Post-processing forecasts

Forecasts of COVID-19, especially forecasts made by humans, often are not perfectly calibrated and tend to be systematically over-confident. Post-processing may be a way to alleviate this problem and improve overall forecast quality.

(Possible) Objectives

- Identify different means of post-processing forecasts in a quantile-based format
- Implement the procedures in R or python
- Evaluate and compare performance of the different procedures
- Identify differences when post-processing data on a logarithmic scale
- Find out whether we can adapt a forecast model using information learned from other forecast models
- Investigate how stable forecast performance is over time. How much training data should be used for re-calibration and is it possible to infer future errors from past errors?

Data

All forecasts are made in a quantile-based format, meaning that forecasters provide a predictive distribution in the form of 23 quantiles (11 prediction intervals plus the median prediction), specifying how likely they think the true observed value will fall in a given range.

Human forecasts of COVID-19 made in the UK COVID-19 Forceasting Challenge

This data includes all forecasts as well as the true observations.

```
uk_data <- fread("data/full-data-uk-challenge.csv")
```

The data has the following columns:

Column name	Column description
location_name	Name of the country
target_end_date	Date for which a forecast was made. This is always a Saturday
target_type	The target variable to be predicted, cases or deaths
true_value	The corresponding true observed value
population	population of the target country
forecast_date	Date on which a forecast was made. This is always a Monday
quantile	quantile-level of the predictive distribution
prediction	Predicted value corresponding to the quantile-level specified in 'quantile'
model	Name of the forecaster
target	Summary of the prediction target variable (redundant information)
horizon	Forecast horizon
expert	Whether or not a forecaster self-identified as an expert

Potential difficulties are:

- there only is one location
- there are not a lot of observations
- many forecasters only submitted a few times and drop in and out

However, it is of particular interest to know

- whether proposed procedures work for small data sets with not a lot of training data
- whether we can e.g. learn anything from other forecasters that we can then apply to a forecaster who submitted for the first time.

Forecasts submitted to the European Forecast Hub

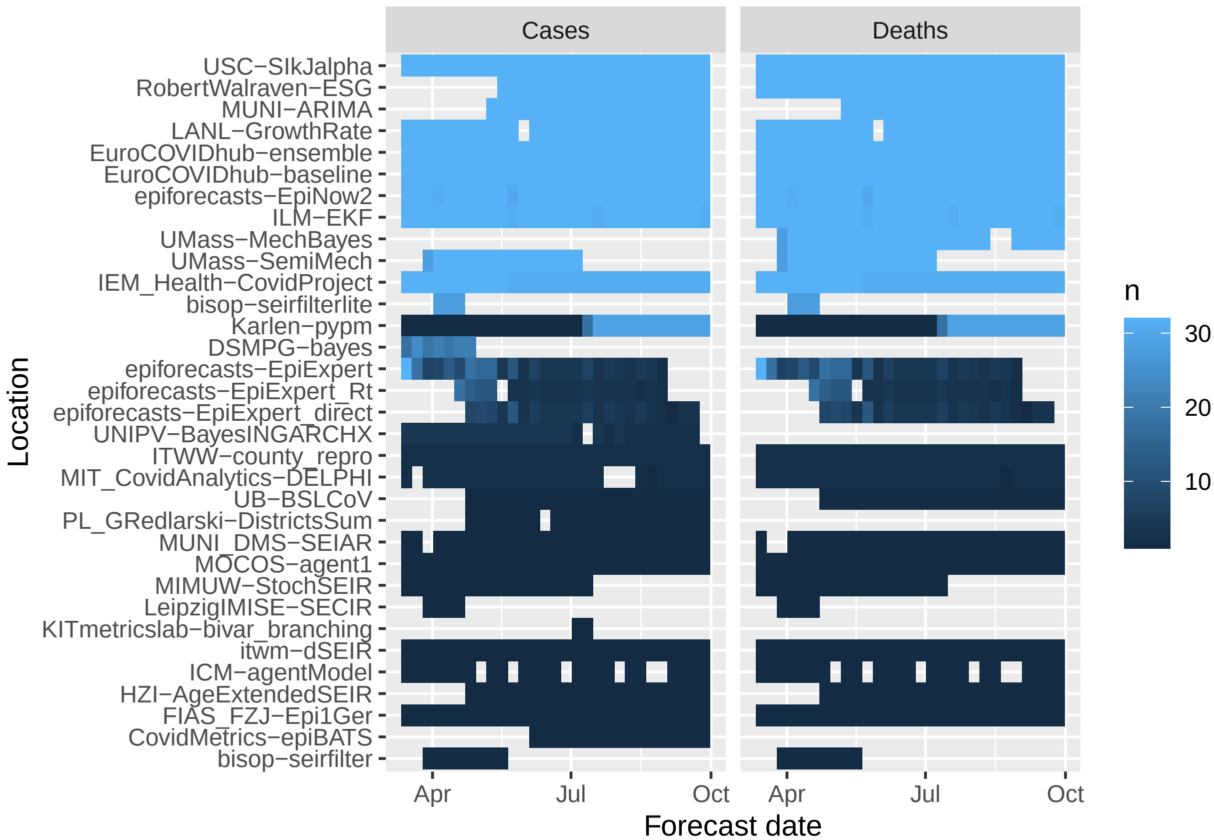
These forecasts were submitted to the European Forecast Hub by different research institutions. The file contains forecasts as well as true observations.

```r
hub_data <- rbindlist(
  list(
    fread("data/full-data-european-forecast-hub-1.csv"),
    fread("data/full-data-european-forecast-hub-2.csv")
  )
)
```

