

Comparing functional Embedded Domain-Specific Languages for hardware description

João Paulo Pizani Flor
Department of Information and Computing Sciences,
Utrecht University - The Netherlands
e-mail: j.p.pizaniflor@students.uu.nl

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1 Introduction

Hardware design has become a very complex activity. The size of circuits has increased, while low-level concerns (power consumption, error correction, parallelization, layout, etc.) have to be incorporated earlier and earlier in the design process. This breaks modularity and makes it harder to validate and verify the correctness of circuits.

In this context, researchers have been suggesting (since the 1980s) the usage of functional programming languages to model circuits. One particular line of research is to create Embedded Domain-Specific Languages for hardware description based on existing functional programming languages, such as Haskell.

There are a multitude of EDSLs for hardware description, but they vary wildly on a number of aspects: host language, level of abstraction, capabilities of simulation, formal verification, synthesis (generation of netlists) and integration with other tools, to name a few. All this variety can make the task of choosing a hardware EDSL for the task at hand daunting and time-consuming.

The main goal of this experimentation project is to establish some order in this landscape, and to perform a practical analysis of some popular functional hardware EDSLs. By reading the materials produced in this project (circuit models, test cases, generated netlists, report), a hardware designer wishing to use a functional hardware EDSL for his next design should gain some insight about the strengths and weaknesses of each language and have an easier time choosing one.

As an additional result of this research, we intend to identify recent, cutting-edge developments in the Haskell language and its implementations from which the analyzed EDSLs could benefit. Also, we intend to discuss to which extent some shortcomings of the EDSLs could be overcome by having them hosted in a dependently-typed language.

2 Methodology

In this project, we compared a number of functional hardware EDSLs that we considered representative (more details on the choice of EDSLs further ahead). The comparison was performed on a number of *aspects* for each EDSL, and the analysis was done by considering a *sample set of circuits* used as case studies.

We tried to model all circuits in all EDSLs considered, and as similarly as possible in each EDSL. To avoid using any of the analyzed EDSLs as “base” for analysis, we provide a neutral, behavioural description of the circuits.

2.1 The languages

The embedded Hardware Description Languages we decided to analyze are:

Lava The Lava[2] language, developed initially at Chalmers University in Sweden. Lava is deeply embedded in Haskell, and provides features such as netlist generation and circuit verification using SAT-solvers. There are several “dialects” of Lava available, and the one used for this project is considered the “canonical” one, originally developed at Chalmers.

ForSyDe The Haskell ForSyDe library is an EDSL based on the “Formal System Design” approach[9], developed at the swedish Royal Institute of Technology (KTH). It offers both shallow and deep embeddings, and provides a significantly different approach to circuit modelling, using *Template Haskell* to allow the designer to describe combinational functions with Haskell’s own constructs.

Coquet The Coquet[3] EDSL differs from the other 2 mainly because it’s embedded in a dependently-typed programming language (the Coq theorem prover). Coquet aims to allow the hardware designer to describe his circuits and then *interactively* prove theorems about the behaviour of whole *families* of circuits (using proofs by induction).

2.2 The aspects evaluated

For each of the hardware description EDSLs we experimented with, a number of *aspects* were evaluated. The evaluated aspects do not necessarily make sense for *all* EDSLs, therefore our presentation follows a language-centric approach, in which we expose the strengths and weaknesses of each EDSL concerning the applicable aspects.

Without further ado, the following aspects are considered in the analysis:

Simulation The capability of simulating circuits modeled in the EDSL (and the ease with which it can be performed). Simulation is understood in this context as *functional* simulation, i.e, obtaining the outputs calculated by the circuit for certain input combinations.

Verification The capability of verifying *formal properties* concerning the behaviour of circuits (and the ease with which verification can be performed). The properties we are interested in are those which are *universally quantified* over the circuit’s inputs. As an example of such a property, we might have:

$$\forall a \forall b \forall sel (MUX(a, b, sel) = a) \vee (MUX(a, b, sel) = b)$$

Genericity Whether (and how well) the EDSL allows the modelling of *generic* or *parameterized* circuits. An example of a generic circuit is a multiplexer with 2 n-bit inputs and 1 n-bit output, or a multiplexer with n 1-bit inputs and 1 1-bit output. Besides parametrization in the *size* of inputs and outputs, we will also analyze whether the EDSL provides chances for parametrization on other functional and/or non-functional attributes.

Depth of embedding Whether the EDSL models circuits with a *shallow* embedding (using predicates or functions of the host language), a *deep embedding* (in which circuits are members of a dedicated data type), or anything in between. The depth of embedding of an EDSL might have consequences for other aspects being analyzed.

Integration with other tools How well does the EDSL allow for interaction with (getting input from / generating output for) other tools in the hardware design process. For example, synthesis tools for FPGAs or ASICs, timing analysis tools, model checkers, etc.

Extensibility The extent to which the user can *add* new interpretations, data types, and combinator forms to the language. For example, the user might want to model circuits that consume and produce custom datatypes, or might be interested in extracting *metrics* from a circuit such as power consumption, number of elementary gates, etc.

3 Modeled circuits

When thinking of which circuits to model using the analyzed EDSLs, some principles guided us. First of all, they shouldn't be too simple but also not too complex. Some very simple circuits (adders, counters, etc.) are very often shown as examples in the papers that define the EDSLs themselves, as well as in tutorials. On the other hand, we also did not want to model too complex circuits; that would require too much effort on the hardware design itself, and diverge from the focus of this project, which is to evaluate and analyze the EDSLs.

Another principle that guided our choice is that the circuits should be immediately familiar to anyone with some minimal experience in hardware design. We avoided, therefore, considering application-specific circuits such as those for Digital Signal Processing (DSP), implementing communication protocols, etc. Having ruled out this class of circuits, we were left to choose from circuits that form a general-purpose computing machine, such as arithmetic units, memory blocks, control units and so forth.

Finally, we wanted to choose among circuits that already had a well-defined, *behavioural* description, to avoid using any of the analyzed EDSLs as “basis” of comparison.

Taking these considerations into account, we chose to implement, in each of the EDSLs analyzed, three circuits originating from the book “The Elements of Computing Systems”[6]. This book aims to give the reader a deep understanding of how computer systems work by taking a hands-on approach, in which the reader is given the most basic logic gates and builds, step-by-step, all the hardware and software components necessary to implement a complete computer system.

From the hardware design part of the book, we took our three circuits to be modeled:

- A simple Arithmetic Logic Unit (ALU), from here onwards referred to as “circuit 1”.
- A RAM memory block with 64 words, from here onwards referred to as “circuit 2”.
- A CPU with an extremely reduced instruction set (capable of executing the *Hack* assembly language defined in the book) from here onwards referred to as “circuit 3”.

3.1 Circuit 1: ALU

The Arithmetic Logic Unit built by us is a 2-input ALU, in which each of the inputs (as well as the output) is a 16-bit long word (interpreted as two's-complement signed integer). It is capable of computing several functions, and the choice of which function to compute is made by setting the ALU's 6 *control bits*. To become more familiar with this circuit, let's first take a look at its block diagram, shown in figure 1

Each of the 6 control bits to the ALU has, in isolation, a well-defined effect on the inputs or outputs to the ALU core. The bits (*zx*, *nx*, *zy*, *ny*) control “pre-processing” steps for the inputs *x* and *y*, with the following behaviour:

zx and zy Zeroes the *x* input (respectively *y*). The ALU core will receive 0 as input.

nx and ny Performs *bitwise negation* of input *x* (respectively *y*).

Therefore, the ALU “core” itself (adder, and gate) has, as inputs, the results of performing these pre-processing steps controlled by (*zx*, *nx*, *zy*, *ny*). Furthermore, the *output* of the ALU core can also be *bitwise negated* as a “post-processing” step, controlled by bit *no*.

