UFV
Universidade Federal de Viçosa

Lecture 07 – Spatial filtering II

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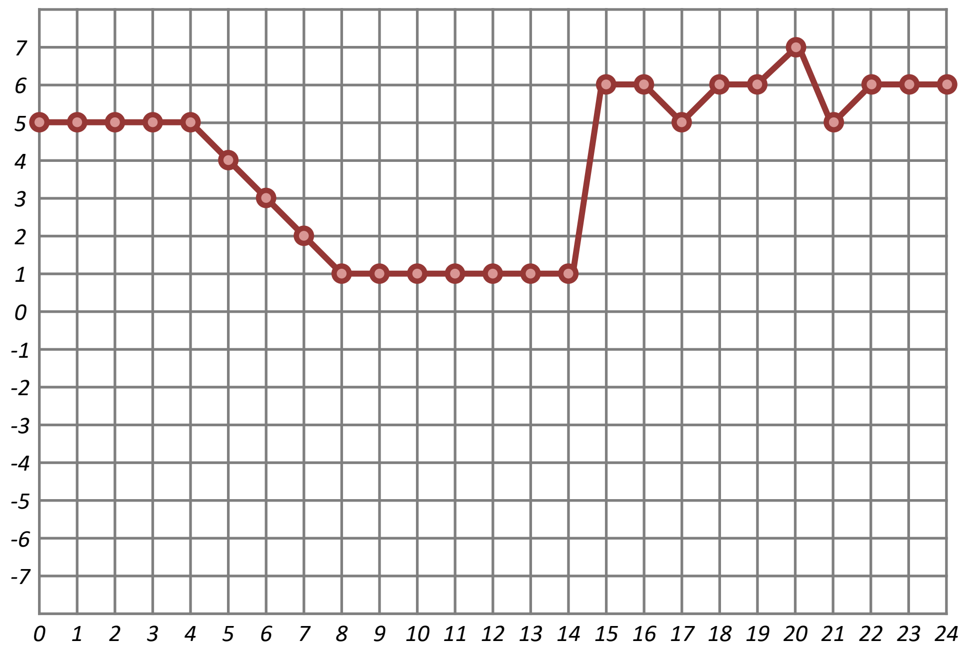
Agenda

- Derivatives of 1D discrete functions
- The Laplacian
- Laplacian variations
- The Gradient
- Roberts cross-gradient operators
- Prewitt and Sobel operators

DERIVATIVES OF 1D DISCRETE FUNCTIONS

Derivatives of 1D discrete functions



First order derivative of a 1D function *f(x)*:

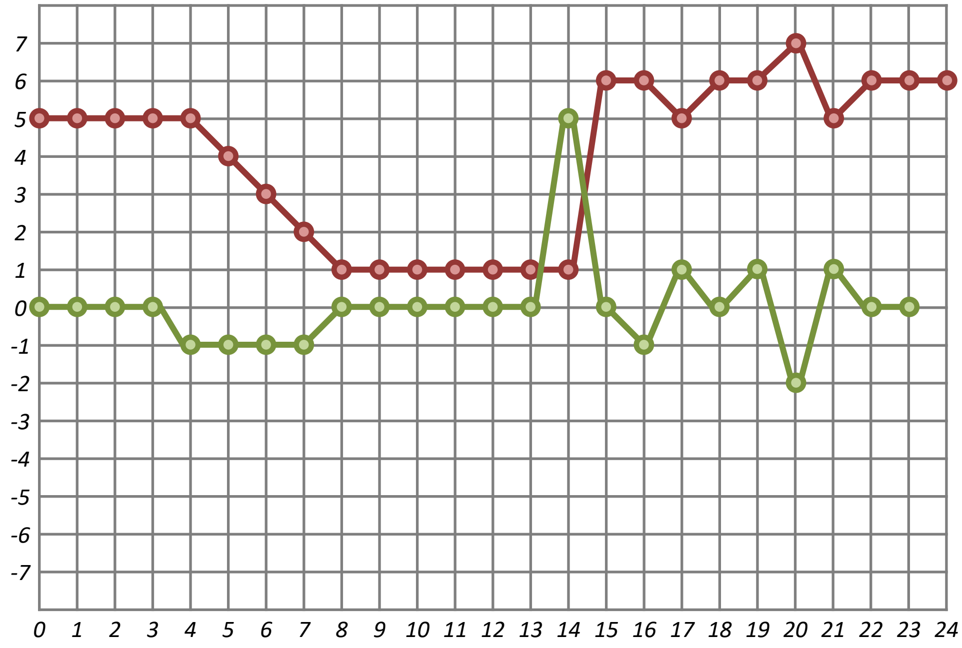
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

