

Assignment 3

1. Let A, B, I, b be the pre-condition, post-condition, invariant and condition of the while-loop such that:

$$A: n \geq 0 \wedge y = 1 \wedge r = 1 \wedge i = 0$$

$$B: r = 2^{n+1} - 1$$

$$I: y = 2^i \wedge r = 2^{i+1} - 1 \wedge i \leq n$$

$$b: i \neq n$$

Now, use the inference rules to prove the correctness of the while-loop on line 5-10, called P . (And let S be the line 7-9)



$$\begin{aligned} \{y = 2^i \wedge r = 2^{i+1} - 1 \wedge i < n\} &\rightarrow \{2y = 2^{i+1} \wedge r + 2y = 2^{i+2} - 1 \wedge i + 1 \leq n\} \\ &\leftrightarrow \{y = 2^i \wedge r = 2^{i+2} - 1 - 2^{i+1} \wedge i \leq n - 1\} \\ &\leftrightarrow \{y = 2^i \wedge r = 2^{i+1} - 1 \wedge i < n\} \\ &\therefore \text{true} \end{aligned}$$

$$\frac{\vdash I \wedge i \neq n \rightarrow I[y/y, r+y/r, i+1/i] \quad \vdash \{I[r+y/r, i+1/i]\} r = r+y \quad \{I[i+1/i]\} i = i+1 \quad \{I\}}{\vdash \{I \wedge b\} y = 2 \times y \quad \{I[r+y/r, i+1/i]\} \quad \{I[r+y/r, i+1/i] \quad r = r+y \quad i = i+1 \quad \{I\}}$$

$$\vdash \{I \wedge b\} y = 2 \times y \quad \{I[r+y/r, i+1/i]\} \quad \{I[r+y/r, i+1/i] \quad r = r+y \quad i = i+1 \quad \{I\}$$

$$\begin{aligned} &\Rightarrow \text{true, clearly} \\ \vdash \{n \geq 0 \wedge y = 1 \wedge r = 1 \wedge i = 0\} &\rightarrow \{y = 2^i \wedge r = 2^{i+1} - 1 \wedge i \leq n\} \end{aligned}$$

$$\Rightarrow \text{true, clearly}$$

$$\vdash \{y = 2^i \wedge r = 2^{i+1} - 1 \wedge i = n\} \rightarrow \{r = 2^{n+1} - 1\}$$

$$\vdash \{A\} \rightarrow \{I\} \quad \vdash \{I \wedge b\} S \{I\} \quad \vdash \{I \wedge \neg b\} \rightarrow \{B\}$$

$$\vdash \{A\} \quad P \quad \{B\}$$