TMF CONSORTIUM

A novel finite element framework for numerical simulation of multiphase granular flow

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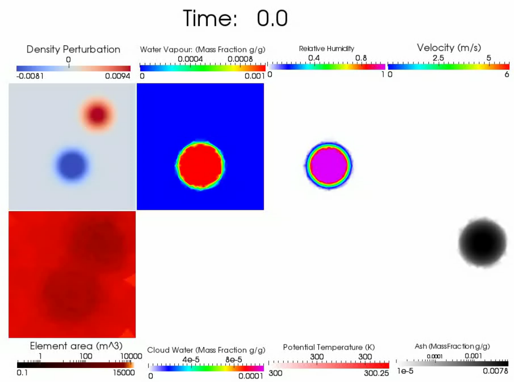
TMF Sponsors meeting
@ Imperial College London
on 8th & 9th April 2014

Overview

- Dispersed multiphase modelling using *Fluidity:* multiphase control-volume finite element Navier-Stokes/ Darcy flow solver.
- Eulerian-Eulerian modelling
- Mesh Adaptivity
- Two (many) Fluid model

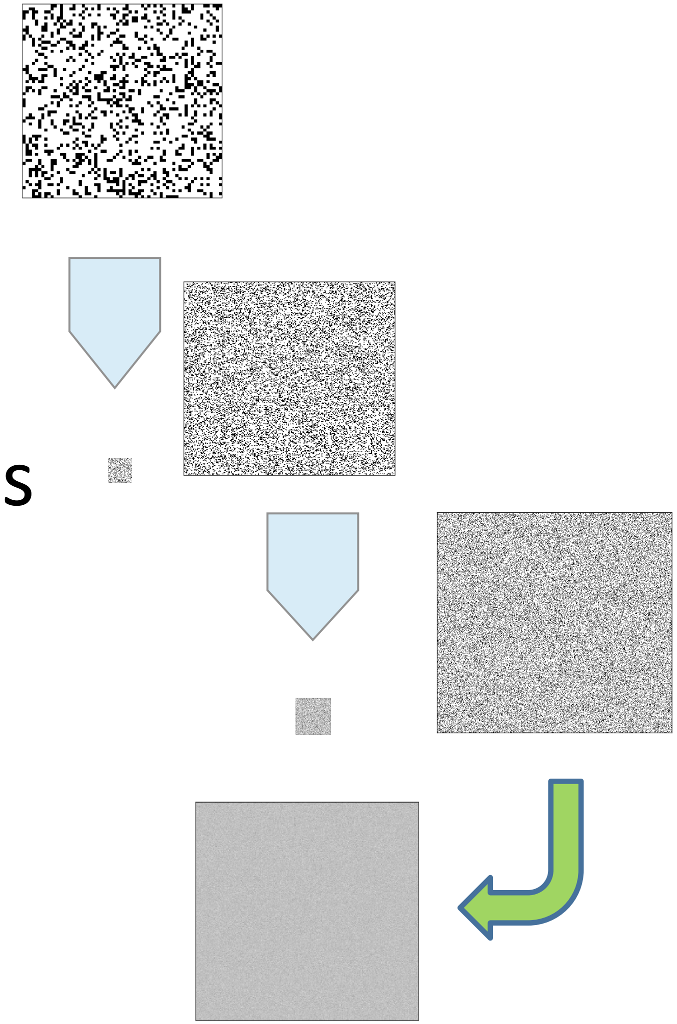


Eulerian-Eulerian modelling

Numerical modelling implies existence minimum resolvable length scale

Droplets/bubbles/solid particles may be far below this scale.

Filter equations to homogenize into “two-fluid” model.



Eulerian-Eulerian modelling

General filtered equations of motion:

momentum:

$$\frac{\partial}{\partial t}(\rho_i \alpha_i \boldsymbol{u_i}) + \nabla \cdot (\rho_i \alpha_i \boldsymbol{u_i} \boldsymbol{u_i}) = -\alpha_i \nabla p_i - \rho_i \alpha_i g \hat{\boldsymbol{z}} + \boldsymbol{F}_i$$

continuity:

$$\frac{\partial}{\partial t}(\rho_i \alpha_i) + \nabla \cdot (\rho_i \alpha_i \boldsymbol{u_i}) = S_i$$

internal energy:

$$\frac{\partial}{\partial t}(\rho_i \alpha_i c_{p,i} T_i) + \nabla \cdot (\rho_i \alpha_i c_{p,i} T_i \boldsymbol{u_i}) + p_i \nabla \cdot \boldsymbol{\alpha_i u_i} = S_i$$

Interphase forces: drag

- Different closures appropriate for different systems

$$K_{\text{drag}} = \frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} (\boldsymbol{u}_1 - \boldsymbol{u}_2) \qquad \frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} = f(\alpha_i, \boldsymbol{u}_i, \rho_i)$$

Example: Wen & Yu drag correlation

$$K_{\text{drag}} = \frac{3}{4} C_D \frac{\alpha_s \rho_g \left| \boldsymbol{u}_g - \boldsymbol{u}_s \right|}{\phi d_s} \alpha_g^{-2.65}$$

$$C_D = \begin{cases} 0.44 & \mathrm{Re}_i \geq 1000, \\ \frac{24}{\mathrm{Re}_s} \left(1 + 0.15 \mathrm{Re}_s^{0.687}\right) & \mathrm{Re}_i < 1000, \end{cases}$$

$$\mathrm{Re}_s = \frac{\alpha_g \rho_g \left| \boldsymbol{u}_g - \boldsymbol{u}_s \right| \phi d_s}{\mu_g}.$$

Interphase forces: granular temperature

- Solid has
  - Max packing density
  - Quasi-elastic collisions
  - Additional kinetic energy in fluctuations
- Kinetic theory closure
- qv. Gidaspow(1994)

$$\boldsymbol{F}_{\text{solid}} = \nabla p_s + K_{\text{drag}} + \boldsymbol{F}_{\text{friction}}$$

$$\Theta_s = \frac{1}{3}\left\langle \left|\boldsymbol{u} - \boldsymbol{v}\right|^2 \right\rangle$$

$$p_s = \rho_s \alpha_s \left(1 + 2\left(1 + e\right) \alpha_s g_0\right) \Theta_s$$

$$g_0 = \left(1 - \left(\frac{\alpha_s}{\alpha_{s,\max}}\right)^{\frac{1}{3}}\right)^{-1}$$

Interphase forces: granular temperature

$$\boldsymbol{F}_{\text{friction}} = \alpha_s \nabla \cdot \left(\mu_s \left(\nabla \boldsymbol{u}_s + \nabla \boldsymbol{u}_s \right) + \left(\lambda_s - \frac{2\mu_s}{3} \nabla \cdot \boldsymbol{u}_s \right) \boldsymbol{I} \right)$$

$$\frac{\partial}{\partial t} \left(\rho_s \alpha_s \Theta \right) + \nabla \cdot \left(\rho_s \alpha_s \Theta \boldsymbol{u}_s \right) = \nabla \boldsymbol{u}_s \cdot \tau_s - \gamma \Theta - \frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} \Theta$$

$$\gamma = \rho_s \alpha_s \left(g_0 \alpha_s \right) (1 - e) (1 + e) \left(\frac{4}{d_s} \sqrt{\frac{\Theta}{\pi}} - \nabla \cdot \boldsymbol{u}_s \right),$$

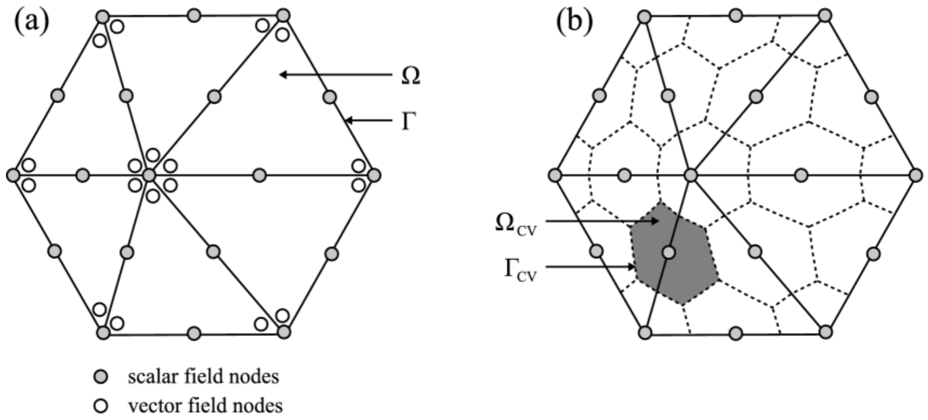
$$\mu_s = \frac{4}{5} \alpha_s d_s g_0 (1 + e) \left(\frac{\Theta_s}{\pi} \right)^{\frac{1}{2}}$$

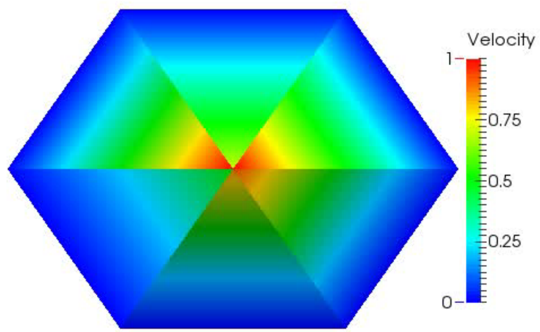
$$\lambda_s = \frac{4}{3} \alpha_s d_s g_0 (1 + e) \left(\frac{\Theta_s}{\pi} \right)^{\frac{1}{2}}$$

CVDFEM methods

Control Volume Finite Element Method

- Solve weak form PDEs
- piecewise smooth fns for unconserved, unbounded variables
- piecewise flat fns for conserved & bounded tracers

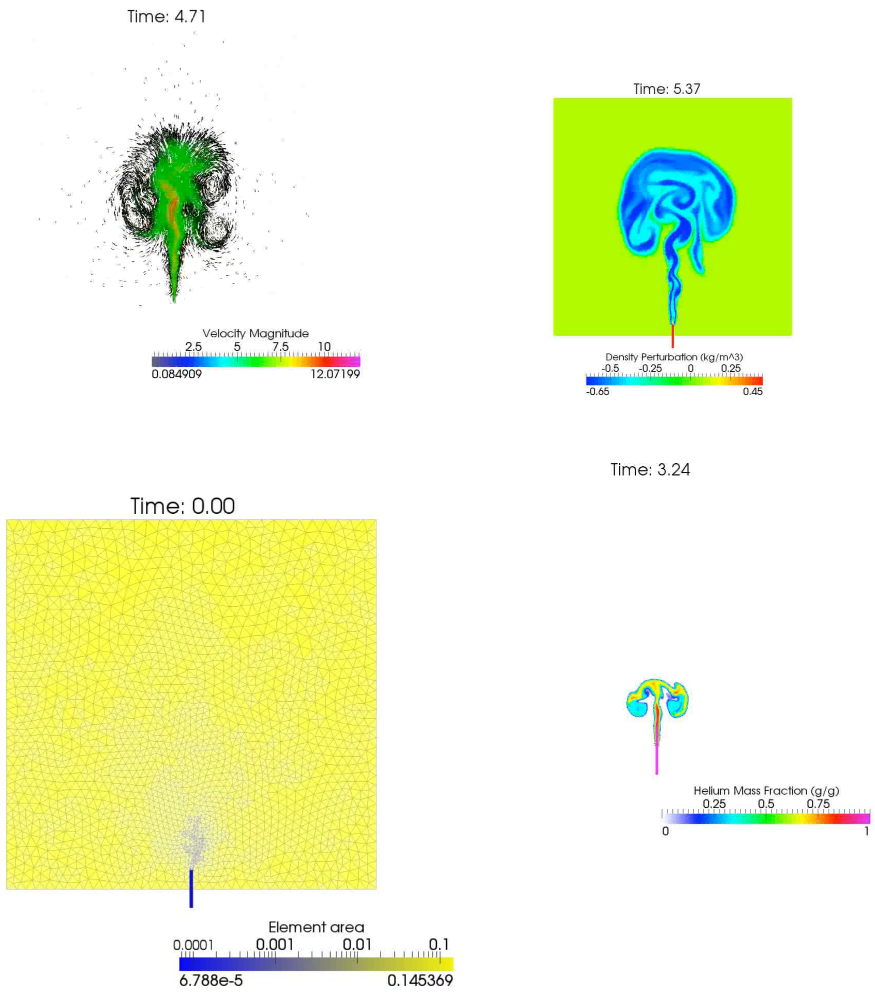




Discontinuous Galerkin Methods
Don't strongly enforce continuity of the finite element variables across finite element boundaries