Second order derivative of a 1D function *f(x)*:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

5	5	5	5	5	4	3	2	1	1	1	1	1	1	1	6	6	5	6	6	7	5	6	6	6	*Signal*
																									First order derivative
																									Second order derivative

Derivatives of 1D discrete functions



5	5	5	5	5	4	3	2	1	1	1	1	1	1	1	6	6	5	6	6	7	5	6	6	6	*Signal*
0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	5	0	-1	1	0	1	-2	1	0	0		*First order derivative*
																									Second order derivative

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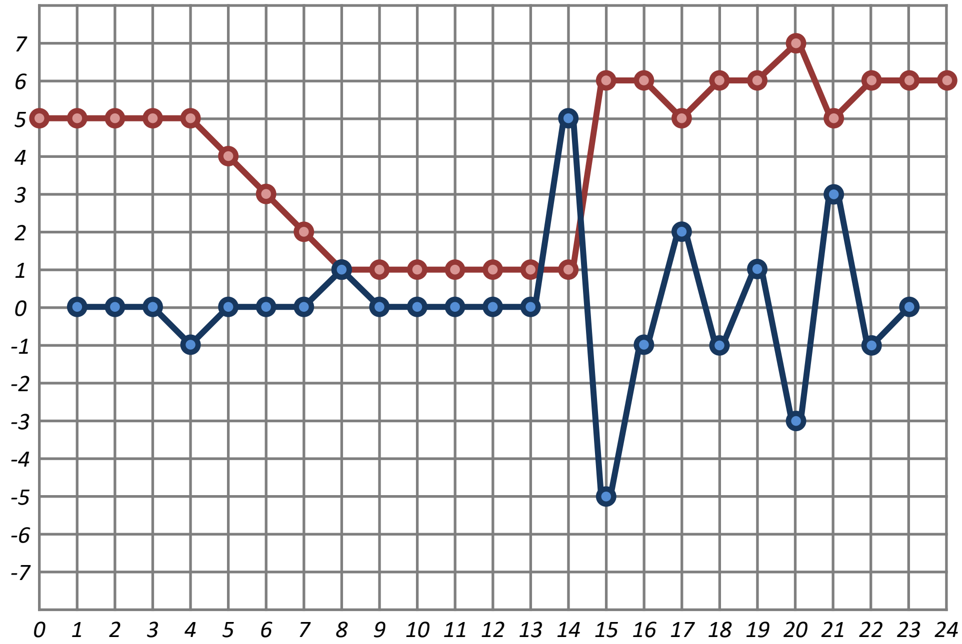
Derivatives of 1D discrete functions



5	5	5	5	5	4	3	2	1	1	1	1	1	1	1	6	6	5	6	6	7	5	6	6	6	*Signal*
0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	5	0	-1	1	0	1	-2	1	0	0		*First order derivative*
	0	0	0	-1	0	0	0	1	0	0	0	0	0	5	-5	-1	2	-1	1	-3	3	-1	0		*Second order derivative*

First order derivative of a 1D function *f(x)*:

