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The educational competition optimizer

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ABSTRACT

In recent research, metaheuristic strategies stand out as powerful tools for complex optimization, capturing widespread attention. This study proposes the Educational Competition Optimizer (ECO), an algorithm created for diverse optimization tasks. ECO draws inspiration from the competitive dynamics observed in real-world educational resource allocation scenarios, harnessing this principle to refine its search process. To further boost its efficiency, the algorithm divides the iterative process into three distinct phases: elementary, middle, and high school. Through this stepwise approach, ECO gradually narrows down the pool of potential solutions, mirroring the gradual competition witnessed within educational systems. This strategic approach ensures a smooth and resourceful transition between ECO's exploration and exploitation phases. The results indicate that ECO attains its peak optimization performance when configured with a population size of 40. Notably, the algorithm's optimization efficacy does not exhibit a strictly linear correlation with population size. To comprehensively evaluate ECO's effectiveness and convergence characteristics, we conducted a rigorous comparative analysis, comparing ECO against nine state-of-the-art metaheuristic algorithms. ECO's remarkable success in efficiently addressing complex optimization problems underscores its potential applicability across diverse real-world domains. The additional resources and open-source code for the proposed ECO can be accessed at https://aliasgharheidari.com/ECO.html and https://github.com/junbolian/ECO.

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1. Introduction

Many industries and sectors encounter a diverse range of optimisation problems. These issues frequently involve complications like non-linear characteristics, discontinuities, uncertainties, large-scale dimensions, multiple objectives, and non-convex shapes (Li & Sun, 2021). This underscores the imperative for advancing more dependable optimisation methodologies, mainly focusing on metaheuristic optimisation algorithms (Faris et al., 2018; Yang, 2013). These methodologies exhibit stochastic characteristics and can approximate optimal solutions across diverse optimisation problems (Kumar et al., 2014; Wang et al., 2020). Significantly, the superiority of metaheuristic optimisation algorithms compared to traditional ones is credited to their lack of reliance on gradient information and proficiency in bypassing local optima (Zhang et al., 2018; Zhao et al., 2018).

Optimisation scenarios may encompass numerous objective functions, addressing multiple criteria simultaneously or a single objective aimed at maximising or minimising a specific performance indicator (Zhang et al., 2023). In multi-objective optimisation, the exploration of Pareto optimal solutions is crucial for effectively balancing competing objectives (Cao et al., 2019; Cao et al., 2020). Conversely, single-objective optimisation concentrates on swiftly identifying the global optimum within the solution space, typically employing algorithms such as gradient-based techniques or metaheuristic

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approaches to attain optimal results (Cao et al., 2019; Duan et al., 2023). Single objective metaheuristic algorithms generally employ two significant search strategies: (i) exploration/diversification and (ii) exploitation/reinforcement. Exploration refers to the ability to explore the search space globally, avoiding local optimality and overcoming local optima traps (Xie et al., 2023). Conversely, exploitation involves exploring nearby promising solutions to improve their local quality (Mou et al., 2023). Achieving superior performance with an algorithm requires a delicate balance between these two strategies (Gholizadeh et al., 2020; Tang et al., 2015). Compared to traditional methods (Lyu et al., 2024), a fundamental characteristic of population-based algorithms is that they employ simple search and exploitation strategies. However, optimisation performance can significantly differ among algorithms that employ distinct operators and mechanisms when confronted with diverse problem scenarios (Sun et al., 2019).

A widely accepted classification of metaheuristic algorithms delineates them into four distinct classes: evolutionary algorithms, swarm intelligence algorithms, physics-inspired methodologies, and human-derived approaches (Moghdani & Salimifard, 2018). Evolutionary algorithms imitate natural evolutionary processes and adopt operators inspired by biological behaviour, such as crossover and mutation. A well-known example of this class of algorithms is the genetic algorithms (GA), which draws inspiration from Darwinian evolutionary principles. Other traditional approaches within this category include evolutionary programming (Fogel, 1998), ant colony optimiser (ACO) (Sun et al., 2018), liver cancer algorithm (Houssein et al., 2023), differential evolution (Storn & Price, 1997), and evolutionary strategies (Hansen et al., 2003).

