



MCV M2 Project

Optimization and inference for
Computer Vision

Pérez, Ferran
Poveda, Jonatan
Group X

 Week 1: Inpainting

Week 2: Poisson editing

Week 3: Segmentation

Week 4: Graphical models

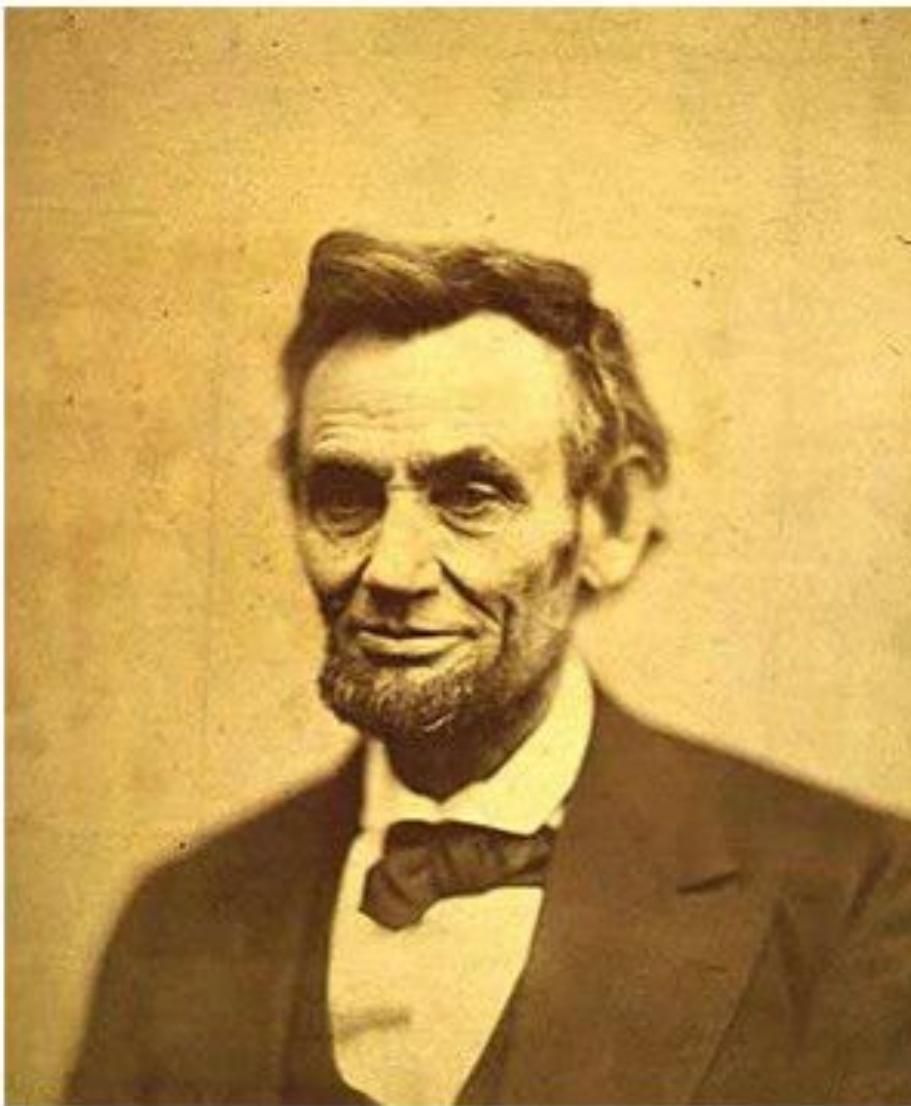
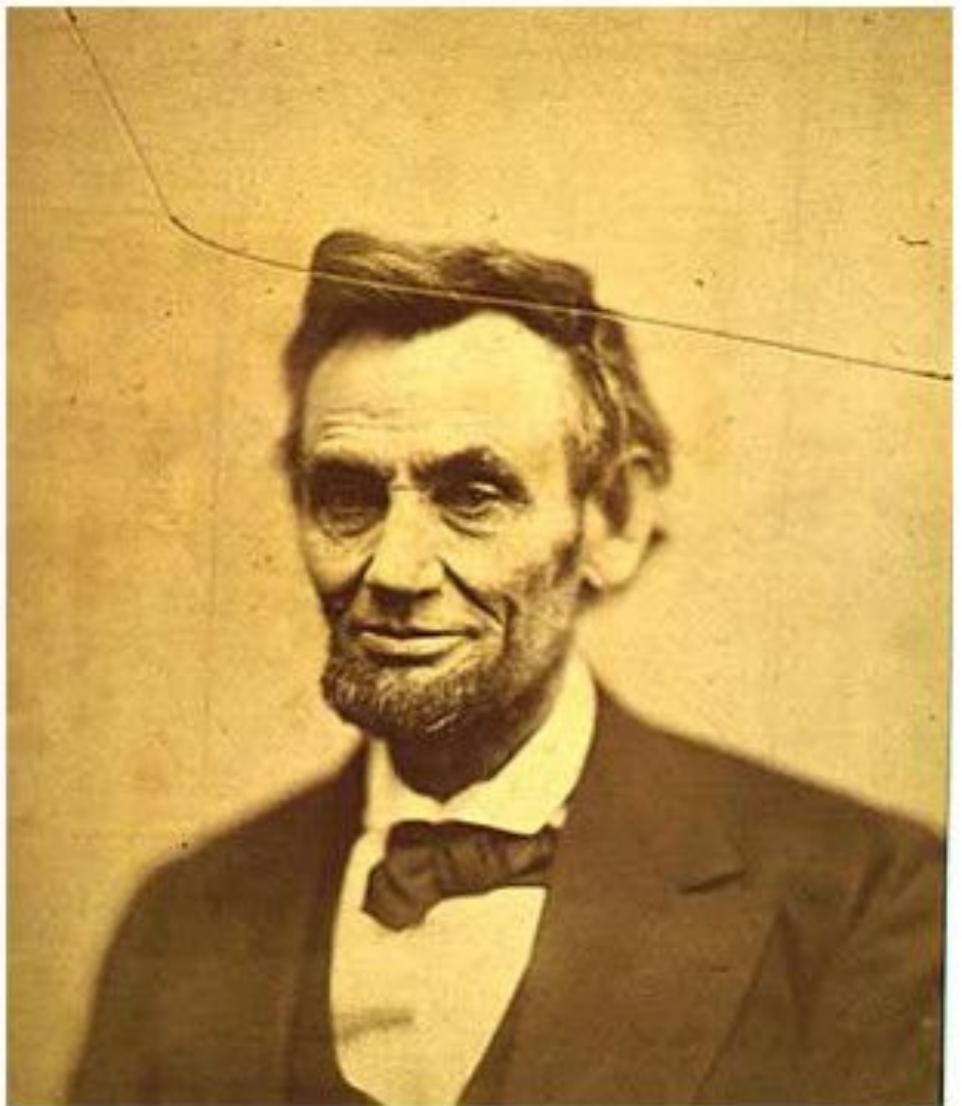
OVERVIEW

Week 1

Image Inpainting

What is image inpainting?