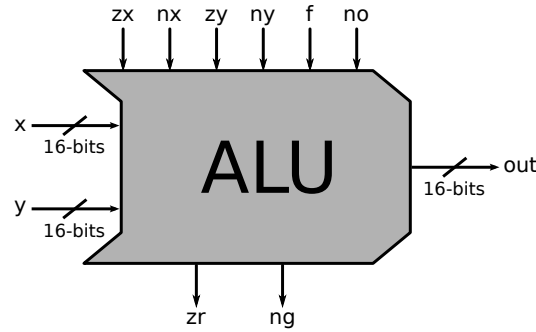


Figure 1: Block diagram of circuit 1, showing its input and output ports.

Finally, the control bit f can be used to select which operation is to be performed by the ALU core: if we wish to add the two inputs, we need to set $f = 1$, and if we want bitwise conjunction, then we need to set $f = 0$.

Besides the main (16-bit wide) output of the ALU, there are two other output *flags*, that indicate predicates over the main output:

zr Is high whenever $out = 0$.

ng Is high whenever $out < 0$.

When the ALU is used in the context of a microprocessor these flags can be used, for example, to facilitate conditional jumps.

Even though there are $2^6 = 64$ possible combinations for the values of the control bits, only 18 of these combinations result in interesting functions – that is because several combinations of control bits can be used to calculate the same function. We show these 18 functions that the ALU can calculate on table 3.1.

zx	nx	zy	ny	f	no	out=
1	0	1	0	1	0	0
1	1	1	1	1	1	1
1	1	1	0	1	1	-1
0	0	1	1	0	0	x
1	1	0	0	0	0	y
0	0	1	1	0	1	$\neg x$
1	1	0	0	0	1	$\neg y$
0	0	1	1	1	1	$-x$
1	1	0	0	1	1	$-y$
0	1	1	1	1	1	$x + 1$
1	1	0	1	1	1	$y + 1$
0	0	1	1	1	0	$x - 1$
1	1	0	0	1	0	$y - 1$
0	0	0	0	1	0	$x + y$
0	1	0	0	1	1	$x - y$
0	0	0	1	1	1	$y - x$
0	0	0	0	0	0	$x \wedge y$
0	1	0	1	0	1	$x \vee y$

Table 1: Functions that the ALU can calculate, given different settings of the control bits

3.2 Circuit 2: RAM64

Circuit 2 is a block of RAM with 64 lines, in which each line is a 16-bit word. Actually, using the term “RAM” to refer to this component is an abuse of terminology, as this circuit is nothing more than a register bank.

All the input and output ports of the circuit are pictured in its block diagram, shown in figure 2.

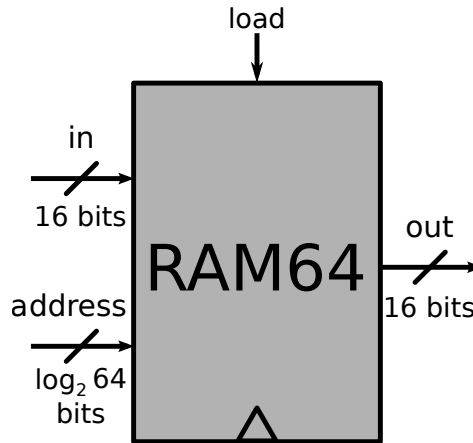


Figure 2: Block diagram of circuit 2, a RAM of 64 lines

The circuit has one 16-bit output, named out, and three inputs (in, address and load). The in port is 16-bit wide and holds a value to be written into the RAM. The address port has a width of $\log_2 64 = 6$ bits and holds the address in which reading or writing is to be performed. Finally, the load bit controls whether the value currently at in should be written to the selected address. There is also one *implicit* input for a clock signal in this component. Implicit, in this case, means that the clock signal is not present in any of the models that we developed for this circuit, but must be present in any physical implementation.

The temporal behaviour of this memory block is as follows: At any point in time, the output out holds the value stored at the memory location specified by address. If the load bit is high, then the value at in is loaded into the memory word specified by address. The loaded value will then be emitted on the output at the **next** clock cycle.

3.3 Circuit 3: The Hack CPU

Circuit 3, the largest and most complex circuit among the ones we have chosen to implement, is the Central Processing Unit for the *Hack* computer, the machine described in the book “The Elements of Computing Systems”[6].

The Hack computer is based on the *Harvard architecture*, that means that it has different storage components and signal pathways for instructions and data. Therefore, the Hack CPU expects to be connected to *two* memory blocks, the instruction memory and the data memory. Having this in mind facilitates the understanding of the CPU’s block diagram, shown in figure 3

The Hack architecture has an *extremely* reduced instruction set, and consists in fact of only two instructions (each 16-bit wide): A (meaning “address”) and C (meaning “compute”). The A instruction can be used as a means to load numerical literals into the data memory, as well as setting a special “cache” register inside the CPU. The C instruction is the one responsible for effectively performing computations using the ALU, testing outputs and jumping. More details about programming in the Hack assembly language can be found in [7].

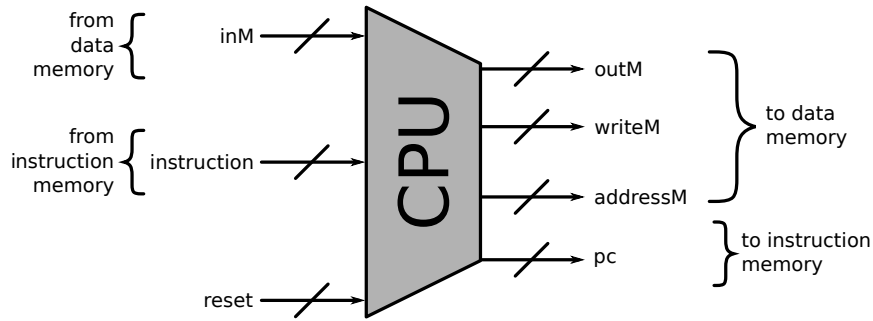


Figure 3: Block diagram of circuit 3, the Hack CPU

The meaning of each of the CPU's input and output ports becomes much clearer when we look at the context in which the CPU is inserted, namely, the memory modules to which it is connected. So, let's analyze the CPU's ports by taking a look at figure 4.

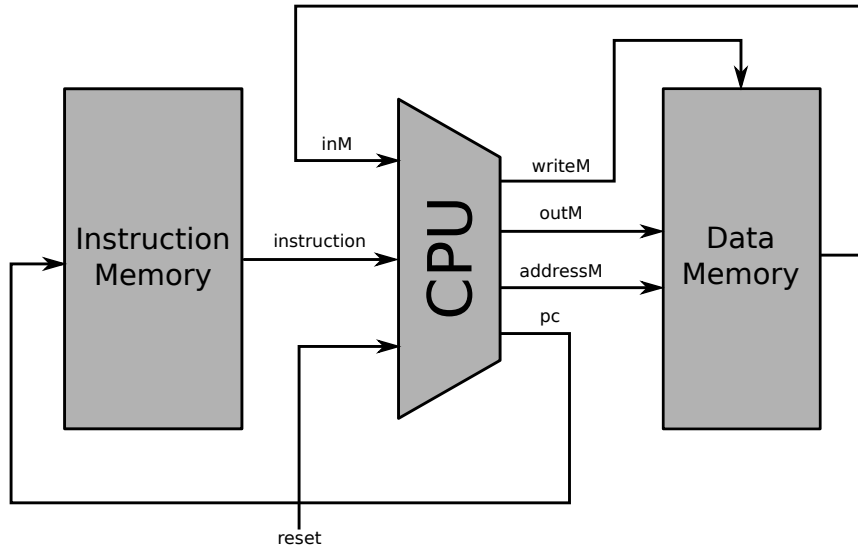


Figure 4: The Hack CPU connected to the data and instruction memory blocks

Finally, the CPU is a circuit which is built mostly from the parts we already defined in circuits 1 and 2. We use the ALU, some registers, multiplexers, an instruction decoder and a counter (the *program counter*). The diagram on figure 5 shows these “parts” of the CPU circuit and how they are connected to each other.

4 Analysis of the EDSLs

4.1 Lava

Lava[2] is an EDSL for hardware description developed originally around 1998 at Chalmers University of Technology, in Sweden. It uses Haskell as the host language, and circuits described in Lava are *deeply embedded*.