Mesh adaptivity

- Hessian approach optimizes estimate of linear interpolation error in chosen variables for given number of degrees of freedom[run time]

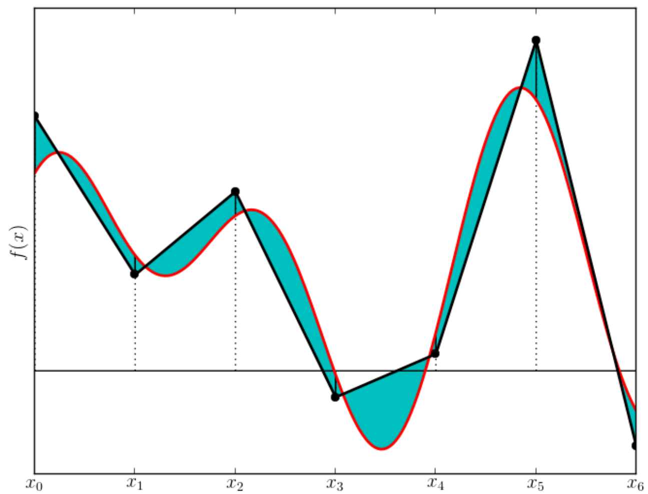


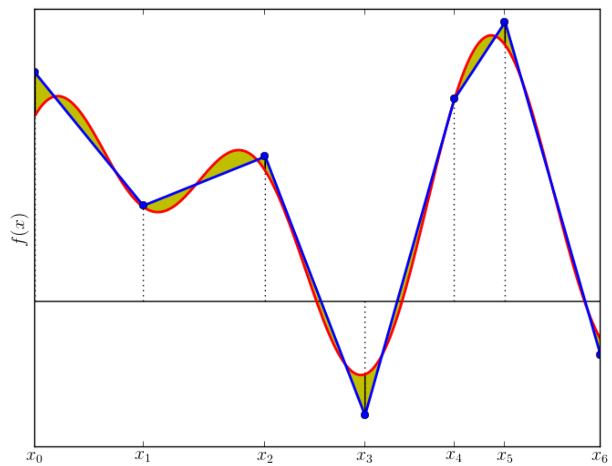
Mesh Adaptivity

$$F = \sum_{i \in \text{edges}} (\boldsymbol{v}_i^T \underline{\boldsymbol{M}}_i \boldsymbol{v}_i - 1)^2$$

$$[\underline{\boldsymbol{M}}]_{ij} = \det |H|^{-1/7} \frac{|H|_{ij}}{\epsilon}$$

$$[H]_{ij} = \frac{\partial^2 \alpha}{\partial x_i \partial x_j}$$

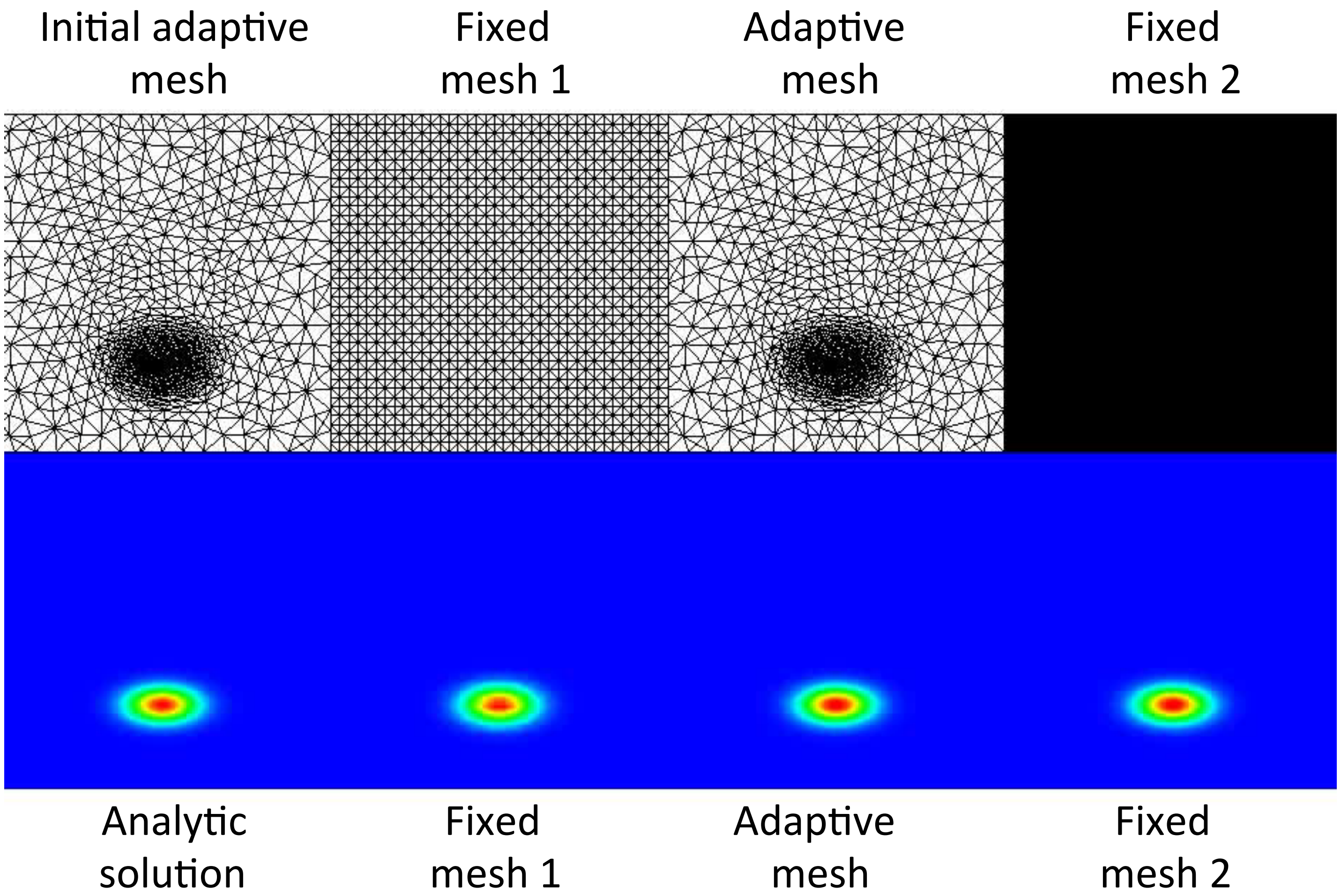




C.C. Pain, A.P. Umpleby,
C.R.E. de Oliveira & A.J.H. Goddard (2001)
Computer Methods in Applied
Mechanics and Engineering

Mesh Adaptivity: Faster & more accurate solutions

Advection of a miscible Gaussian bubble



Mesh Adaptivity: Faster & more accurate solutions

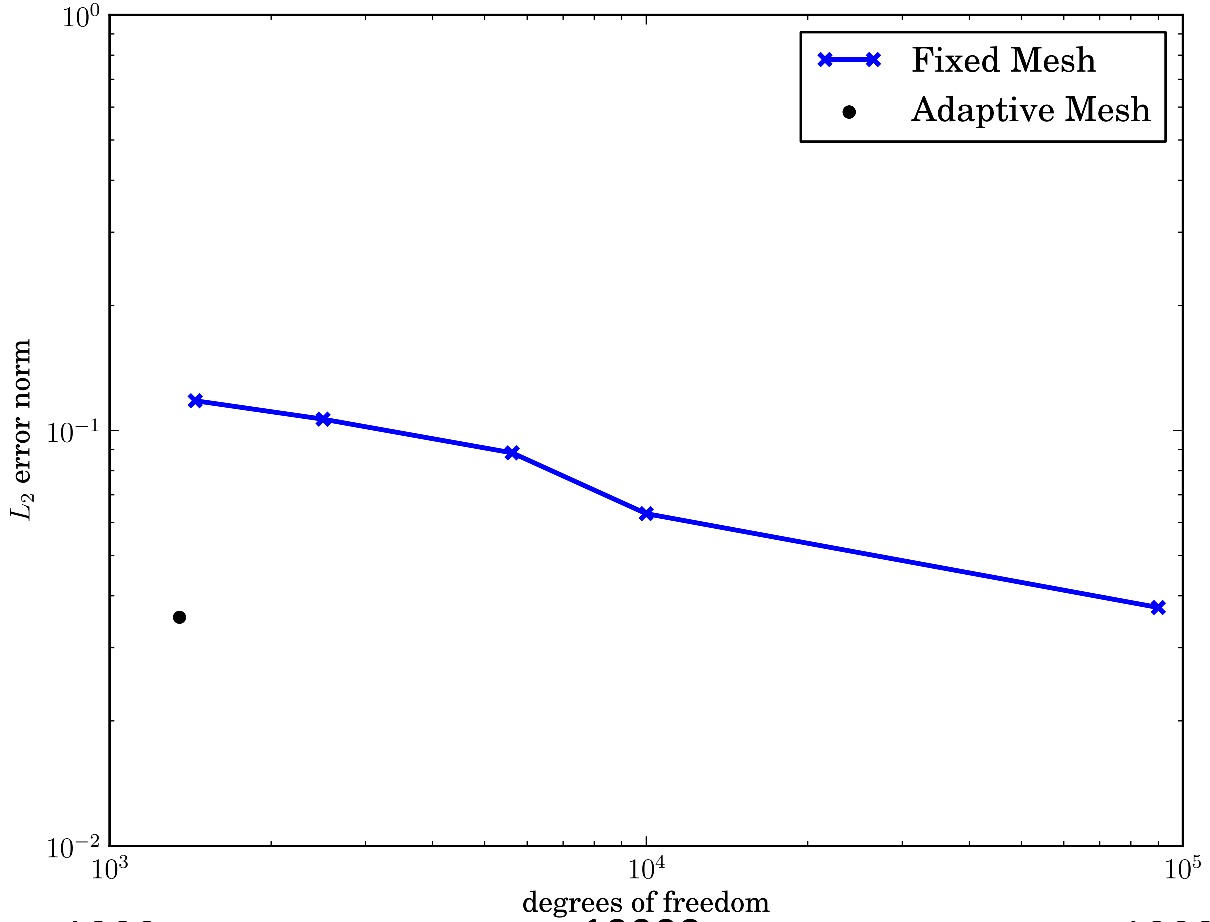
2 orders of magnitude smaller problem/half error

30x speed up

1

0.1

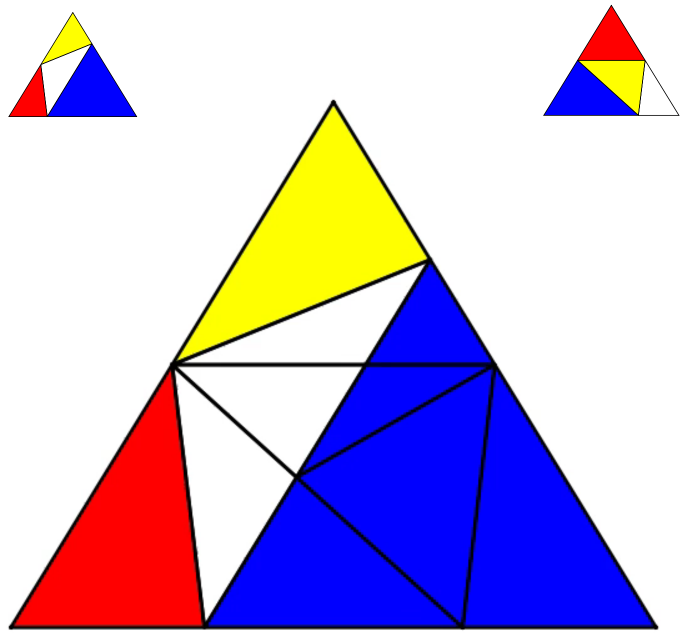
0.01



1000 10000 100000

Supermeshing

FE solutions/test functions piecewise smooth over mesh elements



Elements of supermesh: old variables and new test fns both smooth.
No jumps.