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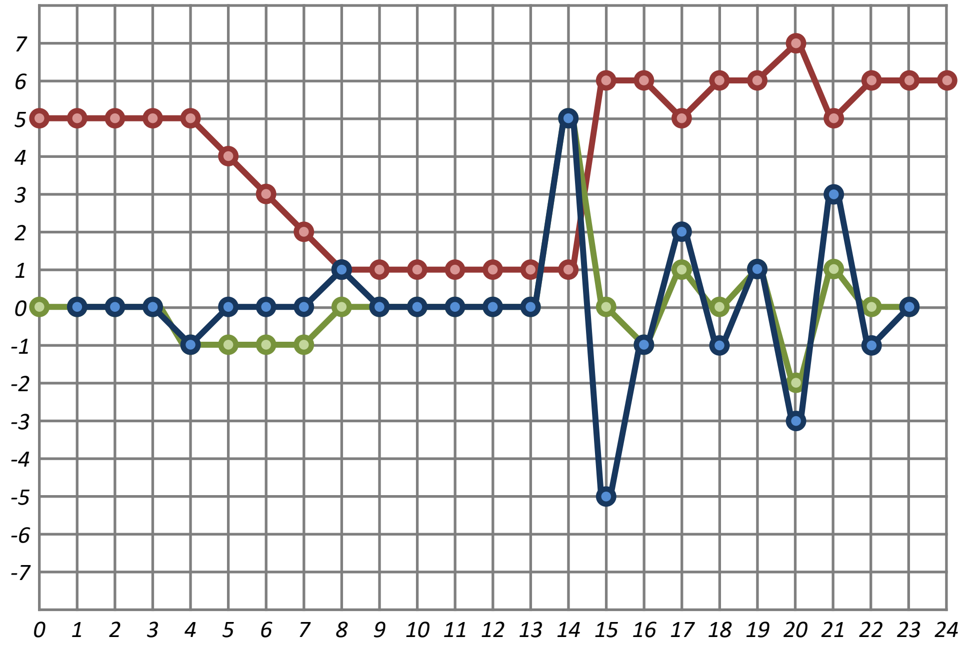
Second order derivative of a 1D function *f(x)*:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Derivatives of 1D discrete functions



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$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

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$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

5	5	5	5	5	4	3	2	1	1	1	1	1	1	1	6	6	5	6	6	7	5	6	6	6	*Signal*
0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	5	0	-1	1	0	1	-2	1	0	0		*First order derivative*
	0	0	0	-1	0	0	0	1	0	0	0	0	0	5	-5	-1	2	-1	1	-3	3	-1	0		*Second order derivative*

THE LAPLACIAN

The Laplacian

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- The Laplacian of a two-dimensional function f(x, y) is:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- If we separate the Laplacian into the x and y directions, we have:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

	-1	*0*	*1*
-1	0	1	0
0	1	-4	1
1	0	1	0

- Thus, the discrete Laplacian of two variables is:

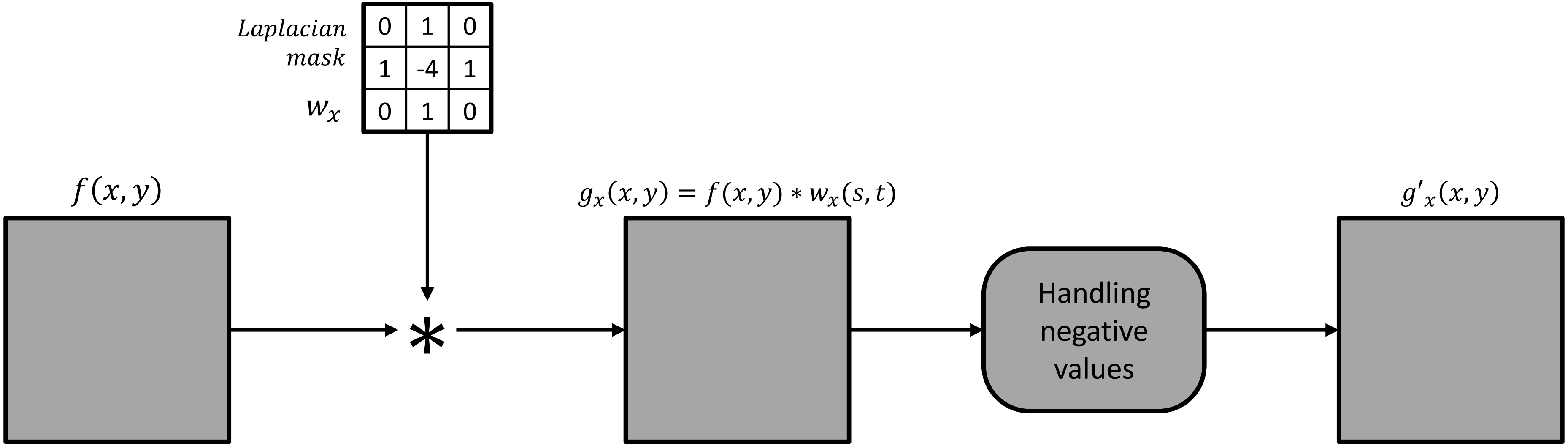
$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

