Lecture 09 – Image segmentation II
Thresholding

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Agenda

- Thresholding
- Basic global thresholding
- Otsu’s method

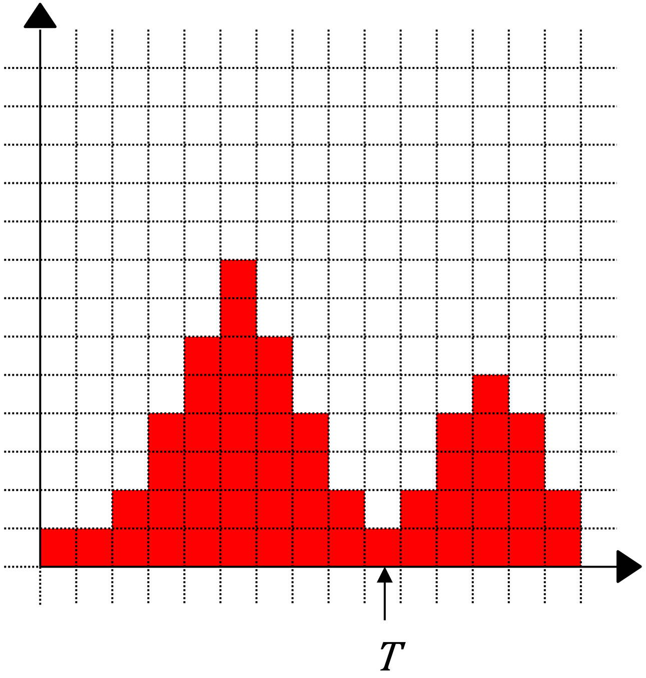
Thresholding

- Image thresholding:
  - Central position in image segmentation applications
  - Ease of implementation
  - Computational speed

- Global thresholding:
  - *T* is a constant applicable to an entire image

- Local threshold (variable or regional):
  - *T* changes throughout the image

$$g(x,y) = \begin{cases} 1, & if\ f(x,y) > T \\ 0, & if\ f(x,y) \leq T \end{cases}$$



Thresholding

- Image thresholding:
  - Central position in image segmentation applications
  - Ease of implementation
  - Computational speed

- Global thresholding:
  - *T* is a constant applicable to an entire image

- Local threshold (variable or regional):
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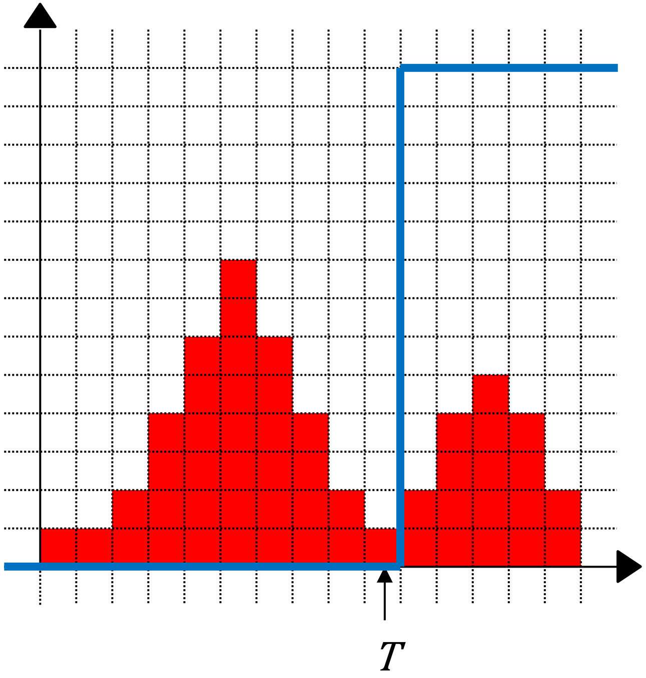
$$g(x,y) = \begin{cases} 1, & if\ f(x,y) > T \\ 0, & if\ f(x,y) \leq T \end{cases}$$



BASIC GLOBAL THRESHOLDING

Basic global thresholding

1. Select an initial guess for the global threshold, T.
2. Segment the image using T:

$$g(x,y) = \begin{cases} 1 & if\ f(x,y) > T \\ 0 & if\ f(x,y) \leq T \end{cases}$$

   – This will result in two groups of pixels:
      - G_1, pixels with intensity values > T;
      - G_2, pixels with intensity values ≤ T.
3. Compute the mean intensity values m_1 and m_2 for the pixels in G_1 and G_2, respectively.
4. Compute a new threshold value:

$$T = \frac{1}{2}(m_1 + m_2)$$

5. Repeat steps 2 to 4 until the difference between the T values in successive iterations is smaller than the predefined parameter ΔT.

