Aula 06 – Filtragem espacial I

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Roteiro

- Convolução e correlação
- Exemplo: Convolução

CONVOLUÇÃO E CORRELAÇÃO

Convolução e Correlação

Correlação

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolução

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

f(x,y)

	0	1	2	3
0	f(0, 0)	f(0, 1)	f(0, 2)	…
1	f(1, 0)	f(1, 1)	f(1, 2)	…
2	f(2, 0)	f(2, 1)	f(2, 2)	…
3	…	…	…	…

w(s,t)

	-1	0	1
-1	w(-1,-1)	w(-1, 0)	w(-1, 1)
0	w(0, -1)	w(0, 0)	w(0, 1)
1	w(1, -1)	w(1, 0)	w(1, 1)

Convolução e correlação

Correlação

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolução

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

f(x,y)

	0	1	2	3
0	***f(0, 0)***	***f(0, 1)***	***f(0, 2)***	…
1	***f(1, 0)***	***f(1, 1)***	***f(1, 2)***	…
2	***f(2, 0)***	***f(2, 1)***	***f(2, 2)***	…
3	…	…	…	…

w(s,t)

	-1	0	1
-1	***w(-1,-1)***	***w(-1, 0)***	***w(-1, 1)***
0	***w(0, -1)***	***w(0, 0)***	***w(0, 1)***
1	***w(1, -1)***	***w(1, 0)***	***w(1, 1)***

Convolução e correlação

Correlação

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolução

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

f(x,y)

	0	1	2	3
0	***f(0, 0)***	***f(0, 1)***	...	...
1	***f(1, 0)***	***f(1, 1)***	...	...
2	...	...	...	...
3	...	...	...	...

w(s,t)

	-1	0	1
-1	***w(-1,-1)***	***w(-1, 0)***	***w(-1, 1)***
0	***w(0, -1)***	***w(0, 0)***	***w(0, 1)***
1	***w(1, -1)***	***w(1, 0)***	***w(1, 1)***

Convolução e correlação

Correlação

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolução

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

f(x,y)

	0	1	2	3
0	***f(0, 0)***	***f(0, 1)***	...	...
1	***f(1, 0)***	***f(1, 1)***	...	...
2	...	...	...	...
3	...	...	...	...

w(s,t)

	-1	0	1
-1	***w(-1,-1)***	***w(-1, 0)***	***w(-1, 1)***
0	***w(0, -1)***	***w(0, 0)***	***w(0, 1)***
1	***w(1, -1)***	***w(1, 0)***	***w(1, 1)***

padding

EXEMPLO: CONVOLUÇÃO

Convolução

f(x,y)

	0	*1*	*2*	*3*
0	1	0	6	4
1	2	1	7	2
2	5	0	2	3
3	5	0	3	2

w(s,t)

	-1	*0*	*1*
-1	1	2	3
0	4	5	6
1	7	8	9

Convolução

$f(x,y)$

	0	1	2	3	
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

padding

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

$f(x,y)$

	0	1	2	3		
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

Convolução

f(x,y)

	0	*1*	*2*	*3*	
	0	*0*	*0*	*0*	*0*	*0*
0	*0*	1	0	6	4	*0*
1	*0*	2	1	7	2	*0*
2	*0*	5	0	2	3	*0*
3	*0*	5	0	3	2	*0*
	0	*0*	*0*	*0*	*0*	*0*

w(s,t)

	-1	*0*	*1*
-1	1	2	3
0	4	5	6
1	7	8	9

g(x,y)

	0	*1*	*2*	*3*	
	0	*0*	*0*	*0*	*0*	*0*
0	*0*	0	0	0	0	*0*
1	*0*	0	0	0	0	*0*
2	*0*	0	0	0	0	*0*
3	*0*	0	0	0	0	*0*
	0	*0*	*0*	*0*	*0*	*0*

Convolução

f(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

g(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

convolução

Convolução

f(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	1	0	6	4	0
2	0	2	1	7	2	0
3	0	5	0	2	3	0
	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180°

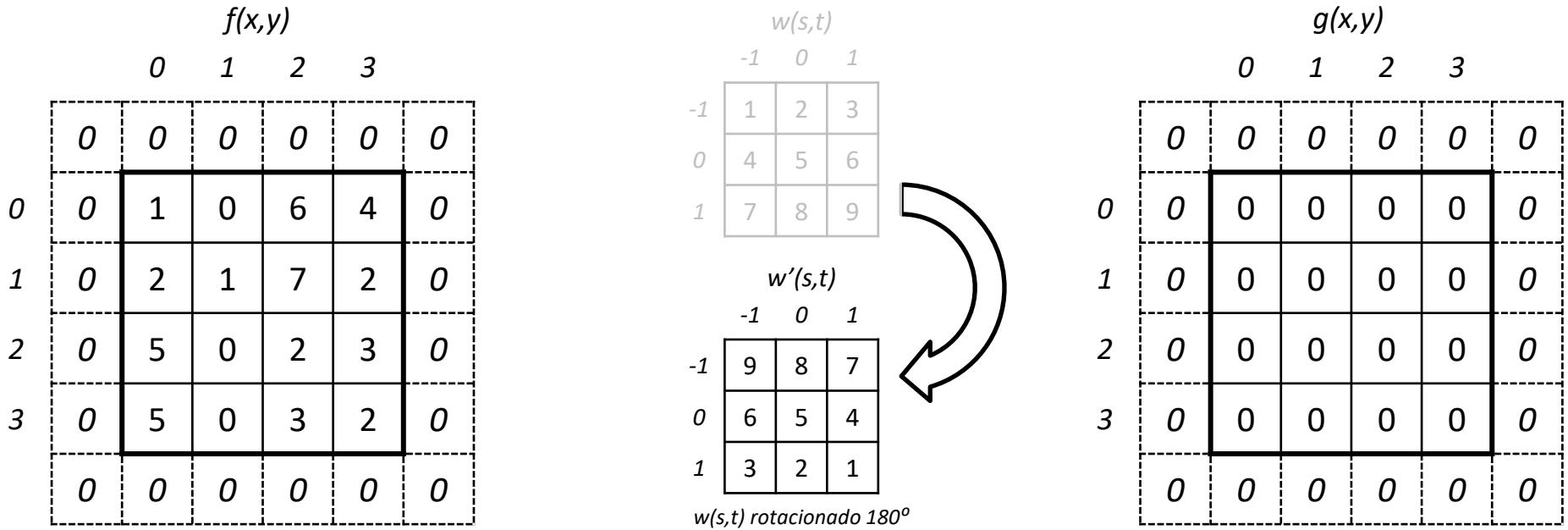
g(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

