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  `https://github.com/julianmak/academic-notes`

  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 3301 :

basic **Data Analysis** in ocean sciences

Session 10: fun with maps

Outline

(Just overview here; for actual content see Jupyter notebooks)

- actual fun with maps through `cartopy`

  → map projections

  examples with GEBCO and WOA13 data
- interpolation/extrapolation via an example
- Empirical Orthogonal Functions (EOFs)

  → basically PCA but with a spatial component (cf. 04 linear regression)

Digression: some geometry and topology

- location depends on choice of co-ordinates
- for the point $\boldsymbol{x} = (1, 1)$ in standard Cartesian co-ordinates, we could have

$$x = 1, \qquad y = 1$$

- but we could also have it in polar co-ordinates

$$r = \sqrt{2}, \qquad \theta = \pi/4$$

with $x = r\cos\theta$, $y = r\sin\theta$

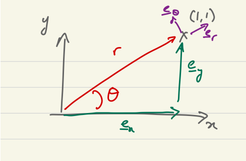

Figure: Polar co-ordinates schematic.

- some things more natural in polar co-ordinates
  $\rightarrow$ unit circle is the set of points with $r = 1$ and $\theta \in [0, 2\pi)$, compared to $y = \pm\sqrt{1 - x^2}$ for $x \in [-1, 1]$

Digression: some geometry and topology

- similarly, on a sphere, it is easier to consider spherical co-ordinates of (r, θ, ϕ) for radius, longitude and latitude

  $\rightarrow$ if on surface of Earth, take $r = R_{\mathrm{Earth}}$, so now 2d data

  $\rightarrow$ otherwise need a tuple (x, y, z) (raw satellite data do this actually)
- represent (lon, lat) data on 2d plane?

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- represent (lon, lat) data on 2d plane?

!! problem: sphere is intrinsically 'curved' while a finite plane is intrinsically 'flat'

$\rightarrow$ unit sphere has Gaussian curvature $\kappa = 1$, while plane has $\kappa = 0$

$\rightarrow$ Gauss–Bonnet theorem tells you κ is related to the Euler characteristic χ (cf. think of χ as the sum of angles of a triangle on the surface)

geometry ('local', from κ) $\leftrightarrow$ **topology** ('global', from χ)

Digression: some geometry and topology

cursed cow

cursed doughnut

- topology studies global properties

  → e.g. how things are connected

  → cow is smoothly deformable into sphere

  → cup is smoothly deformable into doughnut

- cow is **not** smoothly deformable into doughnut

  → difference in genus (cf. 'holes')

Cartopy and map projections

- to go from sphere to plane, need to introduce a 'tear'

  (actually just removing one point will do it)

- main upshot: no global isometry between sphere and plane

  $\rightarrow$ i.e. **no distance preserving transformation possible**

Cartopy and map projections

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A map projection assigns (lon, lat) to some (x, y) on the plane via some formula, but implication from above is that we can at best, for a global map projection:

1. preserve angles (conformal map)
2. preserve areas (area-preserving map)
3. do neither

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`cartopy` package does most of the heavy lifting for you

Cartopy and map projections

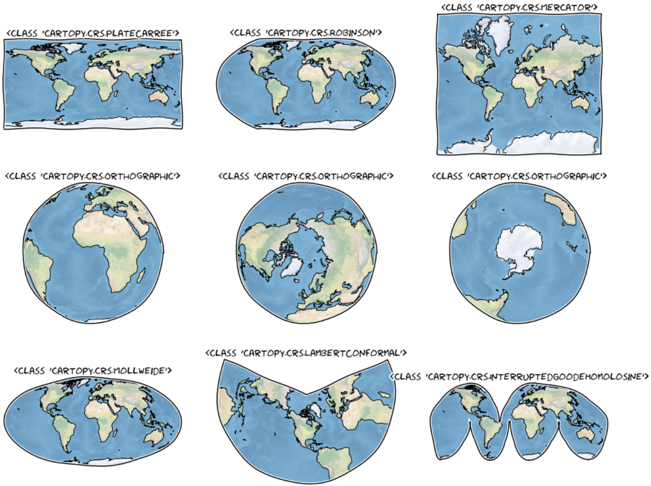


Figure: Sample projections available in `cartopy`. See notebook for `cartopy` usage and syntax.