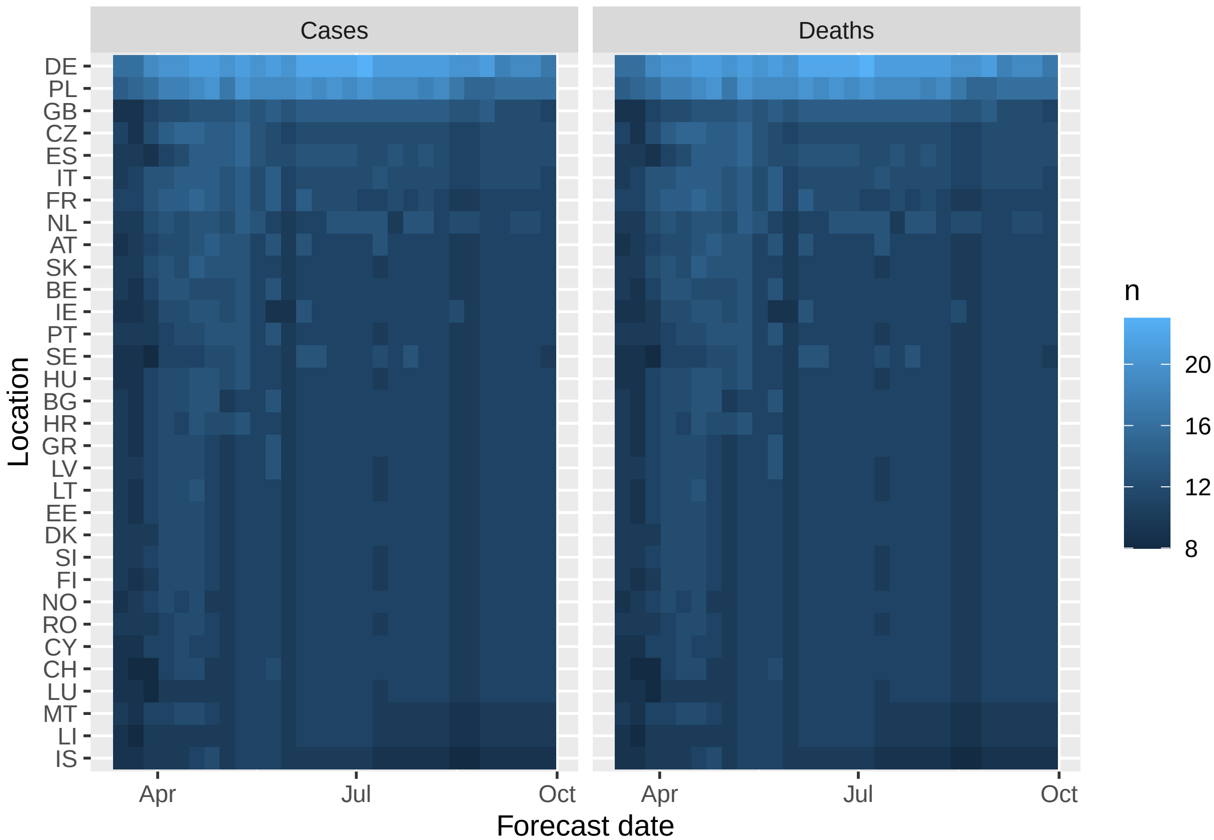
Optionally, the Hub data can be filtered to obtain a complete set of forecasts, as the current data set has missing forecasts:

```r
# helper functions for visualisation
plot_models_per_loc <- function(data) {
  data |>
    group_by(location, forecast_date) |>
    mutate(n = length(unique(model))) |>
    ggplot(aes(y = reorder(location, n), x = as.Date(forecast_date), fill = n)) +
    geom_tile() +
    facet_wrap(~ target_type) +
    labs(y = "Location", x = "Forecast date")
}

plot_locs_per_model <- function(data) {
  data |>
    group_by(model, forecast_date) |>
    mutate(n = length(unique(location))) |>
    ggplot(aes(y = reorder(model, n), x = as.Date(forecast_date), fill = n)) +
    geom_tile() +
    facet_wrap(~ target_type) +
    labs(y = "Location", x = "Forecast date")
}
```

```r
plot_locs_per_model(hub_data)
```