Allows efficient conservative mesh to mesh interpolation via projection methods

P. E. Farrell & J. R. Maddison (2011) Computer Methods in Applied Mechanics and Engineering

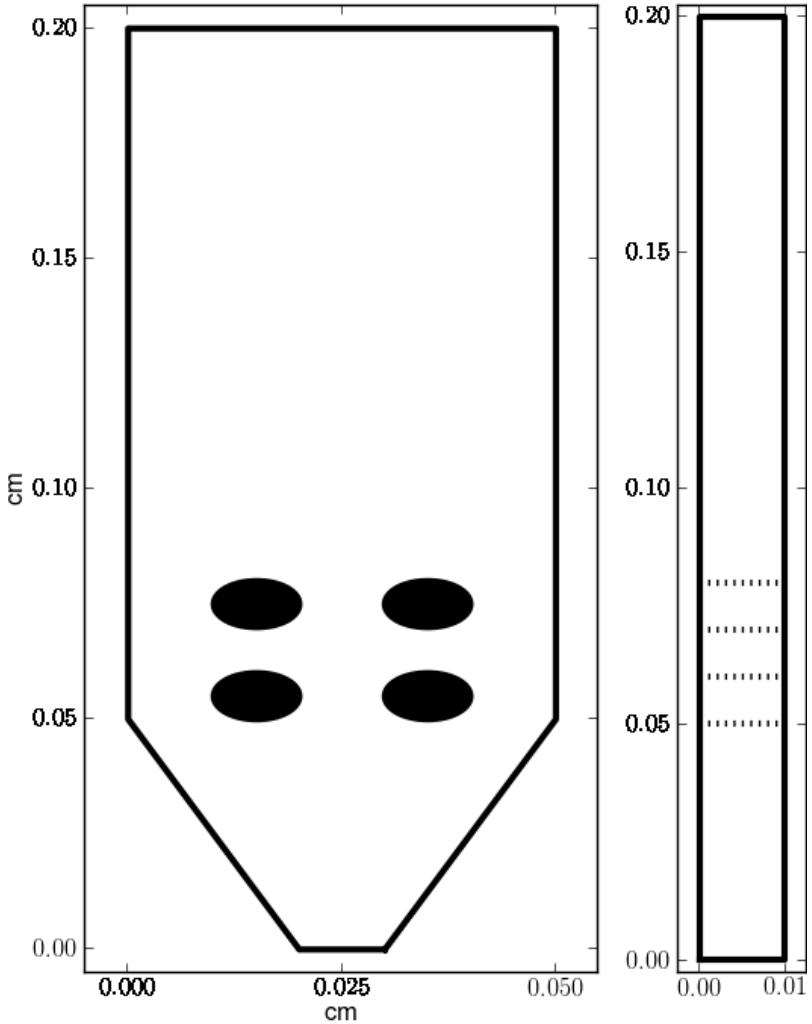
$$\sum_{k\in\{1,\ldots,|T_3|\}} \int_{\Omega_k} N_{\sigma_i(k)} N_{\sigma_j(k)} \hat{\alpha}_{\sigma_j(k)} dA = \sum_{k\in\{1,\ldots,|T_3|\}} \int_{\Omega_k} N_{\sigma_i(k)} N_{\pi_j(k)} \alpha_{\pi_j(k)} dA$$

Progress Since Last Meeting

Progress Since Last Meeting

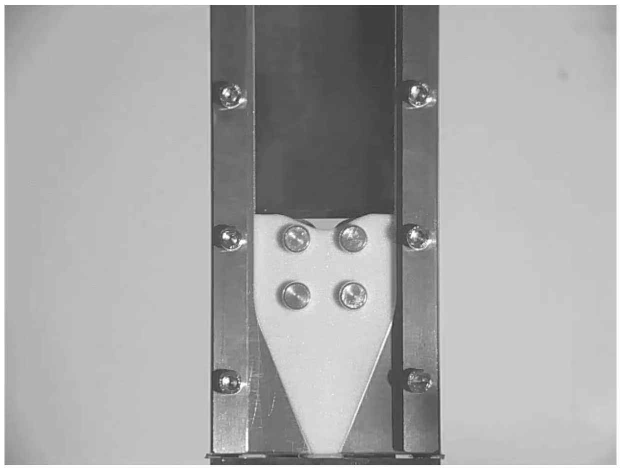
- Model validation + quantification of advantages of adaptivity in real problem
- Polydispersity: Particle binning
  - 3 particle classes

Validation
Fluidised bed device



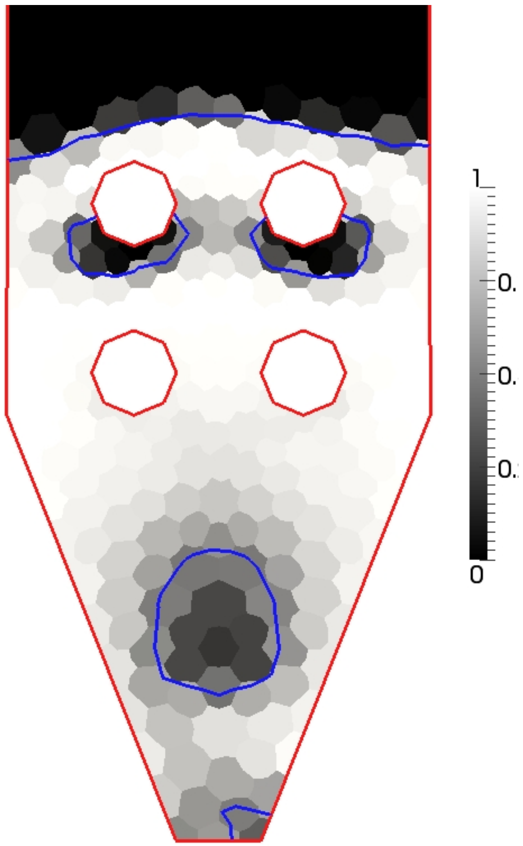
- 20cmx5cmx1cm laboratory device
- Air fluidized through inlet in bottom
- Particles: glass beads of specified size
- Intrusions pass through entire device.

Video : 190 µm beads

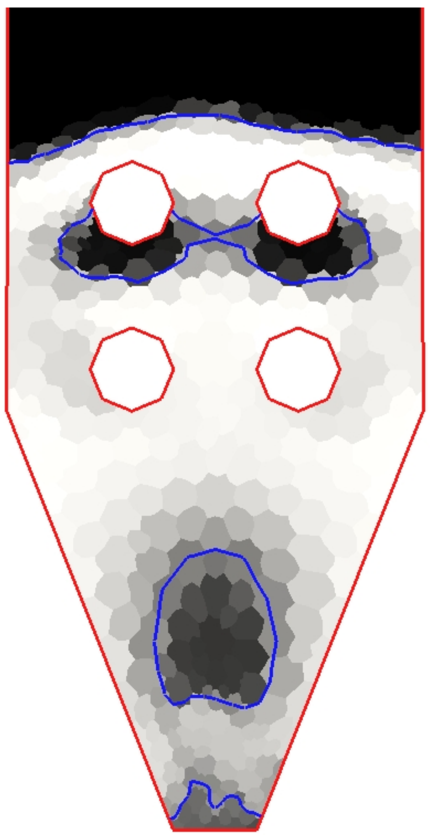


Validation

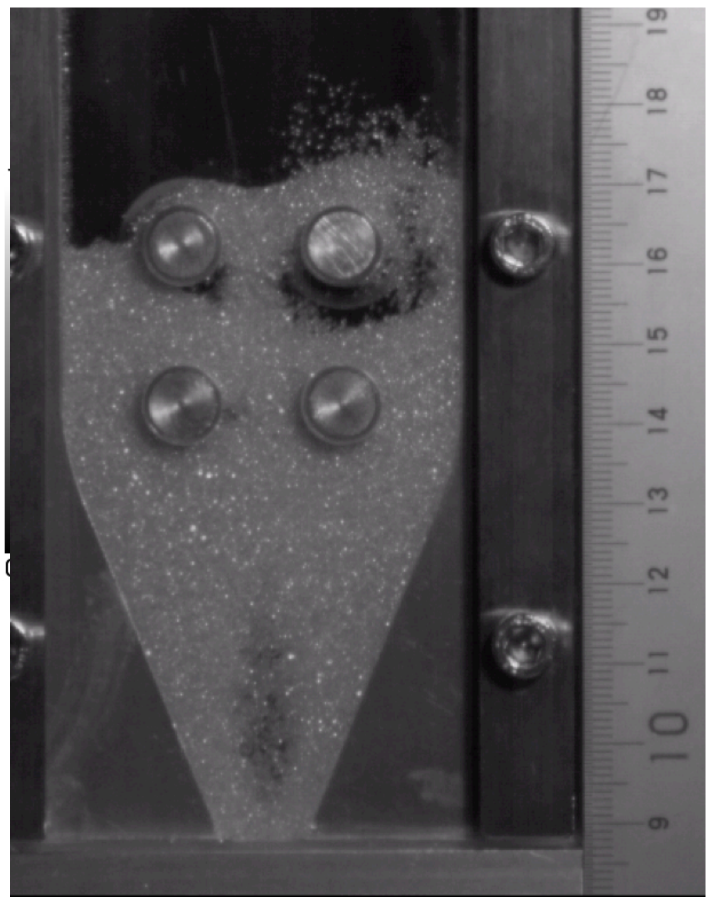
Fixed Mesh



Adaptive Mesh



Laboratory Experiment

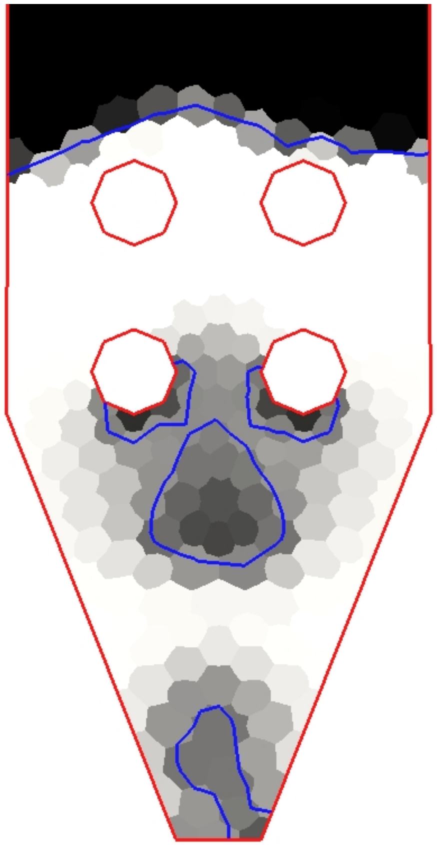


500µm beads

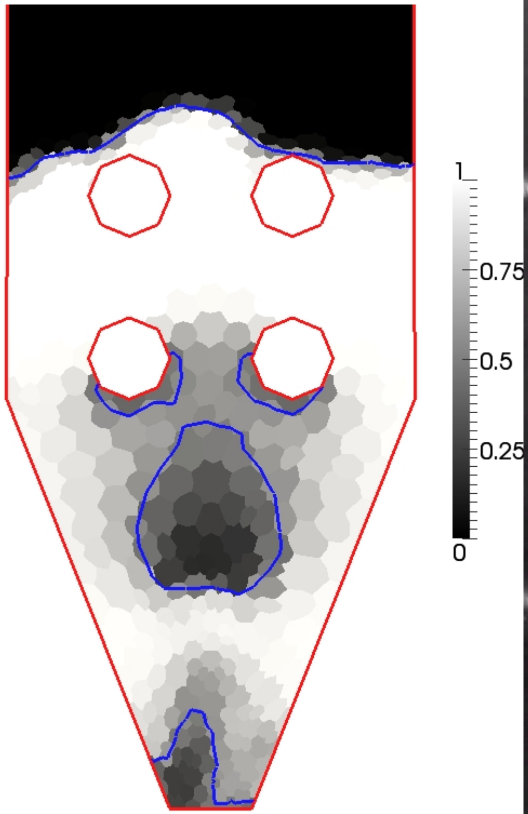
Experimental data courtesy Prof. M. Sakai. Univ. Tokyo