Cartopy examples

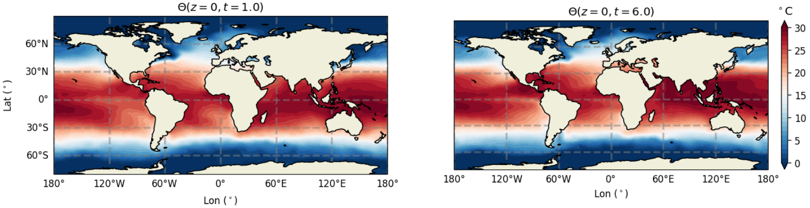


Figure: WOA13 data.

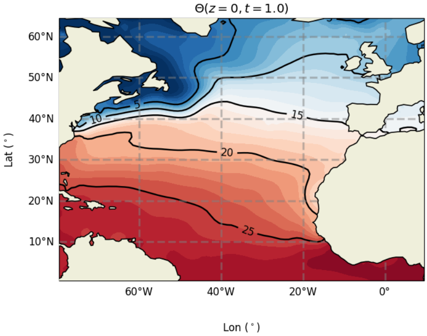
- WOA13 surface temperature from last time in Jan and Jul

  $\rightarrow$ xarray to read

  $\rightarrow$ cartopy Plate Carree projection

  $\rightarrow$ land features from cartopy

Cartopy examples



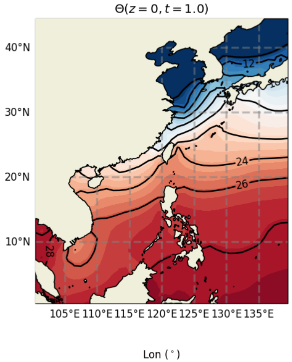


Figure: WOA13 data with zooms.

- WOA13 surface temperature centered over two different regions
  $\rightarrow$ xarray to read and subset
  $\rightarrow$ contours overlaid onto the colour plots

Cartopy examples

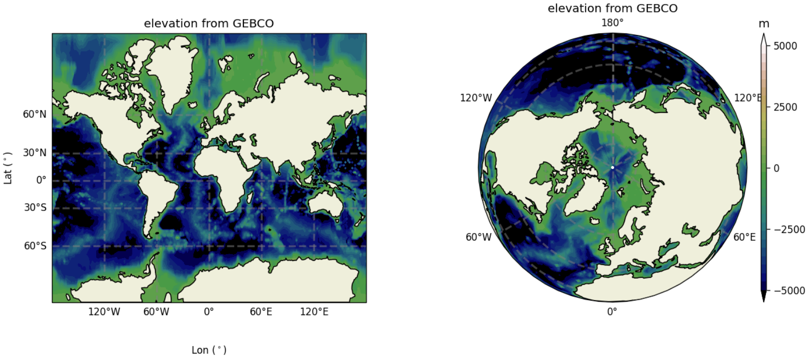


Figure: WOA13 data with zoms.

- GEBCO data using different projections
  → Mercator (conformal but really messes area up)
  → Northern Orthographic (does not show everything)

Interpolation/extrapolation (cf. 08)

- theory basically as before
  - → more caveats when more dimensions are involved
  - → mostly focus on the use, using `scipy.interpolate`
- demonstrative example from the paper below

Figure: ISME paper and steps to reproduce bits of Fig 1a.

Interpolation/extrapolation (cf. 08)

- want the SSS (sea surface salinity)

  $\rightarrow$ satellite products tend to be too coarse ($> 1/2^\circ$ horizontal resolution), and issues near coast

  $\rightarrow$ model output is finer, but other issues

- use the HYCOM dataset here

  $\rightarrow$ global calculation, data assimilated, around $1/12^\circ$ horizontal resolution

  $\rightarrow$ only need the greater bay area data

  $\rightarrow$ only need the month of June (grab all snapshots and time average)

Interpolation/extrapolation (cf. 08)