Swarm intelligence algorithms represent an additional category of metaheuristic algorithms, which simulate the collective behaviour observed in animal herds or hunting packs (Shadravan et al., 2019). The defining characteristic of these algorithms lies in exchanging information among all group members throughout the optimisation process (Li et al., 2023). Notable methods within this category include the RIME algorithm (Su et al., 2023), colony predation algorithm (Tu et al., 2021), Harris Hawks optimisation (Heidari et al., 2019), slime mould algorithm (Chen et al., 2022; Li et al., 2020), whale optimisation algorithm (Mirjalili & Lewis, 2016), weighted mean of vectors (Ahmadianfar et al., 2022), parrot optimiser (Lian et al., 2024), and hunger games search (Yang et al., 2021).

Physics-based methods form a distinct category of optimisation algorithms inspired by the principles of real-world physical laws. These algorithms model the interaction of search solutions through control rules that are anchored in physical processes. Among the notable algorithms in this category are the gravitational search algorithm (Rashedi et al., 2009), multi-verse optimiser (Mirjalili et al., 2016), and charged system search (Kaveh & Talatahari, 2010).

The final category of optimisation techniques includes human-inspired methods, drawing from principles of human cooperation and collective behaviour. One frequently employed algorithm in this group is the social cognitive optimisation (Zhu et al., 2023), imperialist competition algorithm (Atashpaz-Gargari & Lucas, 2007), motivated by human sociopolitical growth practices. Another algorithm within this group is the human evolutionary optimisation algorithm (Lian & Hui, 2024).

While each algorithm contributes importantly to metaheuristic optimisation, they also present specific limitations, which can be summarised as follows:

- **Balancing Exploration and Exploitation**: Metaheuristic algorithms typically involve two main phases: exploration and exploitation. The exploration phase is marked by high randomness, efficient updates, and variable solution quality across iterations. On the other hand, the exploitation phase features algorithmic stability, slower update rates, and consistent solution quality. Achieving the optimal balance between these phases is crucial for maximising the algorithm's overall performance (Xiao et al., 2022).
- **Parameter Sensitivity**: Parameters play a crucial role in the optimisation effectiveness of many algorithms, and identifying the ideal parameters for a specific optimisation challenge can be difficult. The absence of qualitative analysis and consideration of parameter sensitivity in newly introduced algorithms makes the task of effectively addressing complex problems more challenging.
- **Focus on Novelty vs. Computational Performance**: Certain algorithms prioritise novelty by introducing new metaphors without sufficiently

emphasising the computational performance advantages for effectively solving complex problems (Su et al., 2023). This approach can lead to inefficiencies. Additionally, when these algorithms are only tested on a narrow range of problems or a limited set of test cases, they may exhibit high algorithmic complexity and low compatibility. Consequently, these algorithms may not perform effectively when applied to other types of problems, yielding suboptimal results.

Researchers typically do not rely on a single algorithm due to the No Free Lunch theorem (Wolpert & Macready, 1997), which asserts that no single algorithm can effectively address all optimisation problems. Hence, it becomes imperative to consider adopting or proposing adjustments to existing algorithms, or even introducing novel approaches, to better tackle present scenarios or provide solutions to evolving challenges. This motivation underpins our proposal of an effective optimisation method, the educational competition optimiser.

ECO is an innovative metaheuristic algorithm that draws inspiration from competitive dynamics observed in real-world scenarios of educational resource allocation. It leverages this principle to enhance its search process. The algorithm comprises three phases: elementary, middle, and high school. As an effective human-based optimisation model, ECO utilises an innovative roulette-like structure that iteratively cycles through three distinct phases. This step-by-step approach progressively narrows the range of potential solutions, mirroring the gradual competition within the education system. While ensuring population quality, ECO effectively enhances population diversity and strives to avoid local optima.