Validation

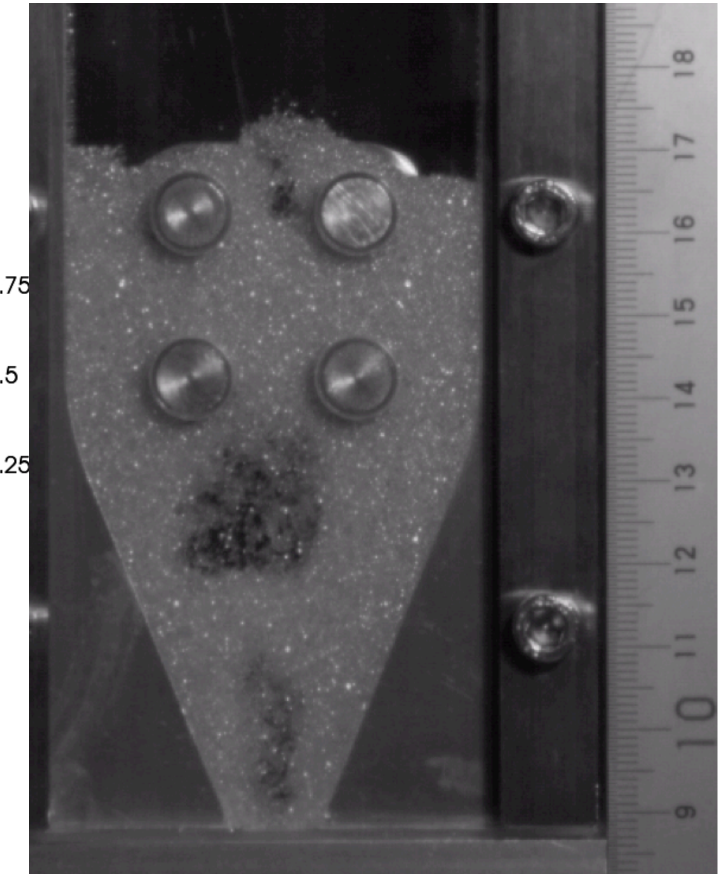
Fixed Mesh



Adaptive Mesh



Laboratory Experiment



500µm beads

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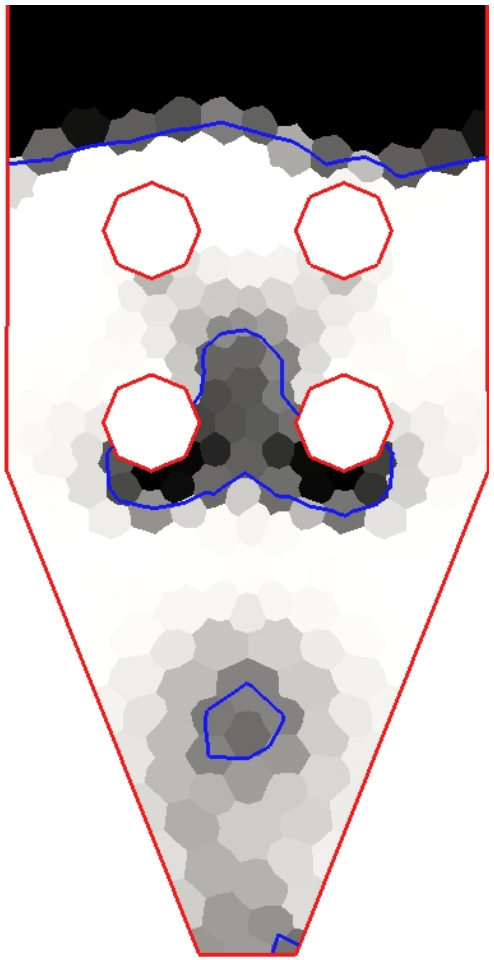
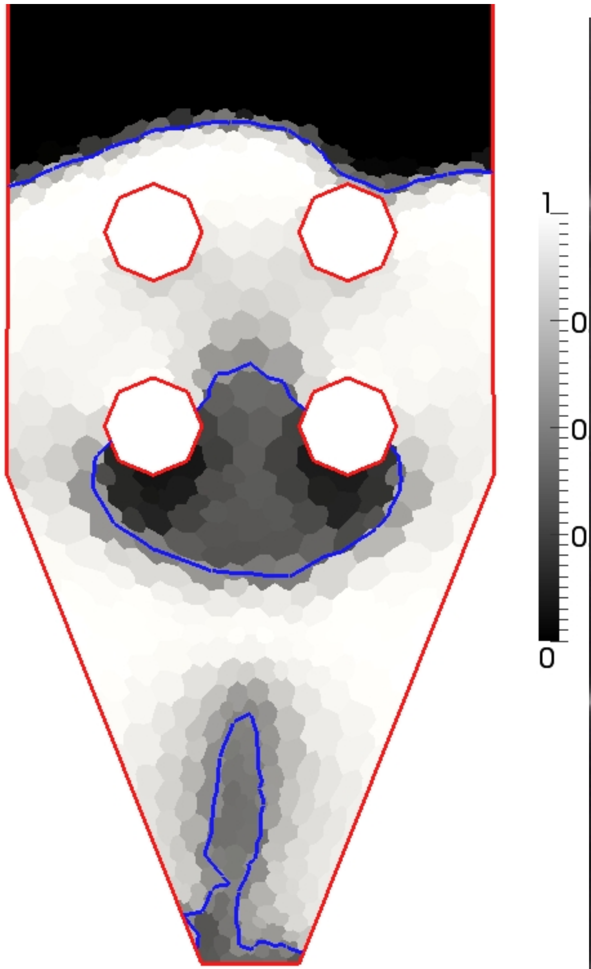
Validation

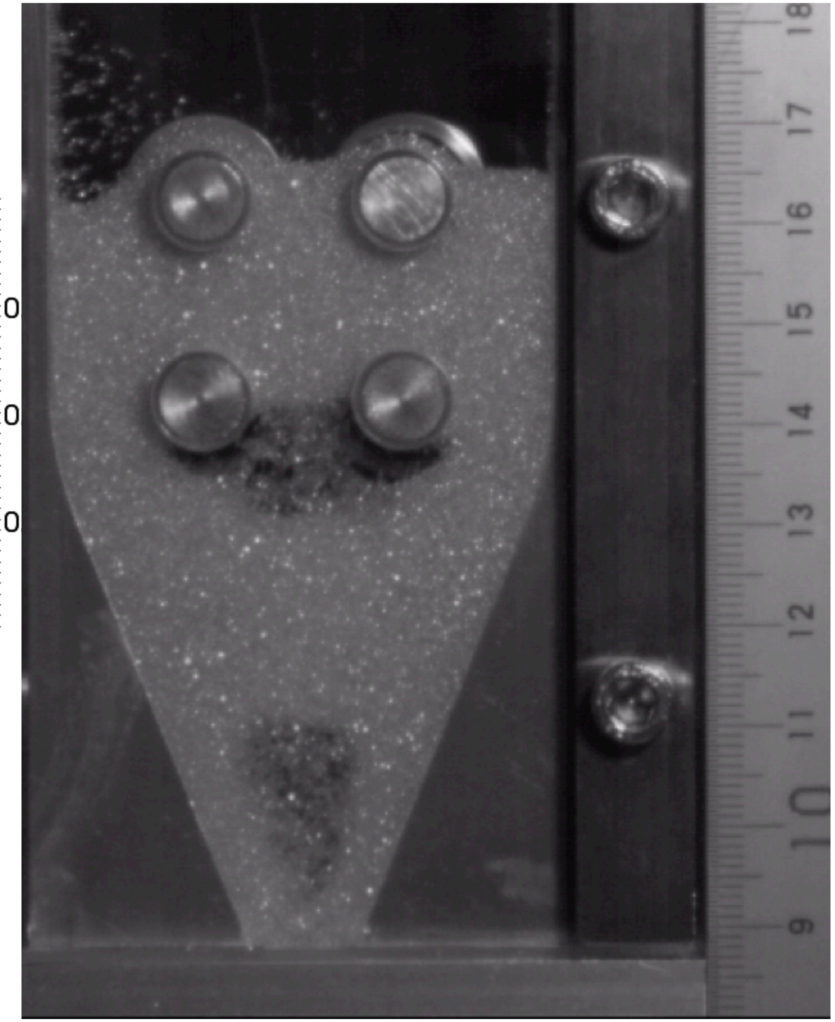
Fixed Mesh



Adaptive Mesh

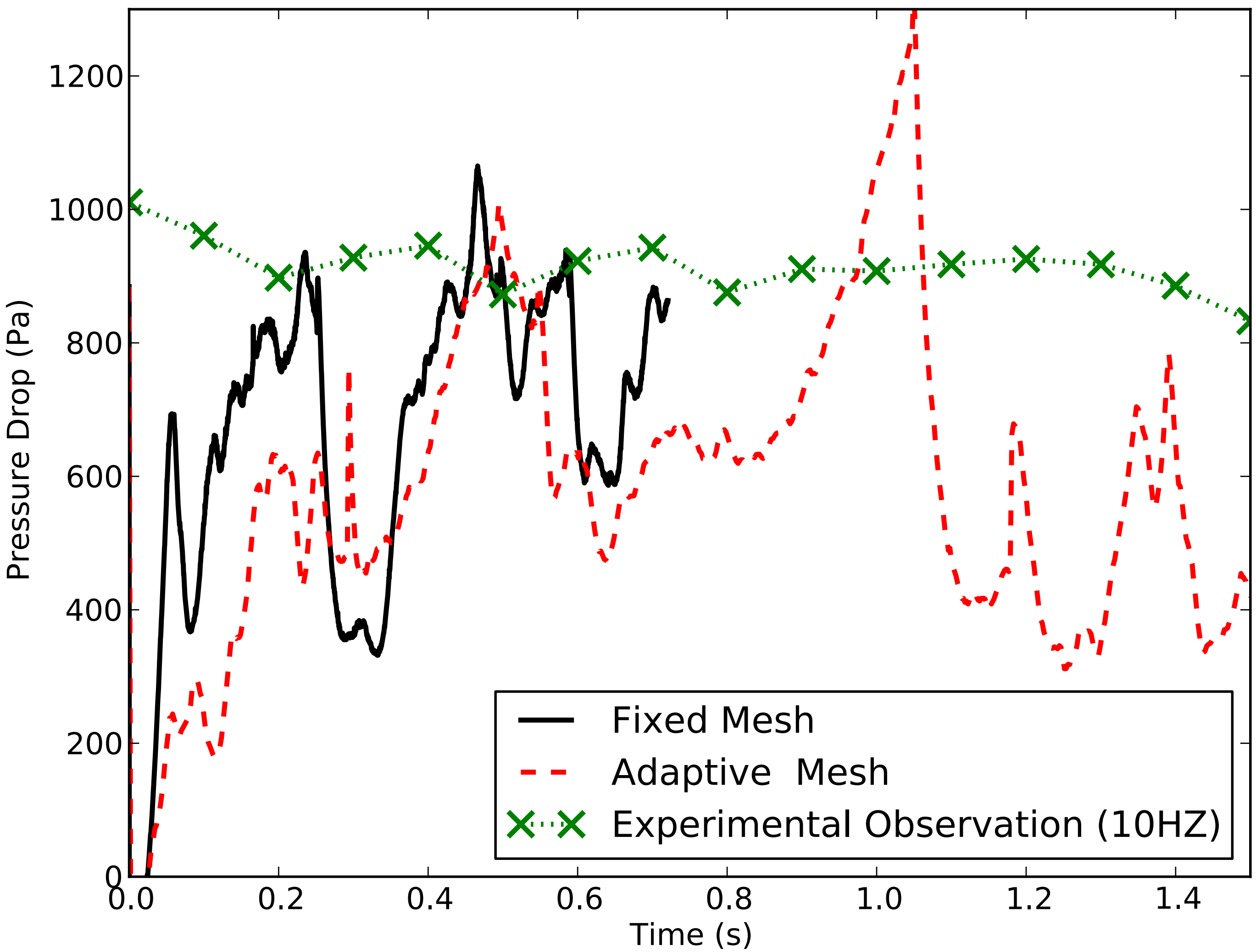


Laboratory Experiment



Experimental data courtesy Prof. M. Sakai.
Univ. Tokyo

Pressure Drop



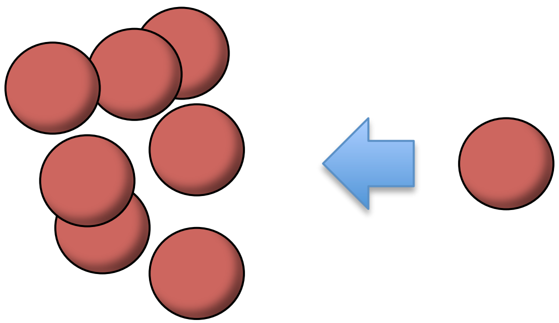
Polydispersity

- Particle binning method (q.v. MUSIG: Multiple Size Groups)
  - Number of classes, each with single:
    - representative length scale
    - sphericity
    - material properties.
- Effectively using delta function as distribution.
- Particle classes may each have their own velocity, or groups may move together.

