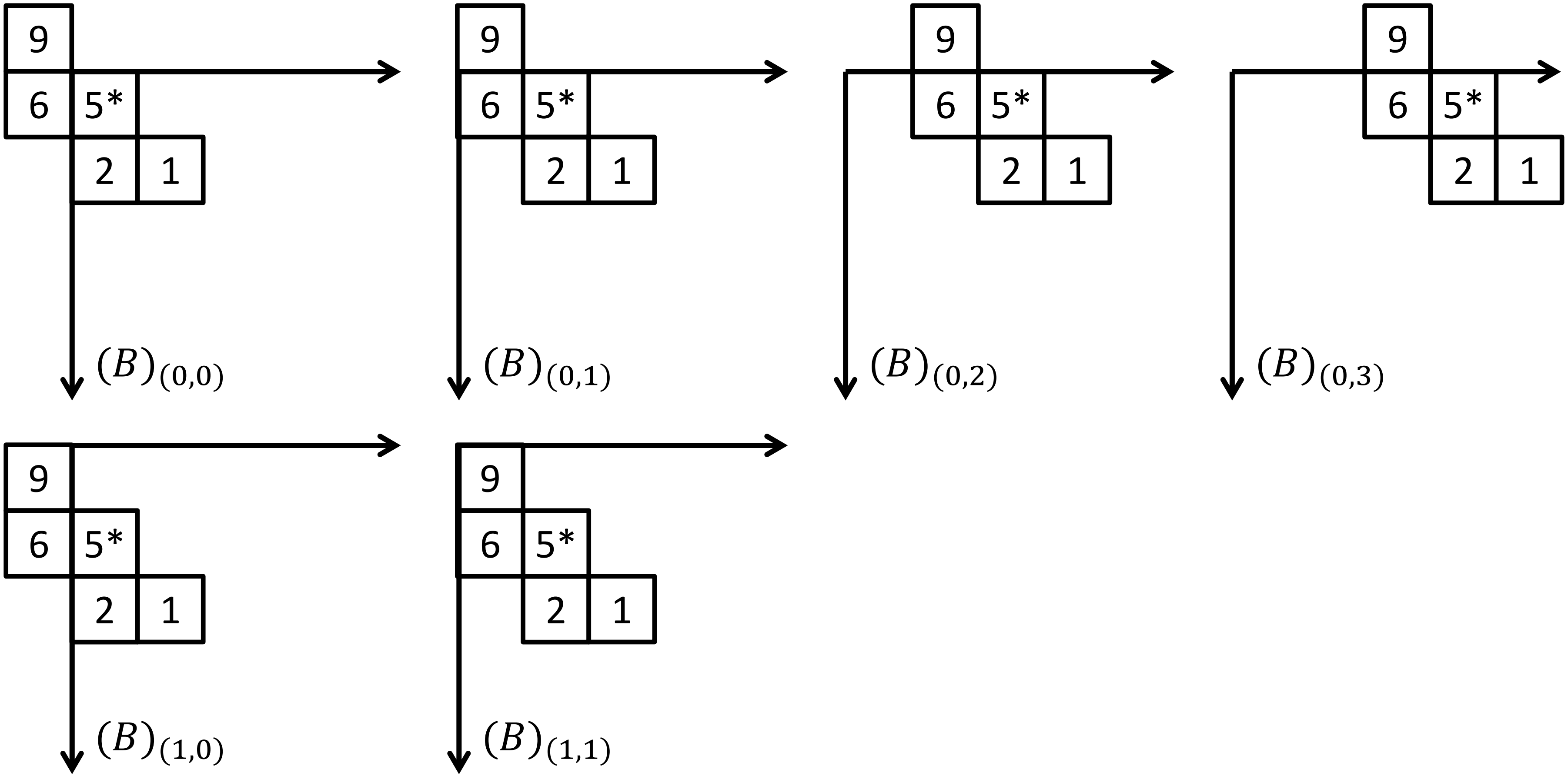
Translation



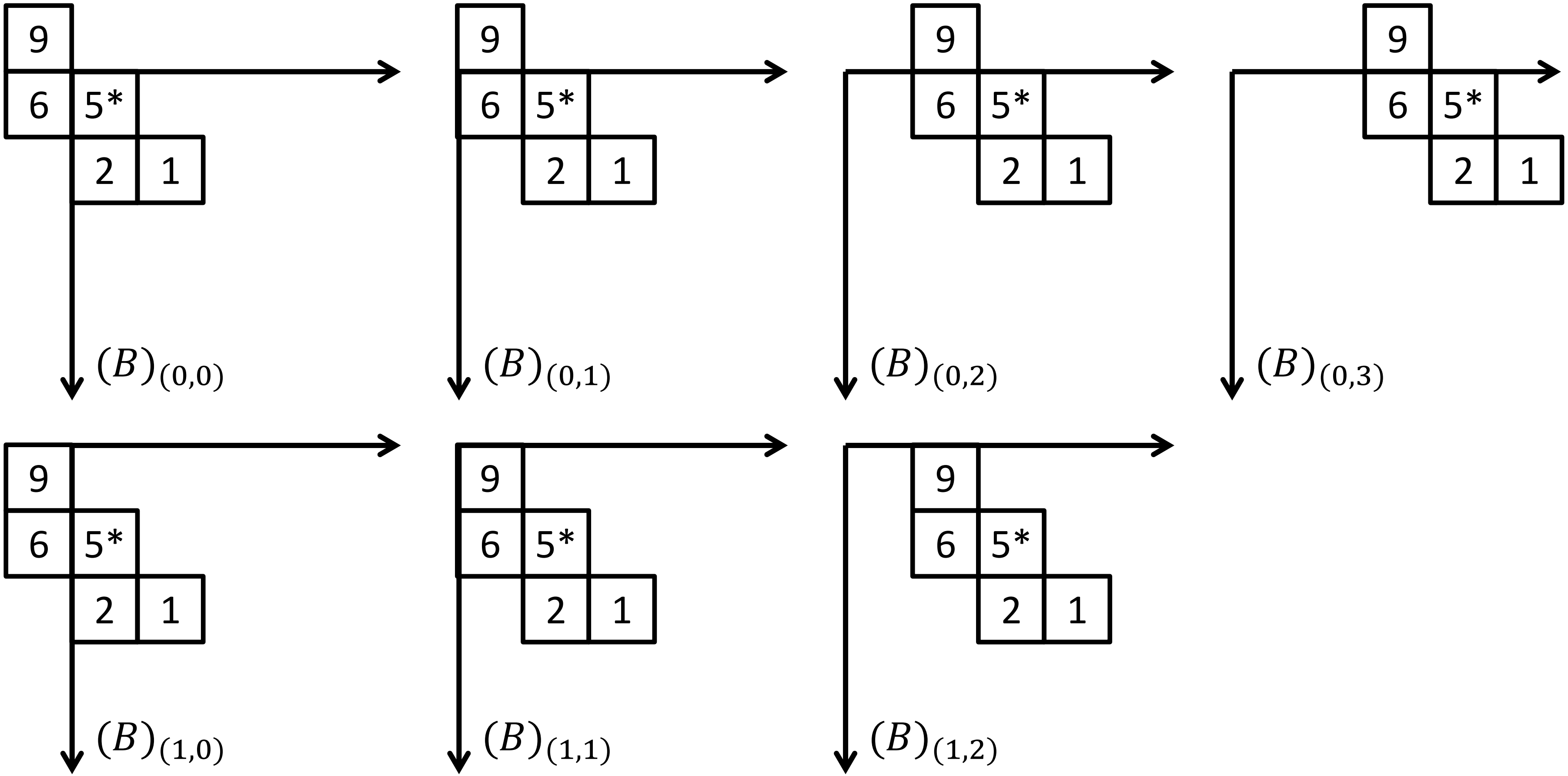
Translation