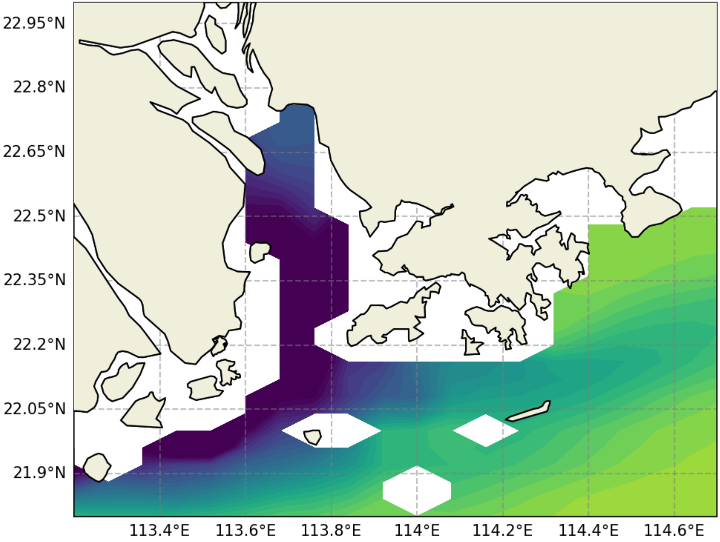
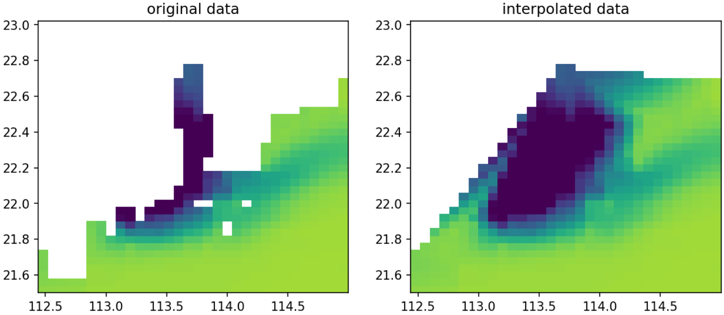


Figure: HYCOM SSS raw output time-averaged over June 2020, with Cartopy land features overlaid. Plot using `contourf`.

Interpolation/extrapolation (cf. 08)

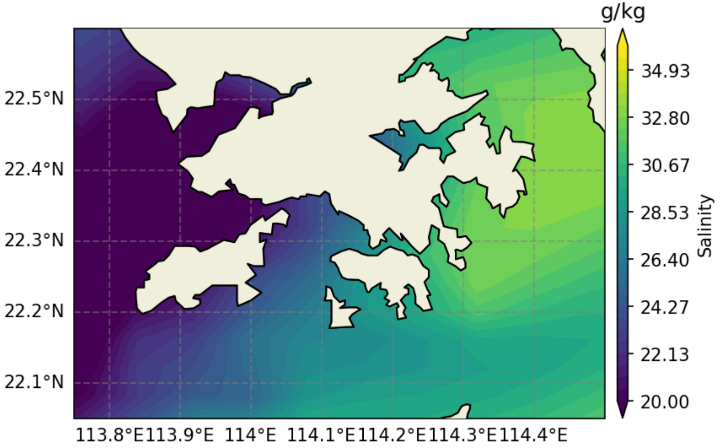


(left) raw plot as pixels (`pcolors`), masked places are land points

(right) interpolated data (used `CloughTocher2DInterpolator`)

Q. only some points are filled, what is the condition for a valid inteprolation?

Interpolation/extrapolation (cf. 08)



- pass to caropy, overlay land features, format accordingly
  → probably fine here as a visual schematic

EOFs

Want to do something like

$$f(t,x,y) = \sum_{k=1}^{N} \mathrm{PC}_k(t)\,\mathrm{EOF}_k(x,y)$$

to pick out spatial patterns that capture the most variability

- sound familiar?

EOFs

Want to do something like

$$f(t,x,y) = \sum_{k=1}^{N} \mathrm{PC}_k(t)\, \mathrm{EOF}_k(x,y)$$

to pick out spatial patterns that capture the most variability

- sound familiar? basically like PCA! (04 regression)

  → $\mathrm{EOF}_k(x,y)$ the Empirical Orthogonal Function (EOF)

  → EOF tagged with a Principal Component $\mathrm{PC}_k(t)$
- algorithm and methodology largely the same

  → actually going to use the Singular Value Decomposition (SVD cf. diagonalisation of covariance matrix for PCA)

  → going to leverage `scikit-learn` package again

- EOF analysis finds you a spatial basis via data (one could choose it in advance also; cf. Fourier analysis)

EOFs: work flow

- start with array containing $f(t, x, y)$, flatten into $f(t, \text{space})$
  $\rightarrow$ "space" is now the categorisation, and t contains the data points
  $\rightarrow$ cf. Iris sepal length vs. entries of Iris sepal length etc.

- preprocessing, but a few choices:
  $\rightarrow$ de-mean, de-trend, Z-score standard scaling, others...

- throw into PCA algorithm (or your SVD algorithm if you want)
  $\rightarrow$ unflatten the resulting EOF(space) back to EOF(x, y)
  $\rightarrow$ EOFs ranked in variance explained

EOFs: idealised example

Try first for (cf. data made for animation)

$$f(x, y) = \sin(x)\cos(y)\sin(t) + \frac{1}{2}\cos(2x)\cos(2t)$$

- 1st term as is, circular blobs
- 2nd term are ‘rolls’
- power spectrum would definitively pick out two peaks

  (which ones? ignoring isolated values of t where individual parts of the function vanish)

Q. what would the EOF analysis do?

EOFs: idealised example

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Q. what would the EOF analysis do?