- We have a deteriorated image
- We want to reconstruct it realistically
- We want to remove some object(s)
- We select the region with a mask
- We use optimization to generate the reconstructed image



Criteria

- The output image,
 - should look natural (**smoothness**)
 - should be similar to the original (**difference**)

Energy functional
to optimize

$$J(u) = \int_{\Omega} \frac{1}{2} |\nabla u|^2 + \frac{1}{2\lambda} |u - f|^2 dx$$

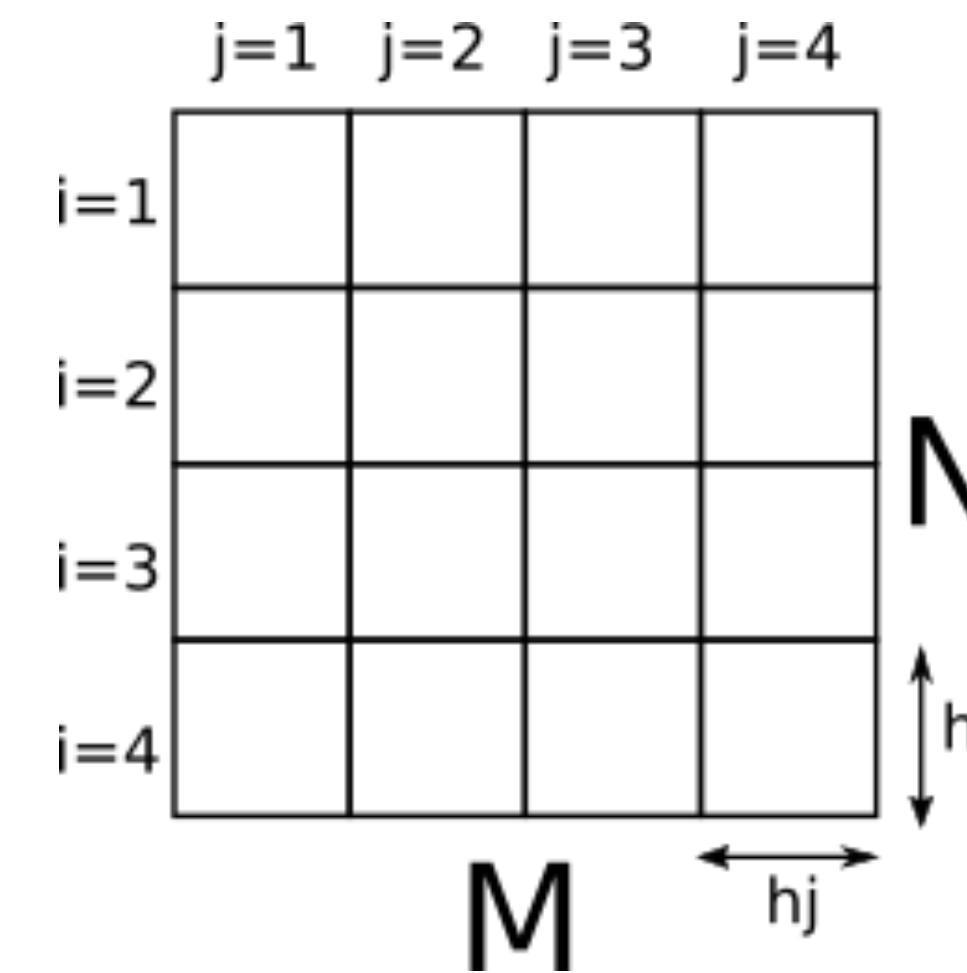
INPAINTING

Computation

- Strategy: Find the necessary condition for the extremum
- We use the Euler-Laplace equation to solve it
- So, we have a system of equations $\mathbf{Au} = \mathbf{b}$ to solve
 - \mathbf{A} : sparse matrix of shape (P,P) ,

where $P = N \cdot M$, and (N,M) the image shape

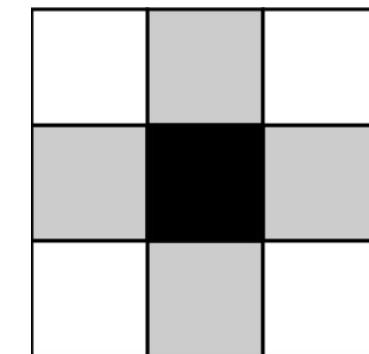
- \mathbf{u} : “answer” image that minimizes the energy
- \mathbf{b} : output vector (0 inside region, $f(i,j)$ otherwise)



$$\nabla F(\mathbf{u}) = 0$$

Setup and parameters

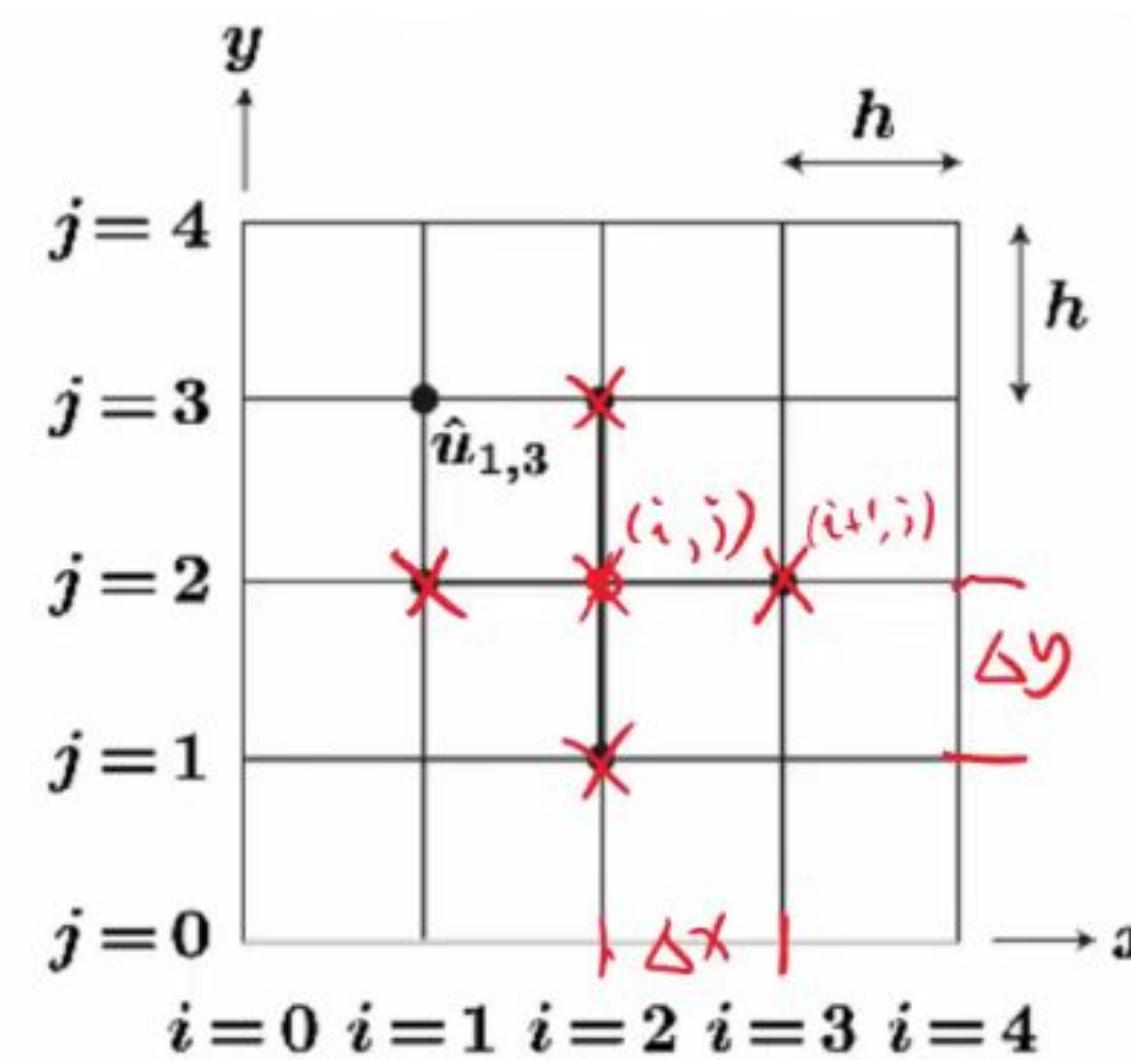
- The output image (\mathbf{u}) should be equal to the original (\mathbf{f}) ,
 - outside the region to be inpainted
 - on the external boundaries of the region to be inpainted
- In the region to be inpainted,
 - we use the Laplacian to approximate $J(\mathbf{u})$ with 5 values (4-connectivity)
- Distance between pixels in i, j direction:
 - $h_i = 1/(N-1)$
 - $h_j = 1/(M-1)$



INPAINTING

Computation

- We use the **implicit** formulation of the Gradient Descent (GD).
- We use the Euler-Laplace equation to solve it.
- We have a system of equations $\mathbf{Au} = \mathbf{b}$
 - \mathbf{A} : sparse matrix of shape (P,P) ,
where $P = N \cdot M$, and (N,M) the image shape
 - \mathbf{u} : “answer” image that minimizes the energy
 - \mathbf{b} : output vector (contains 0’s & values of original image $I(j,i)$)