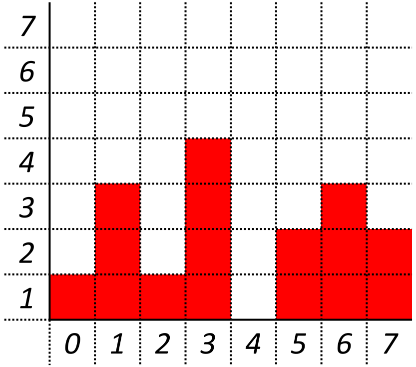
Basic global thresholding

Image I

2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

$T_0 = \min(I) = 0$

$\Delta T = 0.001$



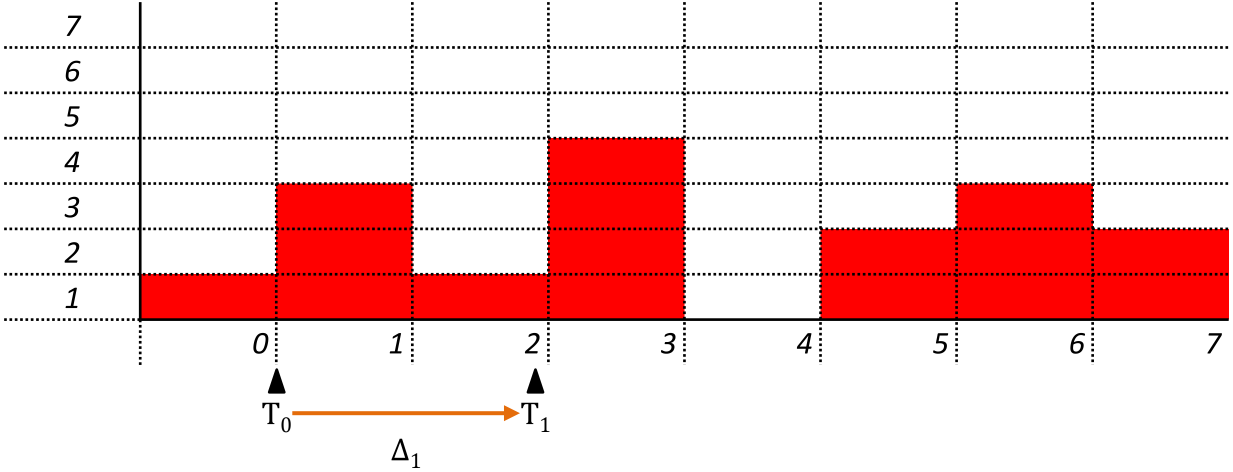
Basic global thresholding

Image I

2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

$T_0 = \min(I) = 0$
$\Delta T = 0.001$

- $T_0 = \min(I) = 0$
- $G_1 = [2, 3, 6, 5, 3, 1, 1, 1, 6, 7, 6, 3, 5, 7, 3]$
- $G_2 = [0]$
- $m_1 = (2 + 3 + 6 + 5 + 3 + 1 + 1 + 1 + 6 + 7 + 6 + 3 + 5 + 7 + 3) / 15$
  $= 59 / 15 = 3.9333$
- $m_2 = 0 / 1 = 0$
- $T_1 = (3.9333 + 0) / 2 = 1.9667$
- $|T_1 - T_0| = |1.9667 - 0| = 1.9667 > \Delta T$. Then, new iteration.