Particle-particle interactions

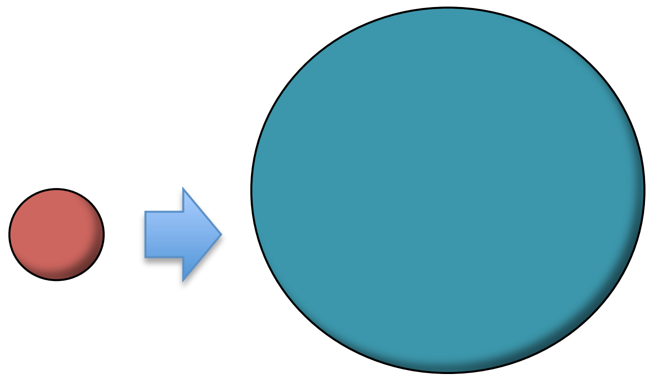
- Divided into intra-class and inter-class interactions

Intra-class interactions



=> Existing granular temperature model

Inter-class collisions



⇒ New term
Bulk momentum dominated

Syamlal 1987

Collisional model for inter-class momentum transfer

Coefficient for collisional friction factor

Quadratic drag term

$$\beta_{ij} = \frac{3\pi}{4}\left(1+e_{ij}\right)\left(1+\mathcal{F}_{ij}\frac{\pi}{4}\right)\left(\frac{\rho_i\alpha_i\rho_j\alpha_j\left(d_i+d_j\right)^2}{\rho_i d_i^3+\rho_j d_j^3}\right)\left|\boldsymbol{u}_j-\boldsymbol{u}_i\right|$$

Coefficient of restitution for material-material collision

Polydispersive radial distribution function
Measure of likelihood of i-j collision based
On volume fractions and respective diameters

Syamlal, M., 1987. The particle-particle drag term in a multiparticle model of fluidization. National Technical Information Service, Springfield, VA 188.

Polydispersity : Single vs multiple granular temperatures

- For single particle class granular temperature is well defined

$$\Theta_s = \frac{1}{3}\left\langle\, |\boldsymbol{u} - \boldsymbol{v}|^2 \,\right\rangle$$

- For multiple particle classes, more options:

$$\Theta_i = \frac{1}{3}\left\langle |\boldsymbol{u}_i - \boldsymbol{v}|^2 \right\rangle$$

$$\Theta = \frac{1}{3\sum \alpha_i} \sum \alpha_i \left\langle |\boldsymbol{u}_i - \boldsymbol{v}|^2 \right\rangle$$

$$\Theta = \frac{1}{3\sum \rho_i d_i^3 \alpha_i} \sum \rho_i d_i^3 \alpha_i \left\langle |\boldsymbol{u}_i - \boldsymbol{v}|^2 \right\rangle$$

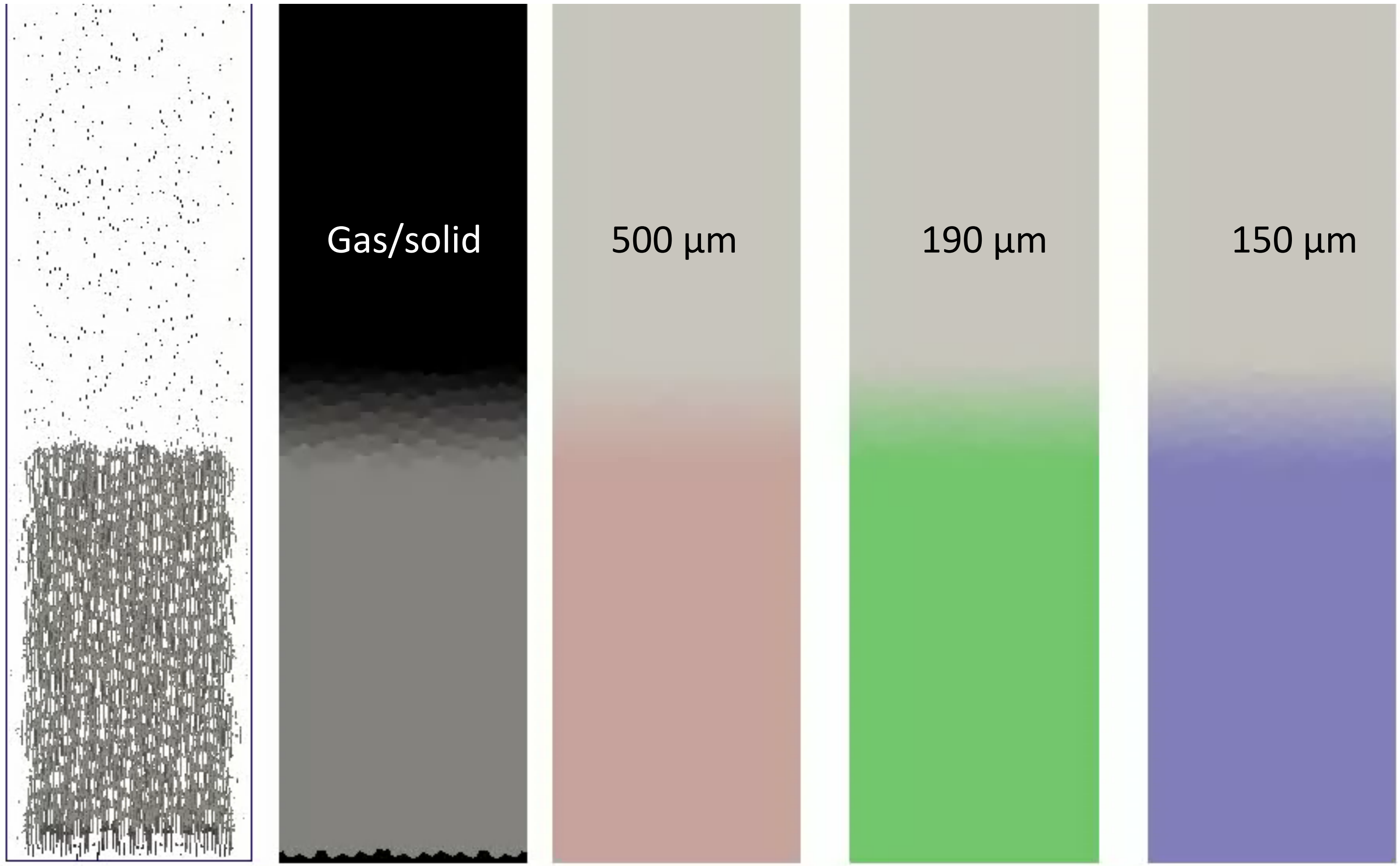
Polydispersity: binary/trinary mixtures

- Radial distribution function measure of probability of collisions

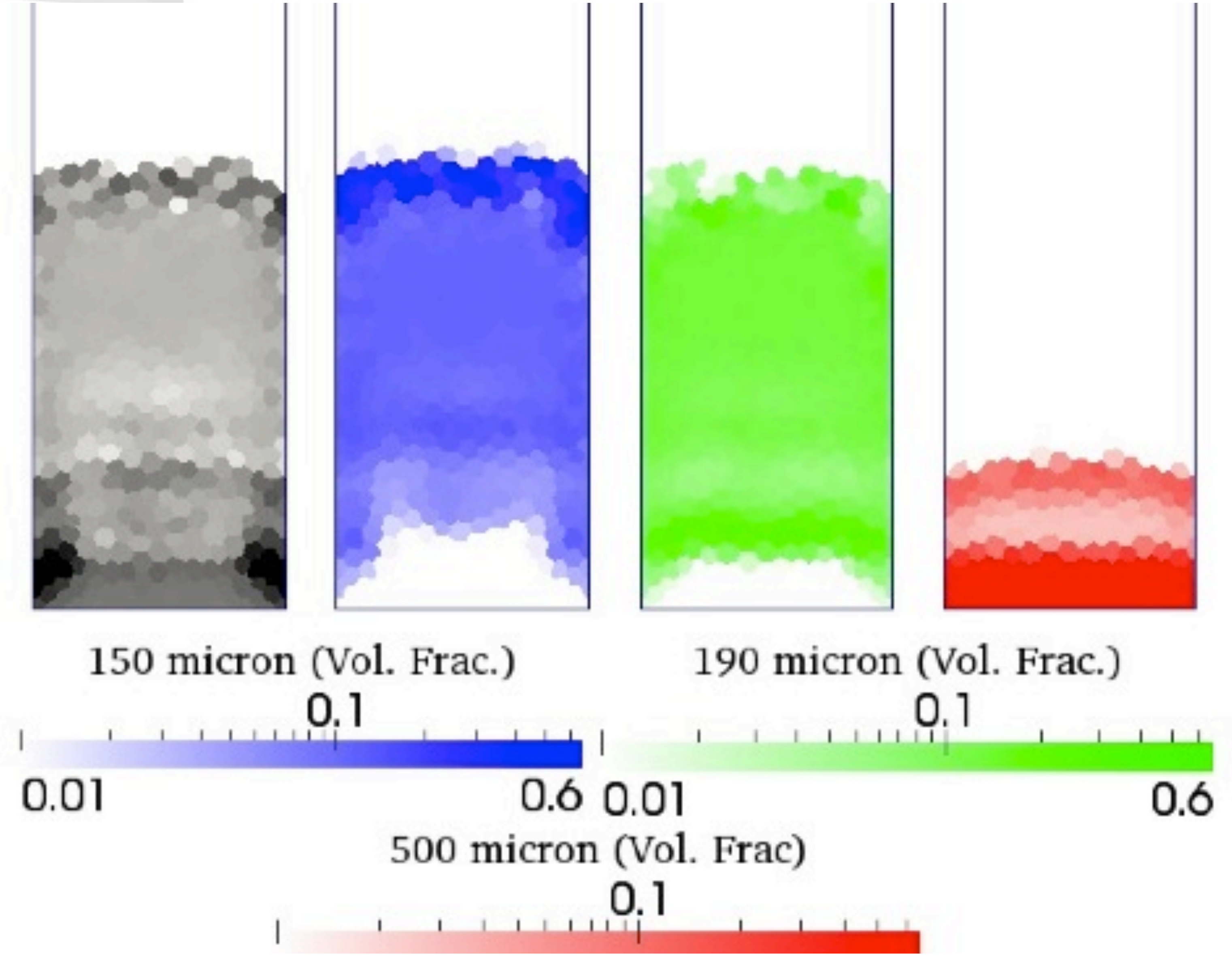
$$g_0 = \left(1 - \left(\frac{\alpha_s}{\alpha_{s,\max}}\right)^{\frac{1}{3}}\right)^{-1}$$

- Binary mixtures have increased maximum packing fraction
- Maximum packing fraction now function of solid volume fractions & sizes.

