

Aula 13 – Transformada de Fourier

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- Transformada de Fourier
- O par de transformadas de Fourier
- Analisando a equação da Transforma de Fourier
- A Transformada Discreta de Fourier
- Cálculo da DFT

Transformada de Fourier

- A transformada de Fourier de uma função contínua $f(t)$ é definida como:

$$\mathfrak{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t} dt$$

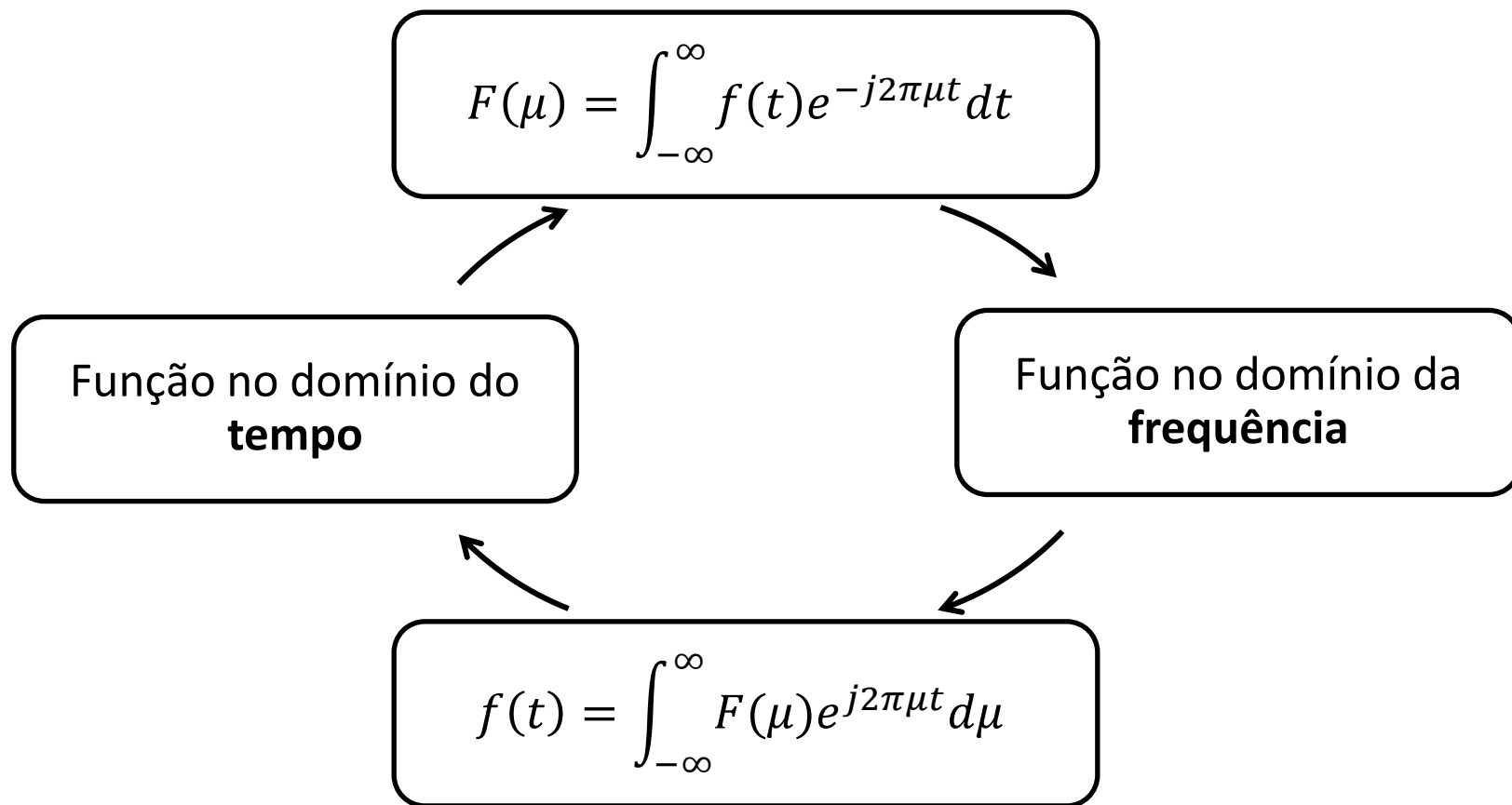
- Como $\mathfrak{F}\{f(t)\}$ é uma função de apenas μ , pois t é eliminada pela integração, a transformada de Fourier de $f(t)$ pode ser expressa como:

$$F(\mu) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t} dt$$

- Dada $F(\mu)$, podemos obter novamente $f(t)$ utilizando a transformada inversa de Fourier, $f(t) = \mathfrak{F}^{-1}\{F(\mu)\}$, expressa como:

$$f(t) = \int_{-\infty}^{\infty} F(\mu)e^{j2\pi\mu t} d\mu$$

O par de transformadas de Fourier



Analizando a equação da Transforma de Fourier

- Utilizando a fórmula de Euler podemos reescrever...