In our experiments, we conducted parameter sensitivity analyses and qualitative experiments to elucidate the characteristics and adaptability of the ECO algorithm. We discussed the algorithm's performance under various parameters and when applied to different problem domains. Additionally, to assess the algorithm comprehensively, we compared and tested it against nine highly cited primitive metaheuristics using the 23 classical benchmark functions (Yao et al., 1999) and 10 CEC2021 test functions (Mohamed et al., 2021) test datasets. Furthermore, we verified the algorithm's capacity to solve real-world problems by applying it to five classical engineering optimisation problems.

In summary, this paper contributes in the following ways:

(1) This research introduces the Educational Competition Optimizer (ECO), an education-inspired meta-heuristic algorithm.
(2) This work constructs new exploration, exploitation, and selection mechanisms within ECO, which can be applied to enhance other peer-to-peer algorithms.
(3) This paper provides detailed insights into the characteristics of the ECO algorithm through parameter sensitivity experiments and qualitative analysis, facilitating its application to various optimisation problems.
(4) This paper validates the algorithm's performance through comparative experiments involving nine famous algorithms. The results demonstrate that ECO either outperforms or shows comparable performance to these algorithms across various problem types.
(5) Demonstrates the applicability of the ECO algorithm to several real-world engineering optimisation problems, establishing its potential for addressing diverse optimisation challenges.

The rest of the paper is organised as follows: Section 2 provides a detailed explanation of our proposed ECO method. Section 3 presents the outcomes of experiments performed on several benchmark functions and real-life issues. Finally, Section 4 concludes the paper and suggests directions for future research.

2. The educational competition optimiser (ECO)

This section elucidates the overall background of ECO and formulates the optimisation models.

2.1. Inspiration

Competition in education has become a prominent and contentious issue in contemporary society. As students continuously strive to enhance their abilities and fulfil the stringent admission criteria of educational institutions, the pursuit of higher education has become a relentless quest (Merten, 1997; Rich & DeVitis, 1992; Wu, 2008; Zhou et al., 2022). This pursuit mirrors a fundamental aspect of optimisation problems: navigating a vast and complex search space to find the optimal solution. As the level of education rises, the intensity of educational competition increases accordingly. The ECO algorithm continuously retains the elite by simulating this competitive advancement, aligning with the principles of greedy selection and balanced exploration and exploitation in optimisation algorithms. This approach not only justifies the methodology but also validates the algorithm's design.

Drawing inspiration from this educational competition, the concept of the educational competition optimiser emerged. This innovative approach offers a fresh perspective on metaheuristic algorithms by metaphorically connecting education and optimisation. Consequently, it opens new avenues for devising improved strategies to tackle demanding real-world challenges.

In the primary school stage, characterised by $t \equiv 1(mod3)$, schools select their optimal educational locations based on the population's average location. Students, in turn, compete by aiming for the closest school as their target (approach). In the middle school stage, when $t \equiv 2(mod3)$, the number of schools decreases, prompting schools to consider the best educational location, factoring in both the population's mean position and the best position. Students continue to compete for the nearest school (proximity). Finally, in the high school stage, when $t \equiv 0(mod3)$, schools exercise more careful consideration. They now consider the population's mean, best, and worst positions to determine their educational location. With only one school as their option, students strive to compete for this singular goal (proximity).

