$$x(t) = e^{-at^2}$$

$$X'(f) = \widehat{(-2i\pi t)e^{-at^2}}(f)$$

$$= \int_{\mathbb{R}} -2i\pi t \, e^{-at^2} e^{-2i\pi f t} \, dt$$

$$\frac{1}{a}\left(e^{-at^2}\right)' = -2t \, e^{-at^2}$$

$$= \frac{i\pi}{a} \int_{\mathbb{R}} \left(e^{-at^2}\right)' e^{-2i\pi f t} \, dt$$

$$= \frac{i\pi}{a} \left(\underbrace{\left[e^{-at^2} e^{-2i\pi f t} \right]_{-\infty}^{+\infty}}_{0} - \int_{\mathbb{R}} e^{-at^2} (-2i\pi f) e^{-2i\pi f t} \, dt \right)$$

$$= \frac{i\pi}{a} \, 2i\pi f \int_{\mathbb{R}} e^{-at^2} e^{-2i\pi f t} \, dt = -\frac{2\pi^2}{a} f \, X(f)$$

$$\boxed{X'(f) = -\frac{2\pi^2}{a} f\, X(f)}$$

$$X(f) = k\, e^{-\frac{2\pi^2}{2} f}$$

$$\frac{X'(f)}{X(f)} = -\frac{2\pi^2}{a} f$$

$$\ln X(f) = -\frac{\pi^2}{a} f^2 + C$$

$$X(f) = K\, e^{-\frac{\pi^2}{a} f^2}$$

$f=0$: $X(0) = K$

$$X(0) = \int_{\mathbb{R}} e^{-at^2}\, dt = \sqrt{\frac{\pi}{a}} \longrightarrow K = \sqrt{\frac{\pi}{a}}$$

$$X(f) = \sqrt{\frac{\pi}{a}}\, e^{-\frac{\pi^2}{a} f^2},$$

ex: $a = \pi$. $x(t) = e^{-\pi t^2}$

$$X(f) = e^{-\pi f^2}$$

$$\boxed{x''(t) - 4\pi^2 t^2 x(t) = 4\pi x(t)}$$

$x \in L^1(\mathbb{R})$, $x \in C^2$, $x', x'' \in L^1$, $tx(t) \in L^1$, $t^2 x(t) \in L^1$

$$\underbrace{\widehat{x''(t)}(f)} - \widehat{4\pi^2 t^2 x(t)}(f) = 4\pi \widehat{x(t)}(f)$$

$$(2j\pi f)^2 \hat{x}(f) + \underbrace{\widehat{(-2j\pi t)^2 x(t)}(f)}_{\hat{x}''(f)} = 4\pi \widehat{x(t)}(f)$$

$$\boxed{(\hat{x})''(f) - 4\pi^2 f^2 \hat{x}(f) = 4\pi \hat{x}(f)}$$

$$\int_{\mathbb{R}} \left(\frac{\sin t}{t}\right)^3 dt = \int \underbrace{\frac{\sin t}{t}}_{\hat{x}(t)} \underbrace{\left(\frac{\sin t}{t}\right)^2}_{\hat{y}(t)} dt$$

$$= \langle \hat{x}, \hat{y} \rangle_{L^2} = \langle x, y \rangle_{L^2} = \int_{\mathbb{R}} \pi^2 \, \Pi_{\frac{1}{\pi}}(t) \, \Lambda_{\frac{1}{\pi}}(t) \, dt$$

$$\frac{1}{T} \widehat{\Pi_T}(f) = \cancel{T} \operatorname{sinc}(\pi T f) = \operatorname{sinc}(f) \quad \text{si } T = \frac{1}{\pi}$$

- $\hat{x}(t) = \dfrac{\sin t}{t} \notin L^1 \longrightarrow x(t) = \cancel{\pi \sin} \; \pi \, \check{\Pi}_{\frac{1}{\pi}}(t)$

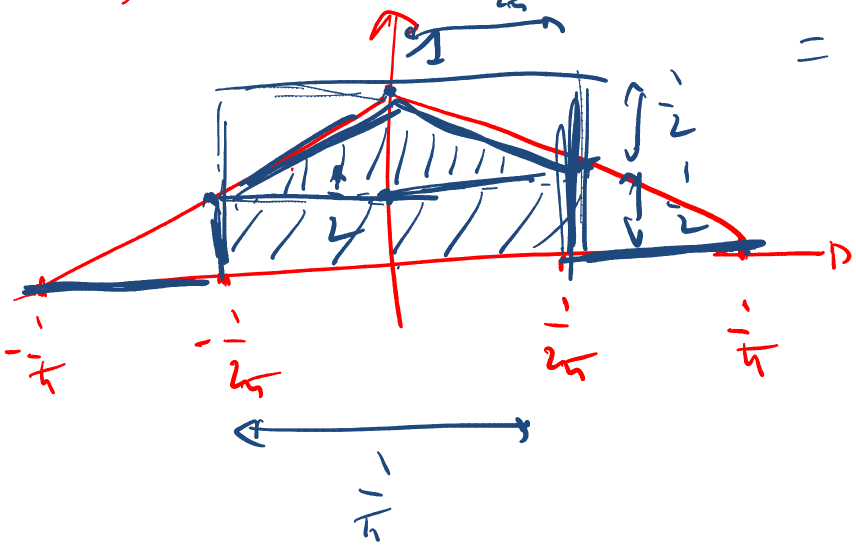
- $\hat{y}(t) = \left(\dfrac{\sin t}{t}\right)^2 \longrightarrow y(t) = \cancel{\pi} \; \pi \, \Lambda_{\frac{1}{\pi}}(f)$