The Lava EDSL has several “dialects”, among which are Xilinx-Lava, York-Lava, Kansas-Lava and Chalmers-Lava. Xilinx-Lava[10] was developed by Satnam Singh and

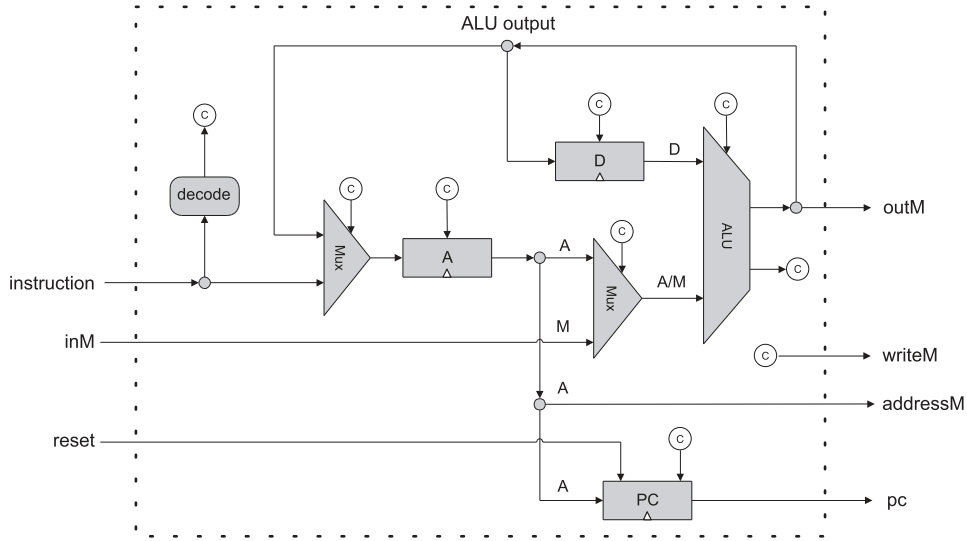


Figure 5: Parts used in building the CPU circuit and how they are connected.

puts a greater emphasis on the *layout* of the described circuits, focusing on their implementation in Xilinx’s FPGAs. York-Lava was developed as part of the Reduceron[5] project, and is a variation of Chalmers-Lava, omitting some features and adding some others, like a “Prelude” of commonly used circuits ((de)multiplexers, (de)coders, RAM memory blocks, etc.). Chalmers-Lava is considered the “standard” dialect, also being the one which was first developed, therefore it was chosen as the one to be studied in this project.

Before diving into the inner workings of the Chalmers Lava library, we first need to make clear that there are two very distinct versions of this library. The original paper that defines the Lava language[2] contains the first version, while the current version is the one defined in a later paper[4] by Koen Claessen and David Sands. This current version of Chalmers Lava is the one in which our case study is implemented.

As already said, Lava uses a *deep embedding*, and the datatype used to represent a circuit is `Signal`, defined in figure 6. As can be noticed from the definition, the *actual* circuit type (`S`) is “wrapped” around the `Ref` type constructor. This has to do with the approach that Lava takes to solving the problem of observable sharing, which is to rely on comparing *references to object* given by the Haskell implementation, to detect cycles in syntax graphs representing circuits. This approach is the cause of some of Lava’s advantages as well as disadvantages, which will be discussed further ahead.

Having defined a circuit operating on values of a type `a` to have type `Signal a`, then there are several circuit combinators provided by Lava, which take circuits as inputs and provide circuits as outputs. For example, on figure 7 we show some boolean circuit combinators:

With Lava, one can also model circuits operating on `Ints` (and there are several interesting integer circuit combinators already included in the Lava library). However, our goal in this project was to model *boolean* circuits and, besides that, integer circuits offer a reduced set of features.

4.1.1 Circuits modeled

In order to be able to describe circuit 1, the ALU (described in more detail on section 3.1), we first needed to model the necessary parts. The “core” of the ALU is composed of a 16-bit ripple-carry adder and a 16-bit AND gate. To model the ripple-carry adder we used

```

newtype Signal a = Signal Symbol

newtype Symbol = Symbol (Ref (S Symbol))

data S s
  = Bool      Bool
  | Inv       s
  | And      [s]
  | Or       [s]
  | Xor      [s]
  | VarBool   String
  | DelayBool s s
  -- other constructors

```

Figure 6: Lava’s Signal datatype, used to represent circuits.

```

bool :: Bool -> Signal Bool
bool b = lift0 (Bool b)

low, high :: Signal Bool
low  = bool False
high = bool True

inv :: Signal Bool -> Signal Bool
inv = lift1 Inv

and1, or1, xor1 :: [Signal Bool] -> Signal Bool
and1 = lift1 And
or1  = lift1 Or
xor1 = lift1 Xor

and2 (x, y) = and1 [x, y]
or2  (x, y) = or1  [x, y]
xor2 (x, y) = xor1 [x, y]

nand2 = inv . and2
nor2  = inv . or2
xnor2 = inv . xor2

```

Figure 7: Some of Lava’s boolean circuit combinators.

full adders as parts, which in turn used half adders. To get used to the way in which circuits are described in Lava, let us first take a look at the definition of the hierarchy of adders in figure 8:

Based on this small model we can already make some observations concerning the aspects that we are analyzing. These observations are:

- All circuits in Lava must be modeled as *uncurried* functions, that is, if multiple inputs are needed, they need to be packed into one tuple, the same “packing” happens also in the case of multiple outputs.
- The *basic* type of input/output for all circuits modeled is `Signal Bool`. This is not coincidental: Lava’s VHDL generation backend can only work with circuits whose


```

type SB = Signal Bool

halfAdder :: (SB, SB) -> (SB, SB)
halfAdder inputs = (xor2 inputs, and2 inputs)

fullAdder :: (SB, (SB, SB)) -> (SB, SB)
fullAdder (cin, (a, b)) = (s, cout)
  where
    (ab, c1) = halfAdder (a, b)
    (s, c2)  = halfAdder (ab, cin)
    cout    = or2 (c1, c2)

rippleCarryAdder :: [(SB, SB)] -> [SB]
rippleCarryAdder ab = s
  where (s, _) = row fullAdder (low, ab)

```

Figure 8: Hierarchy of adders used in circuit 1.

input/output types are `Signal Bool` or any nested combination of tuples and lists thereof. This limitation makes Lava have low **extensibility**, not allowing – for example – user-defined types.

- In Lava, (families of) circuits with variable-sized inputs/outputs are modeled as lists (as can be seen in the definition of `rippleCarryAdder`). This approach has a good **genericity**, but is **not type-safe enough**. For example, we could have a circuit assuming that its inputs are 32-bit wide. There is no way to enforce, at *Haskell compilation time*, that inputs with correct size are provided. Possible problems could only be detected at simulation or VHDL generation.

Now, after having defined all the necessary parts, let's take a look at the ALU circuit itself in figure 9:

```

type ALUControlBits = (SB, SB, SB, SB, SB, SB)

alu :: ([SB], [SB], ALUControlBits) -> ([SB], SB, SB)
alu (x, y, (zx, nx, zy, ny, f, no)) = (out', zr, ng)
  where
    x'  = mux (zx, (x, replicate (length x) low))
    x'' = mux (nx, (x', map inv x'))
    y'  = mux (zy, (y, replicate (length x) low))
    y'' = mux (ny, (y', map inv y'))
    out = let xy'' = zip x'' y'' in mux (f, (and1 xy'', rippleCarryAdder xy''))
    out' = mux (no, (out, map inv out))
    zr   = foldl (curry and2) low out'
    ng   = equalBool high (last out')

```

Figure 9: Top-level model for circuit 1, the ALU.

In the definition of the ALU itself, we would like to have a user-defined datatype to represent the kinds of functions that can be computed by the ALU (functions listed on table 3.1). However, due to the limitations of the VHDL backend already discussed, we have to define `ALUControlBits` as simply a *type synonym* for a 6-tuple of bits.