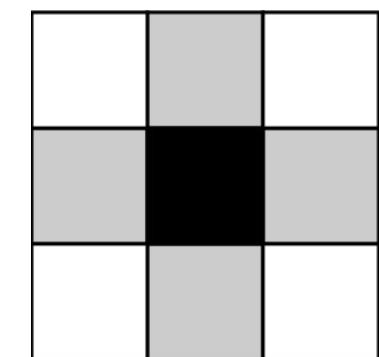


$$2D \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = f$$

$$\Leftrightarrow \left[\frac{u_{i+1,j} + u_{i-1,j} - 2u_{i,j}}{\Delta x^2} + \frac{u_{i,j+1} + u_{i,j-1} - 2u_{i,j}}{\Delta y^2} \right] = f_{i,j}$$

Setup and parameters

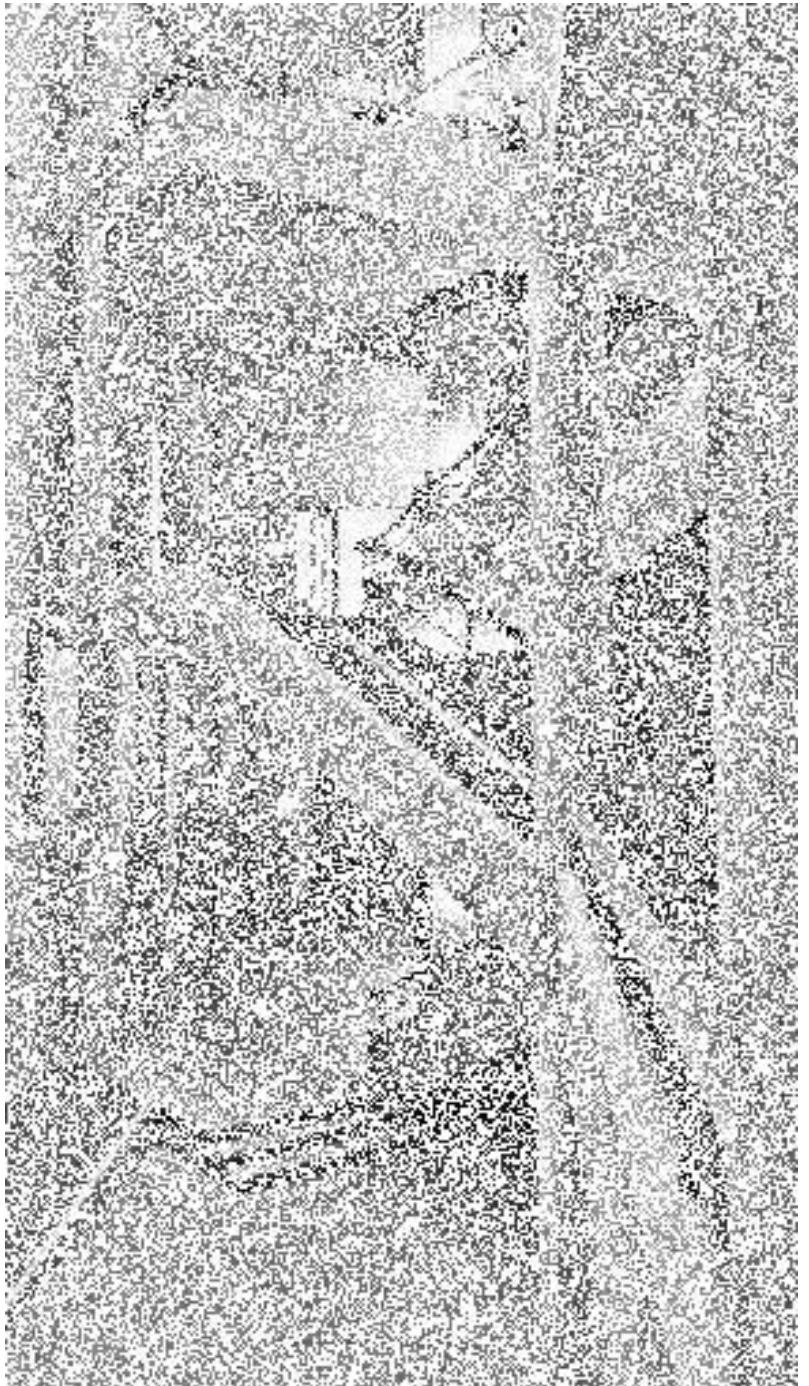
- The output image (\mathbf{u}) should be equal to the original (\mathbf{f}),
 - outside the region to be inpainted
 - on the external boundaries of the region to be inpainted
- In the region to be inpainted,
 - we use the Laplacian to approximate $J(\mathbf{u})$ with 5 values (4-connectivity)
- Distance between pixels in i, j direction:
 - $h_i = 1/(N-1)$
 - $h_j = 1/(M-1)$