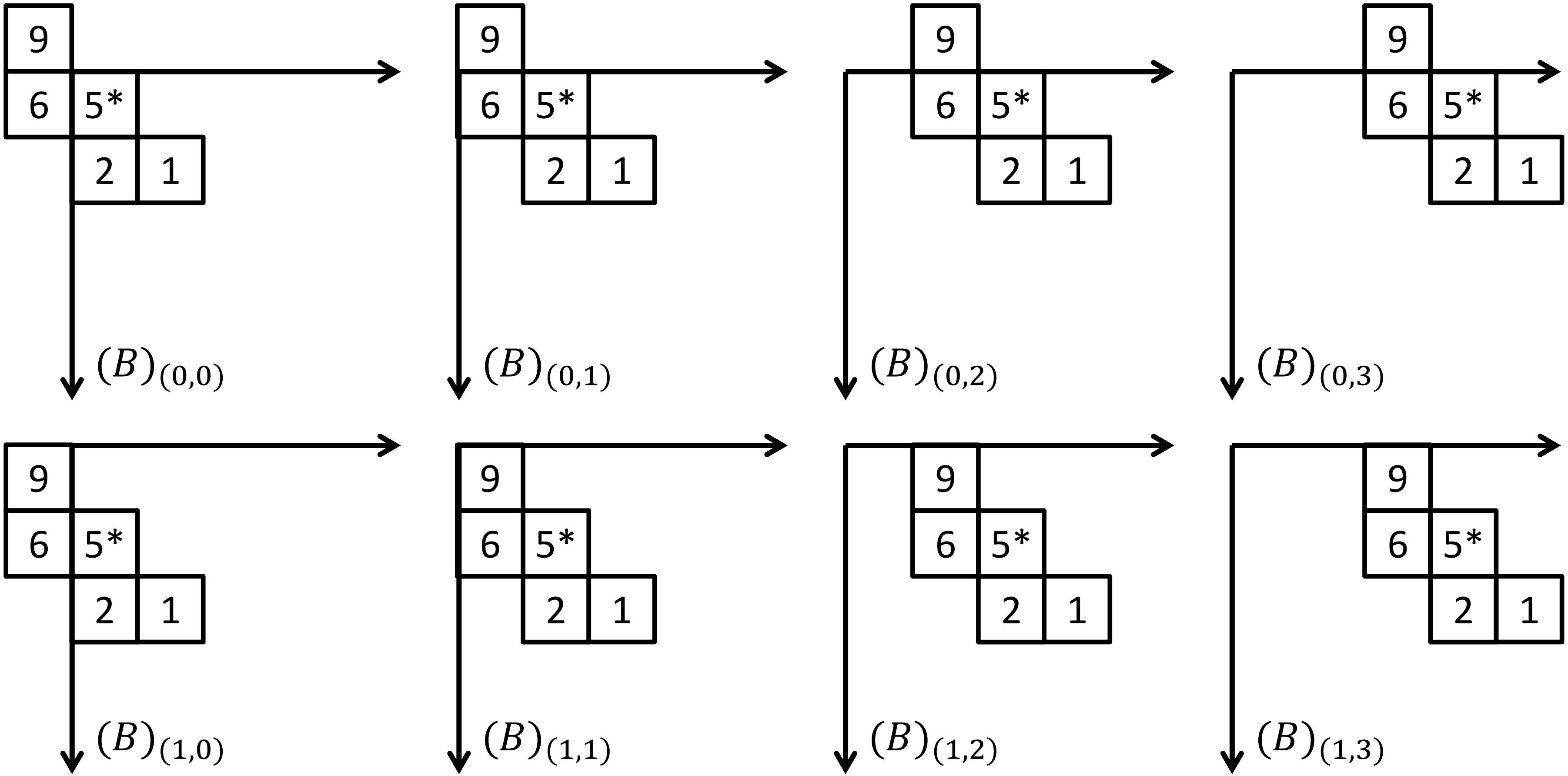
Translation



Translation



Translation



Structuring elements

- Structuring elements (SE)
  - Small sets or sub-images used to examine an image for properties of interest.

