Lecture 06 – Spatial filtering I

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Agenda

- Convolution and correlation
- Example: Convolution

CONVOLUTION AND CORRELATION

Convolution and correlation

Correlation

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolution

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

f(x,y)

	0	1	2	3
0	f(0, 0)	f(0, 1)	f(0, 2)	…
1	f(1, 0)	f(1, 1)	f(1, 2)	…
2	f(2, 0)	f(2, 1)	f(2, 2)	…
3	…	…	…	…

w(s,t)

	-1	0	1
-1	w(-1,-1)	w(-1, 0)	w(-1, 1)
0	w(0, -1)	w(0, 0)	w(0, 1)
1	w(1, -1)	w(1, 0)	w(1, 1)

Convolution and correlation

Correlation

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolution

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

f(x,y)

	0	1	2	3
0	f(0, 0)	f(0, 1)	f(0, 2)	…
1	f(1, 0)	f(1, 1)	f(1, 2)	…
2	f(2, 0)	f(2, 1)	f(2, 2)	…
3	…	…	…	…

w(s,t)

	-1	0	1
-1	w(-1,-1)	w(-1, 0)	w(-1, 1)
0	w(0, -1)	w(0, 0)	w(0, 1)
1	w(1, -1)	w(1, 0)	w(1, 1)

Convolution and correlation

Correlation

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolution

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

f(x,y)

	0	1	2	3
0	***f(0, 0)***	***f(0, 1)***	...	...
1	***f(1, 0)***	***f(1, 1)***	...	...
2	...	...	...	...
3	...	...	...	...

w(s,t)

	-1	0	1
-1	***w(-1,-1)***	***w(-1, 0)***	***w(-1, 1)***
0	***w(0, -1)***	***w(0, 0)***	***w(0, 1)***
1	***w(1, -1)***	***w(1, 0)***	***w(1, 1)***

Convolution and correlation

Correlation

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolution

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

f(x,y)

	0	1	2	3
0	***f(0, 0)***	***f(0, 1)***	...	...
1	***f(1, 0)***	***f(1, 1)***	...	...
2	...	...	...	...
3	...	...	...	...

w(s,t)

	-1	0	1
-1	***w(-1,-1)***	***w(-1, 0)***	***w(-1, 1)***
0	***w(0, -1)***	***w(0, 0)***	***w(0, 1)***
1	***w(1, -1)***	***w(1, 0)***	***w(1, 1)***

padding

EXAMPLE: CONVOLUTION

Convolution

f(x,y)

	0	*1*	*2*	*3*
0	1	0	6	4
1	2	1	7	2
2	5	0	2	3
3	5	0	3	2

w(s,t)

	-1	*0*	*1*
-1	1	2	3
0	4	5	6
1	7	8	9

Convolution

f(x,y)

	0	*1*	*2*	*3*	
	0	*0*	*0*	*0*	*0*	*0*
0	*0*	1	0	6	4	*0*
1	*0*	2	1	7	2	*0*
2	*0*	5	0	2	3	*0*
3	*0*	5	0	3	2	*0*
	0	*0*	*0*	*0*	*0*	*0*

padding

w(s,t)

	-1	*0*	*1*
-1	1	2	3
0	4	5	6
1	7	8	9

Convolution

$f(x,y)$

	0	1	2	3		
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

Convolution

f(x,y)

	0	*1*	*2*	*3*	
0	*0*	1	0	6	4	*0*
1	*0*	2	1	7	2	*0*
2	*0*	5	0	2	3	*0*
3	*0*	5	0	3	2	*0*
	0	*0*	*0*	*0*	*0*	*0*

w(s,t)

	-1	*0*	*1*
-1	1	2	3
0	4	5	6
1	7	8	9

g(x,y)

	0	*1*	*2*	*3*	
0	*0*	0	0	0	0	*0*
1	*0*	0	0	0	0	*0*
2	*0*	0	0	0	0	*0*
3	*0*	0	0	0	0	*0*
	0	*0*	*0*	*0*	*0*	*0*

Convolution

$f(x,y)$

	0	1	2	3	
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

$g(x,y)$

	0	1	2	3	
	0	0	0	0	0	0
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

convolution

Convolution

$f(x,y)$

	0	1	2	3	
0	0	0	0	0	0
0	1	0	6	4	0
1	2	1	7	2	0
2	5	0	2	3	0
3	5	0	3	2	0
0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