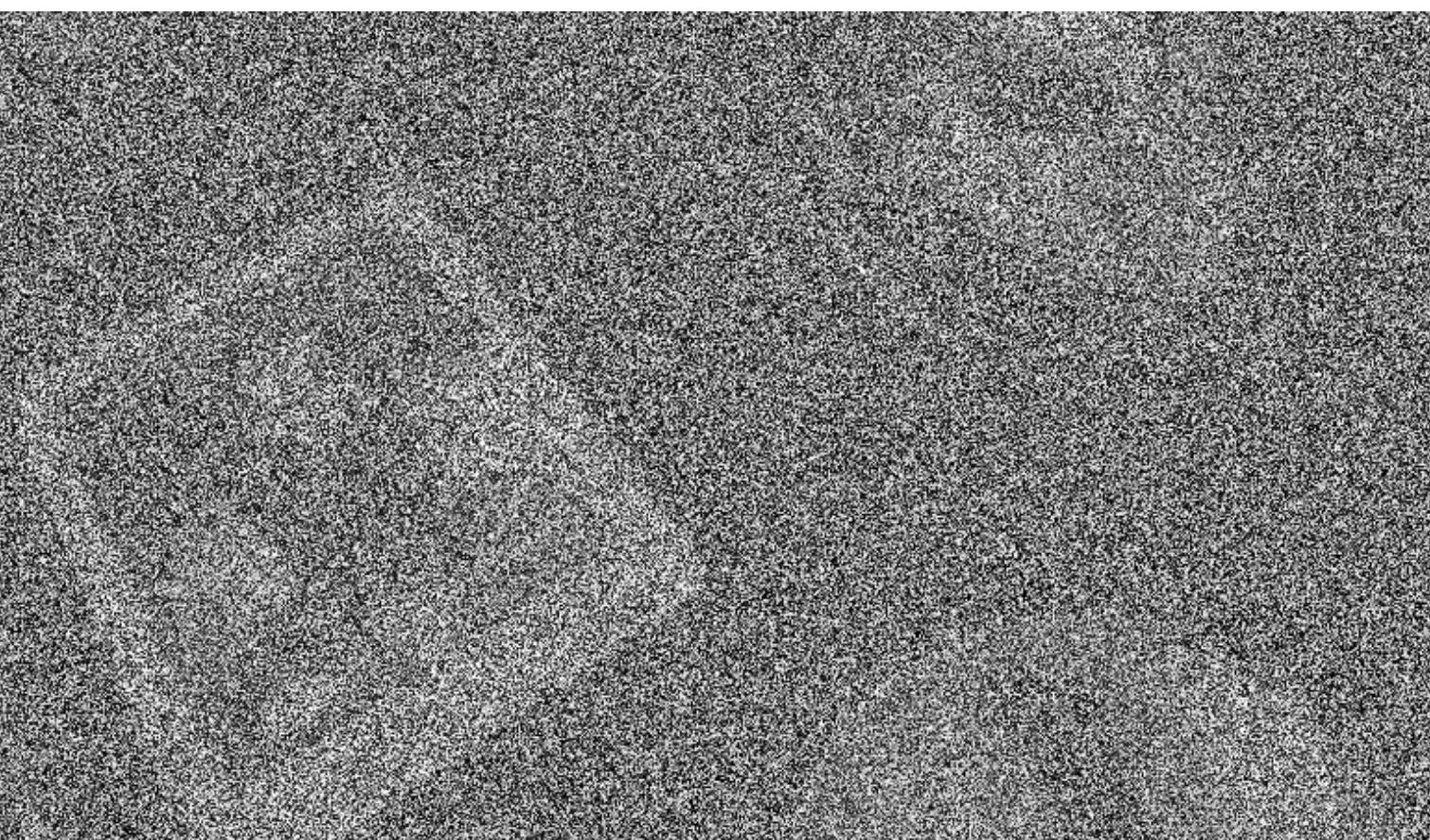
INPAINTING

Results - Image to restore #1-3

IMG1



IMG2



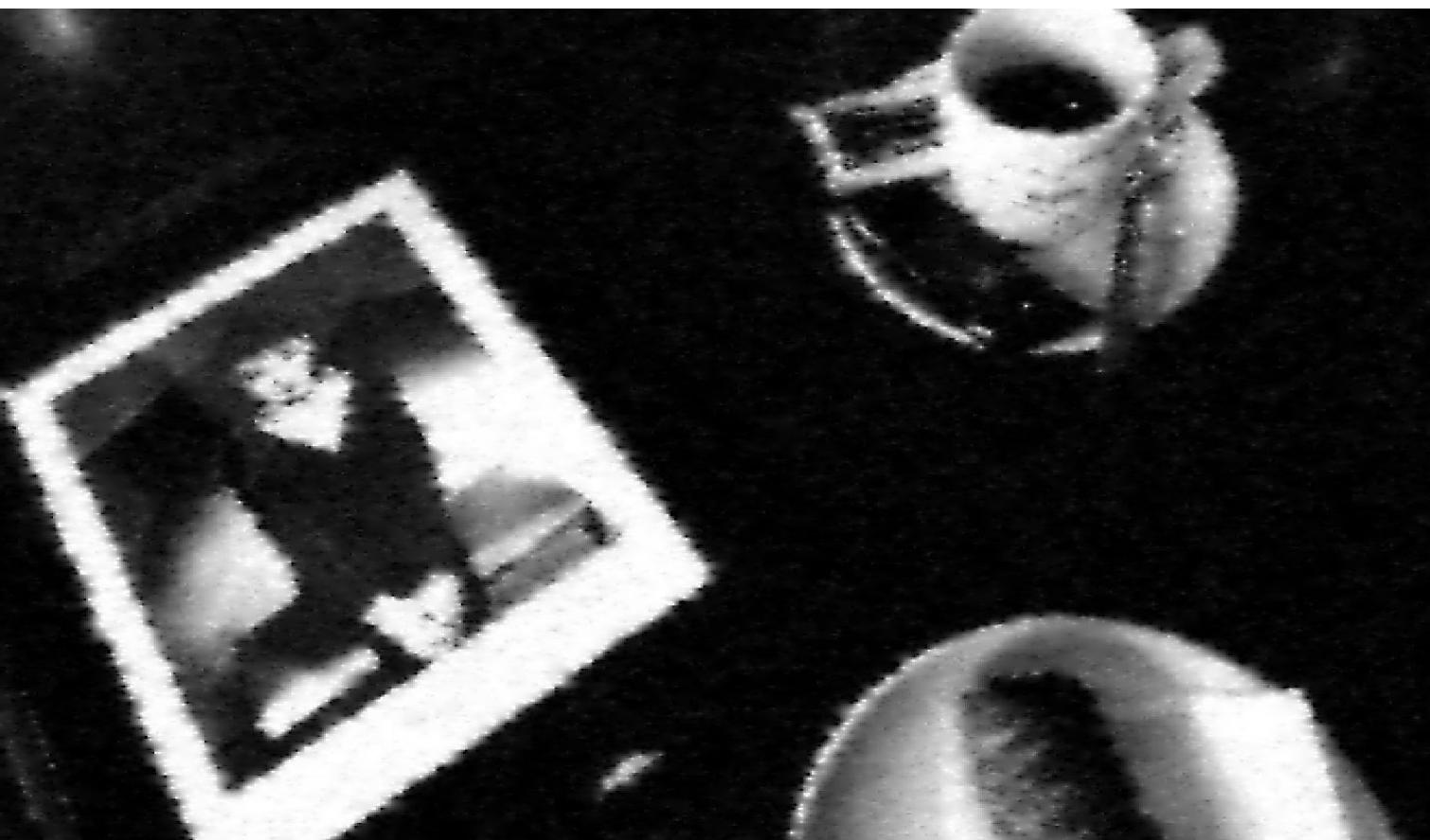
IMG3



IMG1-
RESTORED



IMG2-RESTORED

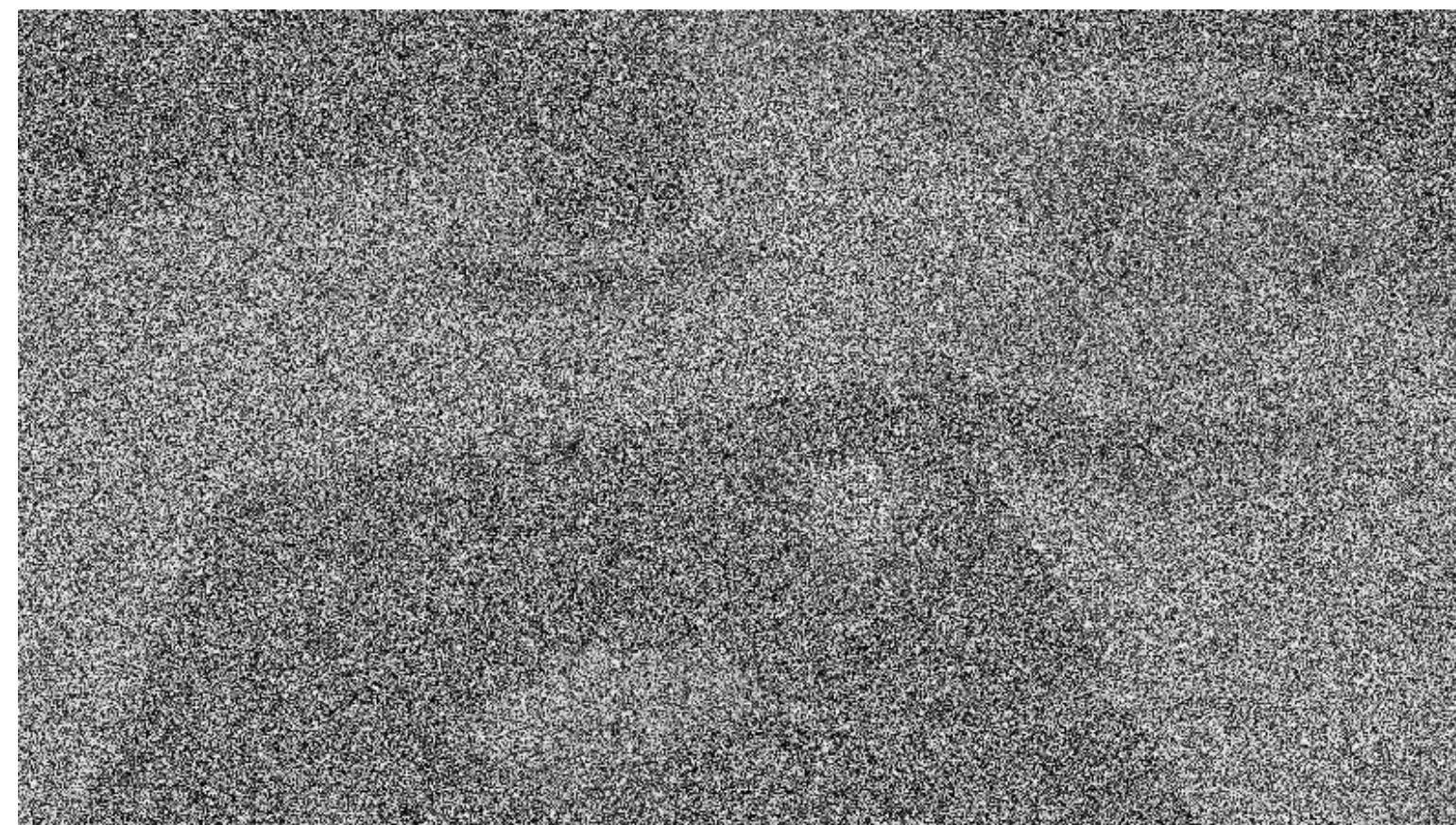


IMG3-RESTORED

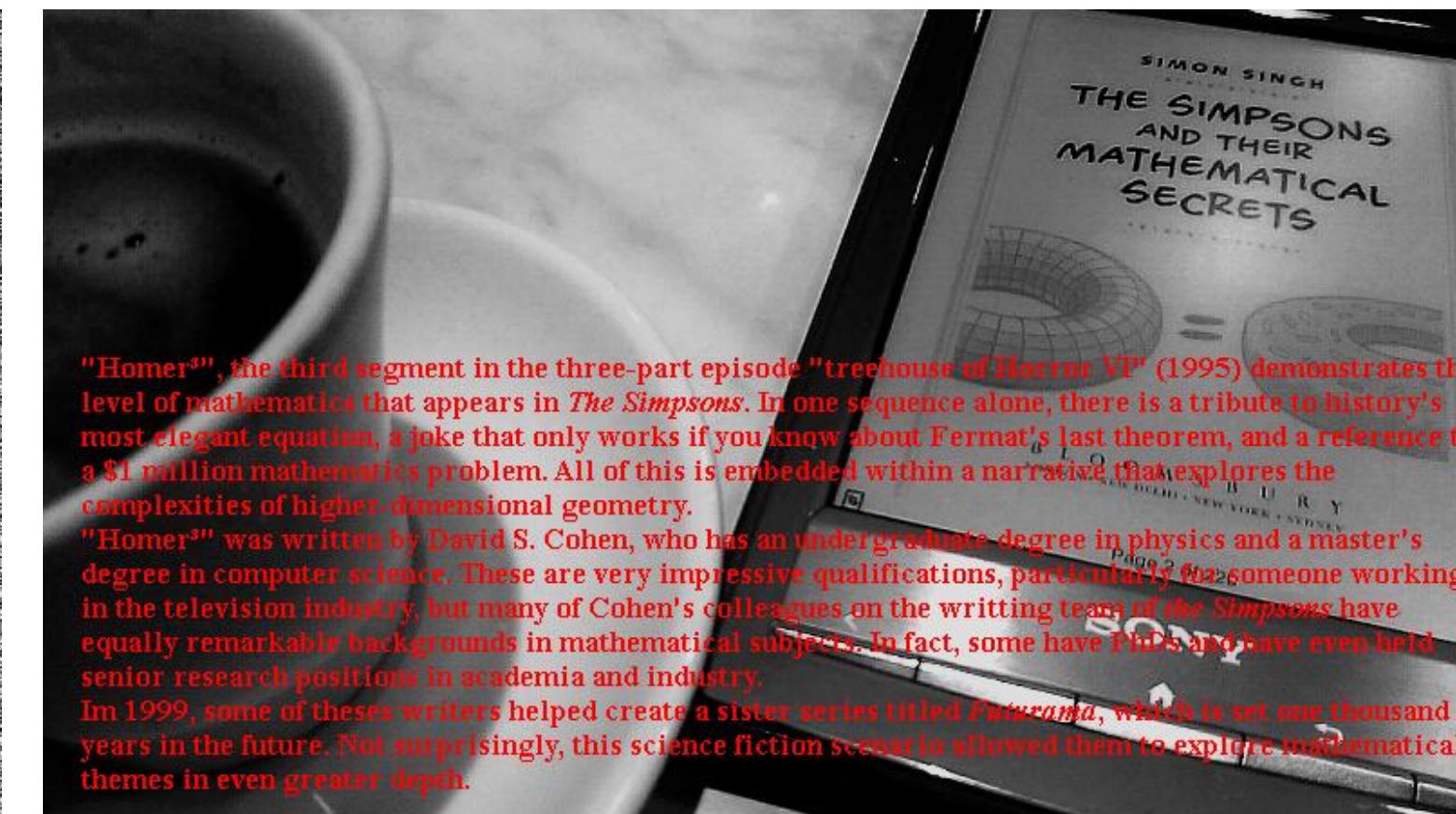


Results - Image to restore #4-6

IMG4



IMG5



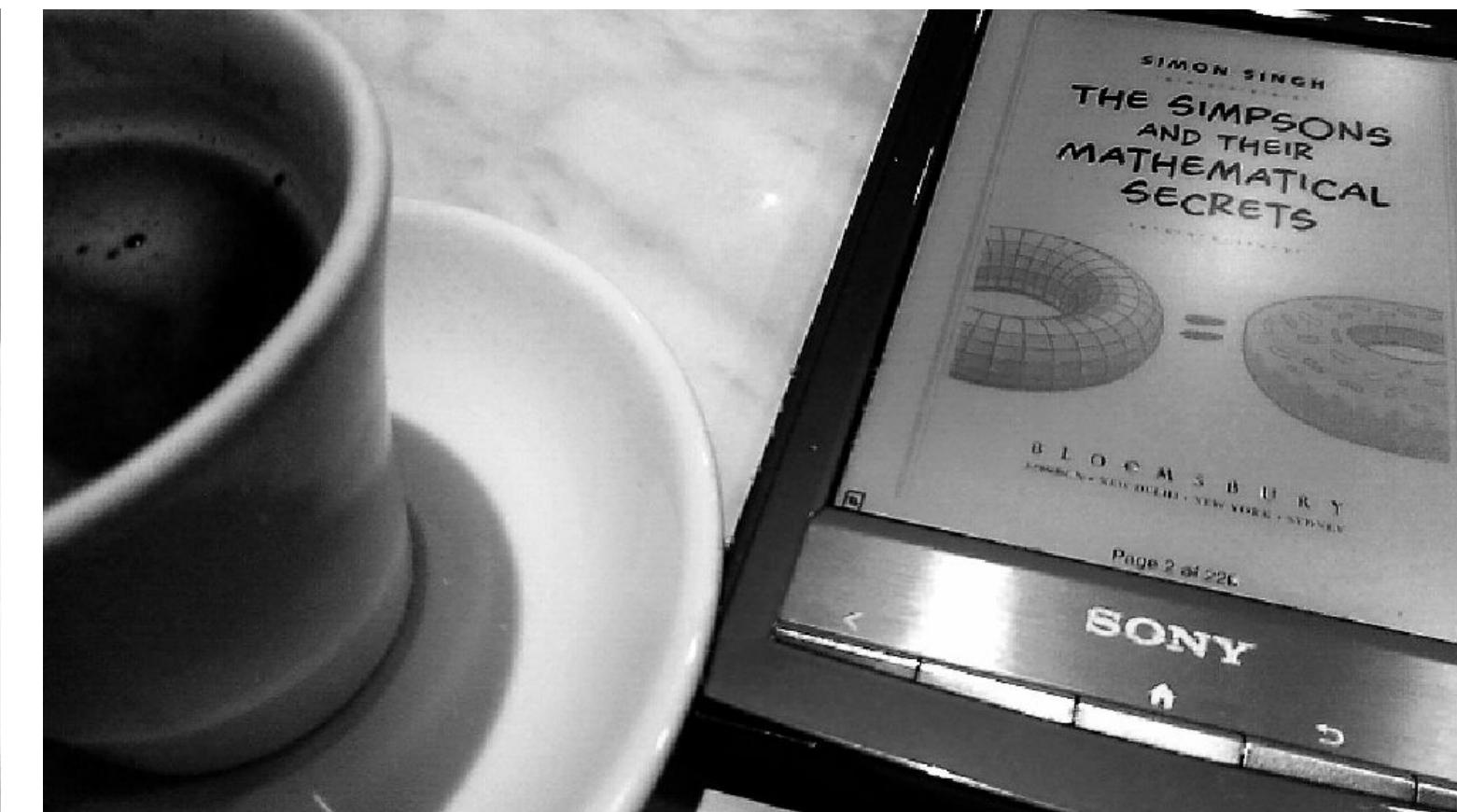
IMG6



IMG4-RESTORED



IMG5-RESTORED



IMG6-RESTORED



Results - Final image to restore

Image

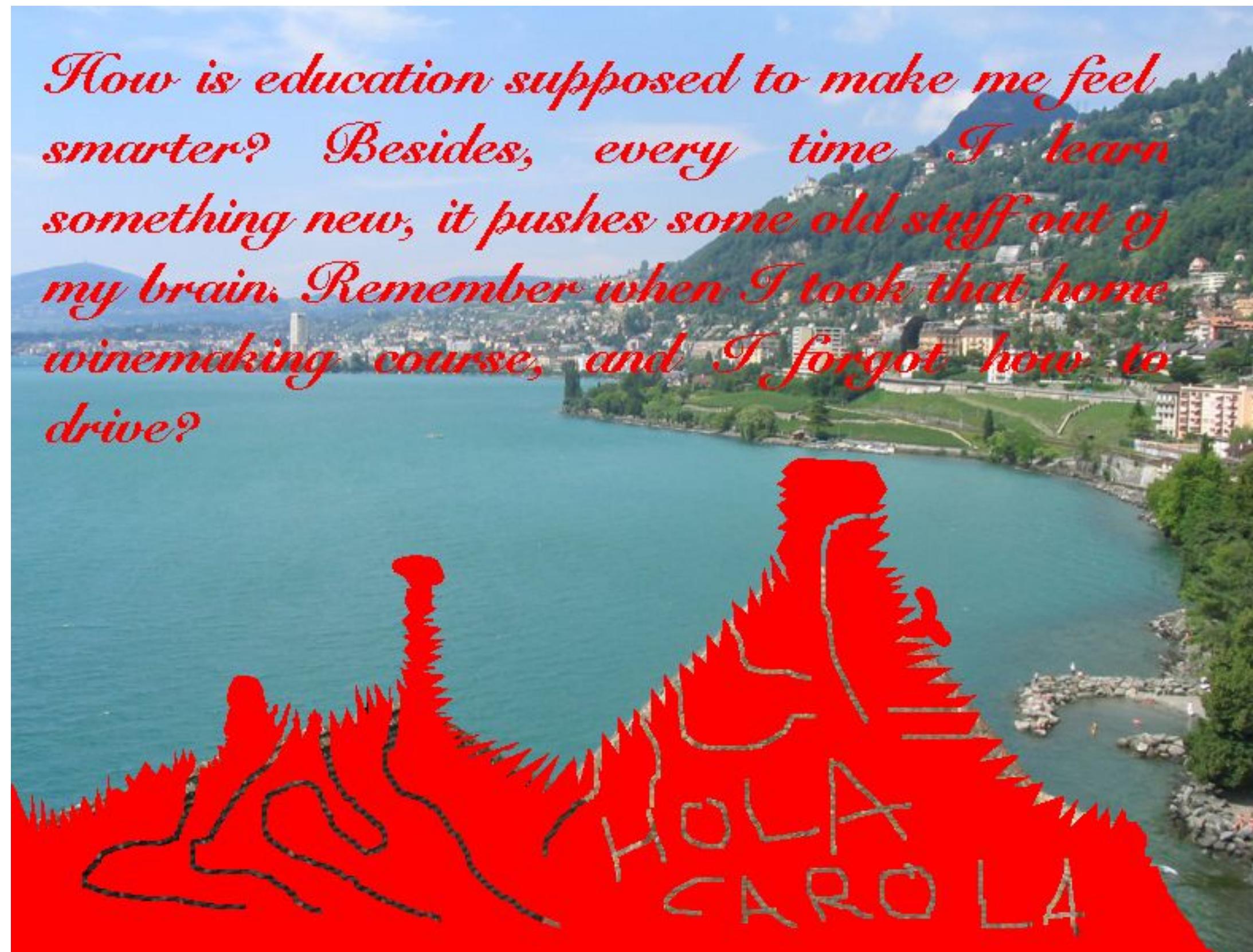


Image restored



Conclusions

- Text-over-background can be removed without noticeable quality loss
- Blur effect due to the effect of the smoothness parameter λ
 - Spatial high frequency cannot be recovered
 - Contours, corners, texture, ... are not recovered
 - More unpleasant with colour images
- Some results are impressive, even with only 1% of original pixels we can recognise objects in the image



Improvements

- Tune λ for each case
- Use an 8-connectivity to get more context (diagonal gradient is not considered on 4-conn)
- Apply some techniques to guess some lost contours beforehand. For example Active Contour Models
- Is inpainting each RGB channel separately and then combining them, the best option?
 - Channel-specific inpainting
 - Combine them in a different manner

CONCLUSIONS