Laplacian variations

$$\begin{array}{c|ccc} & -1 & 0 & 1 \\ \hline -1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 1 \\ 1 & 0 & 1 & 0 \end{array} \qquad \begin{array}{c|ccc} & -1 & 0 & 1 \\ \hline -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 1 & 0 & -1 & 0 \end{array}$$

$$\begin{array}{c|ccc} & -1 & 0 & 1 \\ \hline -1 & 1 & 1 & 1 \\ 0 & 1 & -8 & 1 \\ 1 & 1 & 1 & 1 \end{array} \qquad \begin{array}{c|ccc} & -1 & 0 & 1 \\ \hline -1 & -1 & -1 & -1 \\ 0 & -1 & 8 & -1 \\ 1 & -1 & -1 & -1 \end{array}$$

The Laplacian – how to apply



THE GRADIENT

The Gradient

- The gradient of a two-dimensional function *f(x, y)* is:

$$\nabla f \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix},$$

$$\frac{\partial f}{\partial x} = f(x, y) - f(x+1, y), \qquad \frac{\partial f}{\partial y} = f(x, y) - f(x, y+1)$$

	0	*1*
0	1	0
1	-1	0

- The magnitude (size) of the gradient vector (∇f), *M(x, y)* is:

$$M(x, y) = mag(\nabla f) = \sqrt{{g_x}^2 + {g_y}^2}$$

	0	*1*
0	1	-1
1	0	0

- Or it can be approximated by absolute values:

$$M(x, y) \approx |g_x| + |g_y|$$

The Gradient – Roberts cross-gradient operators

- Roberts cross-gradient operators consider diagonal differences:

$$\frac{\partial f}{\partial x} = f(x, y) - f(x + 1, y + 1), \qquad \frac{\partial f}{\partial y} = f(x + 1, y) - f(x, y + 1)$$