$$\frac{1}{T} \widehat{\Lambda}_T(f) = \cancel{T} \operatorname{sinc}^2(\pi T f) \quad , \quad T = \frac{1}{\pi}$$

$$\tau^2 \int_{\mathbb{R}} \Pi_{\frac{1}{\tau}}(t)\, \Lambda_{\frac{1}{\tau}}(t)\, dt = \tau^2 \left(\frac{1}{2} \times \frac{1}{\tau} + \frac{1}{2} \frac{1}{2\tau} \right)$$

$$= \frac{\tau^2}{\tau} \left(\frac{1}{2} + \frac{1}{4} \right)$$

$$= \frac{3\pi}{4}$$



$$x * y(\theta) = \int_{-\infty}^{\infty} \cancel{\frac{\sin \pi a u}{\pi u} \frac{\sin \pi b(\theta - u)}{\pi(\theta - u)} du} = \int \frac{\sin \pi a u}{\pi u} \frac{\sin \pi b u}{\pi u} du$$

$$= c = \min(a, b)$$

$$x(t) = \frac{\sin \pi a t}{\pi t}, \quad y(t) = b \frac{\sin \pi b t}{\pi t b} = \widehat{\Pi}_b(t)$$

$$= \widehat{\Pi}_a(t) \qquad = b \operatorname{sin_c}(\pi b t)$$

$$x * y(t) = \widehat{\Pi}_a * \widehat{\Pi}_b(t) = \widehat{\Pi_a \Pi_b}(t), \quad c = \min(a, b)$$

$$= \widehat{\Pi}_c(t) = c \operatorname{sin_c}(\pi c t)$$

$$\boxed{x * y(t) = c \frac{\sin \pi c t}{\pi t c}}$$

$$a = b = \frac{1}{\pi}$$

$$\int \frac{(\sin u)^2}{\pi^2 u^2} = \frac{1}{\pi}$$

$$\Rightarrow \boxed{\int \left(\frac{\sin u}{u}\right)^2 du = \pi}$$

$$RCv'(t) + v(t) = x(t)$$

On suppose : $x, v \in L^1$

$v \in C^1$, $v' \in L^1$

$$\Rightarrow \hat{v}(f) = \ldots$$

$$\widehat{e^{-\lambda t} \mathbb{1}_{\mathbb{R}_+}(t)} = \frac{1}{\lambda + 2j\pi f} \qquad \text{pour } \lambda > 0$$

$$RC\,\widehat{v'}(f) + \hat{v}(f) = \hat{x}(f)$$

$$RC(2j\pi f)\hat{v}(f) + \hat{v}(f) = \hat{x}(f)$$

$$\hat{v}(f)\,(1 + 2j\pi RCf) = \hat{x}(f)$$

$$\hat{v}(f) = \frac{1}{1 + 2j\pi RCf}\,\hat{x}(f) = \frac{1}{RC\left(\frac{1}{RC} + 2j\pi f\right)}\,\hat{x}(f)$$

$$\hat{v}(f) = \widehat{\frac{1}{RC}\, e^{-\frac{t}{RC}}\, \mathbb{1}_{\mathbb{R}^+}(t)}(f)\, \hat{x}(f) = \hat{h}(f)\, \hat{x}(f) = \widehat{h * x}(f)$$

$$h(t) = \frac{1}{RC}\, e^{-\frac{t}{RC}}\, \mathbb{1}_{\mathbb{R}^+}(t) \in L^1(\mathbb{R}) \qquad \text{causale}$$

Injectivité de la T.F. :

$$\underset{\text{continue}}{v}(t) = \underbrace{\underset{\in L^\infty}{h} * \underset{\in L^1}{x}}_{\text{continue}}(t) \qquad \text{p.p.}$$

Donc : $\quad v(t) = h * x(t) \quad \forall t \in \mathbb{R}$

$$x(t) \longrightarrow \boxed{} \longrightarrow v(t)$$