● Proof that the solution of the Laplace's equation is a solution of the problem

$$\left\{ \begin{array}{l} \arg \min_{u \in W^{1,2}(\Omega)} \int_D |\nabla u(x)|^2 dx, \\ u|_{\partial D} = f \end{array} \right. \longrightarrow \nabla F(\mathbf{u}) = 0$$

$$u|_{\Omega \setminus D} = f$$

Let

$$\frac{1}{2} \int_D |\nabla u(x)|^2 dx = F(\vec{u})$$

$$\operatorname{argmin}_u F(\vec{u})$$

$$\frac{\delta}{\delta u} F(\vec{u}) = \vec{0}$$

then to minimize

$$\nabla F(\vec{u}) = \vec{0} \quad \blacksquare$$

that by definition is

(1) Applying finite backward difference to approximate Laplacian

$$\Delta_u \approx \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h_i^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{h_j^2}$$

DEFINITIONS

(**) we apply Neumann boundary conditions (i.e.: mirroring)

- Necessary condition for the extremum (minimum in our case)

$$\frac{\partial J}{\partial u} = - \sum_{i=1}^D \frac{\partial}{\partial X_i} \frac{\partial F}{\partial u_{x_i}} + \frac{\partial F}{\partial u} = 0$$

where D : dimension of our space. $D=2$, in our case.

- Our functional has no f terms so, the second term is already 0

The first term can be computed in parts as:

$$\frac{\partial F}{\partial u_x} = \frac{1}{2} 2u_x = u_x ; \quad \frac{\partial F}{\partial u_y} = \frac{1}{2} 2u_y = u_y$$