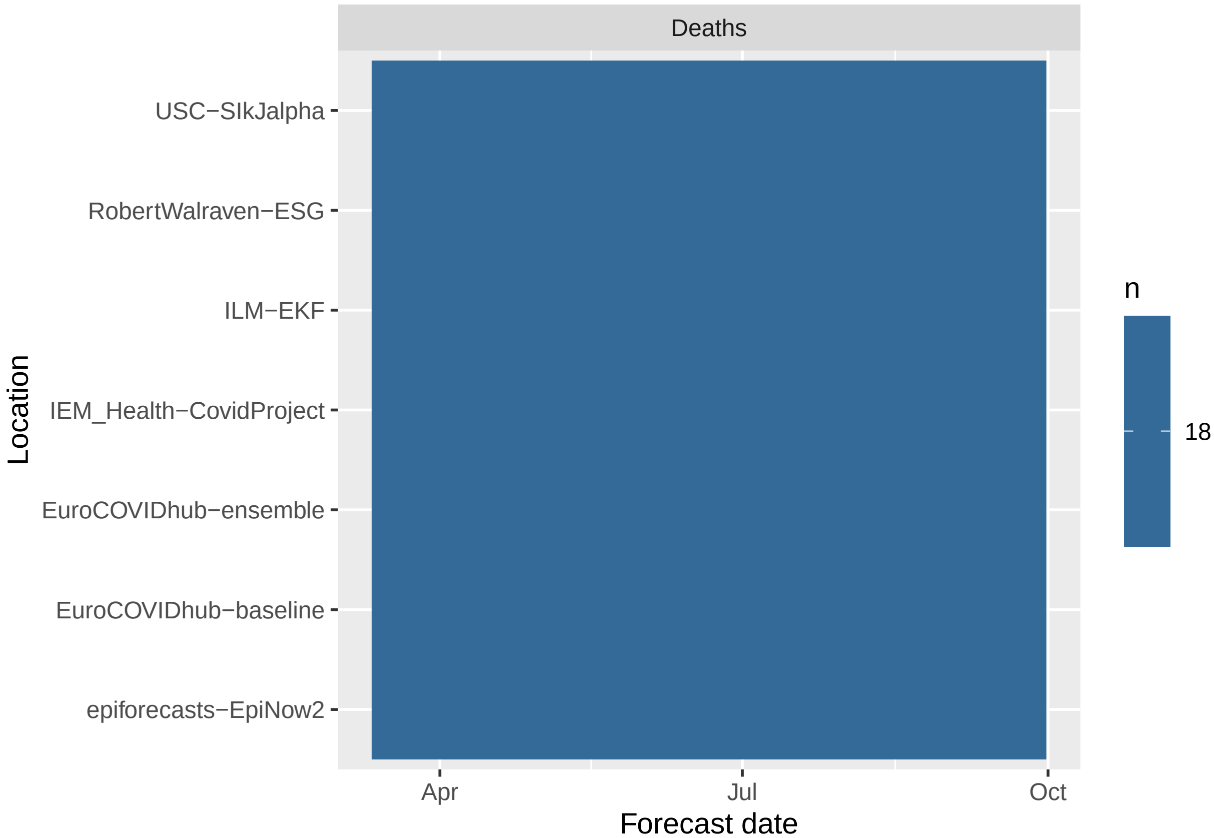



```
plot_models_per_loc(hub_data)
```