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

- como:

$$F(\mu) = \int_{-\infty}^{\infty} f(t) \cos(2\pi\mu t) - j \operatorname{sen}(2\pi\mu t) dt$$

- $F(\mu)$ é a própria função $f(t)$ multiplicada por termos senoidais com frequências definidas pelos valores de μ .
 - A variável t (tempo) é eliminada pela integração.
 - Na verdade t pode representar qualquer variável continua: tempo, espaço, etc.
 - As unidades da variável de frequência dependem da unidade definida para t :
 - Se t representa o tempo e está em segundos: μ representa ciclos/s (Hz)
 - Se t representa o espaço e está em metros: μ representa ciclos/metro

A Transformada Discreta de Fourier

- Dada a natureza continua da transformada de Fourier, ela não pode ser implementada em um computador.

- A transformada discreta de Fourier é:

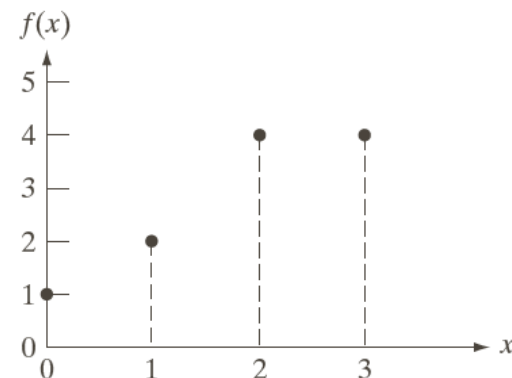
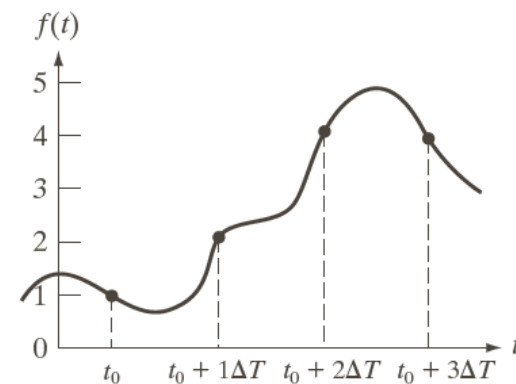
$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M}, \quad u = 0, 1, 2, \dots, M-1$$

- A transformada inversa discreta de Fourier é:

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M}, \quad x = 0, 1, 2, \dots, M-1$$

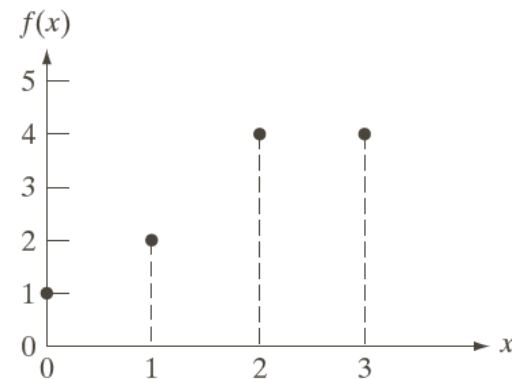
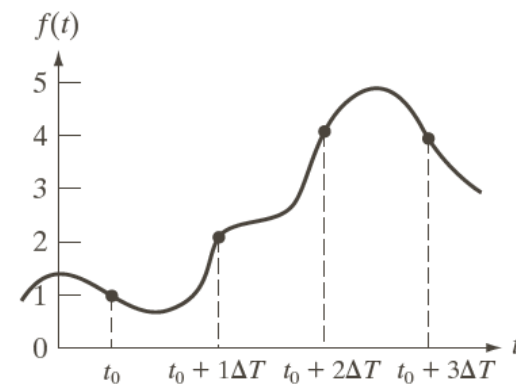
Calculo da DFT

- DFT: $F(u) = \sum_{x=0}^{M-1} f(x) e^{-\frac{j2\pi ux}{M}}$
- $F(0) = \sum_{x=0}^3 f(x) = [f(0) + f(1) + f(2) + f(3)]$
- $F(0) = 1 + 2 + 4 + 4 = 11$
- $|F(0)| = \sqrt{(11)^2 + (0)^2} = 11,0$