$$\frac{\partial J}{\partial u} = - \left(\frac{\partial}{\partial y} u_y + \frac{\partial}{\partial x} u_x \right) = -\Delta_u =^{(1)}$$

$$\frac{-u_{i+1,j} + 2u_{i,j} - u_{i-1,j}}{h_i^2} + \frac{-u_{i,j+1} + 2u_{i,j} - u_{i,j-1}}{h_j^2} \Big|_{h_i=h_j=1} = \\ 4u_{i,j} - u_{i+1,j} - u_{i-1,j} + -u_{i,j+1} - u_{i,j-1} = 0 \quad \forall i, j$$

We can build a sparse matrix of coefficients \mathbf{A} to define, with the other criteria, a system of equations like: $\mathbf{Au} = \mathbf{b}$

METRICS (I)

Dataset	Training		Test		Time (s)	
	Split	Split1	Split2	Split1	Split2	
Method1						
Method2						
Method3						
Method4						
Method5						

METRICS (II)

Dataset	Training		Test		Time (s)	
	Split	Split1	Split2	Split1	Split2	
Method1						
Method2						
Method3						
Method4						
Method5						

Graph 1:





Thanks!

Any questions?

You can contact us at:
jonatan.poveda@e-campus.uab.cat
ferran.perezg@e-campus.uab.cat

REFERENCES

- [1] Ref1
- [2] Ref2
- [3] Ref3
- [4] Ref4
- [5] Ref5
- [6] Ref6
- [7] Ref7
- [8] Ref8

CREDITS

Special thanks to all people who made and share these awesome resources for free:

- Presentation template designed by [Slidesmash](#)
- Photographs by [unsplash.com](#) and [pexels.com](#)
- Vector Icons by [Matthew Skiles](#)

Hello!

I Am John Miller

I am here because I love to design presentations.