0	1	0
1	1	1
0	1	0

1	1	1
1	1	1
1	1	1

1
1
1
1
1

			1			
		1	1	1		
	1	1	1	1	1	
1	1	1	1	1	1	1
	1	1	1	1	1	
		1	1	1		
			1			

1	1	1
0	0	1
0	1	0

0	1	0
1	1	1
0	1	0*

- *The * indicates the center of the structuring element.*
- *When omitted, the center of the SE corresponds to the center of the array.*

EROSION

Erosion

- **Erosion** and **dilation** are fundamental operations of mathematical morphology.
  - Most morphological algorithms are derived from these two operations.

- The erosion of a set A by an SE B is:
  - $\mathrm{A} \ominus B = \{z \mid (B)_z \subseteq A\}$
  - The erosion of *A* by *B* is the set of all *z* so that *B* translated by *z* is contained in *A*.

- An alternative definition for the same case:
  - An alternative definition for the same case:
  - Saying that B is contained in A is equivalent to saying that B has no elements in common with the background.
  - $\mathrm{A} \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$

Erosion

- $A \ominus B = \{z | (B)_z \subseteq A\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	0
2	0	1	1	1	1	1	1	0
3	0	1	1	1	1	1	1	0
4	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0
7	0	0	0	0	0	0	0	0

A

1	1	1
1	1	1
1	1	1

B

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$A \ominus B$

Erosion

- $A \ominus B = \{z | (B)_z \subseteq A\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	0
2	0	1	1	1	1	1	1	0
3	0	1	1	1	1	1	1	0
4	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0
7	0	0	0	0	0	0	0	0

A

1	1	1
1	1	1
1	1	1

B

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$A \ominus B$

Erosion

- $A \ominus B = \{z|(B)_z \subseteq A\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	0
2	0	1	1	1	1	1	1	0
3	0	1	1	1	1	1	1	0
4	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0
7	0	0	0	0	0	0	0	0

A

1	1	1
1	1	1
1	1	1

B

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	0	0	1	1	1	1	0	0
3	0	0	1	1	1	1	0	0
4	0	0	1	1	1	1	0	0
5	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

$A \ominus B$

Erosion

- $A \ominus B = \{z | (B)_z \subseteq A\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	0
2	0	1	1	1	1	1	1	0
3	0	1	1	1	1	1	1	0
4	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0
7	0	0	0	0	0	0	0	0

A

1
1
1
1
1

B

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$A \ominus B$

Erosion

- $A \ominus B = \{z \mid (B)_z \subseteq A\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	0
2	0	1	1	1	1	1	1	0
3	0	1	1	1	1	1	1	0
4	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0
7	0	0	0	0	0	0	0	0

A

1
1
1
1
1

B

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$A \ominus B$

Erosion

- $A \ominus B = \{z | (B)_z \subseteq A\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	0
2	0	1	1	1	1	1	1	0
3	0	1	1	1	1	1	1	0
4	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0
7	0	0	0	0	0	0	0	0

A

1
1
1
1
1

B

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	1	1	1	1	1	1	0
4	0	1	1	1	1	1	1	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

$A \ominus B$

DILATION

Dilation

- The dilation of a set A by an SE B is:
  - $A \oplus B = \left\{ z \middle| (\hat{B})_z \cap A \neq \emptyset \right\}$

- Firstly, B is reflected around its origin.
  - The dilation of *A* by *B* is the set of all displacements *z*, such that $\hat{B}$ (reflection of *B*) and *A* overlap in at least one element.

- An alternative definition for the same case:
  - $A \oplus B = \left\{ z \middle| \left[(\hat{B})_z \cap A \right] \subseteq A \right\}$

Dilation

- $A \oplus B = \left\{ z \middle| (\hat{B})_z \cap A \neq \emptyset \right\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	0	0	1	1	1	1	0	0
3	0	0	1	1	1	1	0	0
4	0	0	1	1	1	1	0	0
5	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

A

1	1	1
1	1	1
1	1	1

B

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$A \oplus B$

Dilation

- $A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	0	0	1	1	1	1	0	0
3	0	0	1	1	1	1	0	0
4	0	0	1	1	1	1	0	0
5	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

A

1	1	1
1	1	1
1	1	1

B

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$A \oplus B$

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- $A \oplus B = \{z \,|\, (\hat{B})_z \cap A \neq \emptyset\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	0	0	1	1	1	1	0	0
3	0	0	1	1	1	1	0	0
4	0	0	1	1	1	1	0	0
5	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