2.2. Population initialisation

Given that the absence of education leads to societal chaos, we employ logistic chaos mapping to simulate this phenomenon. The initialisation formula for logistic chaos mapping, taking into account a population size of N, maximum iterations of Max_{iter}, and search bound of lb (lower bound) and ub (upper bound), can be shown as:

$$x_i = \alpha \cdot x_{i-1} \cdot (1 - x_{i-1}), 0 \leq x_0 \leq 1,$$
$$i = 1, 2, \cdots, N, \alpha = 4 \tag{1}$$

where x_i represents the i^{th} iteration value and x_{i-1} represents the previous iteration value. Map the chaotic value, x_i, to the search space:

$$X_i = lb + (ub - lb) \cdot x_i \tag{2}$$

2.3. Mathematical model of ECO

The ECO algorithm is designed to simulate the dynamics of educational competition, capturing the varying competitive strategies witnessed at different stages: primary school stages, middle school stages, and high school stages. As the competitive pressure heightens and the number of available schools decreases, the optimisation process of ECO can be outlined in three steps. By adhering to these conditions, the ECO algorithm smoothly transitions from the exploration step to the exploitation step, relying on an enriched search strategy. We mathematically model the educational competition process as an optimisation paradigm to identify the best solution while adhering to specific constraints. The mathematical model of ECO is proposed as follows.

2.3.1. Stage 1: primary school stage

During the elementary grades, schools determine suitable teaching locations by considering the average location of the population. On the other hand, students set their individual goals based on the proximity of their neighbourhood school. At each iteration, the top 20% of the population, ranked based on their fitness, is categorised as schools, while the remaining 80% constitutes the students. It is important to note that this assignment of roles to individuals such as schools or students can change dynamically throughout the iterations. w is the adaptive step size. Figure 1 visually illustrates the behavioural strategies both schools and students adopt at the primary school stage. Primary school students often opt for schools near their residences, considering factors like safety and convenience. In turn, educational institutions often adapt their locations to accommodate the average proximity of their student body, facilitating accessibility and attendance. The mathematical representation of this behaviour is denoted by Eq. (3) and Eq. (4).

$$Schools: X_i^{t+1} = X_i^t + w \cdot (X_{imean}^t - X_i^t) \cdot Levy(dim) \tag{3}$$