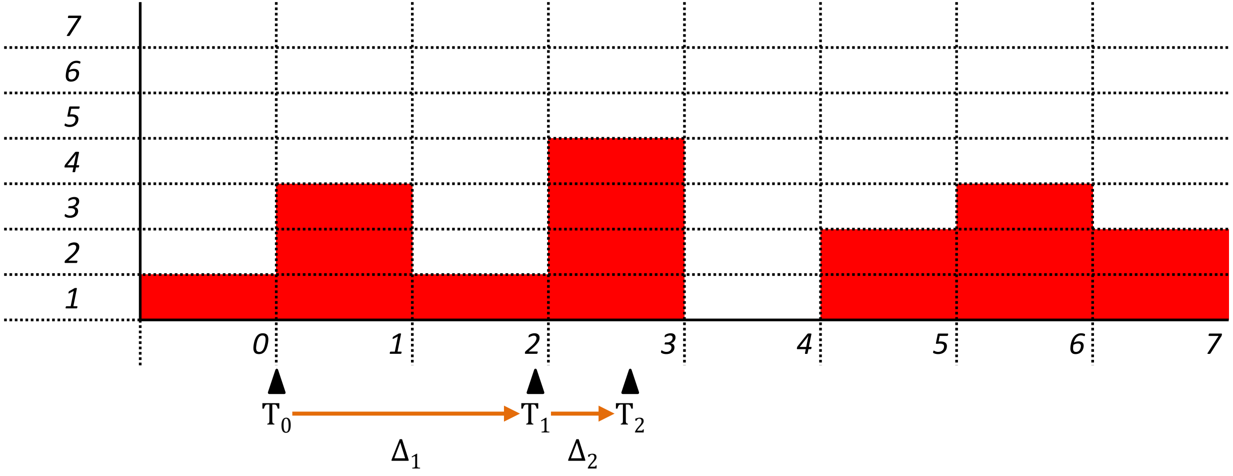
Basic global thresholding

Image I

2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

$T_0 = \min(I) = 0$
$\Delta T = 0.001$

- $T_1 = 1.9667$
- $G_1 = [2, 3, 6, 5, 3, 6, 7, 6, 3, 5, 7, 3]$
- $G_2 = [1, 1, 1, 0]$
- $m_1 = (2 + 3 + 6 + 5 + 3 + 6 + 7 + 6 + 3 + 5 + 7 + 3) / 12$
  $= 56 / 12 = 4.6667$
- $m_2 = (1 + 1 + 1 + 0) / 4 = 3 / 4 = 0.75$
- $T_2 = (4.6667 + 0.75) / 2 = 2.7084$
- $|T_2 - T_1| = |2.7084 - 1.9667| = 0.7417 > \Delta T$. Then, new iteration.



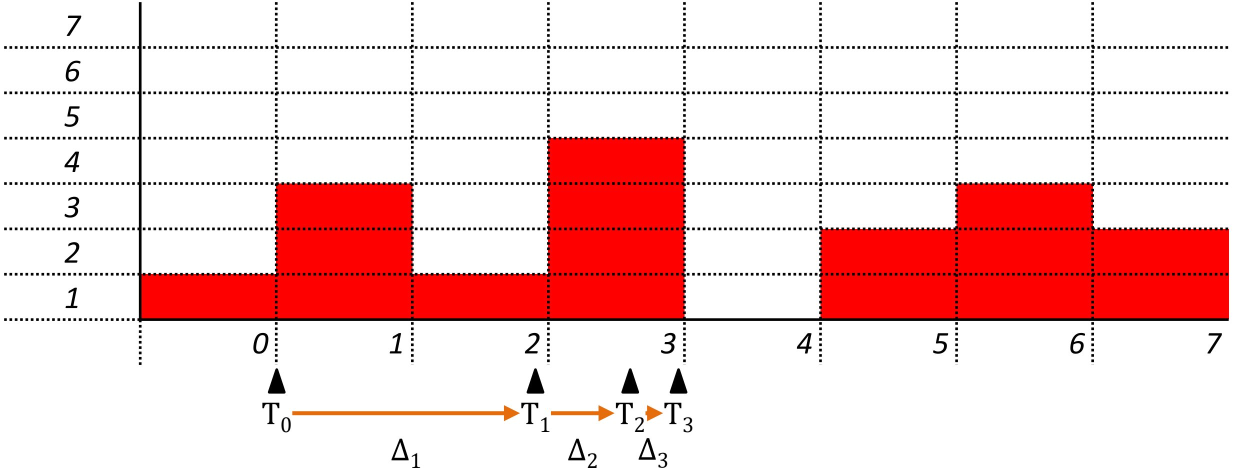
Basic global thresholding

Image I

2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

$T_0 = \min(I) = 0$
$\Delta T = 0.001$

- $T_2 = 2{,}7084$
- $G_1 = [3, 6, 5, 3, 6, 7, 6, 3, 5, 7, 3]$
- $G_2 = [2, 1, 1, 1, 0]$
- $m_1 = (3 + 6 + 5 + 3 + 6 + 7 + 6 + 3 + 5 + 7 + 3) / 11$
  $= 54 / 11 = 4.9091$
- $m_2 = (2 + 1 + 1 + 1 + 0) / 5 = 1$
- $T_3 = (4.9091 + 1) / 2 = 2.9546$
- $|T_3 - T_2| = |2.9546 - 2{,}7084| = 0.2462 > \Delta T$. Then, new iteration.