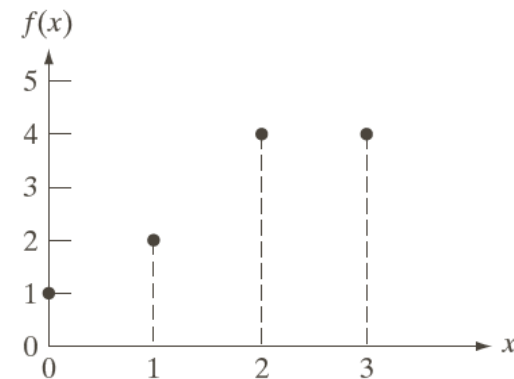
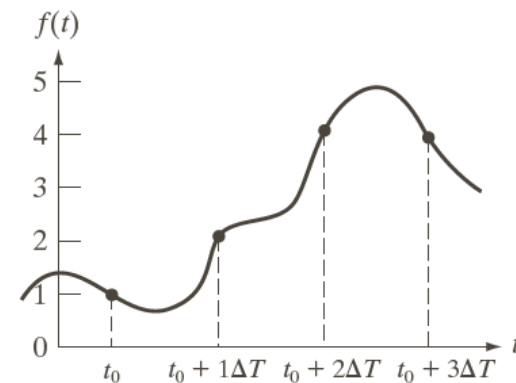
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- $F(1) = \sum_{x=0}^3 f(x) e^{-j2\pi(1)x/4}$
- $F(1) = 1e^{-j2\pi(1)0/4} + 2e^{-j2\pi(1)1/4} + 4e^{-j2\pi(1)2/4} + 4e^{-j2\pi(1)3/4}$
- $F(1) = 1e^0 + 2e^{-j\pi/2} + 4e^{-j\pi} + 4e^{-j3\pi/2} = -3 + 2j$
- $|F(1)| = \sqrt{(-3)^2 + (2)^2} = 3,61$