Besides modelling the three circuits in Lava, we also simulated them. The definition of

the ALU circuit in the book "The Elements of Computing Systems"[6] has a pretty extensive truth table to test the circuit model, which was used to simulate the ALU. However, let's take a look at a simpler simulation case, that of a half-adder:

```
testHalfAdder :: [(SB, SB)]
testHalfAdder = map (simulate halfAdder) input
  where
    input = [ (low, low)
              , (low, high)
              , (high, low)
              , (high, high)
            ]
```

Simulation of combinational circuits is performed by the Lava function `simulate`; it takes as arguments the circuit to simulate and an input combination. In the example of simulation for the `halfAdder`, we *map* the simulation over a list of input combinations, covering all possible cases.

The attentive reader might be asking why is this simulation not an automated test, i.e., why are we **not** comparing the results of the simulation with an *expected* output sequence. This has to do with the way in which Lava handles the problem of observable sharing: values of type `Signal` encapsulate effectively a *runtime reference* to an object of type `a`. Therefore, even though *actual* and *expected* outputs might appear to be equal, they are considered different by Lava. Here is the offending `Eq` instance from the Lava library (module `Lava.Signal`):

```
instance Eq (Signal a) where
  Signal (Symbol r1) == Signal (Symbol r2) = r1 == r2
```

With the drawback of not having *automated* testing, we can say that Lava does provide good **simulation** capabilities, with an interface that is easy to understand for functional programmers.

Now, before moving on to the next circuit studied, let's take a look at how Lava handles *formal verification* with two examples: checking that a full adder is commutative and that the output of an incrementer circuit is always different from its input:

```
prop_FullAdderCommutative :: (SB, (SB, SB)) -> Signal Bool
prop_FullAdderCommutative (c, (a, b)) =
  fullAdder (c, (a, b)) <==> fullAdder (c, (b, a))
```

A property over a circuit in Lava is modeled as a circuit containing *one boolean output*, which – for the property to be true – needs to be *true* for any combination of inputs (these properties are called *safety properties*). Lava performs the verification by converting the circuit model to a CNF logical formula and executing an external SAT solver on the negation of the formula: the property being verified is *valid* if and only if the negated formula is unsatisfiable. The verification for the incrementer introduces another detail of this kind of verification:

```
prop_IncrementIsAlwaysDifferentThanInput :: Int -> Property
prop_IncrementIsAlwaysDifferentThanInput n =
  forAll (list n) (\x -> verifyIncrement x)
  where verifyIncrement x = inv (x <==> increment x)
```

We can see by the type of the verification function that it is a *property generator*, i.e., for each integer `n`, it gives a property. An incrementer is a circuit with generic input/output size, but the SAT-solving approach to verification can only prove properties for *circuits of fixed size*. Therefore, we can only verify a finite number of particular instances of the circuit.

Moving on to circuit 2 (the RAM block), we will take a look at how Lava handles sequential circuits. The “fundamental”¹ sequential circuit in Lava is delay. It takes two boolean signals as input and outputs a single boolean signal. Its semantics is that the output signal will correspond to the input signal *delayed* by one clock cycle, with the other parameter being the first value of the output. Using this fundamental circuit, we modeled the first building block of our hierarchy of memory elements: a 1-bit register with input and load signals:

```
reg :: (Signal Bool, Signal Bool) -> Signal Bool
reg (input, load) = out
  where
    dff = mux (load, (out, input))
    out = delay low dff
```

In this model, we use a mux to control whether the next state of the output will be simply the previous state, or the input value will be “loaded” into the register. Now, a 1-bit register can easily be “lifted” into a generic n-bit circuit:

```
regN :: Int -> ([Signal Bool], Signal Bool) -> [Signal Bool]
regN n (input, load) = map reg $ zip input (replicate n load)
```

The regN definition is *generic*, and parameterized by the size of the input and output (n). This means that *for each value of n, there is a circuit* regN n. In Lava, however, we can only simulate and generate VHDL for specific instances of this family of circuits. The restriction with regards to VHDL generation is not conceptual, that because VHDL has good support for *generic components*, and one could imagine Lava generating generic VHDL from generic circuit models. But, leaving that discussion aside, let’s take a look at the simulation case for regN:

```
testRegN4 :: [[Signal Bool]]
testRegN4 = simulateSeq (regN 4) ins
  where
    lows  = replicate 4 low
    highs = replicate 4 high
    ins   = [(lows,high), (highs,low), (highs,low), (highs,high), (lows,low)]
```

The simulateSeq function, which we already used to test the ALU (a combinational circuit), is actually intended for the simulation of sequential circuits: the list of inputs its given is the sequence of values present at the input ports of the circuit under test – one element of the list for each clock cycle. The list of outputs given by simulateSeq has a similar interpretation.

Having the core sequential component for our memory bank (regN), we modeled some other helper components (such as an *address decoder* and a 64-to-1 *multiplexer*). With all the components at our disposal, we then modeled the RAM block itself:

```
ram64Rows :: Int -> ([SB], (SB,SB,SB,SB,SB,SB), SB) -> [SB]
ram64Rows n (input, addr, load) = mux64WordN n (addr, registers)
  where
    memLine sel = regN n (input, sel <&> load)
    registers   = map memLine (decode6To64 addr)
```

All the registers in the memory bank are connected to the “global” input word for the bank, but the load signal for any particular register is active *iff* the global load signal is active *and* (<&>) that particular memory line is selected. Finally, to be precise, ram64Rows

¹here, *fundamental* means that all sequential circuits have – directly or indirectly – delay as one of its building blocks

actually defines a family of circuits, one for each value of n . The one we are interested in is `ram64Rows 16`, for a RAM block with 64 lines, in which each line is 16 bits wide.

Finally, the last circuit we studied under Lava (circuit 3) is the Hack CPU (described in more detail on section 3.3). The CPU circuit is mostly combinational, as it contains *no form of pipelining* and exactly one instruction is executed per clock cycle. However, there is one sequential component of the CPU: the *program counter*:

```
programCounter :: Int -> (SB, SB, [SB]) -> [SB]
programCounter n (reset, set, input) = out
  where
    incr      = increment out
    out       = delay (replicate n low) increset
    incinput  = mux (set, (incr, input))
    increset  = mux (reset, (incinput, replicate n low))
```

The program counter counts cyclically between 0 and $2^n - 1$, and can have its value reset or set to a particular value at any moment. Having defined the program counter, there are still some helper parts to define before writing the model for the CPU itself: most importantly, we need an *instruction decoder*, responsible for interpreting each Hack instruction and outputting several *control bits* that are used to direct the data flow inside the CPU during each instruction execution cycle:

```
type DestBits = (SB, SB, SB)

type JumpCondBits = (SB, SB, SB)

type CPUControlBits = (SB, SB, DestBits, JumpCondBits, ALUControlBits)

instructionDecoder :: HackInstruction -> CPUControlBits
instructionDecoder (i0,_,_,i3,i4,i5,i6,i7,i8,i9,i10,i11,i12,i13,i14,i15)
  = (aFlag, cAM, cDest, cJump, cALU)
  where
    aFlag = i0
    cAM   = inv i3
    cDest = (i10, i11, i12)
    cJump = (i13, i14, i15)
    cALU  = (i4, i5, i6, i7, i8, i9)
```

Figure 10: The instruction decoder of the Hack CPU.

Here we notice again some limitations of Lava with regards to datatypes: We are limited to lists and tuples of `Signal Bool` (to keep the circuit synthesizable). Here we decided to model the *Hack* instruction itself and the control flags as tuples, to prevent size-related runtime errors. However, using tuples made the model more “cluttered”, as tuples are *not* data structures prone to slicing and regrouping.

Using *fixed-length vectors*, perhaps based on the recent “Type-level Naturals” GHC ² extension [1] (introduced in GHC 7.6 and being improved for GHC 7.8) would make modelling in Lava *safer* and more comfortable.