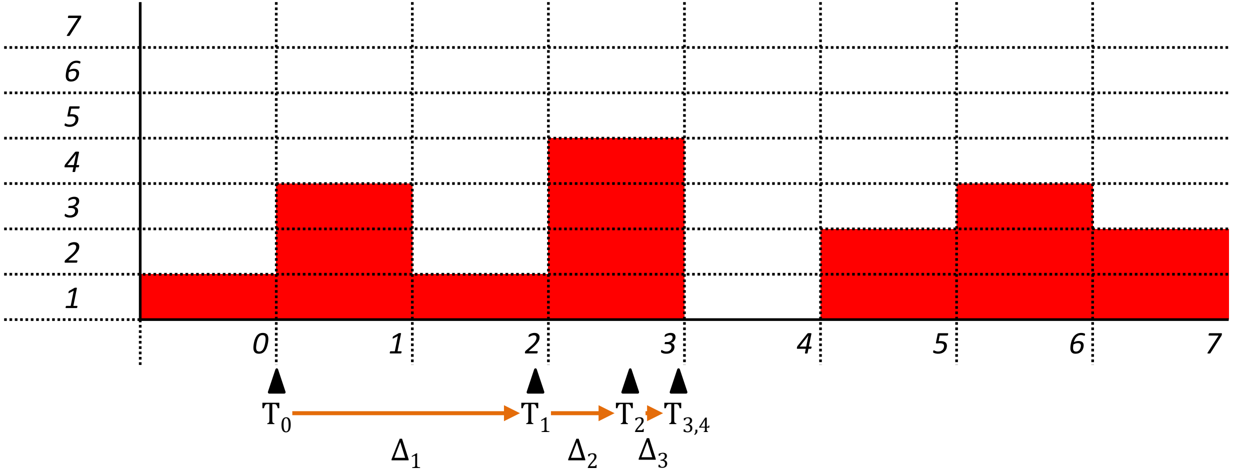
Basic global thresholding

Image I

2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

$T_0 = \min(I) = 0$
$\Delta T = 0.001$

- $T_3 = 2.9546$
- $G_1 = [3, 6, 5, 3, 6, 7, 6, 3, 5, 7, 3]$
- $G_2 = [2, 1, 1, 1, 0]$
- $m_1 = (3 + 6 + 5 + 3 + 6 + 7 + 6 + 3 + 5 + 7 + 3) / 11$
  $= 54 / 11 = 4.9091$
- $m_2 = (2 + 1 + 1 + 1 + 0) / 5 = 1$
- $T_4 = (4.9091 + 1) / 2 = 2.9546$
- $|T_4 - T_3| = |2.9546 - 2.9546| = 0.0 <= \Delta T$. Then, end of the algorithm.



Basic global thresholding

Image I

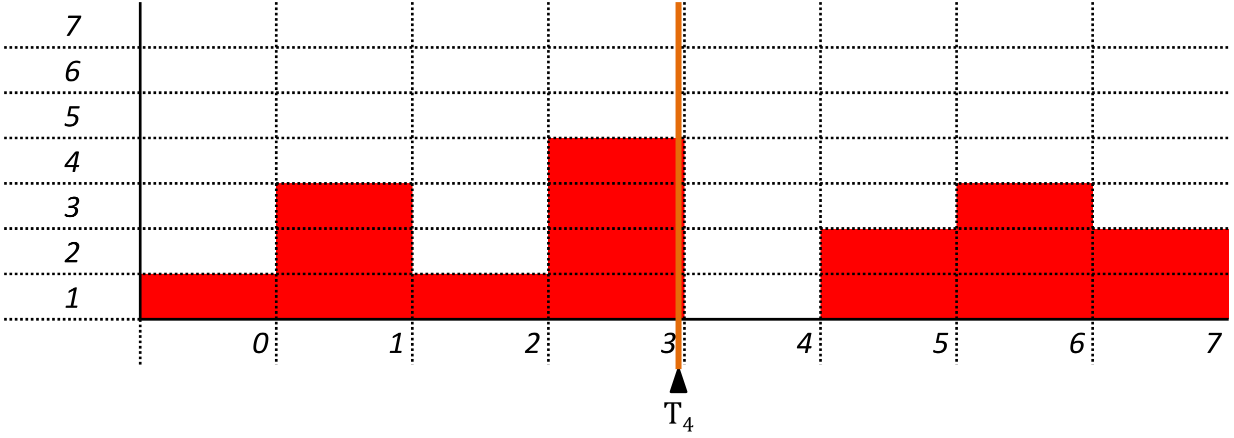
2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

$T_0 = \min(I) = 0$
$\Delta T = 0.001$

Image I'

2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

- $T_3 = 2.9546$
- $G_1 = [3, 6, 5, 3, 6, 7, 6, 3, 5, 7, 3]$
- $G_2 = [2, 1, 1, 1, 0]$
- $m_1 = (3 + 6 + 5 + 3 + 6 + 7 + 6 + 3 + 5 + 7 + 3) / 11$
  $= 54 / 11 = 4.9091$
- $m_2 = (2 + 1 + 1 + 1 + 0) / 5 = 1$
- $T_4 = (4.9091 + 1) / 2 = 2.9546$
- $|T_4 - T_3| = |2.9546 - 2.9546| = 0.0 <= \Delta T$. Then, end of the algorithm.



