Clarifications

Updated: 6 October 2021

This document contains some clarifications that will be helpful to the reader, as well as a list of a few minor (yet sufficiently relevant to be mentioned) typos.

1. Clarification: Equation (28) pertains to $r = R_b(z, \theta, t)$. Its linearization implies that

$$u'(R_b, z, \theta, t) \approx u'(R, z, \theta, t) + \left.\frac{\partial u'}{\partial r}\right|_{r=R} r'(z, \theta, t). \tag{1}$$

   The second term on the rhs of the above equation is of quadratic order in the perturbations and is neglected, so with the linearization, Eq. (29) is evaluated at $r = R$ (rather than at $r = R_b$):

$$u'(R_b, z, \theta, t) \approx u'(R, z, \theta, t) = -ig(R)r'(z, \theta, t). \tag{2}$$

   Equation (33) may thus be written more precisely as

$$\left.\frac{\hat{u}_1}{g_1}\right|_{r=R} = \left.\frac{\hat{u}_2}{g_2}\right|_{r=R}, \tag{3}$$

   which is used to obtain e.g., Eq. (86).

2. Typo: In Eq. (90), Ω needs to be replaced by Ω^2:

$$\left.\frac{dP_2}{dr}\right|_{r=R} = \rho\Omega^2 \left.\frac{R^4}{r^3}\right|_{r=R} \tag{4}$$

3. Clarification on p. 3189, top-right: It is true that the imaginary part of the lhs of Eq. (96) must always be zero. This is trivially fulfilled when $\omega_i = 0$ and $\omega_r \leq (m + 2)\Omega$ (so that β is real). However, for some choices of $\omega = \omega_r + i\omega_i$, the imaginary part of the two terms on the lhs of Eq. (96) could be nonzero, with both terms canceling. This could potentially allow $\omega_i > 0$, which would imply unstable wave growth.

   That such a solution is not possible cannot readily be inferred; however, when considering the solutions on the complex plane, as was done for the Rankine vortex with upward motion in its core and the cylindrical vortex sheet (section 7), it is seen that Eq. (96) indeed is only fulfilled for real ω. While the imaginary part of the lhs of Eq. (96) may be zero for some nontrivial combinations of (ω_r, ω_i), the real part of the equation is not simultaneously fulfilled for these combinations, but only for $\omega_i = 0$. Incidentally, the equation also has no solution for $\omega \geq (m + 2)\Omega$, which corresponds to the requirement that β be real.

4. Captions, Figs. 16 and 18: Delete “and (d) axial group speed as a function of axial wavenumber”.

5. Typo: p. 3209, Acknowledgments: The correct spelling is *Alan Shapiro*.