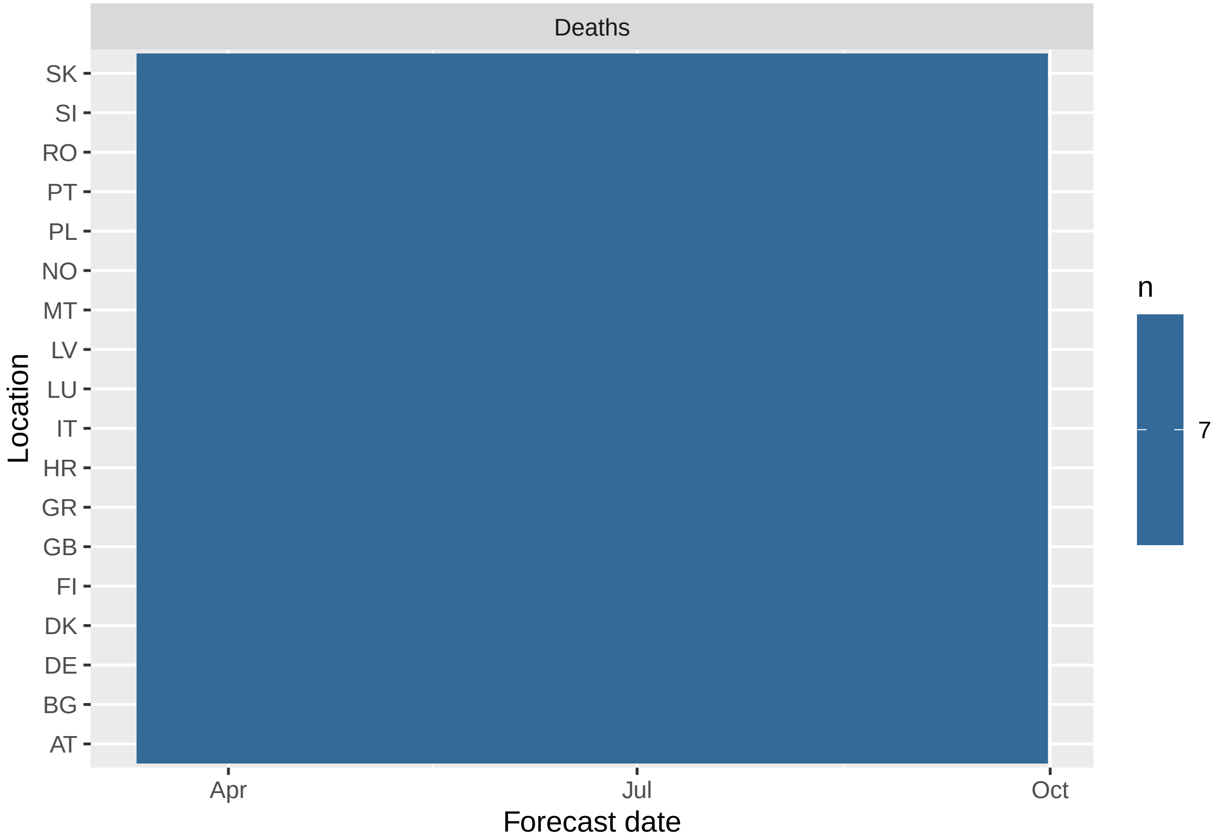



```r
# helper function to make a complete set. The data can be either complete
# per location (meaning that different locations will have different numbers of
# models) or it can be complete overall (removing models and locations)
make_complete_set <- function(hub_data,
                              forecast_dates = c("2021-03-15",
                                                 "2021-09-27"),
                              min_locations = 19,
                              per_location = FALSE) {

  # define the unit of a single forecast
  unit_observation <- c("location", "forecast_date", "horizon",
                        "model", "target_type")

  h <- hub_data |>
    # filter out models that don't have all forecast dates
    filter(forecast_date >= forecast_dates[1],
           forecast_date <= forecast_dates[2]) |>
    group_by_at(c(unit_observation)) |>
    ungroup(forecast_date) |>
    mutate(n = length(unique(forecast_date))) |>
    ungroup() |>
    filter(n == max(n))

  # per_location means a complete set per location, meaning that every location
  # has a complete set, but the numbers of models per location may be different
  # if this is not desired, we need to restrict the models and locations
```

```
  if (!per_location) {
    h <- h|>
      # filter out models that don't have at least min_locations
      group_by_at(unit_observation) |>
      ungroup(location) |>
      mutate(n = length(unique(location))) |>
      ungroup() |>
      filter(n >= min_locations) |>
      # filter out locations that don't have a full set of forecasts
      group_by(location, target_type) |>
      mutate(n = n()) |>
      ungroup() |>
      filter(n == max(n))
  }
  return(h)
}

hub_complete <- make_complete_set(hub_data)
print(plot_locs_per_model(hub_complete))
```