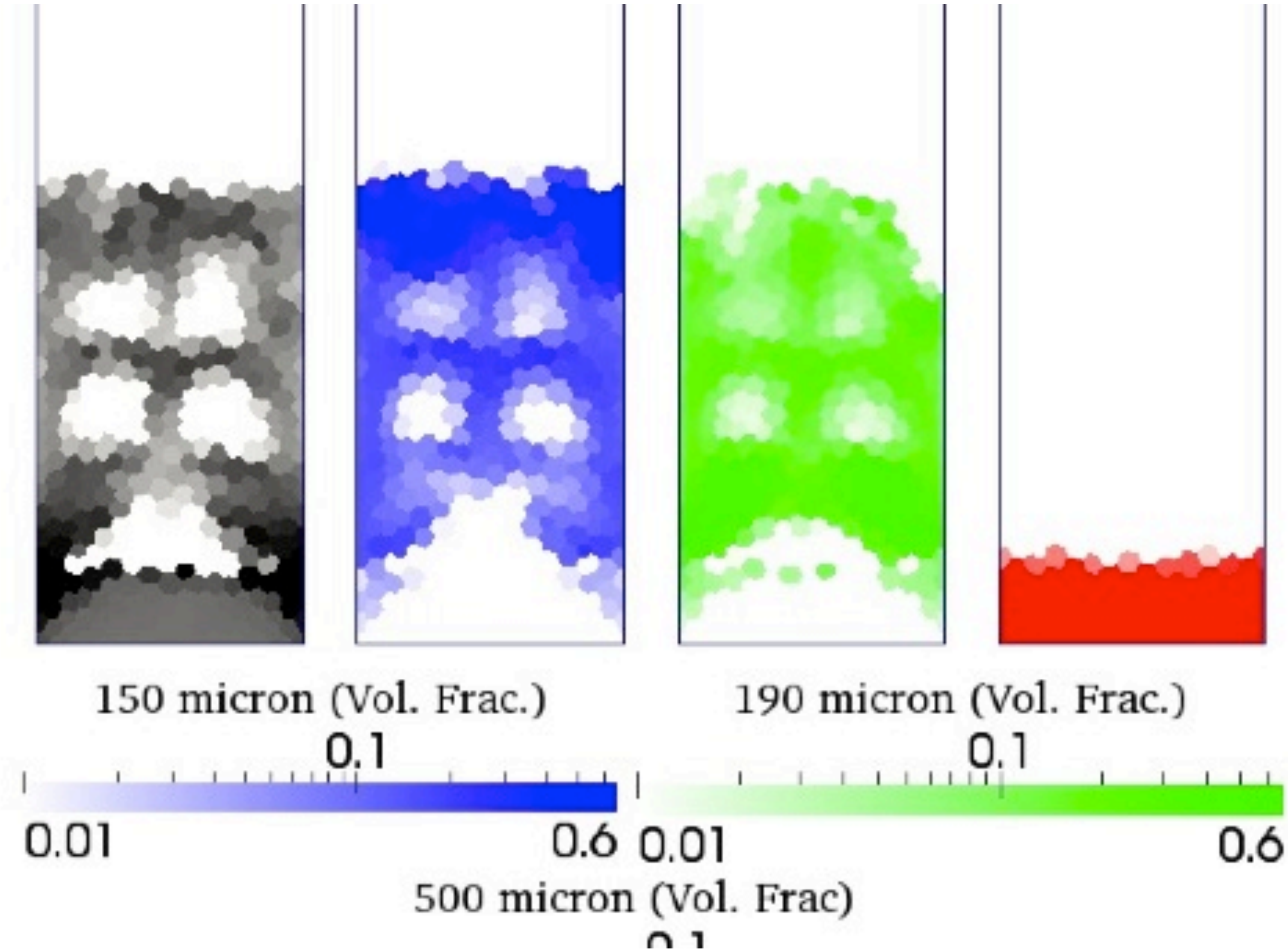
Polydispersity – 3 sizes of particle



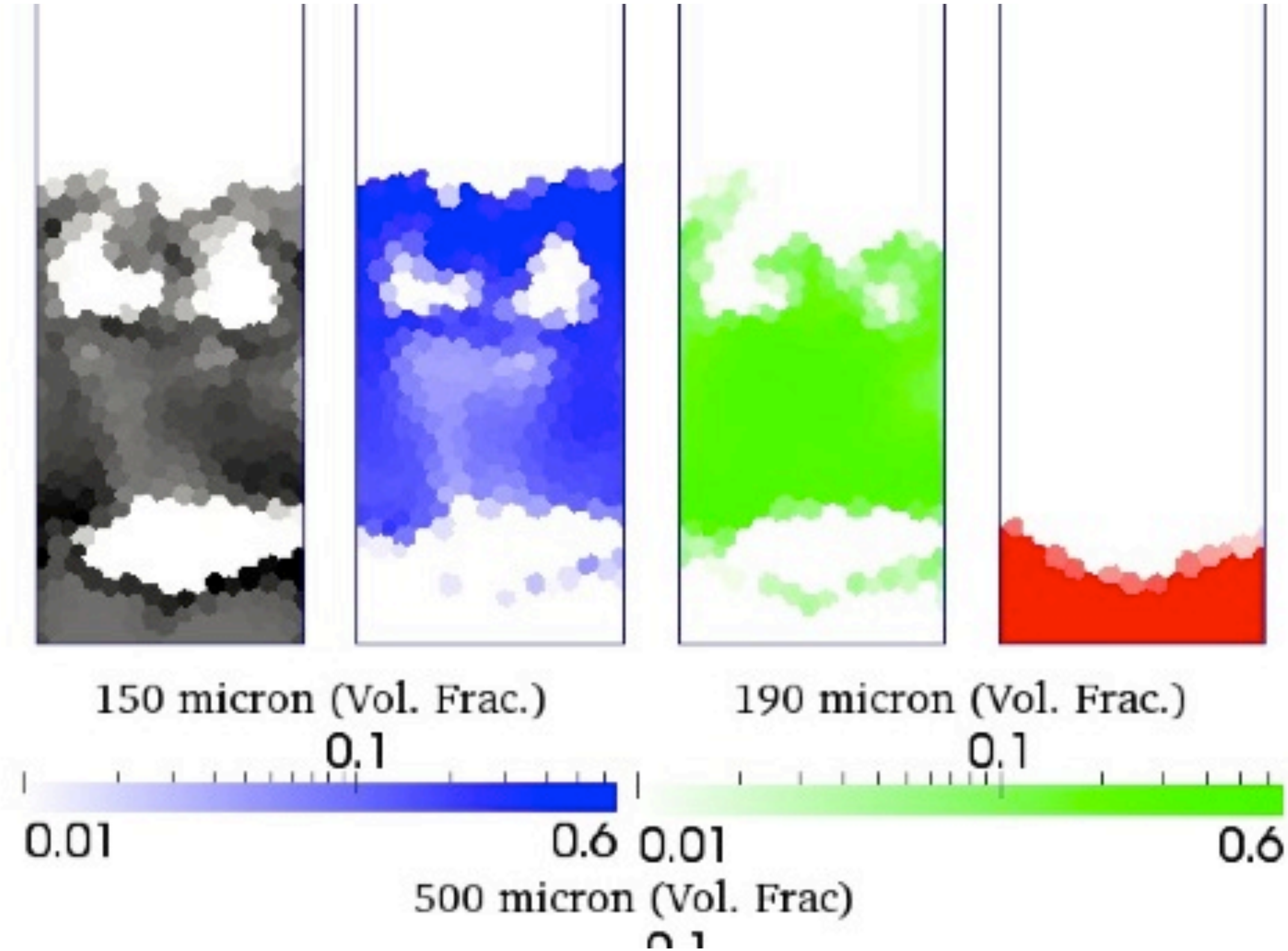
t=0.05 s


150 micron (Vol. Frac.)
0.1
0.01
0.6
190 micron (Vol. Frac.)
0.1
0.01
0.6
500 micron (Vol. Frac)
0.1

T=0.35 s

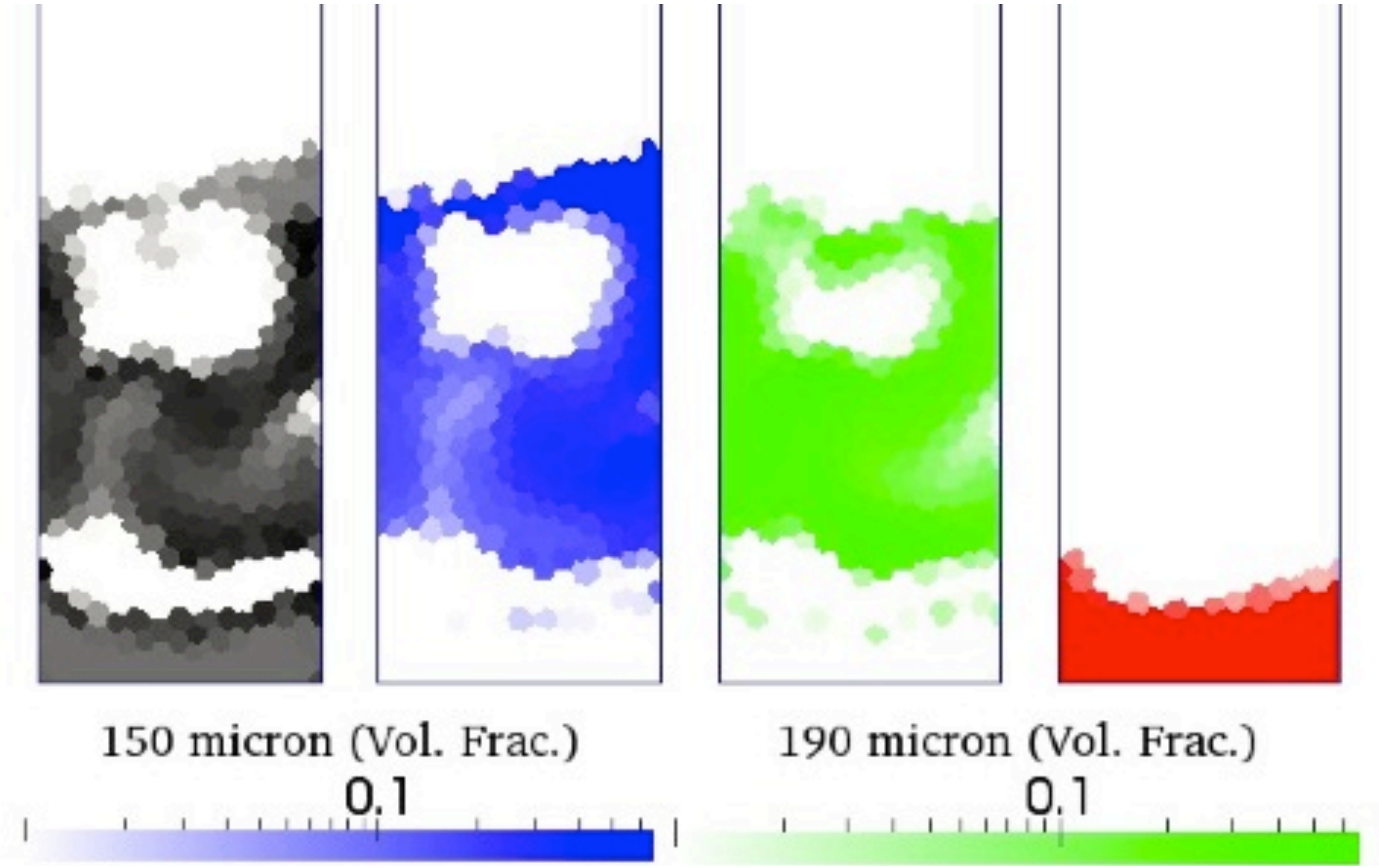

150 micron (Vol. Frac.)
0.1
0.01
0.6
190 micron (Vol. Frac.)
0.1
0.01
0.6
500 micron (Vol. Frac)