$w'(s,t)$

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

$w(s,t)$ rotated 180°

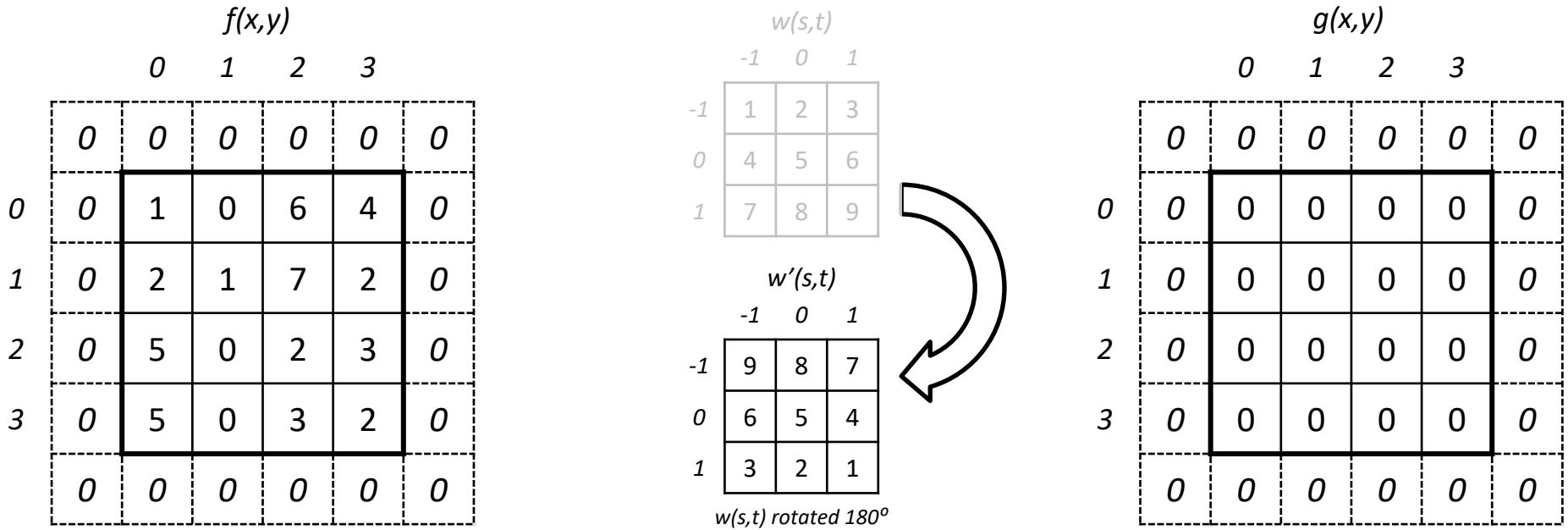
$g(x,y)$

	0	1	2	3	
0	0	0	0	0	0
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

correlation

Convolution



$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

correlation

Convolution

f(x,y)

	0	1	2	3		
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotated 180°

g(x,y)

	0	1	2	3		
	0	0	0	0	0	0
0	0	10	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 0\times 9 + 0\times 8 + 0\times 7 \\ 0\times 6 + 1\times 5 + 0\times 4 \\ 0\times 3 + 2\times 2 + 1\times 1 \end{matrix} = 10$$

Convolution

f(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	1	0	6	4	0
2	0	2	1	7	2	0
3	0	5	0	2	3	0
	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotated 180°

g(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	10	45	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) =$$

$$0 \times 9 + 0 \times 8 + 0 \times 7 + 1 \times 6 + 0 \times 5 + 6 \times 4 + 2 \times 3 + 1 \times 2 + 7 \times 1 = 45$$

Convolution

f(x,y)

	0	1	2	3	
0	0	0	0	0	0
1	0	1	0	6	4	0
2	0	2	1	7	2	0
3	0	5	0	2	3	0
4	0	5	0	3	2	0
5	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotated 180°

g(x,y)

	0	1	2	3	
0	0	0	0	0	0	0
1	0	10	45	65	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{array}{l} 0\times 9 + 0\times 8 + 0\times 7 + \\ 0\times 6 + 6\times 5 + 4\times 4 + \\ 1\times 3 + 7\times 2 + 2\times 1 \end{array} = 65$$