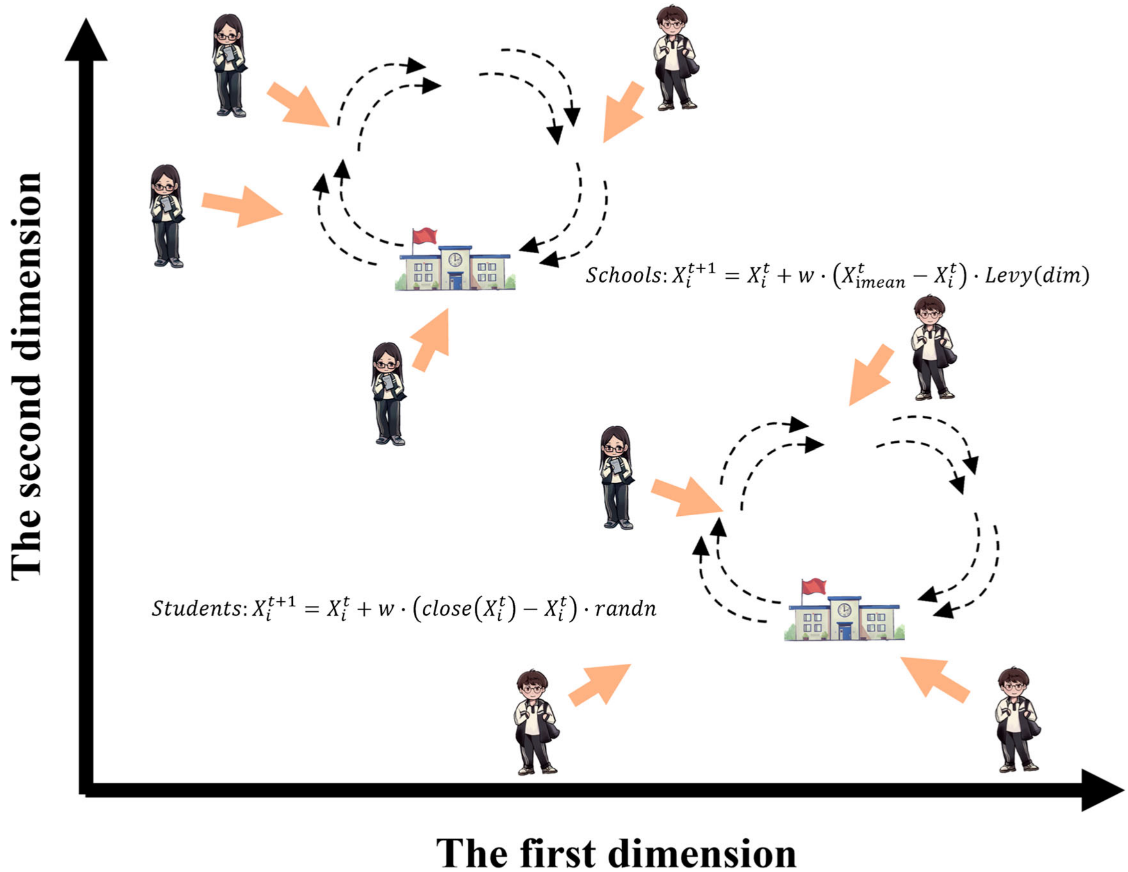

Figure 1. The behaviour at the primary school stage.

$$Students: X_i^{t+1} = X_i^t + w \cdot (close(X_i^t) - X_i^t) \cdot randn \tag{4}$$

$$w = 0.1ln\left(2 - \frac{t}{Max_{iter}}\right) \tag{5}$$

In Eq. (3) and Eq. (4), X_i^t denotes the current position, while X_i^{t+1} signifies the position of the subsequent update. X_{imean}^t represents the average position of each element of the vector for the ith school in the tth round of iteration, and $Levy(D)$ denotes the Levy distribution. $close(X)$ indicates the location of the school closest to X. $Randn$ represents a random variable following a normal distribution. The pertinent parameters and functions can be further elucidated as follows:

Average vector position X_{imean}^t **& Average position** X_{mean}^t: X_{imean}^t represents the average position of each element of the vector for the ith school in the tth round of iteration. X_{mean}^t denotes the average position of the current swarm, denoted as X_{mean}^t. They are calculated as shown in Eq. (6). Where X_{kt} denotes the kth element in the vector X_i^t.

$$\begin{cases} X_{imean}^t = \dfrac{1}{dim}\left(\displaystyle\sum_{k=1}^{dim} X_t^i(k)\right)_{1\times dim} \\ X_{mean}^t = \dfrac{1}{N}\displaystyle\sum_{k=1}^{N} X_k^t \end{cases} \tag{6}$$

Levy distribution: The rule for the Levy distribution is represented in Eq. (7), where γ is assigned the value of 1.5.

$$\begin{cases} Levy(dim) = \dfrac{\mu \cdot \sigma}{|\nu|^{\frac{1}{\gamma}}} \\ \mu \sim N(0, dim) \\ \nu \sim N(0, dim) \\ \sigma = \left(\dfrac{\Gamma(1+\gamma) \cdot \sin\left(\frac{\pi\gamma}{2}\right)}{\Gamma\left(\frac{1+\gamma}{2}\right) \cdot \gamma \cdot 2^{\frac{1+\gamma}{2}}}\right)^{\gamma+1} \end{cases} \tag{7}$$

2.3.2. Stage 2: middle school stage

Schools adopt a more sophisticated approach to choosing their teaching location during the middle school