OTSU’S METHOD

Otsu's method

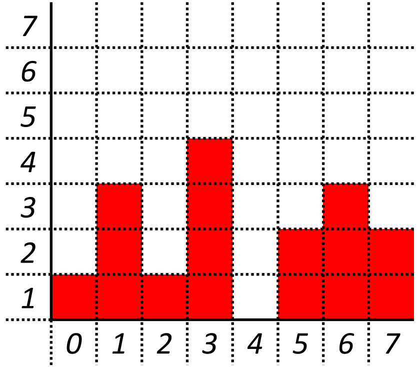
- Calculate the normalized histogram of the input image:
  - Assign the histogram components as p_i, i = 0, 1, ..., L-1.
- Calculate the accumulated sums, $P_1(k)$, for k=0, 1, 2, ..., L-1, according to:
  - $P_1(k) = \sum_{i=0}^{k} p_i$
- Calculate the accumulated means $m(k)$, for k=0, 1, 2, ..., L-1, according to :
  - $m(k) = \sum_{i=0}^{k} i p_i$
- Calculate the global mean intensity, m_G, according to :
  - $m_G = \sum_{i=0}^{L-1} i p_i$
- Calculate the variance between classes, $\sigma_B^2(k)$, para k=0, 1, 2, ..., L-1, according to:
  - $\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2$, rewritten as: $\sigma_B^2(k) = \frac{[m_G P_1(k) - m(k)]^2}{P_1(k)[1 - P_1(k)]}$
- The Otsu threshold, k*, is the value of k for which $\sigma_B^2(k)$ is maximum.
  - If more than one maximum occurs, k* is the average of the corresponding k values
- Obtain the separability measure, η*, considering k = k* in the equation:
  - $\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2}$, in which: $\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 p_i$

Otsu’s method

Image I

2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

16 pixels (4 x 4)
3 bits = 8 gray levels.
[0, ..., 7]



i	h_i	p_i	$P_1(k)$	$m(k)$	$\sigma_B^2(k)$	$(i - m_G)^2 p_i$
0	1	0.0625				
1	3	0.1875				
2	1	0.0625				
3	4	0.2500				
4	0	0.0000				
5	2	0.1250				
6	3	0.1875				
7	2	0.1250				
	16	1.0				

Otsu's method

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$$P_1(k) = \sum_{i=0}^{k} p_i$$

i	h_i	p_i	$P_1(k)$	$m(k)$	$\sigma_B^2(k)$
0	1	0.0625	0.0625		
1	3	0.1875	0.2500		
2	1	0.0625	0.3125		
3	4	0.2500	0.5625		
4	0	0.0000	0.5625		
5	2	0.1250	0.6875		
6	3	0.1875	0.8750		
7	2	0.1250	1.0000		

$(i - m_G)^2 p_i$

Otsu's method

$$P_1(k) = \sum_{i=0}^{k} p_i$$

$$m(k) = \sum_{i=0}^{k} i p_i$$

i	h_i	p_i	$P_1(k)$	$m(k)$	$\sigma_B^2(k)$
0	1	0.0625	0.0625	0.0	
1	3	0.1875	0.2500	0.1875	
2	1	0.0625	0.3125	0.3125	
3	4	0.2500	0.5625	1.0625	
4	0	0.0000	0.5625	1.0625	
5	2	0.1250	0.6875	1.6875	
6	3	0.1875	0.8750	2.8125	
7	2	0.1250	1.0000	3.6875	

$(i - m_G)^2 p_i$

Otsu's method

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$$P_1(k) = \sum_{i=0}^{k} p_i$$

$$m(k) = \sum_{i=0}^{k} i p_i$$

$$m_G = \sum_{i=0}^{L-1} i p_i$$

i	h_i	p_i	$P_1(k)$	$m(k)$	$\sigma_B^2(k)$
0	1	0.0625	0.0625	0.0	
1	3	0.1875	0.2500	0.1875	
2	1	0.0625	0.3125	0.3125	
3	4	0.2500	0.5625	1.0625	
4	0	0.0000	0.5625	1.0625	
5	2	0.1250	0.6875	1.6875	
6	3	0.1875	0.8750	2.8125	
7	2	0.1250	1.0000	3.6875	