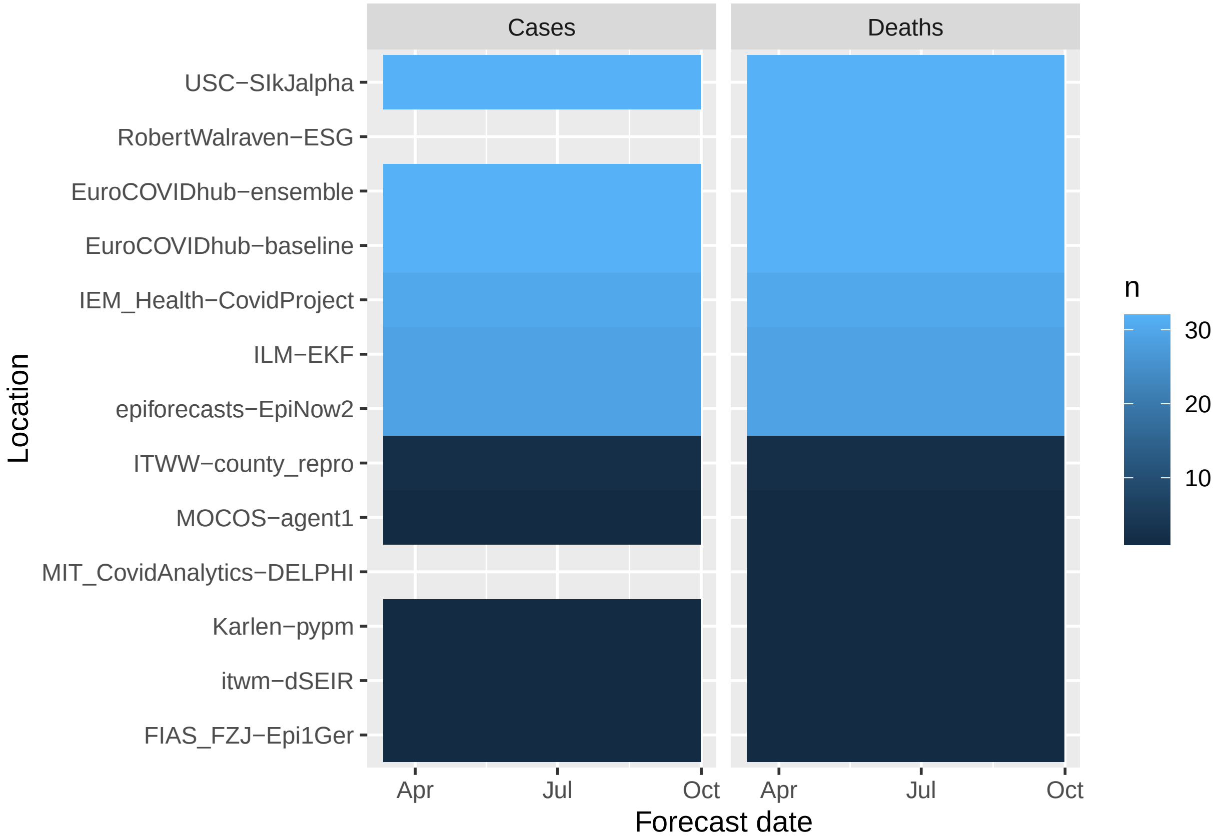



```
print(plot_models_per_loc(hub_complete))
```