$\rightarrow$ EOF_1 to be the 1st term, with $\mathrm{PC}_1 \sim \sin(t)$?

(because of imposed larger amplitude)

$\rightarrow$ EOF_2 to be the 2nd term, with $\mathrm{PC}_2 \sim \cos(2t)$?

EOFs: idealised example, no preprocessing

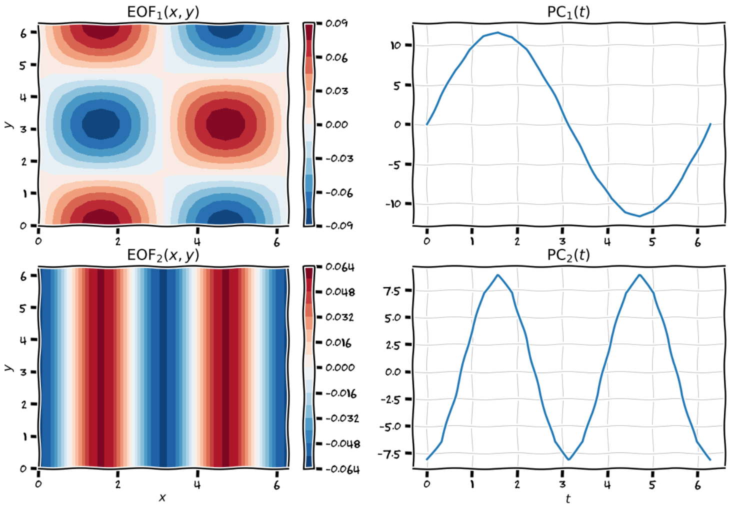

Figure: EOF of idealised data, no preprocessing.

EOFs: idealised example, demean

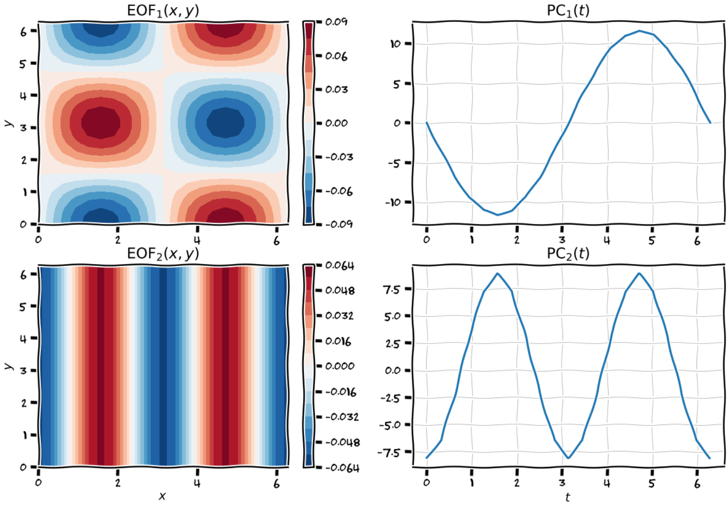

Figure: EOF of idealised data, remove time average per point. Notice a sign flip of EOF and PC 1, but that's ok.

EOFs: idealised example, detrend

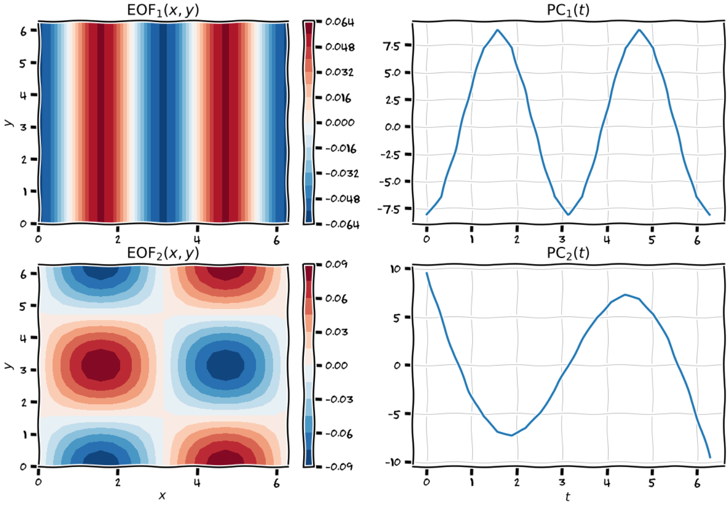

Figure: EOF of idealised data, remove linear trend per point (`scipy.signal.detrend` or use `numpy.polyfit`). Notice 'swapping' of EOF ordering relative to previous case, and PC 2 looks a bit weird.