4.2 ForSyDe

The Haskell ForSyDe library is an implementation of the “Formal System Design” approach to hardware modelling[9]. The ForSyDe approach per se has several significant

²The Glorious Glasgow Haskell Compilation System: <http://www.haskell.org/ghc>

differences when compared to Lava, and even when the two EDSLs agree on *what to do*, sometimes they differ on how to achieve those goals.

To better understand what characterizes the ForSyDe methodology, we first have to establish some vocabulary:

System In ForSyDe, a system or circuit is a network of processes interconnected by *signals*.

Signal A signal is, intuitively, a stream of information that flows between processes. It carries events of some type, and each event has an associated *tag*. The meaning of the tag is defined by the *model of computation* used.

Process A process is nothing more than a pure function on signals. A process *is able* to hold internal state. But, given the same input (possibly infinite) signals, it produces the same output signals.

Process constructor Every circuit in ForSyDe (even simplest combinational ones) is built using a *process constructor*. A process constructor can be seen as a skeleton of behaviour, and it clearly separates computation from synchronization aspects. A process constructor takes a combinational function (called *process function*) as parameter – expressing the computation aspect of the process, and possibly some extra values. There are combinational and sequential process constructors, and some representative examples from each class will be described in more detail in a specific subsection (4.2.2).

4.2.1 Models of Computation

The definition of signal given above is purposefully “vague” mainly because the precise definition of signals depends on the *model of computation* being used. ForSyDe has (currently) process constructors for the following models of computation:

Synchronous All processes in this MoC have a global, implicit clock input, and the tags in the signals are increasing natural numbers. Therefore, a signal can be viewed as a *stream* of values, one for each clock cycle. At each clock cycle, *all* processes consume exactly one value from each of its inputs and produce one value at each of its outputs.

Untimed In the untimed MoC, the processes fire individually and there is no notion of global clock. A process only evaluates when its inputs have a minimum number of values *ready* to be read. The number of needed values can vary per input, but is constant throughout execution.

Continuous The Continuous MoC interprets signals as continuous, one-variable piecewise functions of time. It can be used to model some forms of analog circuits, for example.

Among all MoCs, perhaps the most “notable” one is the Synchronous MoC, because it reflects the usual interpretation of signals as wires and the vast majority of digital designs nowadays having a global clock. Also, all of our studied circuits were modelled in ForSyDe using the Synchronous MoC. Therefore, it is interesting to take a deeper look at it.

First, let's take a look at the behaviour of a system which has an one integer input port and one integer output, and in which the value of the output is equal to the input plus 4. The interface and internal architecture of this system (*addFour*) is depicted in figure 11.

In this system, each of the constituent processes is built using the mapSY process constructor, a constructor of the synchronous model of computation (its name ends in “SY”). It takes a combinational function (in this case “+1”) and evaluates it for each event in the input signal, generating a corresponding event in the output signal.

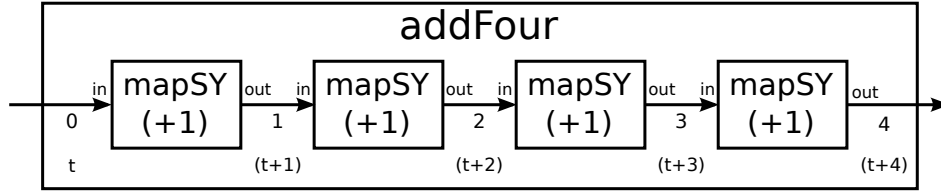


Figure 11: The addFour circuit, example of usage of the Synchronous MoC

Another characteristic of ForSyDe which makes the synchronous MoC even more significant is that only systems that are modeled **exclusively** with process constructors of the synchronous model can be translated into VHDL by ForSyDe.

ForSyDe is a *deeply embedded* EDSL, but it takes a significantly different approach than the one taken by Lava: instead of having some set of “atomic” circuits (they correspond to the constructors of the S type in Lava), ForSyDe uses Template Haskell to *reify* Haskell source code into a syntax tree, and use this syntax tree in order to simulate and/or translate the circuit model into VHDL.

On section ?? We take a closer look at synchronous process constructors, as well as the mechanism by which ForSyDe translates (small fragments of) Haskell source code into the “building blocks” of synchronous systems in VHDL.

On section ?? we proceed to expose the circuits we modeled in ForSyDe, using the models to make a comparative analysis of ForSyDe with the other EDSLs.

4.2.2 Synchronous Process Constructors

TODO: Example of usage of mapSY, with a simple ProcFun. Mention that limitations to ProcFun will be discussed as the case study circuits are analyzed.

In the ForSyDe Haskell package, the module `ForSyDe.Process.SynchProc` provides the list of process constructors that can be used to build systems in ForSyDe’s Synchronous Model of Computation (4.2.1). The functions provided in that module can be divided in 2 big groups:

Combinational Combinations process constructors build processes in which the value on a certain output at time t depends *only* on the input values at time t . A simple example of a combinational process constructor is:

```
mapSY :: (ProcType a, ProcType b) => ProcId
      -> ProcFun (a -> b) -> Signal a -> Signal b
```

Sequential On the other hand, sequential process constructors build processes which can *maintain state*, i.e, the value on a certain output at a moment in time can depend on the value of previous inputs and outputs of the circuit. A simple example of a sequential process constructor is:

```
sourceSY :: (ProcType a, ProcType b) => ProcId
         -> ProcFun (a -> a) -> a -> Signal a
```

The mapSY constructor can be seen as the equivalent of the usual map function, but in the context of Signals: For each element of the input signal (at each clock cycle), it applies the function to it and then produces as its output the result of the function application.

Instead of being passed a “normal” Haskell function (with type $a \rightarrow b$), mapSY is passed a ProcFun (Process Function). ForSyDe has instances of ProcFun which allow for it to be processed with different *interpretations*, such as simulation, generation of VHDL or

generation of graph diagrams. Let's take a look at how one could use the `mapSY` constructor to model a simple incrementer process:

```
incrFunc :: ProcFun (Int16 -> Int16)
incrFunc = \$\ (newProcFun [d | f :: Int16 -> Int16
                             f x = x + 1 |])

incrementer :: Signal Int16 -> Signal Int16
incrementer = mapSY "incrementerProc" incrFunc
```

As already mentioned, ForSyDe makes heavy use of Template Haskell, and this example already clarifies how. First of all, the “innermost” expression `(f x = x + 1)` is *reified* by the “[d]” quasi-quoter into a list of declarations. This list of declarations is then transformed by `newProcFun` into an object of type `ExpQ` (Template Haskell’s reified *expression*). Finally, this reified expression is then *spliced into place* and results in an object of type `ProcFun`.

A `ProcFun` represents, intuitively, the syntax tree of the function, and by traversing this `ProcFun` ForSyDe can perform simulation and VHDL generation. There is, however, one big restriction on `ProcFuns`: As already seen in the type signature of `mapSY` above, the input and output types for the `ProcFun` have to be members of the `ProcType` class. Instances of `ProcType` are provided only for:

Primitive types `Int`, `Int8`, `Int16`, `Int32`, `Bool`, `ForSyDe.Bit`.

Enumerated types User-defined enumerations, with derived instances for `Data` and `Lift`.

Containers Tuples and fixed-length vectors (`Data.Param.FSVec`), holding a type of the above two categories and unrestrictedly nested.

For VHDL to be generated from the system definition, ForSyDe imposes a series of extra restrictions on the form that all `ProcFuns` can take. Upon calling the `writeVHDL` function, the `ProcFun` objects are traversed, and a *runtime* error occurs if any of them does not comply with the restrictions. These restrictions are:

“Point-full” notation Declarations with points-free notation are not accepted as synthesizable

Single-clause To be synthesizable, a `ProcFun` cannot have multiple clauses, and it cannot have `let` or `where` blocks. This essentially forbids recursion inside `ProcFuns`. Pattern matching is still possible by using the `case` construct.

Further details on these restrictions and how they constrain circuit design are shown further ahead, when analyzing the studied circuits.