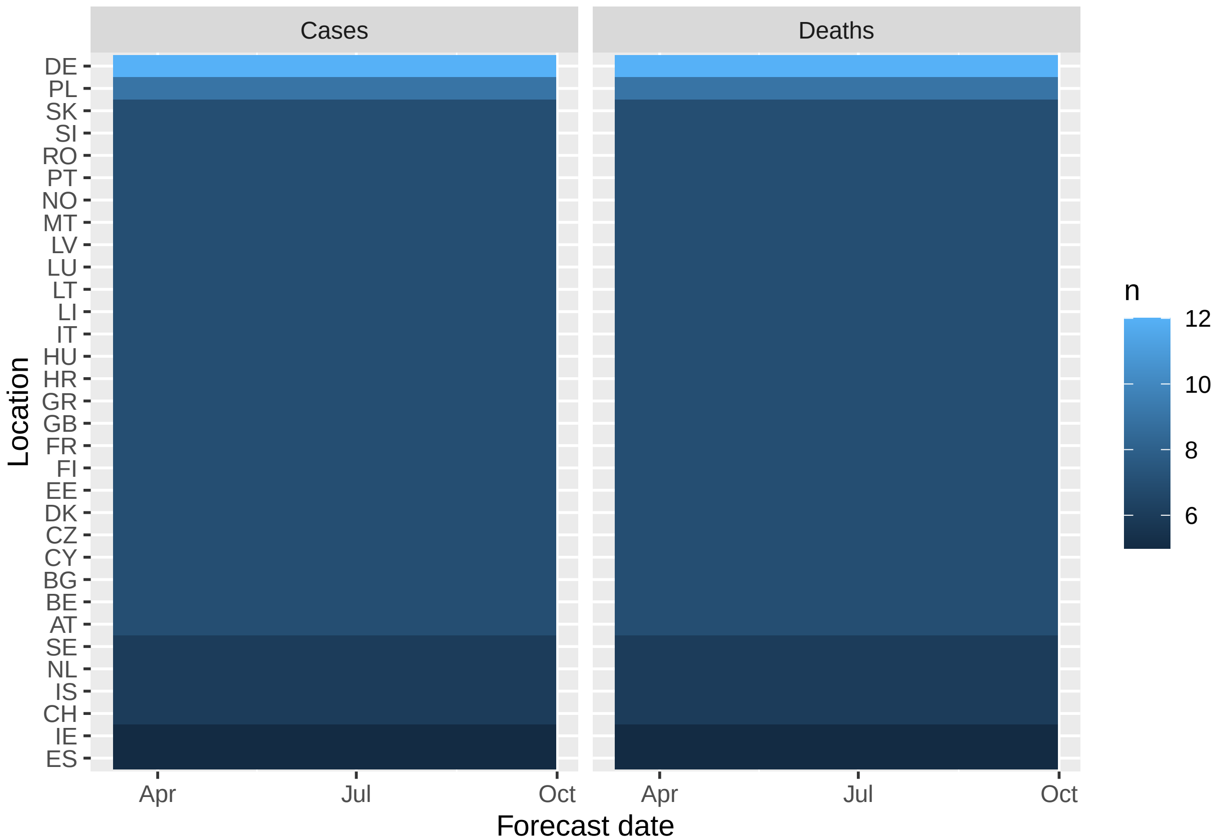


Or allowing different numbers of models per location:

```
hub_complete_loc <- make_complete_set(hub_data, per_location = TRUE)
print(plot_locs_per_model(hub_complete_loc))
```




```
plot_models_per_loc(hub_complete_loc)
```



Updating the European Forecast Hub data (probably not necessary) To update the data, clone the whole repository or use subversion (svn) to only download the relevant folder. Run

```
git clone https://github.com/epiforecasts/covid19-forecast-hub-europe/
```

or

```
svn checkout https://github.com/epiforecasts/covid19-forecast-hub-europe/trunk/data-processed
```