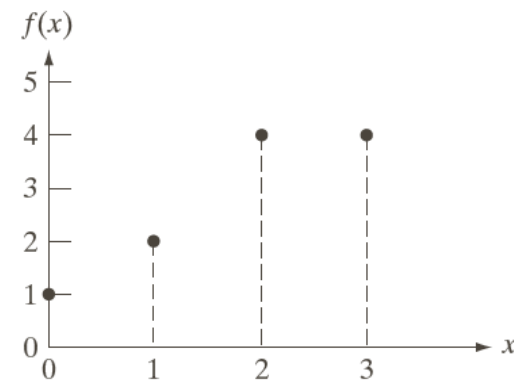
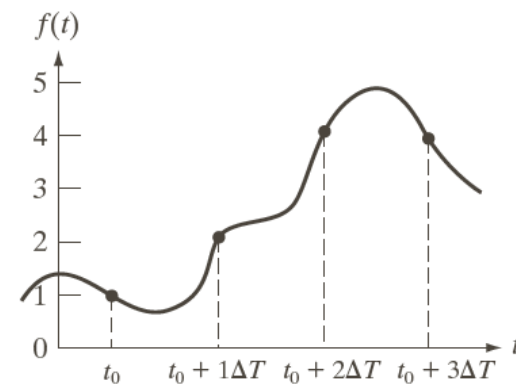
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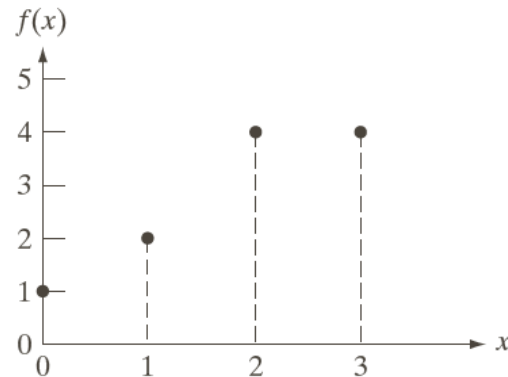
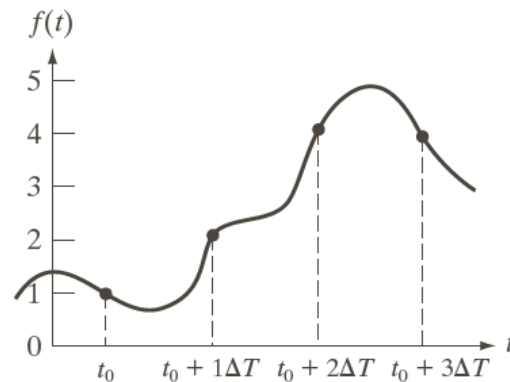
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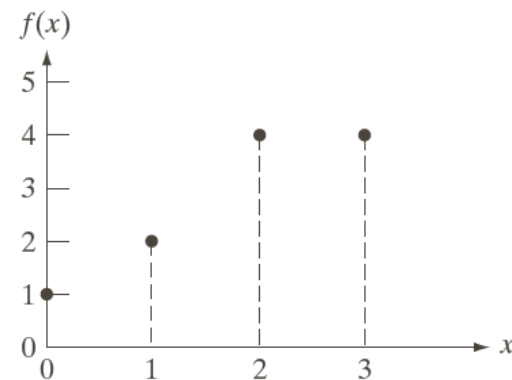
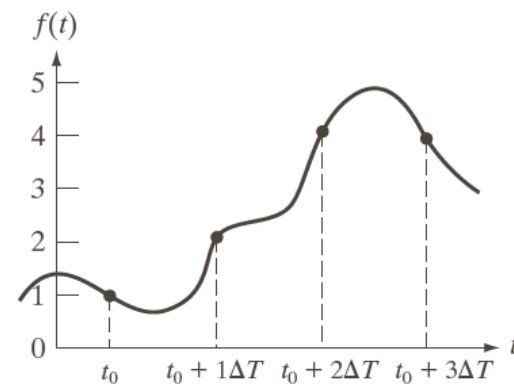
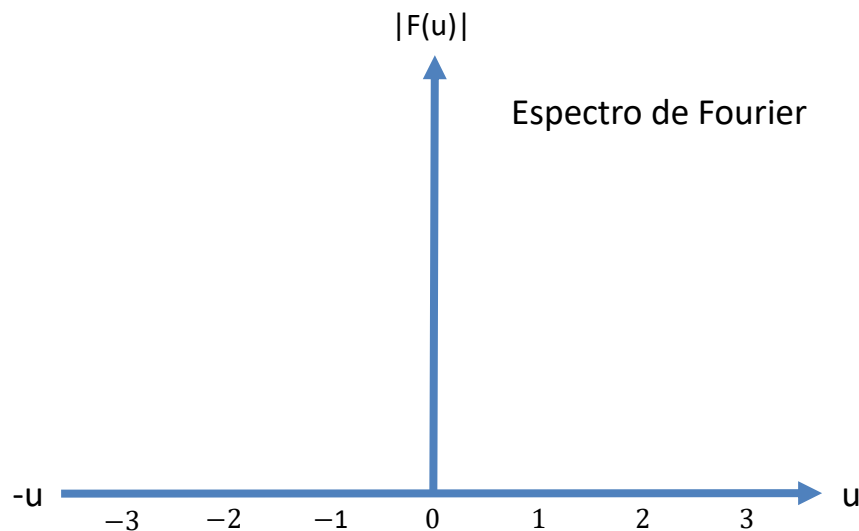