EOFs: idealised example, Z-score standardisation

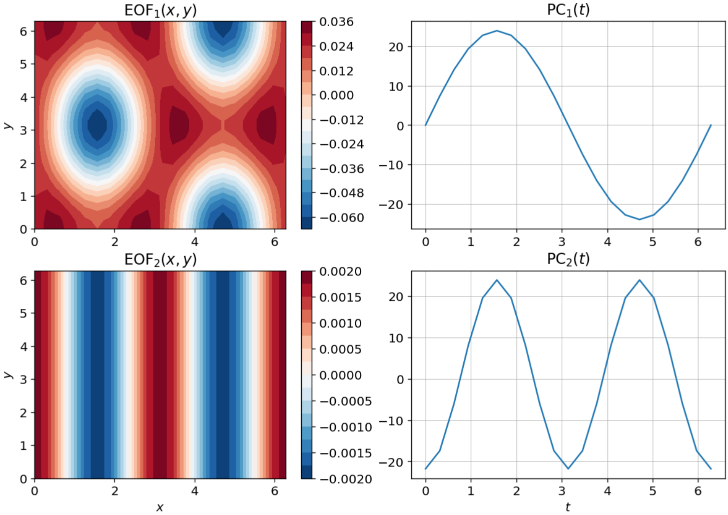

Figure: EOF of idealised data, using Z-score standardisation (`StandardScaler` in `scikit-learn`). Note that while EOF 1 no longer looks like circular blobs, and EOF 2 sign has changed, these are still valid choices of basis.

EOFs: 'real' example

- provided Extended Reconstructed SST data (full and anomaly version)

  → monthly, from mid 1800s to present day

  → global, 2° spatial resolution

  → already masked

  → anomaly relative to some climatology (!?)

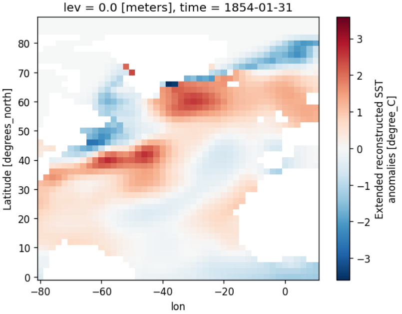

Figure: Raw data plot of SST **anomalies** over Atlantic. Using the anomalies file directly.

EOFs: ‘real’ example

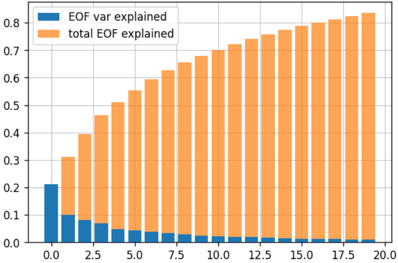

Figure: Percentage and cumulative percentage of variance explained by EOFs.

- EOF analysis as usual

  $\rightarrow$ using anomalies file directly, not detrending or demeaning here
- variance explained of EOFs are generally pretty low actually...

EOFs: ‘real’ example

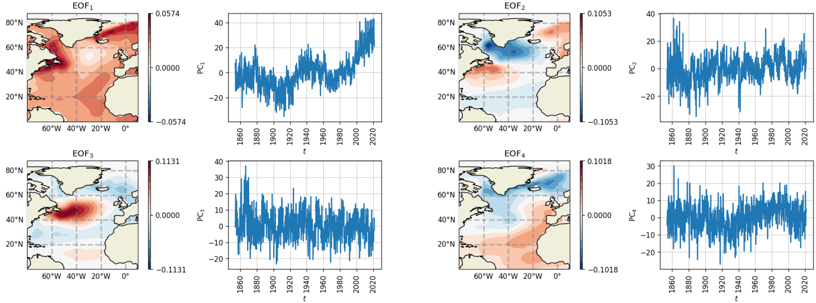

Figure: EOFs. Probably (EOF1) global warming with some Atlantic Multi-decadal Variability (AMV), (EOF2) North Atlantic Oscillation (NAO), (EOF3) looks like EOF 2 of Fig. 3 in Buckley et al. 2013, and (EOF4) looks a bit like the AMV?

Jupyter notebook

go to 10 Jupyter notebook to get some code practise

- exercises and code I haven't demonstrated here

  $\rightarrow$ EOFs with different locations and/or different pre-procesing

  $\rightarrow$ other choices of interpolators

  $\rightarrow$ analysis of PCs computing its power spectrum

  $\rightarrow$ …