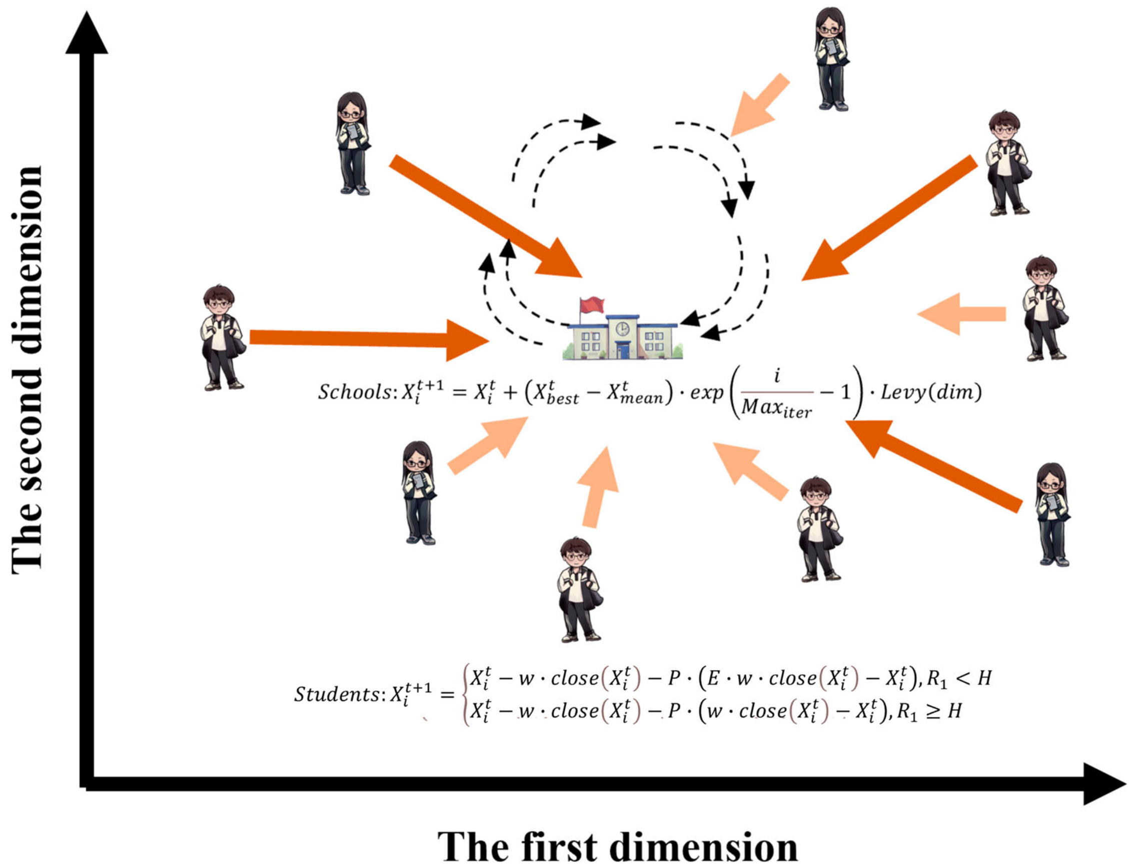

Figure 2. The behaviour at the middle school stage.

stage. They consider a combination of the average and optimal population locations. Similarly, students at this level set their personal goals based on the proximity of neighbouring schools. In each iteration, the top 10% of the population, ranked by their fitness, takes on the role of schools, while the remaining 90% constitutes students.

As middle school academic pressure gradually increases, students' patience in learning is denoted by P. Students are further categorised into two groups based on whether they are academically gifted or not. The judgmental threshold H is set at 0.5 for this classification. For academically gifted students, their motivation to learn is represented by E, while those who are not academically talented have a fixed motivation value of $E = 1$. w is the adaptive step size. Figure 2 visually presents the behavioural strategies both schools and students adopt at the middle school stage. Like elementary school, the competition among students for better educational resources intensifies. The mathematical representation of these behaviours is expressed by Eq. (8) – Eq. (11).