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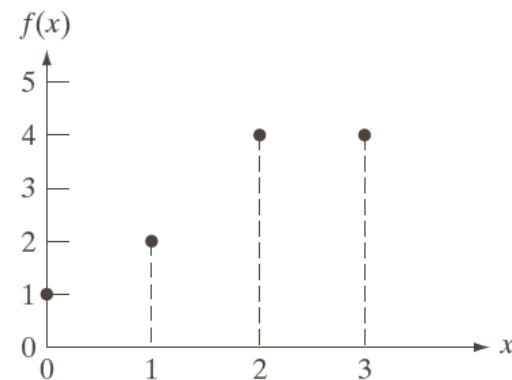
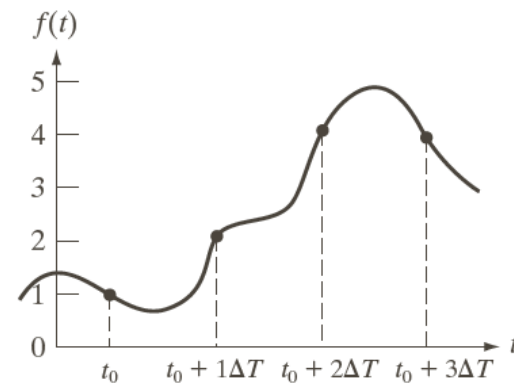
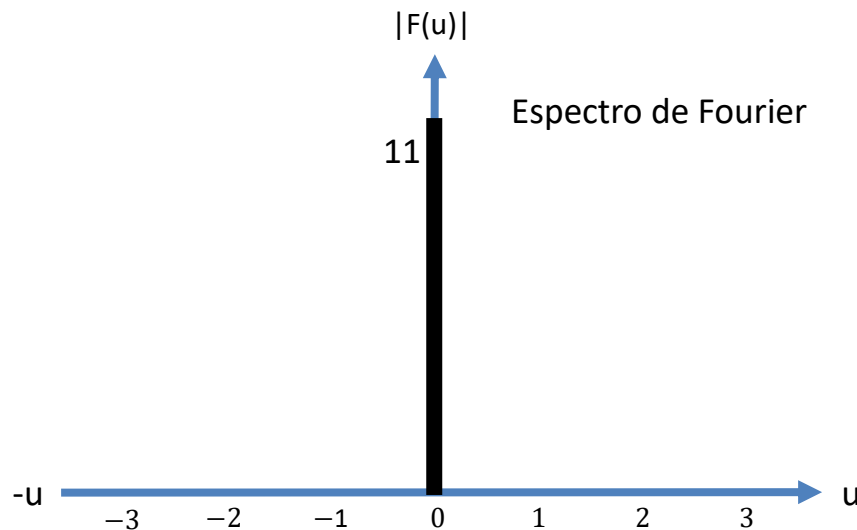
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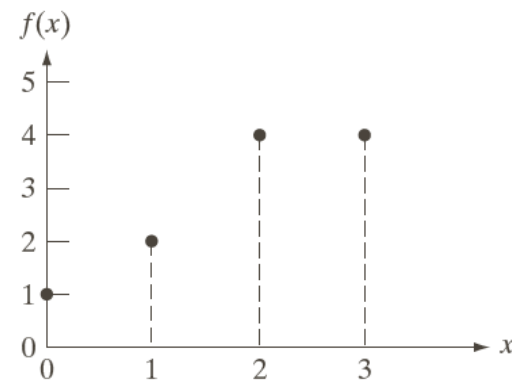
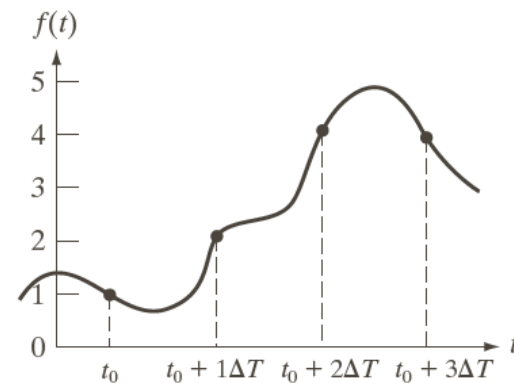
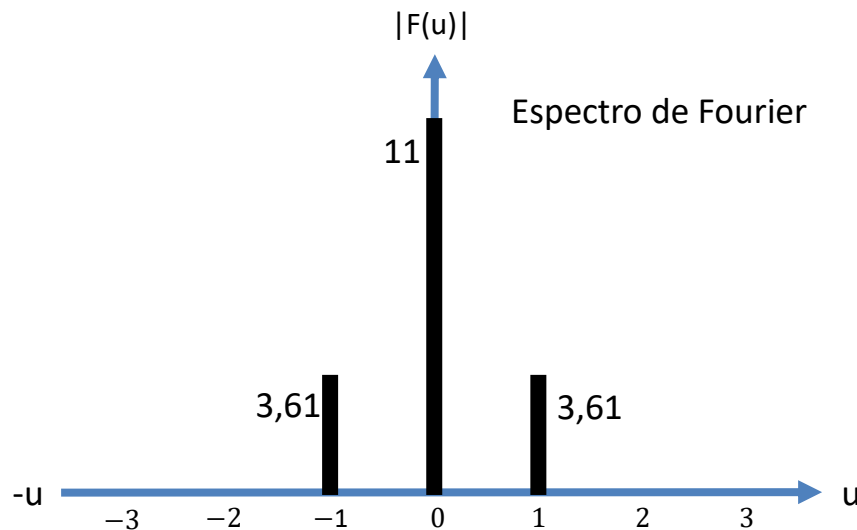
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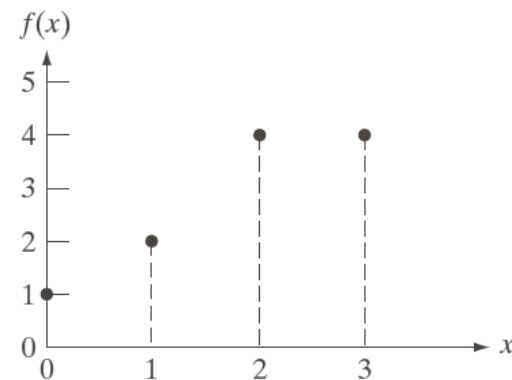
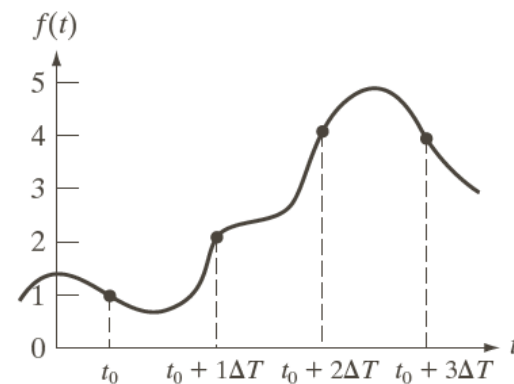
