Assignment 3

1. Let A, B, I, b be the pre-condition, post-condition, invariant and condition of the while-loop such that:

$A: n \geq 0 \wedge y=1 \wedge r=1 \wedge i=0$

$B: r = 2^{n+1} - 1$

$I: y=2^i \wedge r=2^{i+1}-1 \wedge i \leq n$

$b: i \neq n$

Now, use the inference rules to prove the correctness of the while-loop on line 5-10, called P. (And let s be the line 7-9)

$$\{y=2^i \wedge r=2^{i+1}-1 \wedge i<n\} \to \{2y=2^{i+1} \wedge r+2y=2^{i+2}-1 \wedge i+1 \leq n\}$$
$$\leftrightarrow \{y=2^i \wedge r=2^{i+2}-1-2^{i+1} \wedge i \leq n-1\}$$
$$\leftrightarrow \{y=2^i \wedge r=2^{i+1}-1 \wedge i<n\}$$

$\therefore$ true

$$\dfrac{\vdash I \wedge i \neq n \to I[2y/y, r+y/r, i+1/i] \qquad \vdash \dfrac{\{I[r+y/r, i+1/i]\}\ r=r+y\ \{I[i+1/i]\} \quad \{I[i+1/i]\}\ i=i+1\ \{I\}}{\{I[r+y/r, i+1/i]\}\ r=r+y;\ i=i+1\ \{I\}}}{\vdash \{I \wedge b\}\ y=2\times y\ \{I[r+y/r, i+1/i]\}}$$

true, clearly

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$$\dfrac{\dfrac{\vdash \{n \geq 0 \wedge y=1 \wedge r=1 \wedge i=0\} \to \{y=2^i \wedge r=2^{i+1}-1 \wedge i \leq n\}}{\vdash \{A\} \to \{I\}} \qquad \vdash \{I \wedge b\}\ s\ \{I\} \qquad \dfrac{\vdash \{y=2^i \wedge r=2^{i+1}-1 \wedge i=n\} \to \{r=2^{n+1}-1\}}{\vdash \{I \wedge \neg b\} \to \{B\}}}{\vdash \{A\}\ P\ \{B\}}$$

Q4 b is in /
wlang/
inv_test.prg