To load the forecasts and truth data and to update the csv files, run

```
# load truth data using the covidHubutils package ------------------------------
devtools::install_github("reichlab/covidHubUtils")
library(covidHubUtils)

truth <- covidHubUtils::load_truth(hub = "ECDC") |>
  filter(target_variable %in% c("inc case", "inc death")) |>
  mutate(target_variable = ifelse(target_variable == "inc case",
                                  "Cases", "Deaths")) |>
  rename(target_type = target_variable,
         true_value = value) |>
  select(-model)

fwrite(truth, "data/weekly-truth-Europe.csv")

# get the correct file paths to all forecasts ----------------------------------
folders <- here("data-processed", list.files("data-processed"))
folders <- folders[
```

```r
  !(grepl("\\.R", folders) | grepl(".sh", folders) | grepl(".csv", folders))
]

file_paths <- purrr::map(folders,
                         .f = function(folder) {
                           files <- list.files(folder)
                           out <- here::here(folder, files)
                           return(out)}) %>%
  unlist()
file_paths <- file_paths[grepl(".csv", file_paths)]

# load all past forecasts ----------------------------------------------------
# ceate a helper function to get model name from a file path
get_model_name <- function(file_path) {
  split <- str_split(file_path, pattern = "/")[[1]]
  model <- split[length(split) - 1]
  return(model)
}

# load forecasts
prediction_data <- map_dfr(file_paths,
                           .f = function(file_path) {
                             data <- fread(file_path)
                             data[, `:=`(
                               target_end_date = as.Date(target_end_date),
                               quantile = as.numeric(quantile),
                               forecast_date = as.Date(forecast_date),
                               model = get_model_name(file_path)
                             )]
                             return(data)
                           }) %>%
  filter(grepl("case", target) | grepl("death", target)) %>%
  mutate(target_type = ifelse(grepl("death", target),
                              "Deaths", "Cases"),
         horizon = as.numeric(substr(target, 1, 1))) %>%
  rename(prediction = value) %>%
  filter(type == "quantile",
         grepl("inc", target)) %>%
  select(location, forecast_date, quantile, prediction,
         model, target_end_date, target, target_type, horizon)

# merge forecast data and truth data and save
hub_data <- merge_pred_and_obs(prediction_data, truth,
                               by = c("location", "target_end_date",
                                      "target_type")) |>
  filter(target_end_date >= "2021-01-01") |>
  select(-location_name, -population, -target)

# split forecast data into two to reduce file size
split <- floor(nrow(hub_data) / 2)

# harmonise forecast dates to be the date a submission was made
hub_data <- mutate(hub_data,
```

```r
                    forecast_date = calc_submission_due_date(forecast_date))

# function that performs some basic filtering to clean the data
filter_hub_data <- function(hub_data) {

  # define the unit of a single forecast
  unit_observation <- c("location", "forecast_date", "horizon", "model", "target_type")

  h <- hub_data |>
    # filter out unnecessary horizons and dates
    filter(horizon <= 4,
           forecast_date > "2021-03-08") |>
    # filter out all models that don't have all quantiles
    group_by_at(unit_observation) |>
    mutate(n = n()) |>
    ungroup() |>
    filter(n == max(n)) |>
    # filter out models that don't have all horizons
    group_by_at(unit_observation) |>
    ungroup(horizon) |>
    mutate(n = length(unique(horizon))) |>
    ungroup() |>
    filter(n == max(n))

  return(h)
}

hub_data <- filter_hub_data(hub_data)

fwrite(hub_data[1:split, ],
       file = "data/full-data-european-forecast-hub-1.csv")
fwrite(hub_data[(split + 1):nrow(hub_data), ],
       file = "data/full-data-european-forecast-hub-2.csv")
```

(Possible) Methods

Post-processing

Conformalized Quantile Regression (CQR) CQR is a two-step process where in step 1 you obtain quantiles and in step 2 you post-process these quantiles to achieve better calibration. What is neat about this workflow is that we already have quantiles and therefore don't need to worry about step 1.

Re-calibration is achieved by computing so-called conformity scores on a training data set and using these conformity scores to adapt future prediction intervals.