T=0.5 s

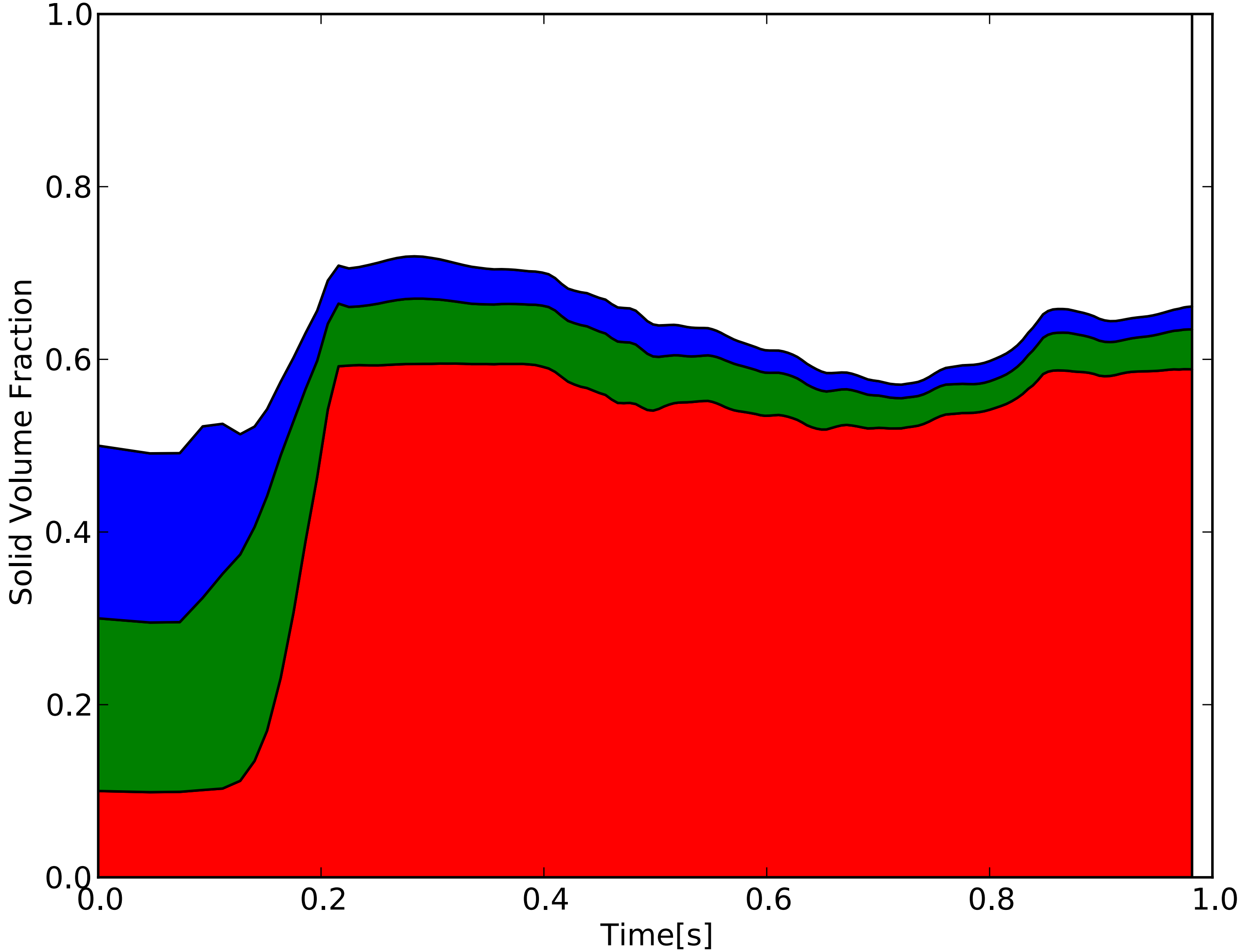


150 micron (Vol. Frac.)
0.1
0.01
0.6
190 micron (Vol. Frac.)
0.1
0.01
0.6
500 micron (Vol. Frac)