$(i - m_G)^2 p_i$

$$m_G = 3.6875$$

Otsu's method

$$P_1(k) = \sum_{i=0}^{k} p_i$$

$$m(k) = \sum_{i=0}^{k} ip_i$$

$$m_G = \sum_{i=0}^{L-1} ip_i$$

$$\sigma_B^2(k) = \frac{[m_G P_1(k) - m(k)]^2}{P_1(k)[1 - P_1(k)]}$$

i	h_i	p_i	$P_1(k)$	$m(k)$	$\sigma_B^2(k)$
0	1	0.0625	0.0625	0.0	0.906510
1	3	0.1875	0.2500	0.1875	2.876302
2	1	0.0625	0.3125	0.3125	3.283026
3	4	0.2500	0.5625	1.0625	4.159288
4	0	0.0000	0.5625	1.0625	4.159288
5	2	0.1250	0.6875	1.6875	3.344389
6	3	0.1875	0.8750	2.8125	1.567522
7	2	0.1250	1.0000	3.6875	----

$(i - m_G)^2 p_i$

$\boldsymbol{m_G} = 3.6875$

Otsu's method

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$$P_1(k) = \sum_{i=0}^{k} p_i$$

$$m(k) = \sum_{i=0}^{k} ip_i$$

$$m_G = \sum_{i=0}^{L-1} ip_i$$

$$\sigma_B^2(k) = \frac{[m_G P_1(k) - m(k)]^2}{P_1(k)[1 - P_1(k)]}$$

$$\boldsymbol{k^*} = \frac{1}{2}(3+4) = \mathbf{3.5}$$

i	h_i	p_i	$P_1(k)$	$m(k)$	$\sigma_B^2(k)$
0	1	0.0625	0.0625	0.0	0.906510
1	3	0.1875	0.2500	0.1875	2.876302
2	1	0.0625	0.3125	0.3125	3.283026
3	4	0.2500	0.5625	1.0625	4.159288
4	0	0.0000	0.5625	1.0625	4.159288
5	2	0.1250	0.6875	1.6875	3.344389
6	3	0.1875	0.8750	2.8125	1.567522
7	2	0.1250	1.0000	3.6875	----

$(i - m_G)^2 p_i$

$$\boldsymbol{m_G} = 3.6875$$

Otsu's method

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$$P_1(k) = \sum_{i=0}^{k} p_i$$

$$m(k) = \sum_{i=0}^{k} i p_i$$

$$m_G = \sum_{i=0}^{L-1} i p_i$$

$$\sigma_B^2(k) = \frac{[m_G P_1(k) - m(k)]^2}{P_1(k)[1 - P_1(k)]}$$

$$\boldsymbol{k^*} = \frac{1}{2}(3+4) = \mathbf{3.5}$$

$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2},$$

i	h_i	p_i	$P_1(k)$	$m(k)$	$\sigma_B^2(k)$
0	1	0.0625	0.0625	0.0	0.906510
1	3	0.1875	0.2500	0.1875	2.876302
2	1	0.0625	0.3125	0.3125	3.283026
3	4	0.2500	0.5625	1.0625	4.159288
4	0	0.0000	0.5625	1.0625	4.159288
5	2	0.1250	0.6875	1.6875	3.344389
6	3	0.1875	0.8750	2.8125	1.567522
7	2	0.1250	1.0000	3.6875	----

$$\boldsymbol{m_G} = 3.6875$$

$(i - m_G)^2 p_i$

Otsu's method

$$P_1(k) = \sum_{i=0}^{k} p_i$$

$$m(k) = \sum_{i=0}^{k} i p_i$$

$$m_G = \sum_{i=0}^{L-1} i p_i$$

$$\sigma_B^2(k) = \frac{[m_G P_1(k) - m(k)]^2}{P_1(k)[1 - P_1(k)]}$$

$$\boldsymbol{k^*} = \frac{1}{2}(3+4) = \mathbf{3.5}$$

$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2},$$ em que:

$$\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 p_i$$

i	h_i	p_i	$P_1(k)$	$m(k)$	$\sigma_B^2(k)$	$(i - m_G)^2 p_i$
0	1	0.0625	0.0625	0.0	0.906510	0.84985
1	3	0.1875	0.2500	0.1875	2.876302	1.35425
2	1	0.0625	0.3125	0.3125	3.283026	0.17798
3	4	0.2500	0.5625	1.0625	4.159288	0.11816
4	0	0.0000	0.5625	1.0625	4.159288	0.00000
5	2	0.1250	0.6875	1.6875	3.344389	0.21533
6	3	0.1875	0.8750	2.8125	1.567522	1.00269
7	2	0.1250	1.0000	3.6875	----	1.37158

$$\boldsymbol{m_G} = 3.6875$$

$$\boldsymbol{\sigma_G^2} = 5.08984$$

Otsu's method

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$$P_1(k) = \sum_{i=0}^{k} p_i$$

$$m(k) = \sum_{i=0}^{k} i p_i$$

$$m_G = \sum_{i=0}^{L-1} i p_i$$

$$\sigma_B^2(k) = \frac{[m_G P_1(k) - m(k)]^2}{P_1(k)[1 - P_1(k)]}$$

$$\boldsymbol{k^*} = \frac{1}{2}(3+4) = \boldsymbol{3.5}$$

$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2},$$ em que:

$$\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 p_i$$

i	h_i	p_i	$P_1(k)$	$m(k)$	$\sigma_B^2(k)$
0	1	0.0625	0.0625	0.0	0.906510
1	3	0.1875	0.2500	0.1875	2.876302
2	1	0.0625	0.3125	0.3125	3.283026
3	4	0.2500	0.5625	1.0625	4.159288
4	0	0.0000	0.5625	1.0625	4.159288
5	2	0.1250	0.6875	1.6875	3.344389
6	3	0.1875	0.8750	2.8125	1.567522
7	2	0.1250	1.0000	3.6875	----

$$\boldsymbol{m_G} = 3.6875$$

$(i - m_G)^2 p_i$
0.84985
1.35425
0.17798
0.11816
0.00000
0.21533
1.00269
1.37158

$$\boldsymbol{\sigma_G^2} = 5.08984$$

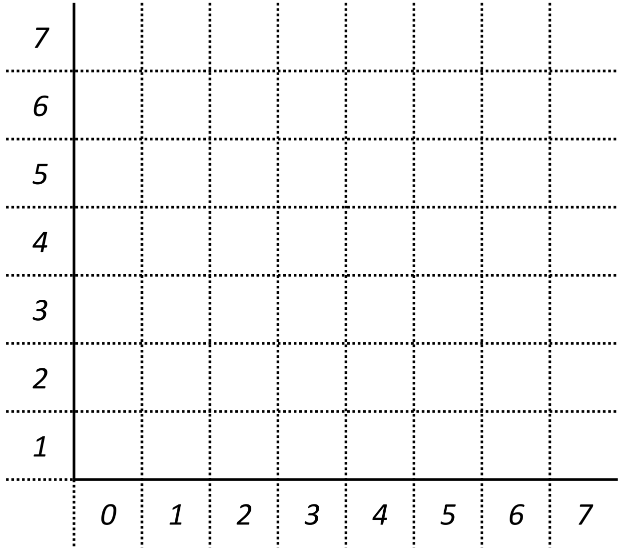
$$\boldsymbol{\eta(k^*)} = 0.81717$$

Otsu's method

2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

i	h_i	p_i	$\sigma_B^2(k)$
0	1		
1	3		
2	1		
3	4		
4	0		
5	2		
6	3		
7	2		

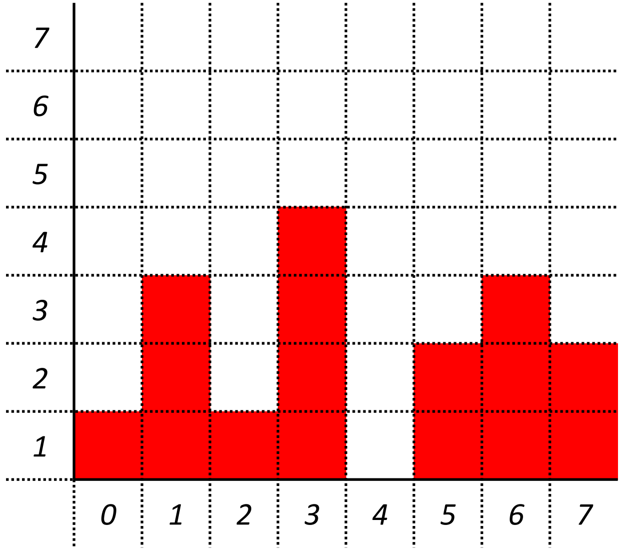


Otsu’s method

2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

i	h_i	p_i	$\sigma_B^2(k)$
0	1	0.0625	
1	3	0.1875	
2	1	0.0625	
3	4	0.2500	
4	0	0.0000	
5	2	0.1250	
6	3	0.1875	
7	2	0.1250	