A

1	1	1
1	1	1
1	1	1

B

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	0
2	0	1	1	1	1	1	1	0
3	0	1	1	1	1	1	1	0
4	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0
7	0	0	0	0	0	0	0	0

$A \oplus B$

Dilation

- $A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	0	0	1	1	1	1	0	0
3	0	0	1	1	1	1	0	0
4	0	0	1	1	1	1	0	0
5	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

A

1
1
1
1
1

B

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$A \oplus B$

Dilation

- $A \oplus B = \left\{ z \middle| (\hat{B})_z \cap A \neq \emptyset \right\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	0	0	1	1	1	1	0	0
3	0	0	1	1	1	1	0	0
4	0	0	1	1	1	1	0	0
5	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

A

1
1
1
1
1

B

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$A \oplus B$

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- $A \oplus B = \{z \,|\, (\hat{B})_z \cap A \neq \emptyset\}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	0	0	1	1	1	1	0	0
3	0	0	1	1	1	1	0	0
4	0	0	1	1	1	1	0	0
5	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

A

1
1
1
1
1

B

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0	0	0	1	1	1	1	0	0
1	0	0	1	1	1	1	0	0
2	0	0	1	1	1	1	0	0
3	0	0	1	1	1	1	0	0
4	0	0	1	1	1	1	0	0
5	0	0	1	1	1	1	0	0
6	0	0	1	1	1	1	0	0
7	0	0	1	1	1	1	0	0

$A \oplus B$

DUALITY

Duality

- Dilation and erosion are dual operations:
  - $(A \ominus B) = A^c \oplus \hat{B}$
  - $(A \oplus B) = A^c \ominus \hat{B}$

  - The **erosion** of *A* by *B* is the complement of the dilation of *Ac* by $\hat{B}$
  - The **dilation** of *A* by *B* is the complement of the erosion of *Ac* by $\hat{B}$
  - When the SE is symmetrical, dilation can be obtained through erosion of the image background.
    - As well as, obtaining erosion through dilation of the image background

GRAY LEVEL MATHEMATICAL MORPHOLOGY

Gray level mathematical morphology

- Mathematical morphology in gray levels using thresholding decomposition:
  1. Decompose the intensity image f(x, y) by thresholding into all possible gray levels.
     - Each thresholding will generate a binary image
  2. Apply the morphological operation on each binary image
  3. Reconstruct the output image g(x, y) by “stacking” the processed binary images.

Gray level mathematical morphology

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	1	0	5	7	5
2	0	0	2	3	0	0	0	6
3	0	0	0	0	0	0	4	7
4	0	0	0	0	0	0	0	0
5	0	1	2	0	0	3	0	0
6	0	1	3	0	0	0	0	0
7	0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1	1
2	0	0	1	1	0	0	0	1
3	0	0	0	0	0	0	1	1
4	0	0	0	0	0	0	0	0
5	0	1	1	0	0	1	0	0
6	0	1	1	0	0	0	0	0
7	0	0	0	0	0	0	0	0

0

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	1	1
2	0	0	1	1	0	0	0	1
3	0	0	0	0	0	0	1	1
4	0	0	0	0	0	0	0	0
5	0	0	1	0	0	1	0	0
6	0	0	1	0	0	0	0	0
7	0	0	0	0	0	0	0	0

1

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	1	1
2	0	0	0	1	0	0	0	1
3	0	0	0	0	0	0	1	1
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	1	0	0
6	0	0	1	0	0	0	0	0
7	0	0	0	0	0	0	0	0

2

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	1	1
2	0	0	0	0	0	0	0	1
3	0	0	0	0	0	0	1	1
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

3

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	1	1
2	0	0	0	0	0	0	0	1
3	0	0	0	0	0	0	0	1
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

4

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	1	0
2	0	0	0	0	0	0	0	1
3	0	0	0	0	0	0	0	1
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

5

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	1	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	1
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

6

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	howpublished = {\url{https://github.com/joaofmari/SIN392_Introduction-to-digital-image-processing_2023}}
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THE END