You can contact me at @username



A Picture Is Worth A Thousand Words

Itself is what the end-user derives value from also can refer is what the end-user derives value from also can refer to the information provided through the medium.



A photograph of a person sitting on a rocky outcrop, looking out over a vast, rolling landscape under a sunset sky.

Use Big Images
To Show Ideas

USE SHAPE TO EXPLAIN IDEAS



“

“A person who never made a
mistake never tried anything
new”

56,790,500

Write here your big numbers

FUNNY FACTS

56,790,500

Revenue from sales

130%

Project Achievements

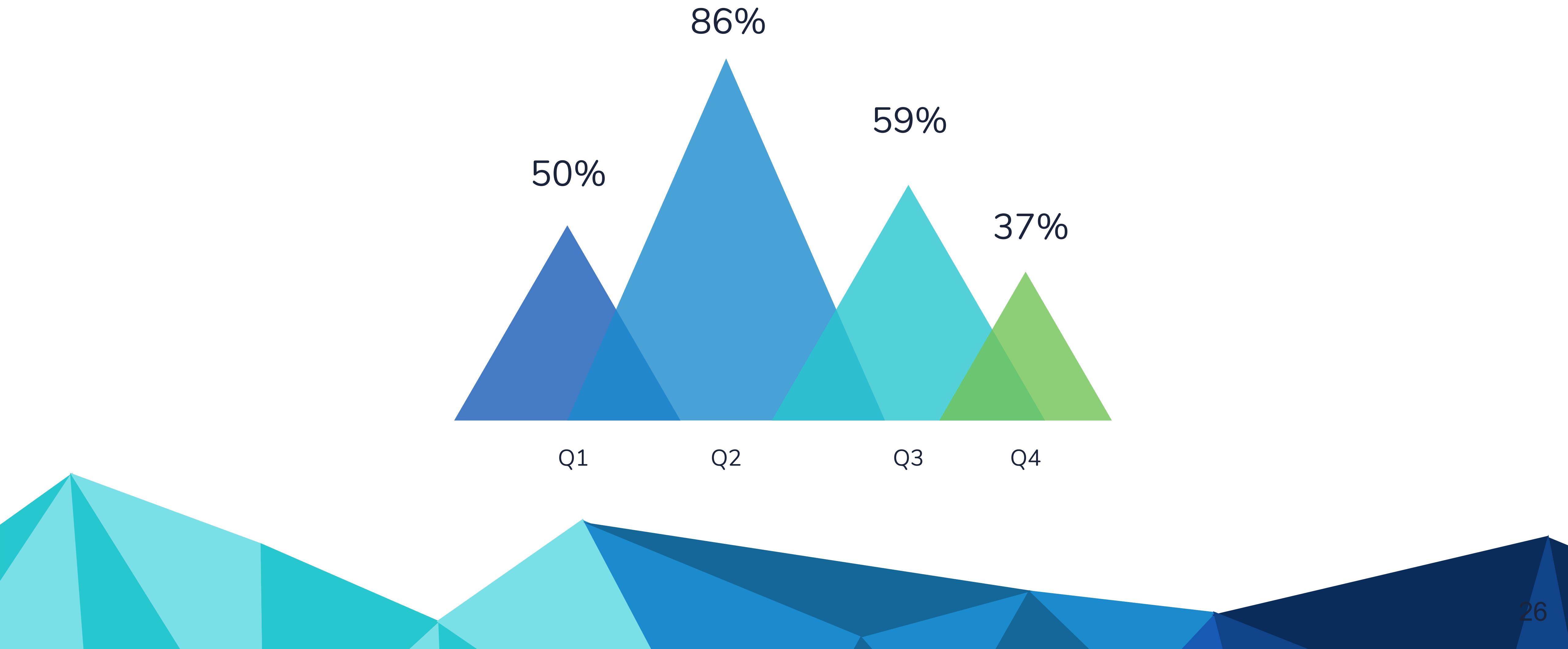
56,790,500

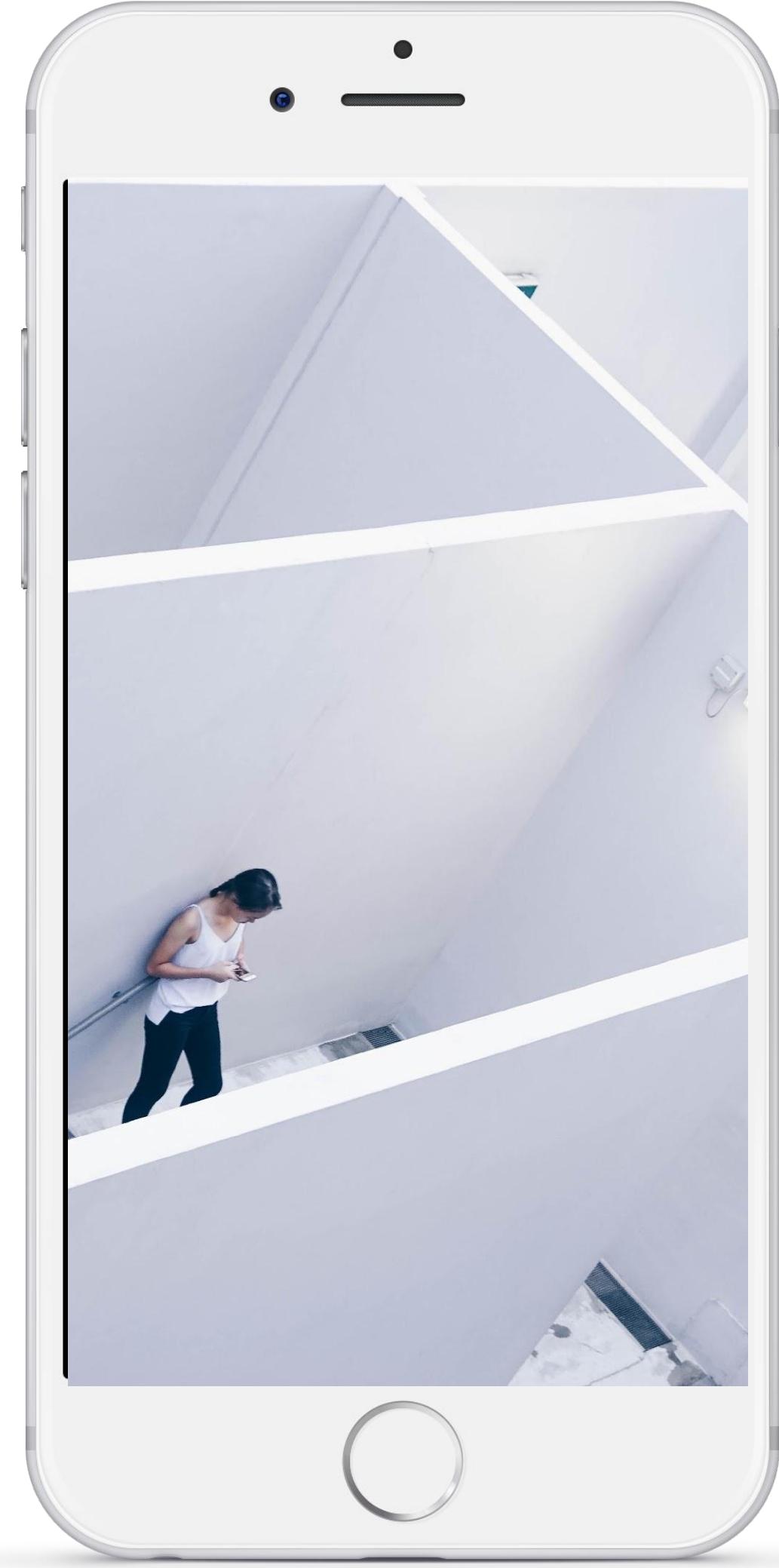
Users around the world

