

- LA DISTANCIA MEDIA ENTRE DOS SEÑALES PERIODICAS $x_1(t) \in \mathbb{R}, \mathbb{C}$ Y $x_2(t) \in \mathbb{R}, \mathbb{C}$; SE PUEDE EXPRESAR A PARTIR DE LA POTENCIA MEDIA DE LA DIFERENCIA ENTRE ELLAS

$$d^2(x_1, x_2) = \bar{P}_{x_1 - x_2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt$$

- SEA $x_1(t)$ Y $x_2(t)$ DOS SEÑALES COMO SE MUESTRA A CONTINUACION

$$x_1(t) = A e^{j\omega_0 t}$$

$$x_2(t) = B e^{j5\omega_0 t}$$

- CON $\omega_0 = \frac{2\pi}{T}$; $T, A, B \in \mathbb{R}^+$, DETERMINE LA SEÑAL ENTRE LAS DOS SEÑALES

5/

- SEGUN LA INTEGRAL DE POTENCIA SE TIENE QUE:

$$\frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt = \frac{1}{T} \left[\underbrace{\int_T |x_1(t)|^2 dt}_{E_{x_1}} - 2 \int_T x_1(t) x_2(t) dt + \dots \right. \\ \left. \dots + \int_T |x_2(t)|^2 dt \right]$$

$$= \bar{P}_{x_1} - \frac{2}{T} \int_T x_1(t) x_2(t) dt + \bar{P}_{x_2}$$

- PARA \bar{P}_{x_1} SE TIENE QUE, COMO $x_1(t) \in \mathbb{C} \Rightarrow |x_1(t)|^2 = x_1(t) x_1(t)^*$

$$\bar{P}_{x_1} = \frac{1}{T} \int_T |x_1(t)|^2 dt = \frac{A^2}{T} \int_0^T e^{j\omega_0 t} \cdot e^{-j\omega_0 t} dt = \frac{A^2}{T} \int_0^T e^0 dt$$

$$= \frac{A^2}{T} \int_0^T dt = \frac{A^2}{T} \left[t \right]_0^T = \frac{A^2}{T} [T]$$

$$\therefore \bar{P}_{x_1} = A^2$$

• PARA \bar{P}_{x_2} SE TIENE QUE, COMO $x_2(t) \in \mathbb{C} \Rightarrow |x_2(t)|^2$
 $= x_2(t) x_2(t)^*$

$$\begin{aligned}\bar{P}_{x_2} &= \frac{1}{T} \int_0^T |x_2(t)|^2 dt = \frac{B^2}{T} \int_0^T e^{j5\omega_0 t} \cdot e^{-j5\omega_0 t} dt \\ &= \frac{B^2}{T} \int_0^T dt = \frac{B^2}{T} \left[t \right]_0^T = \frac{B^2}{T} [T]\end{aligned}$$

$$\therefore \bar{P}_{x_2} = B^2$$

• PARA EL OTRO TERMINO

$$-\frac{2}{T} \int_0^T x_1(t) x_2(t) dt = -\frac{2}{T} \int_0^T A e^{j\omega_0 t} \cdot B e^{j5\omega_0 t} dt$$

$$= -\frac{2}{T} \int_0^T A e^{j\frac{2\pi}{T} t} \cdot B e^{j5\frac{2\pi}{T} t} dt$$

$$= -\frac{2}{T} \int_0^T A \cdot B \cdot e^{j\frac{12\pi}{T} t} dt = -\frac{2}{T} A \cdot B \int_0^T e^{j\frac{12\pi}{T} t} dt$$

$$= -\frac{2}{T} A \cdot B \left[\frac{T}{j12\pi} e^{j\frac{12\pi}{T} t} \right]_0^T$$

$$= -\frac{2}{T} A \cdot B \left[\frac{T}{j12\pi} e^{j\frac{12\pi}{T} T} - \frac{T}{j12\pi} e^{j\frac{12\pi}{T} \cdot 0} \right]$$