Convolution

f(x,y)

	0	1	2	3		
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

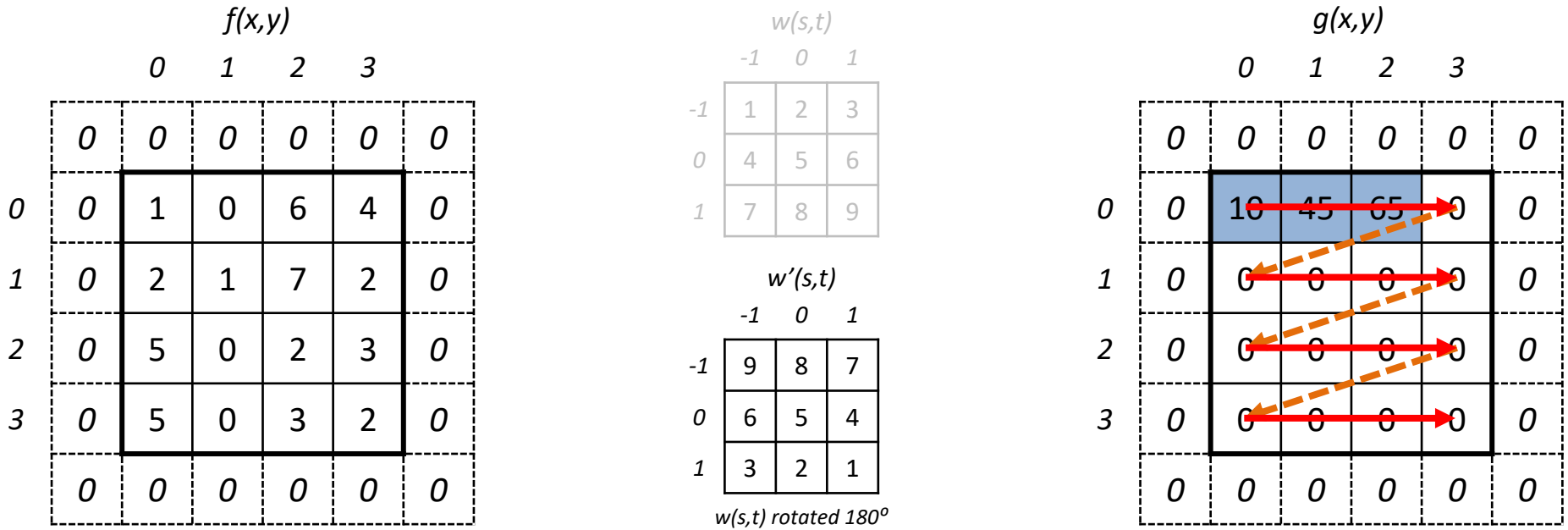
w(s,t) rotated 180°

g(x,y)

	0	1	2	3		
	0	0	0	0	0	0
0	0	10	45	65	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) =$$

Convolução



$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) =$$

Convolution

f(x,y)

	0	1	2	3		
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotated 180°

g(x,y)

	0	1	2	3		
	0	0	0	0	0	0
0	0	10	45	65	81	0
1	0	32	113	132	150	0
2	0	58	131	109	119	0
3	0	65	0	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) =$$

Convolution

$f(x,y)$

	0	1	2	3	
0	0	0	0	0	0
1	0	1	0	6	4	0
2	0	2	1	7	2	0
3	0	5	0	2	3	0
	0	5	0	3	2	0
	0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

$w'(s,t)$

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

$w(s,t)$ rotated 180°

$g(x,y)$

	0	1	2	3	
0	0	0	0	0	0
1	0	10	45	65	81	0
2	0	32	113	132	150	0
3	0	58	131	109	119	0
	0	65	101	0	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{matrix} 5\times 9 + 0\times 8 + 2\times 7 + \\ 5\times 6 + 0\times 5 + 3\times 4 + \\ 0\times 3 + 0\times 2 + 0\times 1 \end{matrix} = 101$$

Convolution

$f(x,y)$

	0	1	2	3	
0	0	0	0	0	0
1	0	1	0	6	4	0
2	0	2	1	7	2	0
3	0	5	0	2	3	0
	0	5	0	3	2	0
	0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