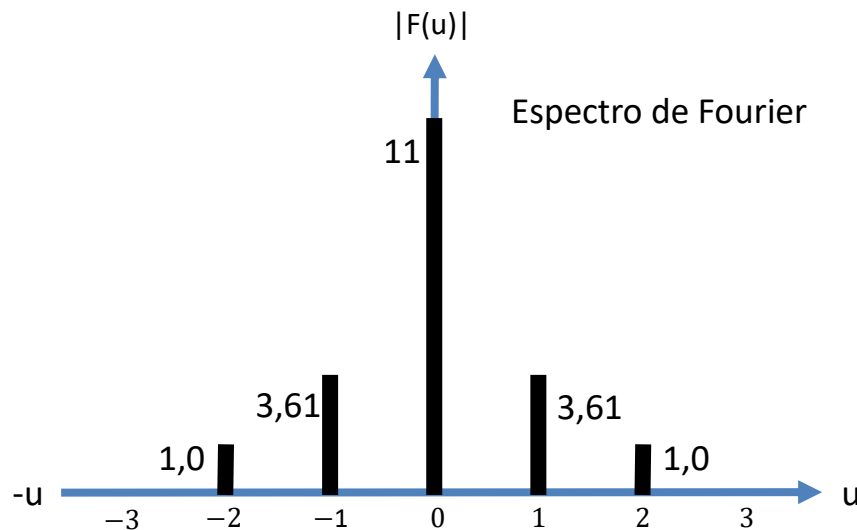
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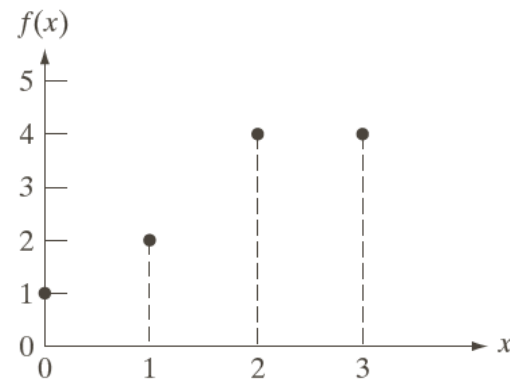
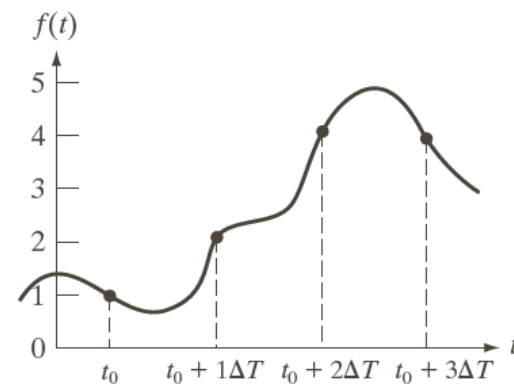
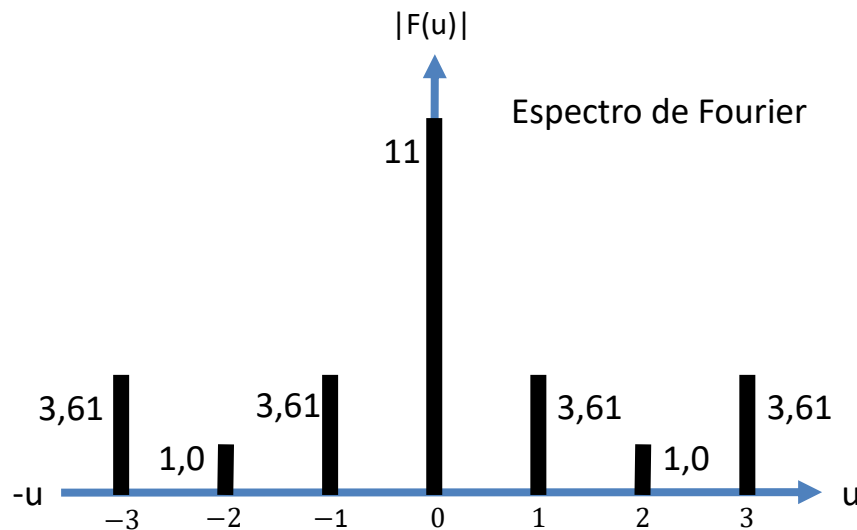
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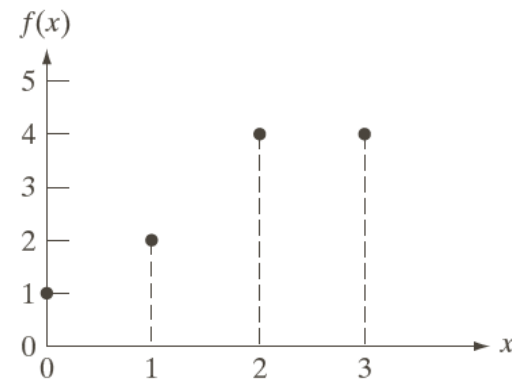
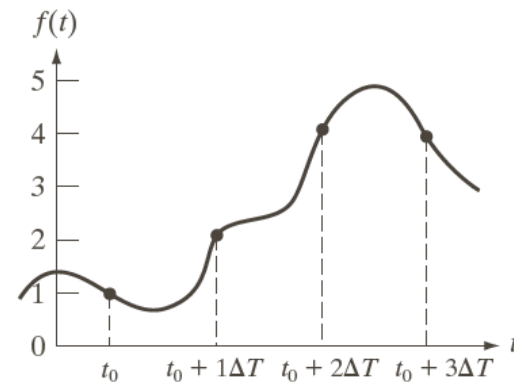
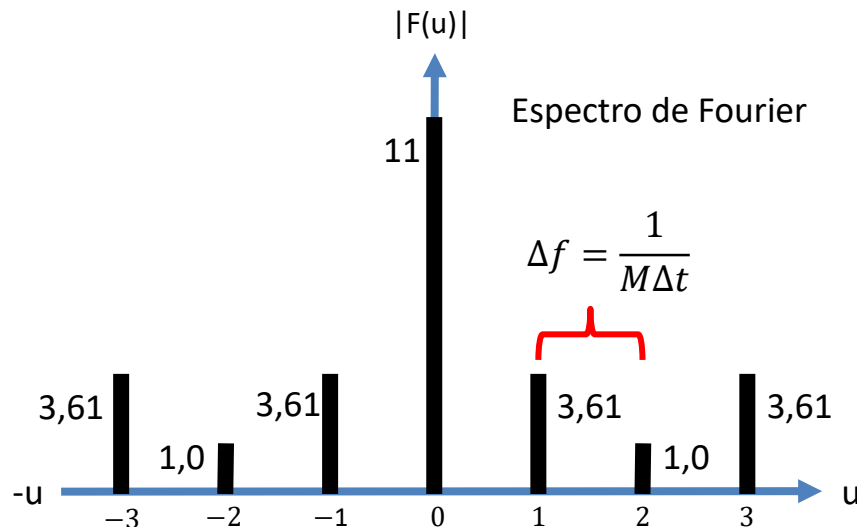