• como $e^{j12\pi} = \cos(12\pi) + j\sin(12\pi) = 1$

$$= -\frac{2}{T} A \cdot B \left[\frac{T}{j12\pi} - \frac{T}{j12\pi} \right]$$

$$= -\frac{2}{T} A \cdot B [0] = 0$$

• POR LO TANTO DE LA INTEGRAL QUEDA

$$\frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt = \bar{P}_{x_1} - 0 + \bar{P}_{x_2}$$

$$= \bar{P}_{x_1} + \bar{P}_{x_2} = A^2 + B^2$$

• PARA LO CUAL SE OBTIENE $d(x_1, x_2)$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} A^2 + B^2$$

$$\therefore d(x_1, x_2) = \sqrt{A^2 + B^2}$$

SOLUCION PREGUNTA #2

JUAN JOSE CARDONA H.
C.C 1002 609 346

- CUAL ES LA SEÑAL OBTENIDA EN TIEMPO DISCRETO AL UTILIZAR UN CONVERTOR ANALOGO DIGITAL CON FRECUENCIA DE MUESTREO DE 5 KHz, APLICADO A LA SEÑAL CONTINUA

$$x(t) = 3 \cos 1000\pi t + 5 \sin 2000\pi t + 10 \cos 11000\pi t$$

5/

HACIENDO $t = n/F_s$ $\rightarrow F_s = 5000$

$$x_1[n] = 3 \cos \left[1000\pi \frac{n}{5000} \right]$$

$$x_1[n] = 3 \cos \left[\frac{\pi}{5} n \right] \rightarrow \boxed{\Omega_1 = \frac{\pi}{5}}$$

$$x_2[n] = 5 \sin \left[2000\pi \frac{n}{5000} \right]$$

$$x_2[n] = 5 \sin \left[\frac{2\pi}{5} n \right] \rightarrow \boxed{\Omega_2 = \frac{2\pi}{5}}$$

$$x_3[n] = 10 \cos \left[11000\pi \frac{n}{5000} \right]$$

$$x_3[n] = 10 \cos \left[\frac{11\pi}{5} n \right] \rightarrow \boxed{\Omega_3 = \frac{11\pi}{5} \rightarrow \text{COPIA}}$$

- como $\Omega_3 > \pi$, HAY QUE HANAR LA ORIGINAL

$$\Omega_{\text{OR3}} = \Omega_3 - 2\pi = \frac{\pi}{5}$$

- REEMPLAZANDO

$$x[n] = 3 \cos \left[\frac{\pi}{5} n \right] + 5 \sin \left[\frac{2\pi}{5} n \right] + 10 \cos \left[\frac{\pi}{5} n \right]$$

$$\therefore x[n] = 13 \cos \left[\frac{\pi}{5} n \right] + 5 \sin \left[\frac{2\pi}{5} n \right]$$

- COMO SE GENERA ALIAS, APLICAMOS NYQUIST PARA UNA DISCRETIZACION APROPIADA

$$F_s' \geq 2 F_{\text{MAX}} \rightarrow \boxed{F_{\text{MAX}} = \frac{N/5}{2\pi} = \frac{11000\pi}{2\pi} = 5500}$$

$$F_s' \geq 2(5500)$$

$$\boxed{F_s' \geq 11000}$$

Norma

• LA SEÑAL OBTENIDA USANDO LA F_s' ES:

$$x[n] = 3 \cos \left[1000 \pi \frac{n}{11000} \right] + 5 \sin \left[2000 \pi \frac{n}{11000} \right] + 10 \cos \left[4000 \pi \frac{n}{11000} \right]$$

$$\therefore x[n] = 3 \cos \left[\frac{\pi}{11} n \right] + 5 \sin \left[\frac{2\pi}{11} n \right] + 10 \cos \left[\pi n \right]$$

SOLUCION PREGUNTA #3

JUAN JOSE CARDONA H.
C.C 1002 609 346

$$X(t) = 20[\cos(t/3) + \cos(t/4)]$$

$$\frac{\omega_1}{\omega_2} = \frac{1}{3} \div \frac{1}{4} = \frac{4}{3}$$

$$\begin{aligned} f_1 &= \frac{\omega_1}{2\pi} = \frac{1}{6\pi} \\ f_2 &= \frac{\omega_2}{2\pi} = \frac{1}{8\pi} \end{aligned}$$

$$T_1 = 6\pi$$

$$T_2 = 8\pi$$

$$T = \text{MCM}\{6, 8\} = 24$$

$$T = 24\pi$$