Now that the concept of a *process function* is clear, let's take a look at how to use a sequential process constructor. In this example, we are using the `sourceSY` constructor to build a counter that counts in ascending order starting from 0:

```
counter :: Signal Int16
counter = sourceSY "counterProc" incrFunc 0
```

Notice that we *reuse* the `incrFunc` process function, as it does exactly what we need. The `sourceSY` constructor takes as parameters a process function f and an initial value x , and has as output signal the sequence $[x, f(x), f(f(x)), f(f(f(x))), \dots]$. This behaviour is equivalent to what the function `iterate` from the Haskell Prelude does.

4.2.3 Circuits modeled

Our comparative analysis of ForSyDe’s strengths and weaknesses was done, as usual, by modelling the 3 circuits used as case-studies. ForSyDe has a peculiar “dual” nature, as it supports both shallow *and* deep embedded models, and models written with netlist generation in mind can look *very* different than models which do not comply with the restrictions that allow synthesis.

Because of this dual nature of ForSyDe, when modelling the case-study circuits we considered 2 kinds of models:

High-level A model that uses Haskell constructs inside the process functions (ProcFun) as close as possible to what a functional programmer would normally use. These models do not comply with ForSyDe’s constraints on the syntax tree of process functions for synthesization, and therefore **can not be translated to VHDL**.

Synthesizable These models are more fine-grained, and use **exclusively** constructs that allow them to be synthesized by ForSyDe’s VHDL backend. They “look” much less like functional programs and more like traditional pen-and-paper diagrams of circuits.

Let’s start our analysis by looking at the *high-level* model for circuit 1, the ALU:

```
type WordType = Int16

data ALUOp = ALUSum | ALUAnd
  deriving (Typeable, Data, Show)

$(deriveLift1 ''ALUOp)

type ALUControl = (Bit, Bit, Bit, Bit, ALUOp, Bit)

type ALUFlags = (Bit, Bit)

bo, bb :: Bit -> Bool
bo = bitToBool
bb = boolToBit

aluFunc :: ProcFun (ALUControl -> WordType -> WordType -> (WordType, ALUFlags))
aluFunc =
  $(newProcFun
    [d|
      aluFunc' :: ALUControl -> WordType -> WordType -> (WordType, ALUFlags)
      aluFunc' (zx,nx,zy,ny,f,no) x y = (out, (bb (out == 0), bb (out < 0)))
      where
        zf z w = if bo z then 0 else w
        nf n w = if bo n then complement w else w
        (xn, yn) = (nf nx $ zf zx $ x, nf ny $ zf zy $ y)
        out = nf no $ case f of
          ALUSum -> xn + yn
          ALUAnd -> xn .&. yn |] )

aluProc :: Signal ALUControl -> Signal WordType -> Signal WordType
  -> Signal (WordType, ALUFlags)
aluProc = zipWith3SY "aluProc" aluFunc
```


The first thing to notice is that the system is working over *16-bit integers*, as by the definition of `WordType`. This is not exclusive of the high-level model, however, as ForSyDe can also produce VHDL models working with integers.

We defined an enumeration type (`ALUOp`) that encodes possible ALU operations, and derived instances of the `Data` and `Lift` classes for it, as is required for `ALUOp` to be `ProcType`. In the body of the `aluFunc` process function, we perform pattern matching on the `f` value to discover which operation to perform. Also, the body of `aluFunc` has a `where` block where all the “parts” that constitute the logic of the ALU are defined. Contrast this definition with the *synthesizable* model of the ALU:

```

type WordType = Int16
type ALUOp = Bit
type ALUControl = (Bit, Bit, Bit, Bit, ALUOp, Bit)
type ALUFlags = (Bit, Bit)

zProc :: ProcId -> Signal Bit -> Signal WordType -> Signal WordType
zProc name = zipWithSY name $(newProcFun [d| f :: Bit -> WordType -> WordType
                                           f z w = if z == H then 0 else w |])

nProc :: ProcId -> Signal Bit -> Signal WordType -> Signal WordType
nProc name = zipWithSY name $(newProcFun [d| f :: Bit -> WordType -> WordType
                                           f n w = if n == H then 42 else w |])

compProc :: Signal ALUOp -> Signal WordType -> Signal WordType -> Signal WordType
compProc = zipWith3SY "compProc"
           $(newProcFun [d| f :: ALUOp -> WordType -> WordType -> WordType
                           f o x y = if o == H then x + y else x .&. y |])

tzProc :: Signal WordType -> Signal Bit
tnProc :: Signal WordType -> Signal Bit

aluProc :: Signal ALUControl -> Signal WordType -> Signal WordType
        -> Signal (WordType, ALUFlags)
aluProc c x y = zipSY "aluProc" out (zipSY "flagsProc" (tzProc out) (tnProc out))
  where
    (zx,nx,zy,ny,f,no) = unzip6SY "ctrlProc" c
    out = nProc "no" no comp
    comp = compProc f (nProc "nx" nx $ zProc "zx" zx $ x)
                  (nProc "ny" ny $ zProc "zy" zy $ y)

```

This model suffers from two consequences of the restrictions imposed by ForSyDe to enable synthesis:

- It is too **fine grained**. As synthesizable `ProcFuns` cannot have local definitions, *every single* step in the datapath inside the ALU has to be a process of its own.
- The parallel and serial combination of processes require several steps of “zipping/unzipping” which have nothing to do with the actual computation. They only adapt the interfaces of the processes to “fit” together, and transform between **tuples of signals and signals of tuples**. We will see the same problem when handling vectors of signals.

An extra weakness of ForSyDe that becomes more problematic in fine-grained models is the need for manual *name management*. Each process in ForSyDe must have a user-provided unique identifier: on the one hand it results in readable and modular VHDL netlists, but on the other hand it forces the hardware designer to work at a lower level than desired, making the design more error-prone.

The issue of name management, along with the usage of fixed-length vectors, will become clearer as we analyze our second circuit: a RAM block of 64 lines.

First of all, we model a n-bit register, which is not so different from the n-bit register we modeled in Lava:

```
type WordType = Int16

reg :: Signal WordType -> Signal Bit -> Signal WordType
reg input load = out
  where out = delaySY "delay" (0 :: WordType) dff
        dff = (instantiate "mux2" mux2SysDef) load out input
```

The next needed part for the RAM is a 64-to-1 multiplexer, to choose which of the RAM lines to select as output, depending on the address. We modeled a whole hierarchy of multiplexers upto the one we needed (64-to-1): starting with a 2-to-1 multiplexer, then building a 4-to-1 using 2-to-1 as components, 16-to-1 using 4-to-1 and finally 64-to-1 using 16-to-1 and 4-to-1. We will only show the first two degress in the hierarchy:

```
mux2 :: Signal Bit -> Signal WordType -> Signal WordType
      -> Signal WordType
mux2 = zipWith3SY "zipWith3SY"
      $(newProcFun [d] f :: Bit -> WordType -> WordType -> WordType
                  f s x y = if s == L then x else y [])

mux2SysDef :: SysDef ( Signal Bit -> Signal WordType -> Signal WordType
                    -> Signal WordType)
mux2SysDef = newSysDef mux2 "mux2Sys" ["sel", "in1", "in2"] ["out"]

mux4 :: Signal (FSVec D2 Bit) -> Signal (FSVec D4 WordType) -> Signal WordType
mux4 ss is = (mux2' "m1") (sv ! d1) m00 m01
  where mux2' l = instantiate l mux2SysDef
        sv      = unzipxSY "unzipSel" ss
        iv      = unzipxSY "unzipInp" is
        m00     = (mux2' "m00") (sv ! d0) (iv ! d0) (iv ! d1)
        m01     = (mux2' "m01") (sv ! d0) (iv ! d2) (iv ! d3)

mux4SysDef :: SysDef ( Signal (FSVec D2 Bit) -> Signal (FSVec D4 WordType)
                    -> Signal WordType)
mux4SysDef = newSysDef mux4 "mux4Sys" ["sel", "inputs"] ["out"]
```

The hierarchy of multiplexers is a perfect example to illustrate the aforementioned issue of zipping/unzipping: as an input to mux4 we get a signal of binary vectors (each with length 2). But we want to use mux2 as a component, therefore we need to **unzip** the signal of vectors into a vector of signals, and then **index** the vector to get each individual signal. In the case of mux16 (not shown here), this situation becomes even worse as, besides unzipping, we need to **re-zip** the constituent signals into “groups” of the right size to be used with the subcomponents.