The procedure is nicely explained in this poster and in this paper.

Quantile Regression Averaging (QRA) Quantile regression averaging is a procedure commonly used to create ensembles from different quantile-based forecasts. The idea is to represent every quantile of the ensemble forecast as a linear combination of quantiles from the individual member distributions. Weights for different quantiles in the linear combination are chosen to minimise the weighted interval score (see below) on a training data set.

This procedure could be adapted such that every quantile of the resulting distribution is a 'linear combination' of the quantiles of the raw predictive distribution. Essentially, this would mean e.g. shifting every quantile of the raw distribution by an additive value or multiplying it with a certain value.

This could presumably done using the `quantgen` package. However, I have not yet have had the time to check this is easily possible.

Adjusting the quantile spread **General idea** For example, the 90% quantile of a predictive distribution can be understood as the 80% quantile times some value (let's call it quantile spread), the 80% quantile can be understood as the 70% quantile times some value etc. If a forecast is over-confident that implies that the quantile-spread is too small on average.

Future forecasts could potentially be improved by multiplying every quantile spread with some constant (or even a varying) factor. This factor could be esimated by writing an `optim` function that finds the value of that factor which optimises the weighted interval score (see below) on a training set. This factor could then be used to expand or contract prediction intervals for future forecasts.

Potentially, something similar could also be implemented using the QRA-framework.

Alternatively, the quantile spread could also be defined as a difference, rather than a ratio.

Other possible methods and keywords

- quantile mapping (e.g. https://journals.ametsoc.org/view/journals/clim/30/9/jcli-d-16-0652.1.xml)
- coherent forecasting
- adjusting nominal quantile levels based on empirical coverage in the past

Forecast evaluation

Proper scoring rules Forecasts in a quantile-based forecast can be evaluated using the weighted interval score (WIS, lower values are better), a proper scoring rule. Proper scoring rules incentivise the forecaster to state their true best belief and cannot be 'cheated'.

The WIS can be decomposed into three components: a penalty for dispersion (i.e. large forecast uncertainty) as well as penalties for over- and underprediction.

WIS = Dispersion + Over-prediction + Under-prediction

For a single prediction interval, the interval score is computed as

$$IS_\alpha(F, y) = (u - l) + \frac{2}{\alpha} \cdot (l - y) \cdot 1(y \leq l) + \frac{2}{\alpha} \cdot (y - u) \cdot 1(y \geq u),$$

where 1() is the indicator function, y is the true value, and l and u are the $\frac{\alpha}{2}$ and $1 - \frac{\alpha}{2}$ quantiles of the predictive distribution F, i.e. the lower and upper bound of a single prediction interval. For a set of K prediction intervals and the median m, the WIS is computed as a weighted sum,

$$WIS = \frac{1}{K + 0.5} \cdot (w_0 \cdot |y - m| + \sum_{k=1}^{K} w_k \cdot IS_\alpha(F, y)),$$

where w_k is a weight for every interval. Usually, $w_k = \frac{\alpha_k}{2}$ and $w_0 = 0.5$

The WIS is closely related to the absolute error. Over- and under-prediction should therefore also be understood as a form of absolute forecasting error.

Forecast calibration **Coverage** In addition to the WIS it makes sense to look at forecast calibration in terms of interval coverage. On average, all 50% or 90% prediction intervals should ideally cover 50% or 90% of the true observations. By comparing nominal and empirical coverage we can quickly determine whether a model is over- or underconfident.

Bias The over- and under-prediction penalties of the WIS capture absolute forecasting errors. In addition we can examine whether a forecast has relative tendency to over- or under-predict. This is less susceptible to large outlier predictions

PIT histograms

Probability integral transform (PIT) histograms are another way of examining forecast calibration

Evaluating forecasts in R Forecasts can be evaluated in R using the `scoringutils` package.

```
scores <- eval_forecasts(uk_data,
                         summarise_by = c("model", "target_type"))

scores
```

Example plot for empirical vs. nominal coverage

```
scores <- eval_forecasts(uk_data,
                         summarise_by = c("model", "target_type", "range"))[]

scores[model == "seb"] |>
  ggplot(aes(y = coverage, x = range)) +
  geom_point() +
  geom_line() +
  facet_wrap(~ target_type)
```