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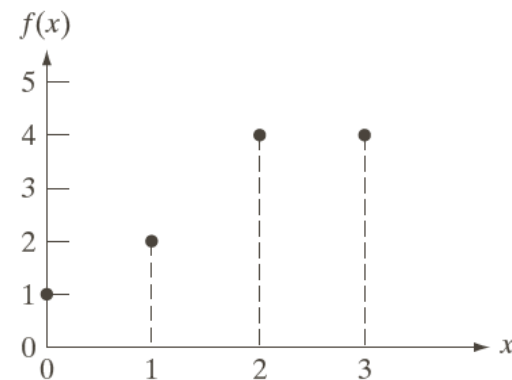
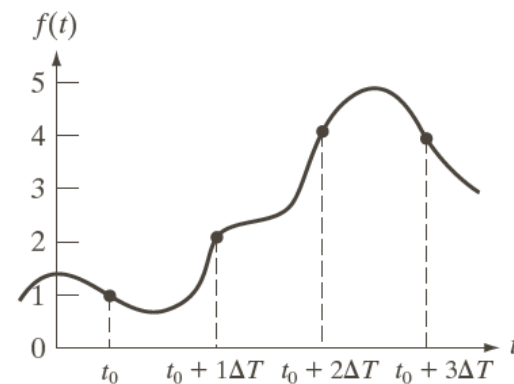
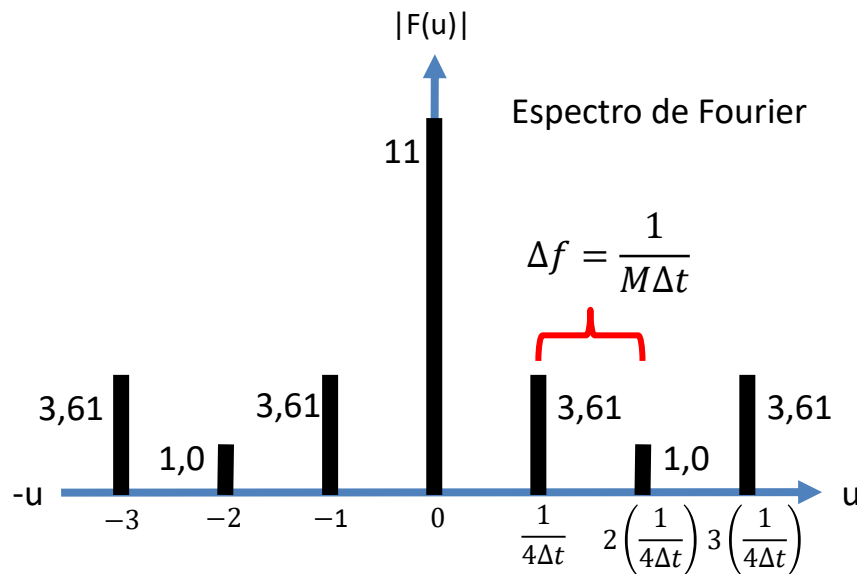
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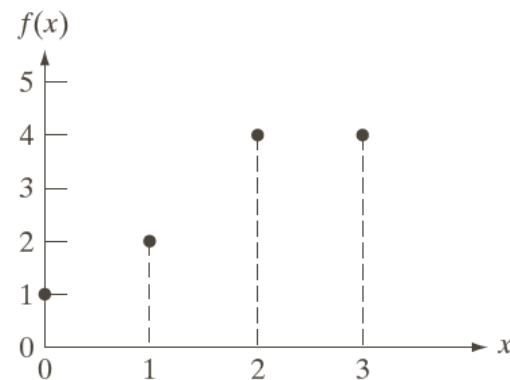
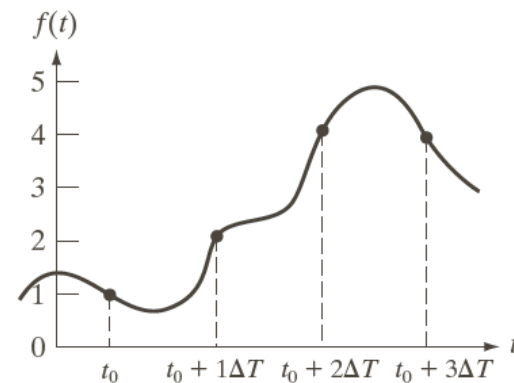
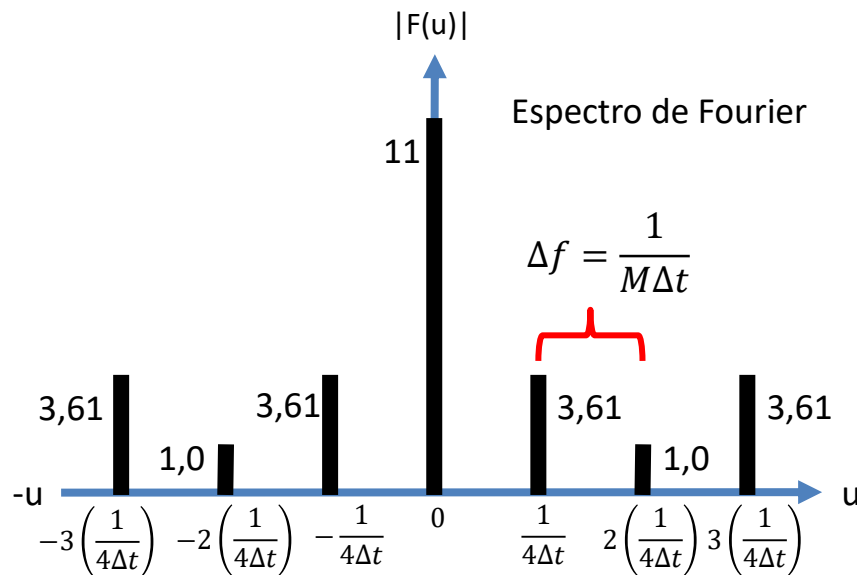
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 - Disponível para download no site do autor (Exclusivo para uso pessoal)
 - <http://dainf.ct.utfpr.edu.br/~hvieir/pub.html>
- GONZALEZ, R.C.; WOODS, R.E.; **Processamento Digital de Imagens**. 3ª edição. Editora Pearson, 2009.
- J. E. R. Queiroz, H. M. Gomes. **Introdução ao Processamento Digital de Imagens**. RITA. v. 13, 2006.
 - <http://www.dsc.ufcg.edu.br/~hmg/disciplinas/graduacao/vc-2016.2/Rita-Tutorial-PDI.pdf>

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FIM