As we already mentioned, the requirement of user-given unique names for processes in ForSyDe results in a much more readable VHDL output for the models. Another factor that helps in this direction is the ForSyDe concept of *component instantiation*.

When finished modelling a circuit in ForSyDe, we “wrap it up” in a *system definition* (a value of type SysDef) by calling the function newSysDef. When we call writeVHDL on this system definition, a VHDL top-level entity is generated. If we then want to use this “finished” model as a subcomponent in another circuit, we can use the *instantiate* function to create a named process out of the component’s SysDef. When the VHDL code for the

bigger circuit is generated, the ForSyDe instantiation is mapped to a VHDL component declaration (with accompanying port map statements), which makes for pretty modular VHDL code.

Let's now finally look at the top-level ForSyDe model for circuit 2, the RAM block:

```
ram64 :: Signal WordType -> Signal (FSVec D6 Bit) -> Signal Bit
      -> Signal WordType
ram64 input addr load = mux' addr (zipxSY "zipRows" rs)
  where
    -- parts declarations
    mux'      = instantiate "mux" mux64SysDef
    decoder'  = instantiate "decoder" decode6To64SysDef
    reg' 1    = instantiate 1 regSysDef
    and' 1    = instantiate 1 andSysDef
    -- using the parts
    r (s, 1) = (reg' 1) input ((and' (1 ++ ":and")) load s)
    rs'      = unzipxSY "unzipAddr" $ decoder' addr
    rs       = V.map r $ V.zip rs' (V.map (\n -> "r" ++ show n)
                                          (V.unsafeVector d64 [0..63]))

ram64SysDef :: SysDef ( Signal WordType -> Signal (FSVec D6 Bit) -> Signal Bit
                      -> Signal WordType)
ram64SysDef = newSysDef ram64 "ram64" ["input", "addr", "load"] ["outWord"]
```

This model is also similar to the one wrote in Lava, and that is the reason for why we don't have separate *high-level* and *synthesizable* models for circuit 2. The “natural” model, i.e, the one that came to mind immediately reading the description of the circuit, happens to also be synthesizable. In this model, we use the parts already defined before (register, 64-to-1 multiplexer), as well as some simple gates (and, or) and an *address decoder* (decoder'). We omit here the code for the address decoder, as it consists simply of an enumeration of all *minterms* (all possible boolean products involving the 6 input bits and their negation).

Lastly, let's analyze and discuss the ForSyDe model for circuit 3, the Hack CPU. We built the CPU using mostly already defined circuits (ALU, registers, multiplexers) as building blocks, which made the model also look very similar to the one written in Lava. This modular approach is not coincidental: in the book “The Elements of Computing Systems”[6], great care is taken to make each circuit in the hierarchy add only a small step in complexity when compared to its already defined subcomponents.

In the case of the CPU, we needed to model three main additional components: a program counter, an instruction decoder, and a component that decides when to perform a jump. Let's first start by looking at the program counter:

```
type AddrType = Int16

pc :: Signal Bit -> Signal Bit -> Signal AddrType -> Signal AddrType
pc = scanld3SY "programCounter" nextStateFun 0
  where
    nextStateFun =
      $(newProcFun [d| f :: AddrType -> Bit -> Bit -> AddrType -> AddrType
                      f cur reset set new = if reset == H then 0
                                              else if set == H then new
                                              else cur + 1 |])

pcSysDef :: SysDef ( Signal Bit -> Signal Bit -> Signal AddrType
                    -> Signal AddrType)
pcSysDef = newSysDef pc "pcSys" ["reset", "set", "addr"] ["out"]
```

The address type `AddrType` is defined as `Int16` because the specification of circuit 3 requires so. The program counter is a simple counter with reset and set inputs. Presenting a high value at the reset input will cause the program counter to output 0 at the next clock cycle, which will make the CPU fetch the instruction from memory address 0, effectively rebooting the computer. Presenting a high value at the set input will cause the program counter to have as its next output the value currently present at input `addr`. This is the way in which *jumps* are performed in the Hack architecture.

Having defined the model for the program counter, we proceeded to test its behaviour, according to the table of test cases present in chapter 5[8] of the book defining the circuits:

```
testPC3 :: Bool
testPC3 = (simulate pcSysDef) resets sets vals == expected
  where
    (r, s) = (H, H) -- nicknames for reset and set
    x      = 0      -- nickname for "don't care"
    expected = [0, 1, 2, 3, 1, 2, 3, 0, 1, 2, 3, 4]
    (resets, sets, vals) = unzip3 inputs
    inputs = [ (L,L,x), (L,L,x), (L,L,x), (L,s,1), (L,L,x), (L,L,x)
              , (r,L,x), (L,L,x), (L,L,x), (L,L,x), (L,L,x), (L,L,x) ]
```

An important aspect of simulation with ForSyDe is that we can actually *compare* the outputs of simulation for equality with an *expected* sequence of inputs, which could *not* be done in Lava.

With the program counter defined and tested, we now analyze the instruction decoder:

```
type HackInstruction = FSVec D16 Bit
type DestType = (Bit, Bit, Bit)

instructionDecoder :: Signal HackInstruction
                  -> Signal (Bit, Bit, DestType, JumpType, ALUControl)
instructionDecoder = mapSY "mapSYdecoder" decoderFun
  where
    decoderFun =
      $(newProcFun [d] f :: HackInstruction
        -> (Bit, Bit, DestType, JumpType, ALUControl))
    f i = ( i!d0
          , not (i!d3)
          , (i!d10, i!d11, i!d12)
          , (i!d13, i!d14, i!d15)
          , (i!d4, i!d5, i!d6, i!d7, i!d8, i!d9)
          ) []

decoderSysDef :: SysDef ( Signal HackInstruction
                        -> Signal (Bit, Bit, DestType, JumpType, ALUControl))
decoderSysDef = newSysDef instructionDecoder "decoder" ["instruction"] ["ctrl"]
```

TODO: discuss how – on the one hand – ForSyDe’s `FSVec`’s can help make modelling more type safe but on the other hand the current implementation is very “hackish” and could ALSO make some use of recent developments in GHC.

TODO: closing remark for ForSyDe: not actively maintained, relies on very old GHC version and doesn’t build with newer ones. But the design methodology in itself is very interesting and could surely benefit from modern developments in Haskell implementations.

4.3 Coquet

The third analyzed EDSL for hardware description, Coquet[3], is strikingly different from both others. Most of these differences can be explained in one way or another by its choice of host “language” – Coq³.

Coq is an interactive theorem prover based on *intuitionistic type theory*. In the context of Coq, the concepts of “term” and “type” are far more intertwined than, say, in Haskell. Types in Coq can contain references to terms and vice-versa. A very typical example of these so-called *dependent types* is the type(-family) of vectors with a certain length:

```
Inductive vec A : nat -> Type :=
| nil   : vec A 0
| cons  : forall (h : A) (n : nat), vec A n -> vec A (S n).
```

By having the length of the vector being part of the type, we can *enforce* several useful properties of functions operating on vectors. In fact, the type-system of Coq is so expressive that it can encode any proposition of *intuitionistic propositional logic*.

Given such expressive power, one can imagine that it might be useful to express circuits in Coq, and use it to prove interesting properties about these circuits. This is exactly the goal of Coquet. How this goal is achieved and the modelling of our studied circuits in Coquet is discussed in the following subsections.