	0	*1*
0	1	0
1	0	-1

	0	*1*
0	0	-1
1	1	0

The Gradient – Prewitt and Sobel operators

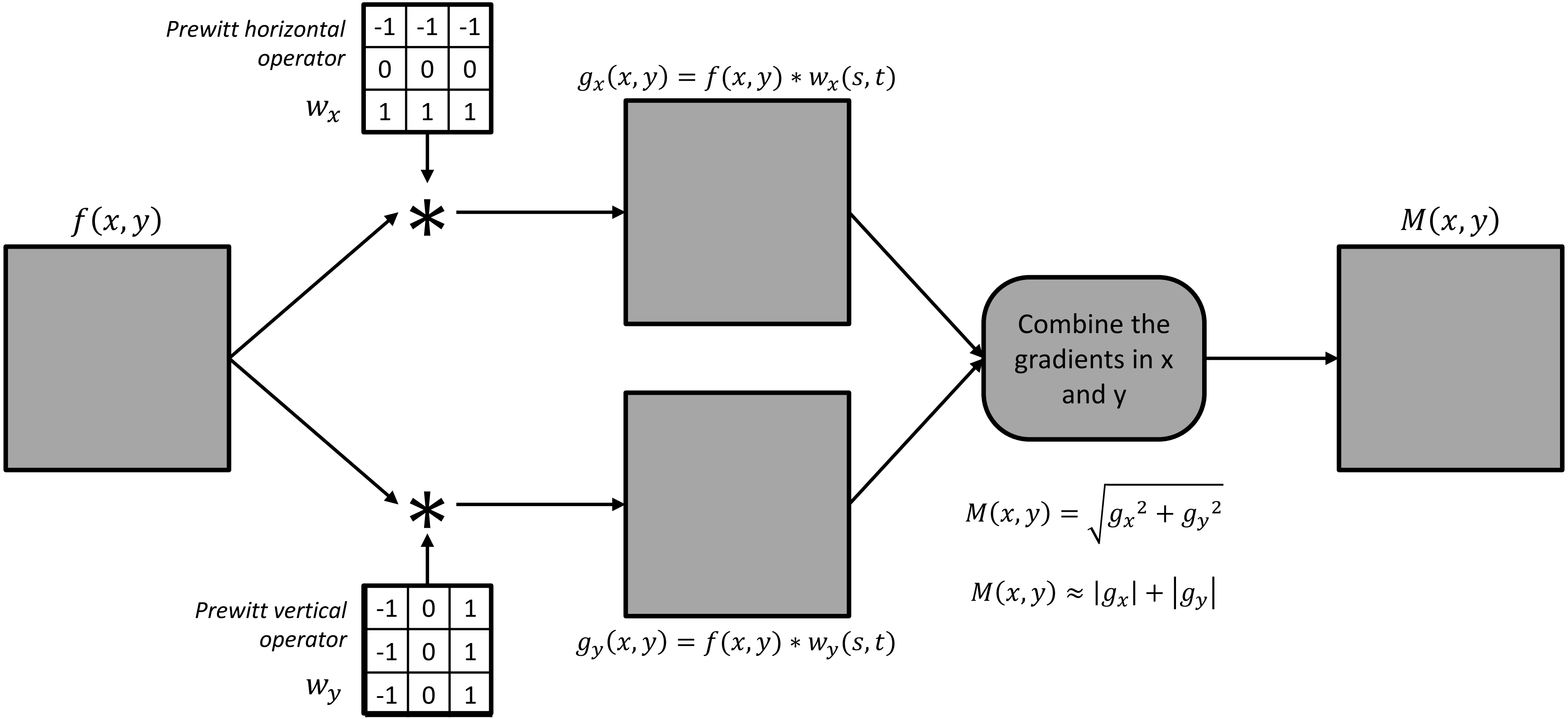
Prewitt:

$$g_x \quad \begin{array}{c|c|c|c|} & \textit{-1} & \textit{0} & \textit{1} \\ \hline \textit{-1} & -1 & -1 & -1 \\ \hline \textit{0} & 0 & 0 & 0 \\ \hline \textit{1} & 1 & 1 & 1 \\ \hline \end{array} \qquad g_y \quad \begin{array}{c|c|c|c|} & \textit{-1} & \textit{0} & \textit{1} \\ \hline \textit{-1} & -1 & 0 & 1 \\ \hline \textit{0} & -1 & 0 & 1 \\ \hline \textit{1} & -1 & 0 & 1 \\ \hline \end{array}$$

Sobel:

$$g_x \quad \begin{array}{c|c|c|c|} & \textit{-1} & \textit{0} & \textit{1} \\ \hline \textit{-1} & -1 & -2 & -1 \\ \hline \textit{0} & 0 & 0 & 0 \\ \hline \textit{1} & 1 & 2 & 1 \\ \hline \end{array} \qquad g_y \quad \begin{array}{c|c|c|c|} & \textit{-1} & \textit{0} & \textit{1} \\ \hline \textit{-1} & -1 & 0 & 1 \\ \hline \textit{0} & -2 & 0 & 2 \\ \hline \textit{1} & -1 & 0 & 1 \\ \hline \end{array}$$

The Gradient – how to apply



Bibliography

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	howpublished = {\url{https://github.com/joaofmari/SIN392_Introduction-to-digital-image-processing_2023}}
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THE END