$w'(s,t)$

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

$w(s,t)$ rotated 180°

$g(x,y)$

	0	1	2	3	
0	0	0	0	0	0
1	0	10	45	65	81	0
2	0	32	113	132	150	0
3	0	58	131	109	119	0
	0	65	101	60	0	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{array}{l} 0\times 9 + 2\times 8 + 3\times 7 + \\ 0\times 6 + 3\times 5 + 2\times 4 + \\ 0\times 3 + 0\times 2 + 0\times 1 \end{array} = 60$$

Convolution

$f(x,y)$

	0	1	2	3	
0	0	0	0	0	0
1	0	1	0	6	4	0
2	0	2	1	7	2	0
3	0	5	0	2	3	0
	0	5	0	3	2	0
	0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

$w'(s,t)$

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

$w(s,t)$ rotated 180°

$g(x,y)$

	0	1	2	3	
0	0	0	0	0	0
1	0	10	45	65	81	0
2	0	32	113	132	150	0
3	0	58	131	109	119	0
	0	65	101	60	70	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{array}{l} 2\times 9 + 3\times 8 + 0\times 7 + \\ 3\times 6 + 2\times 5 + 0\times 4 + \\ 0\times 3 + 0\times 2 + 0\times 1 \end{array} = 70$$

Convolution

$f(x,y)$

	0	1	2	3	
	0	0	0	0	0	0
0	0	1	0	6	4	0
1	0	2	1	7	2	0
2	0	5	0	2	3	0
3	0	5	0	3	2	0
	0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

$w'(s,t)$

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

$w(s,t)$ rotated 180°

$g(x,y)$

	0	1	2	3	
	0	0	0	0	0	0
0	0	10	45	65	81	0
1	0	32	113	132	150	0
2	0	58	131	109	119	0
3	0	65	101	60	70	0
	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = \begin{array}{l} 2\times 9 + 3\times 8 + 0\times 7 + \\ 3\times 6 + 2\times 5 + 0\times 4 + \\ 0\times 3 + 0\times 2 + 0\times 1 \end{array} = 70$$

Convolution

$f(x,y)$

	0	1	2	3	
0	0	0	0	0	0
1	0	1	0	6	4	0
2	0	2	1	7	2	0
3	0	5	0	2	3	0
4	0	5	0	3	2	0
5	0	0	0	0	0	0

$w(s,t)$

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

$w'(s,t)$

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

$w(s,t)$ rotated 180°

$g(x,y)$

	0	1	2	3	
0	0	0	0	0	0
1	0	10	45	65	81	0
2	0	32	113	132	150	0
3	0	58	131	109	119	0
4	0	65	101	60	70	0
5	0	0	0	0	0	0

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolution

f(x,y)

	0	1	2	3
0	1	0	6	4
1	2	1	7	2
2	5	0	2	3
3	5	0	3	2

w(s,t)

	-1	0	1
-1	1	2	3
0	4	5	6
1	7	8	9

w'(s,t)

	-1	0	1
-1	9	8	7
0	6	5	4
1	3	2	1

w(s,t) rotated 180°

g(x,y)

	0	1	2	3
0	10	45	65	81
1	32	113	132	150
2	58	131	109	119
3	65	101	60	70

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Bibliography

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- MARQUES FILHO, O.; VIEIRA NETO, H. **Processamento digital de imagens**. Brasport, 1999.
  - (*in Brazilian Portuguese*)
  - Available on the author's website (for personal use only)
  - http://dainf.ct.utfpr.edu.br/~hvieir/pub.html

- J. E. R. Queiroz, H. M. Gomes. **Introdução ao Processamento Digital de Imagens**. RITA. v. 13, 2006.
  - (*in Brazilian Portuguese*)
  - http://www.dsc.ufcg.edu.br/~hmg/disciplinas/graduacao/vc-2016.2/Rita-Tutorial-PDI.pdf

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@misc{mari_im_proc_2023,
       author = {João Fernando Mari},
       title = {Spatial filtering I},
       year = {2023},
       publisher = {GitHub},
       journal = {Introduction to digital image processing - UFV},
       howpublished = {\url{https://github.com/joaofmari/SIN392_Introduction-to-digital-image-processing_2023}}
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THE END