4.3.1 Modelling circuits

Coquet is a deep-embedded DSL, thus it represents circuits as a datatype. By using dependent types, a designer modelling a circuit in Coquet is able to prevent certain kinds of “errors” much earlier in the design process, because the *well-formedness* is guaranteed by construction, i.e., every circuit built using the constructors provided by Coquet are well-formed by definition. Let’s take a look at the *Circuit* data type declaration:

```
Context {tech : Techno}
Inductive Circuit : Type -> Type -> Type :=
| Atom : forall {n m : Type} {Hfn : Fin n} {Hfm : Fin m}, techno n m -> Circuit n m
| Plug : forall {n m : Type} {Hfn : Fin n} {Hfm : Fin m} (f : m -> n), Circuit n m
| Ser  : forall {n m p : Type}, Circuit n m -> Circuit m p -> Circuit n p
| Par  : forall {n m p q : Type}, Circuit n p -> Circuit m q -> Circuit (n + m) (p + q)
| Loop : forall {n m p : Type}, Circuit (n + p) (n + p) -> Circuit n m
```

Figure 12: The Circuit (\mathbb{C}) datatype in Coquet

The \mathbb{C} type is parameterized by two types. These types are the *input* and *output* types of the circuit, respectively. They **do not** represent what is “carried” on the wires, but the *structure* of the circuit’s input and output ports: How many of them there are, how are they grouped and how are they named.

There are 2 *atomic* constructors from which an element of \mathbb{C} can be built and 3 *combinators*, which build a circuit based on other circuit(s). By observing the serial and parallel composition combinators (Ser and Par, respectively), we notice that the input and output types match exactly as expected.

The 2 atomic constructors constrain the types n and m by requiring them to have instances of the “Fin” type class, i.e., they have to be *finite* types (types from which a finite list of unique elements can be obtained). This constraint is important given the interpretation that these types (n and m) have: each element of n (respectively m) stands for an input (respectively output) “wire” in the circuit interface.

³“Coq” is not the name of a language, but a theorem-proving system that uses different languages for defining terms, interactive commands, and user-defined tactics

The case of the “Atom” constructor is particularly revealing of how Coquet works: this constructor is parameterized by an *instance* of the type class `Techno` for the types `n` and `m`. What this instance provides (in the code fragment that reads “`techno n m`”) is the *type of the fundamental gate* in the technology being used. We could choose our modeled circuits to have, for example, NAND, NOR, or other (more exotic) gates as fundamental.

As an “usage example” of Coquet, we show two simple circuits (NOT and HALFADD), along with proofs that they implement the expected functions over booleans. Let’s start with NOT:

```
Definition NOT x nx : Circuit [:x] [:nx] := Fork2 _ |> (NOR x x nx).
```

```
Instance NOT_Implement {x nx} : Implement (NOT x nx) _ _ negb.
```

```
Proof.
```

```
  intros ins outs H.
```

```
  unfold NOT in H.
```

```
  tac.
```

```
Qed.
```

The input type of NOT is a *tagged unit* with tag `x`, similarly, the output type has tag `nx`. There is some *notation* introduced by Coquet to make the creation of tagged units more convenient. The NOT circuit is a serial composition (denoted as `|>` of a Fork2 circuit (which simply splits the input into two identical copies) and a NOR circuit, which is the underlying fundamental gate in this case.

Below the definition of the circuit itself we state and prove the fact that our circuit implements the desired function (boolean negation, `negb`). More details on exactly what is meant by “implements”, as well as the workings of plugs and tags, are exposed further ahead. Now let’s take a look at a half-adder described in Coquet:

```
Definition HADD a b s c : Circuit ([:a] + [:b]) ([:s] + [:c]) :=
  Fork2 ([:a] + [:b]) |> (XOR a b s & AND a b c).
```

```
Instance HADD_Implement {a b s c} :
```

```
  Implement (HADD a b s c) _ _
```

```
  (fun (x : bool * bool) => match x with (a,b) => (xorb a b, andb a b) end).
```

```
Proof.
```

```
  unfold HADD; intros ins outs H; tac.
```

```
Qed.
```

First of, the sum types that are given as parameters to `ℂ` indicate that we have two input ports and two output ports. By using Fork2 on a binary sum `([:a] + [:b])`, we create as output a sum type in which each of the components is in itself a sum: that matches exactly the interface of the component after the `|>` operator. On the right side of the serial composition, we have a parallel composition of XOR and AND, giving two outputs: respectively the sum `([:s])` and carry-out `([:c])`

Together with the definition, we prove that the HADD circuit implements the boolean function we would expect, and the proof is similar to the case of NOT. It makes use of a Coquet custom tactic (`tac`), but a more throughout example of proof of functional correctness will be given further ahead. Also, in the case of the adder used in our case study (ripple-carry adder), we prove that the circuit implements the *actual* addition function on binary integers, and not some boolean equivalent.

As a last detail on the “user interface” of Coquet, there is the definition of what exactly are the input and output types (a circuit of type `ℂ n m` has input type `n` and output type `m`). Usually, in the Coquet paper[3] and in the examples provided with the library, input and output types are *sum types* in which the terms of the sum are *tagged units*. Using tags serves a form of “documentation”, giving someone reading the circuit model an idea of

what role does each input/output port play. The *type family* of tagged unit types is defined as follows:

```
Inductive tag (t : string) : Type := _tag : tag t
```

```
Notation "[: x ]" := (tag x).
```

```
Notation "[! x ]" := (_tag x).
```

```
Notation "[!!]" := (_tag _).
```

For each string t , there is a type $\text{tag } t$, and this type has exactly one inhabitant. There are also, as part of Coquet, some definitions to make working with sum types less tiresome. For example, there is the function `sumn` which, given a type t and a natural n , returns a sum type with n elements and in which each element has type t . This might be useful if we are defining a n -bit adder:

```
Definition RIPPLE cin a b cout s n :
```

```
  Circuit ([:cin] + sumn [:a] n + sumn [:b] n) (sumn [:s] n + [:cout]) := ...
```

While the provided examples use sums of tagged units as the input/output types, they can be more general: as seen in the definition of the circuit type (Fig. 12), the only requirement is that they belong to the `Fin` type class, which is defined as follows:

```
Class Fin A := {
  eq_fin : eqT A;
  enum   : list A;
  axiom  : forall (x : A), count (equal x) enum = 1
}.
```

4.3.2 Circuit semantics in Coquet

TODO: Explain what the meaning relation for a circuit in Coquet is, give the (inductive) definition of the meaning relation and comment on it.

TODO: Explain how the meaning relation is too “low-level”, and how Coquet uses *data abstraction* and *function abstraction* to prove the correctness of circuits. The proofs of correctness are proofs that a circuit either **Realizes a high-level relation** or **Implements a high-level function** over the high-level input/output types.

We give the definitions of `Realizes` and `Implements`, as well as walk over a small example of functional correctness proof.

4.3.3 Circuits modeled

TODO: Expose the studied circuits modeled in Coquet, highlighting advantages and disadvantages of the Coquet approach along the way.

TODO: circuit 1 (ALU) – Model, proof of some of its components.

TODO: circuit 2 (RAM) – Model.

4.3.4 Possible improvements and future work

The approach to circuit modelling is *very* generic, and leaves room for extension in several aspects, even though the library is already provided with some reasonable defaults.

For example, the definition of the *meaning relation* for circuits (by induction on circuit structure) is parameterized by the *type T of what is carried in the wires*. Even though the library already provides examples for booleans and streams of booleans, it could also be interesting to add cases for dealing with some three-valued logics, or a case for IEEE1164’s `std_logic` type, used often in VHDL.

Another interesting point is that Coquet defines a `stream` type family, and then proceeds to define several interesting function over `stream`, as well as an instance for the meaning relation. The type family is defined as follows:

Definition `stream A := nat -> A.`

If instead of using `nat` in the above definition, we generalized it a bit more (abstracting a new type parameter), we arrive at the concept of an “event stream”:

Definition `events tag A : tag -> A.`

This is exactly how ForSyDe defines its “signals”, so we could model circuits in models of computation other than the synchronous model (which is the one allowed by the current `stream` definition).

5 Conclusions

TODO: Reiterate the analysis of the EDSLs spread throughout the analysis of each of the studied circuits, but in a more summarized manner, actually **comparing** the EDSLs among themselves, **by aspect** (the ones we defined in section 2.2).

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