correlação

Convolução



$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

correlação

Convolução

f(x,y)

	0	1	2	3		
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180º

g(x,y)

	0	1	2	3		
	0	0	0	0	0	0
0	0	10	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 0\times 9 + 0\times 8 + 0\times 7 \\ 0\times 6 + 1\times 5 + 0\times 4 \\ 0\times 3 + 2\times 2 + 1\times 1 \end{matrix} = 10$$

Convolução

f(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	1	0	6	4
2	0	2	1	7	2
3	0	5	0	2	3
4	0	5	0	3	2
5	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180º

g(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	10	45	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{array}{l} 0\times 9 + 0\times 8 + 0\times 7 + \\ 1\times 6 + 0\times 5 + 6\times 4 + \\ 2\times 3 + 1\times 2 + 7\times 1 \end{array} = 45$$

Convolução

f(x,y)

	0	1	2	3	
0	0	0	0	0	0
0	1	0	6	4	0
1	2	1	7	2	0
2	5	0	2	3	0
3	5	0	3	2	0
0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180º

g(x,y)

	0	1	2	3	
0	0	0	0	0	0
0	10	45	65	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{array}{l} 0\times 9 + 0\times 8 + 0\times 7 + \\ 0\times 6 + 6\times 5 + 4\times 4 + \\ 1\times 3 + 7\times 2 + 2\times 1 \end{array} = 65$$

Convolução

f(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180º

g(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	10	45	65	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) =$$

Convolução

$f(x,y)$

	0	1	2	3		
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

$w'(s,t)$

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

$w(s,t)$ rotacionado 180^o

$g(x,y)$

	0	1	2	3		
	0	0	0	0	0	0
0	0	10	45	65	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) =$$

Convolução

f(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180º

g(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	10	45	65	81	0
1	0	32	113	132	150	0
2	0	58	131	109	119	0
3	0	65	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) =$$

Convolução

f(x,y)

	0	1	2	3		
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180º

g(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	10	45	65	81	0
1	0	32	113	132	150	0
2	0	58	131	109	119	0
3	0	65	101	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 5\times 9 + 0\times 8 + 2\times 7 + \\ 5\times 6 + 0\times 5 + 3\times 4 + \\ 0\times 3 + 0\times 2 + 0\times 1 \end{matrix} = 101$$

Convolução

f(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	1	0	6	4	0
2	0	2	1	7	2	0
3	0	5	0	2	3	0
	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180º

g(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	10	45	65	81	0
2	0	32	113	132	150	0
3	0	58	131	109	119	0
	0	65	101	60	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 0\times 9 + 2\times 8 + 3\times 7 + \\ 0\times 6 + 3\times 5 + 2\times 4 + \\ 0\times 3 + 0\times 2 + 0\times 1 \end{matrix} = 60$$

Convolução

f(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	1	0	6	4	0
2	0	2	1	7	2	0
3	0	5	0	2	3	0
	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180º

g(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	10	45	65	81	0
2	0	32	113	132	150	0
3	0	58	131	109	119	0
	0	65	101	60	70	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{array}{l} 2\times 9 + 3\times 8 + 0\times 7 + \\ 3\times 6 + 2\times 5 + 0\times 4 + \\ 0\times 3 + 0\times 2 + 0\times 1 \end{array} = 70$$

Convolução

f(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180º

g(x,y)

	0	1	2	3	
	0	0	0	0	0	0
0	0	10	45	65	81	0
1	0	32	113	132	150	0
2	0	58	131	109	119	0
3	0	65	101	60	70	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{array}{l} 2\times 9 + 3\times 8 + 0\times 7 + \\ 3\times 6 + 2\times 5 + 0\times 4 + \\ 0\times 3 + 0\times 2 + 0\times 1 \end{array} = 70$$

Convolução

$f(x,y)$

	0	1	2	3	
0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

$w'(s,t)$

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

$w(s,t)$ rotacionado 180^o

$g(x,y)$

	0	1	2	3	
0	0	0	0	0	0
0	0	10	45	65	81	0
1	0	32	113	132	150	0
2	0	58	131	109	119	0
3	0	65	101	60	70	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolução

f(x,y)

	0	1	2	3
0	1	0	6	4
1	2	1	7	2
2	5	0	2	3
3	5	0	3	2

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotacionado 180º

g(x,y)

	0	1	2	3
0	10	45	65	81
1	32	113	132	150
2	58	131	109	119
3	65	101	60	70

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Bibliografia

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FIM