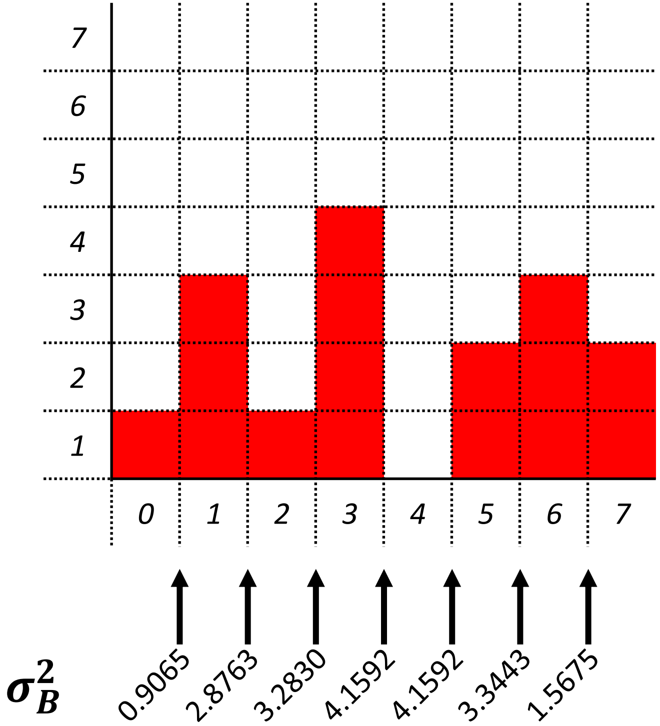


Otsu's method

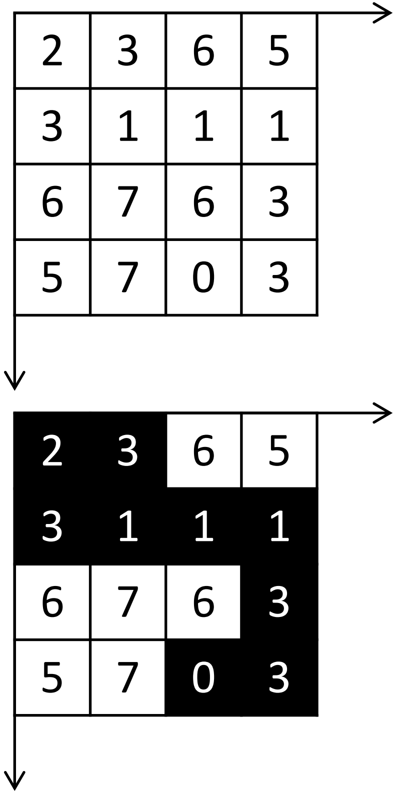
2	3	6	5
3	1	1	1
6	7	6	3
5	7	0	3

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

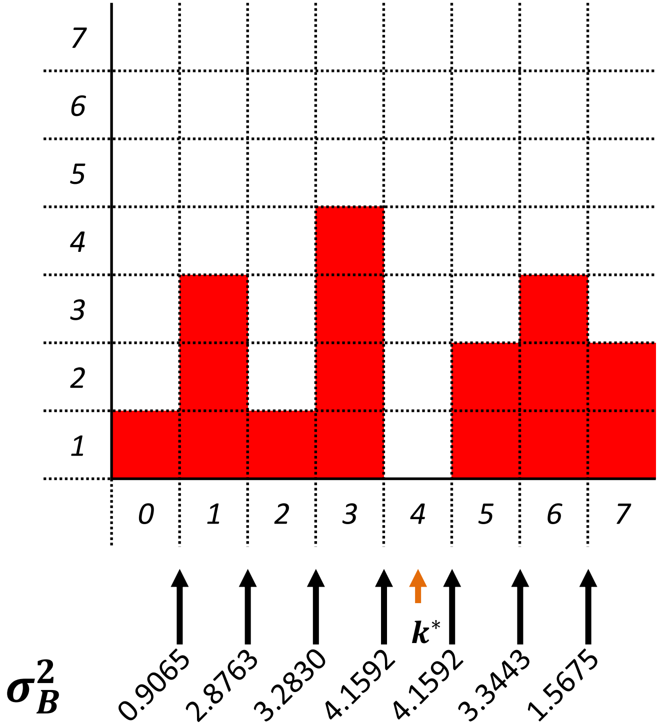
i	h_i	p_i	$\sigma_B^2(k)$
0	1	0.0625	0.906510
1	3	0.1875	2.876302
2	1	0.0625	3.283026
3	4	0.2500	4.159288
4	0	0.0000	4.159288
5	2	0.1250	3.344389
6	3	0.1875	1.567522
7	2	0.1250	----



Otsu's method



i	h_i	p_i	$\sigma_B^2(k)$
0	1	0.0625	0.906510
1	3	0.1875	2.876302
2	1	0.0625	3.283026
3	4	0.2500	4.159288
4	0	0.0000	4.159288
5	2	0.1250	3.344389
6	3	0.1875	1.567522
7	2	0.1250	----



Bibliography

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  - http://www.dsc.ufcg.edu.br/~hmg/disciplinas/graduacao/vc-2016.2/Rita-Tutorial-PDI.pdf

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@misc{mari_im_proc_2023,
	author = {João Fernando Mari},
	title = {Image segmentation II – Thresholding},
	year = {2023},
	publisher = {GitHub},
	journal = {Introduction to Digital Image Processing - UFV},
	howpublished = {\url{https://github.com/joaofmari/SIN392_Introduction-to-digital-image-processing_2023}}
}
```

THE END