OUR PROCESS



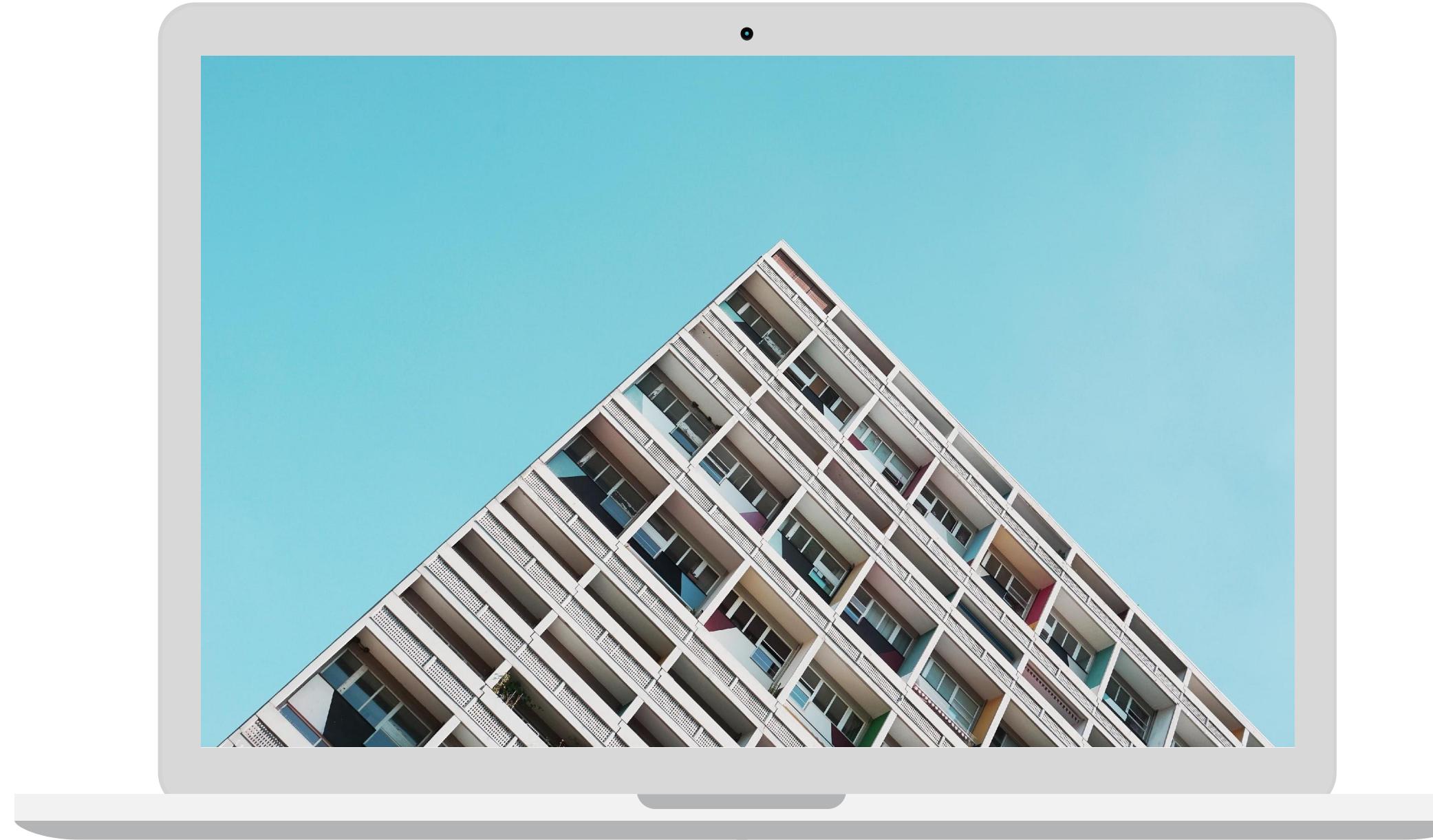
CHARTS TO PRESENT DATA





iPhone App Project

Itself is what the end-user derives value from
also can refer to the information



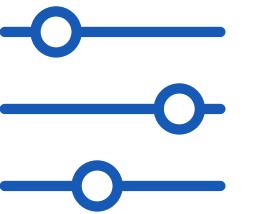
Laptop Project

Itself is what the end-user derives value from
also can refer to the information



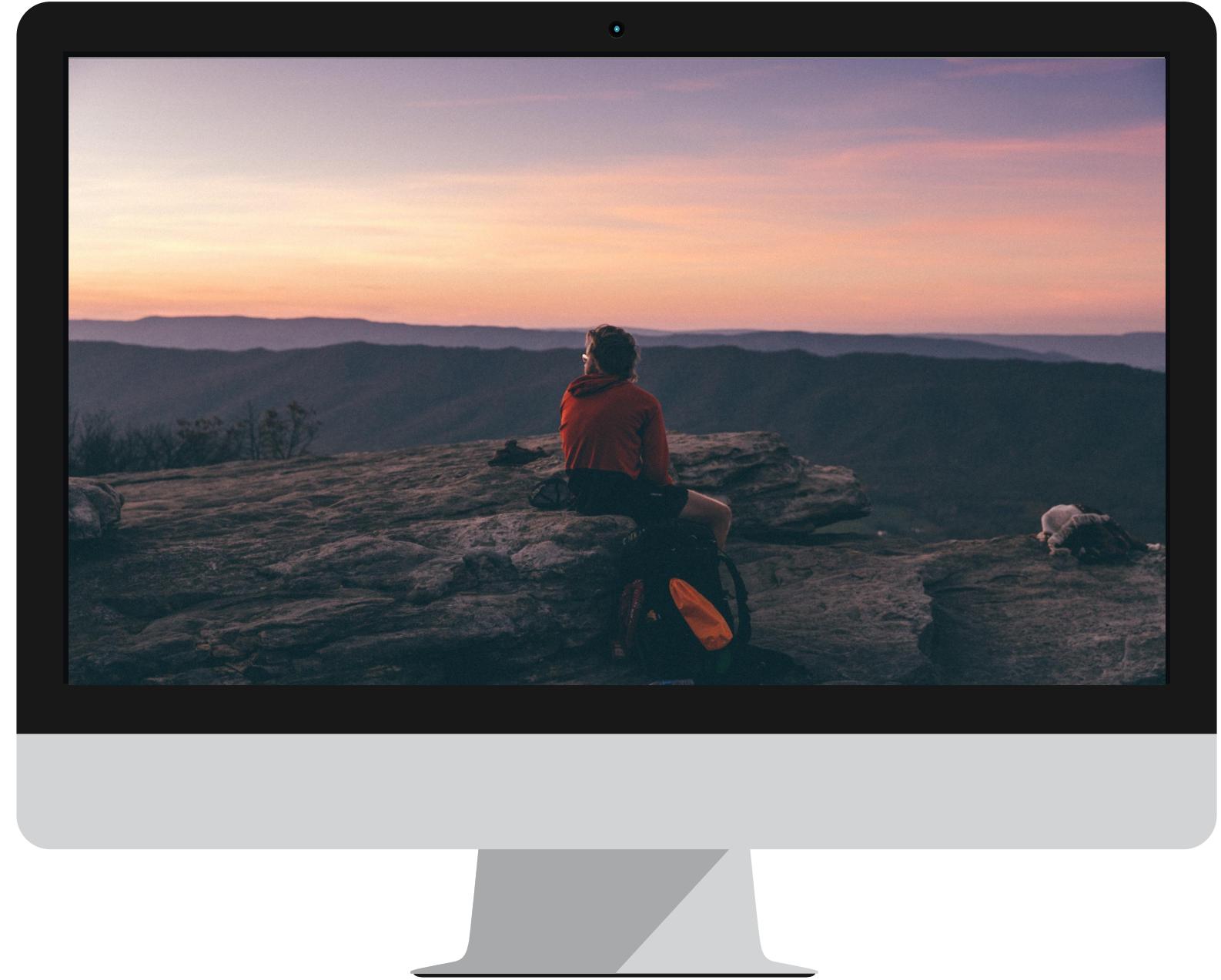
Content B

Itself is what the end-user
derives value from also can
refer to the information



Content D

Itself is what the end-user
derives value from also can
refer to the information



Presentation Design

This presentation uses the following typographies and colors:

Free Fonts used:

<https://www.fontsquirrel.com/fonts/nunito>

Colors used