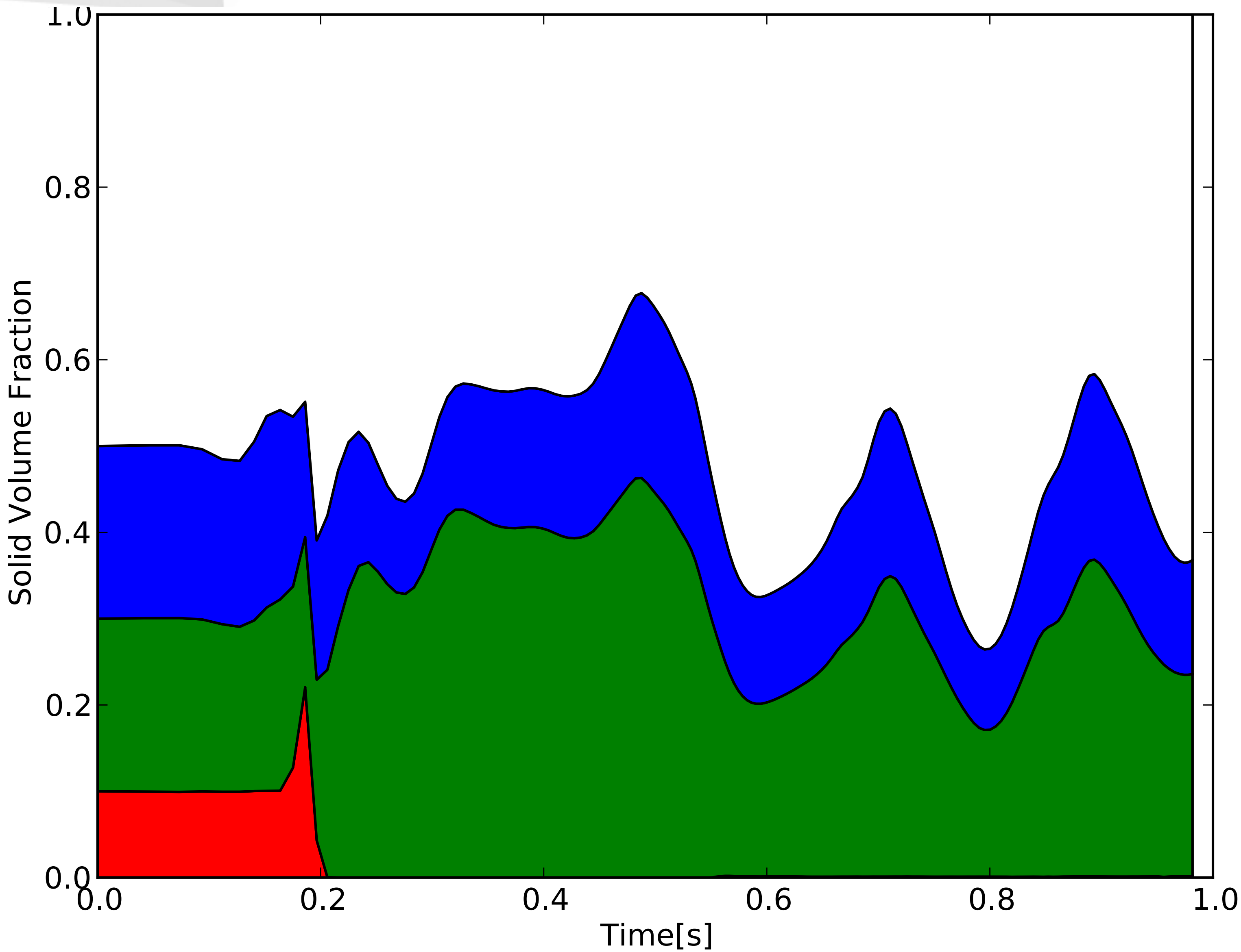
T=0.75



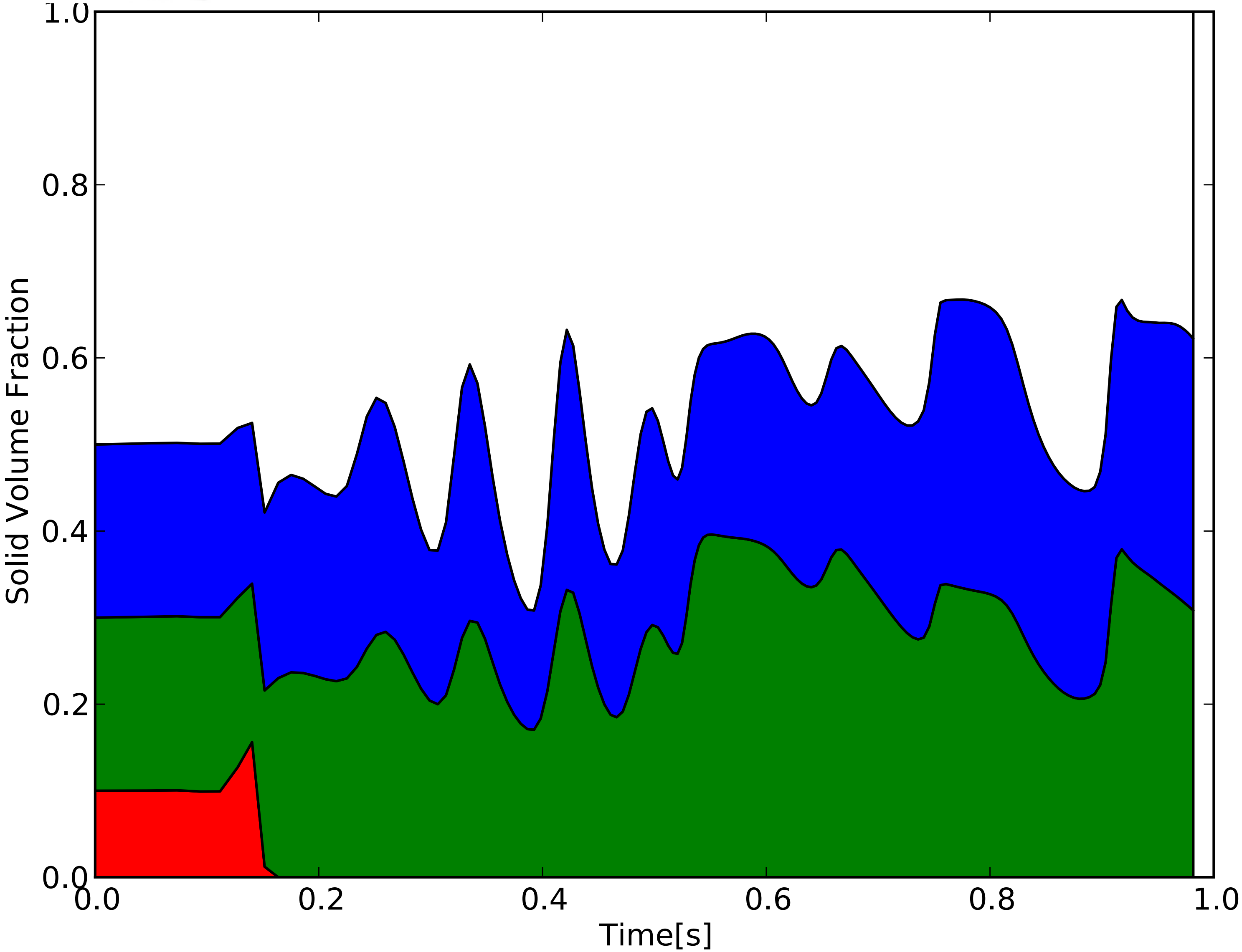
z=1cm



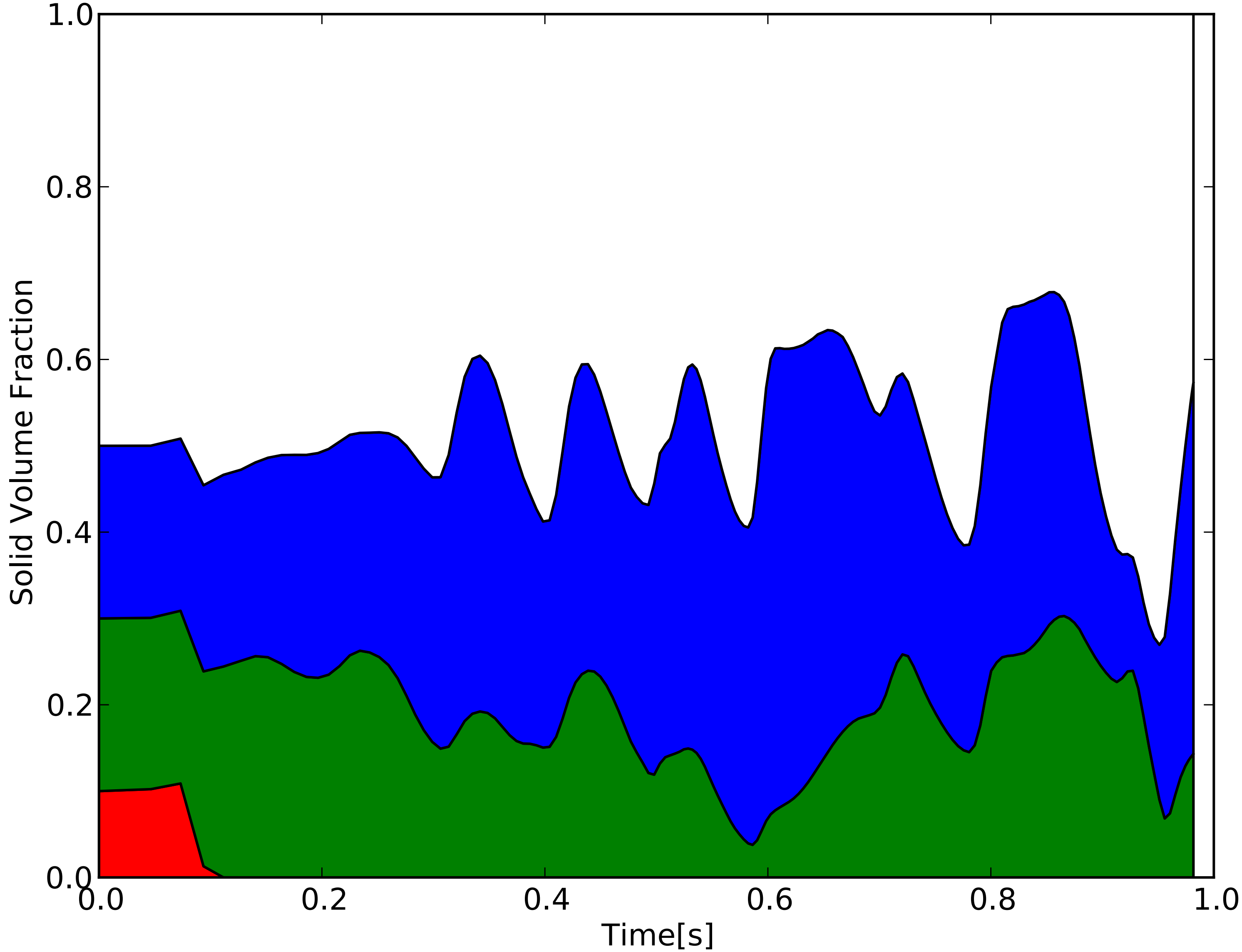
z=2.5cm



z=5cm

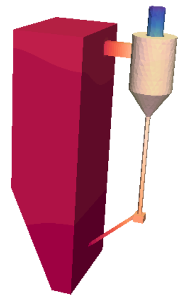


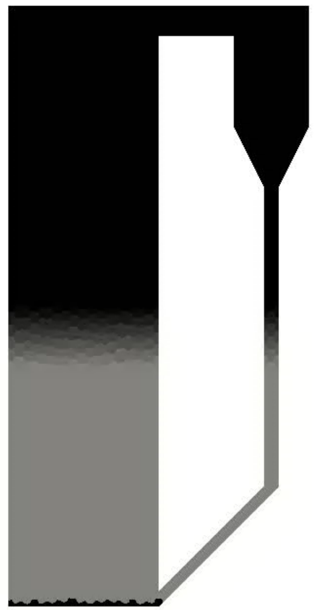
z=7.5cm



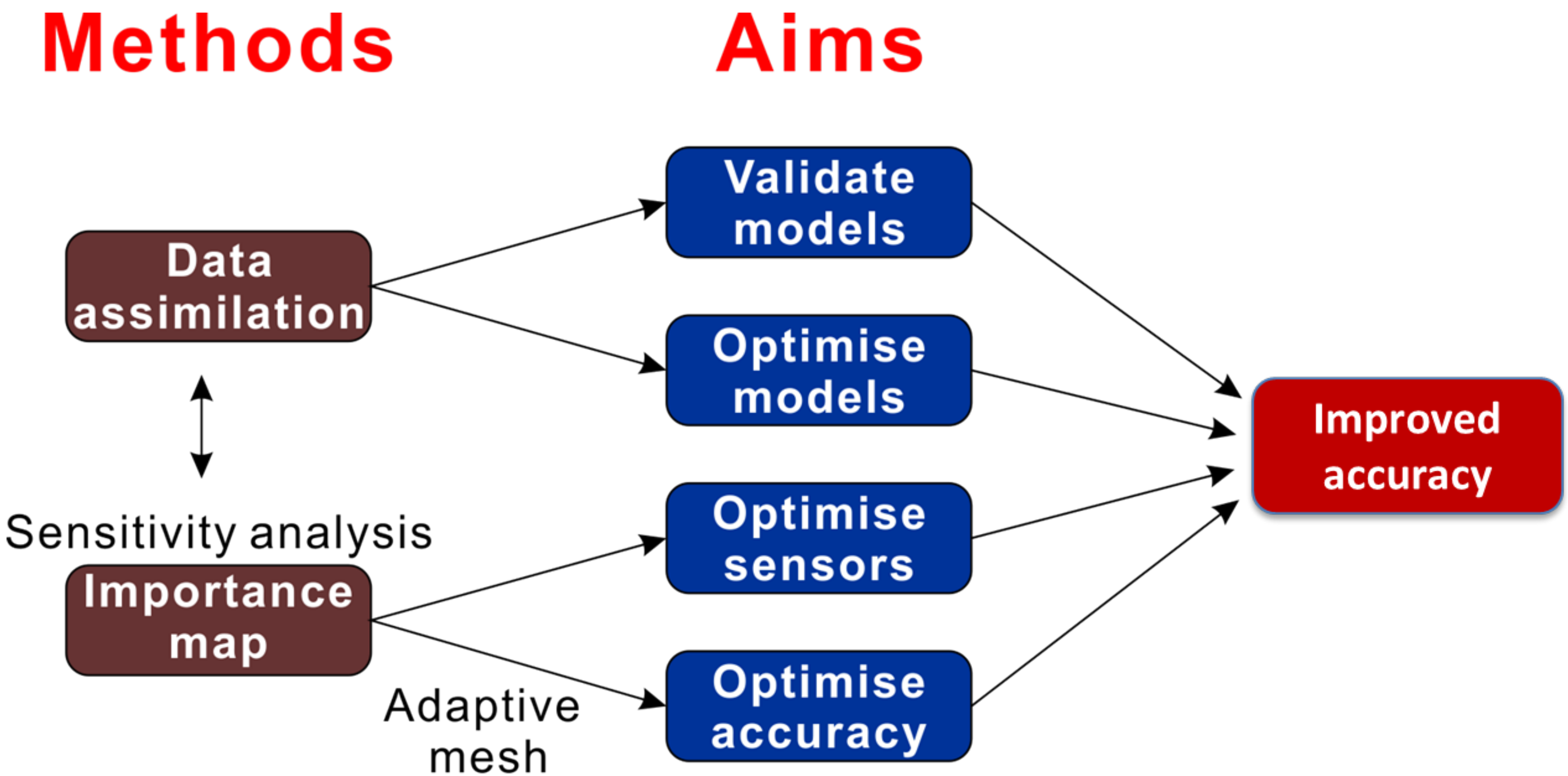
Future Work

- Flows in pipes and risers
- Three dimensional runs
- Investigate sphericity & deformation
- MuSiG for granular & bubbly flows
- Systematic investigation of drag model skill in the context short term system forecasting

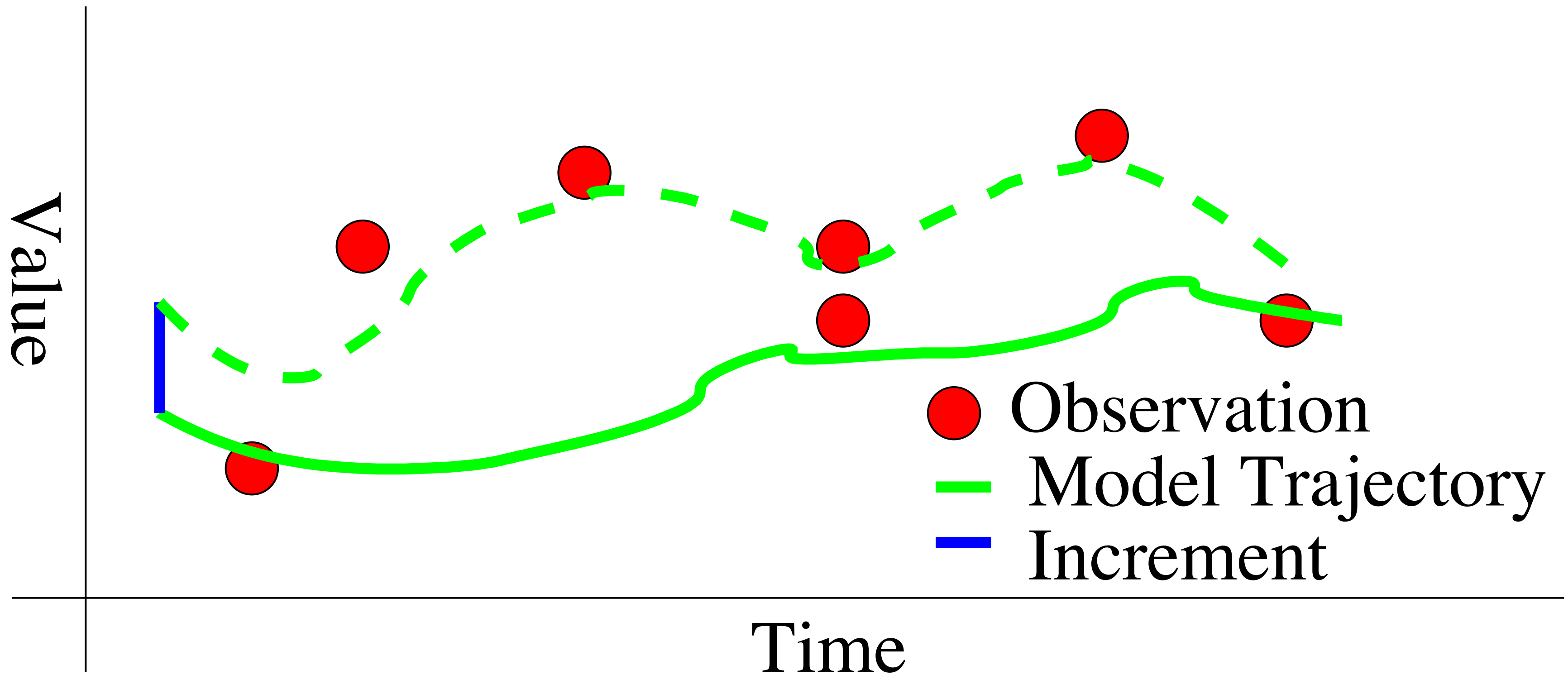




Models & Experiments



Data Assimilation



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