$$P = 4 \cdot randn \cdot \left(1 - \frac{i}{Max_{iter}}\right) \tag{8}$$

$$E = \frac{\pi}{P} \cdot \frac{i}{Max_{iter}} \tag{9}$$

$$Schools : X_i^{t+1} = X_i^t + (X_{best}^t - X_{mean}^t) \cdot exp\left(\frac{i}{Max_{iter}} - 1\right) \cdot Levy(dim) \tag{10}$$

$$Students : X_i^{t+1} = \begin{cases} X_i^t - w \cdot close(X_i^t) \\ \quad -P \cdot (E \cdot w \cdot close(X_i^t) - X_i^t), & R_1 < H \\ X_i^t - w \cdot close(X_i^t) \\ \quad -P \cdot (w \cdot close(X_i^t) - X_i^t), & R_1 \geq H \end{cases} \tag{11}$$

The talent values of different students are simulated using the random numberR_1, which takes on a value within the range of [0, 1].

2.3.3. Stage 3: high school stage

At the high school level, schools adopt a meticulous approach to selecting their teaching locations. They consider not only the average population location but also the best and worst locations within their population. This comprehensive assessment helps them make informed decisions about their educational location. In contrast, all students converge toward the current best location, which is identified as the best high school location. The optimisation process motivates every student to strive for admission to this best high school. During each iteration, the top 10% of the population, determined by their fitness, are designated schools, while the remaining 90% continue as students. Figure 3 provides a visual representation of the behavioural strategies adopted by both schools and students at the high school level. High schools adapt their locations based on student demographics while students vie for superior educational opportunities, transcending geographical constraints in their pursuit of excellence. Eq. (12) and Eq. (13) represent the mathematical expressions for this behaviour.

$$Schools : X_i^{t+1} = X_i^t + (X_{best}^t - X_i^t) \cdot randn - (X_{best}^t - X_i^t) \cdot randn \tag{12}$$

$$Students : X_i^{t+1} = \begin{cases} X_{best}^t - P \cdot (E \cdot X_{best}^t - X_i^t), & R_2 < H \\ X_{best}^t - P \cdot (X_{best}^t - X_i^t), & R_2 \geq H \end{cases} \tag{13}$$

The talents of individual students are represented by a random number denoted as R_2, which falls within the range of [0, 1].

2.4. Pseudo-code of the ECO algorithm

In ECO, the optimisation process commences with the random generation of a predetermined set of candidate solutions, known as the population. Through iterative trajectories, ECO's search strategy explores regions proximate to the optimal solution or where the best solution has been identified. Each solution dynamically updates its position based on the best solution attained during ECO's optimisation process.

Algorithm 1: Pseudo-code of the ECO algorithm

```
Initialize the ECO parameters
Initialize the solutions' positions randomly (Logistic Chaos Mapping)
For i = 1:Max_iter do
    Calculate the fitness function
    Find the best position and worst position
    Calculate R1, R2, P, E
    For j = 1:N do
        Stage 1: Primary school competition
        If mod(i, 3) == 1 Then
            If j = 1:G1Number Then
                Update schools position by Eq. (3)
            Elseif j = G1Number + 1:N Then
                Update students position by Eq. (4)
            End
        Stage 2: middle school competition
        Elseif mod(i, 3) == 2 Then
            If j = 1:G2Number Then
                Update schools position by Eq. (10)
            Elseif j = G2Number + 1:N Then
                Update students position by Eq. (11)
            End
        Stage 3:High school competition
        Elseif mod(i, 3) == 0 Then
            If j = 1:G2Number Then
                Update schools position by Eq. (12)
            If j = G2Number + 1:N Then
                Update students position by Eq. (13)
            End
        End
        If X_i^{t+1} > X_i^t Then
            Select the optimal solution using the positive greedy
            selection mechanism
        End
    End
    Return the best solution
End
```

ECO places significant emphasis on maintaining a balance between its search strategies: exploration and exploitation. Six distinct exploration and exploitation search strategies are introduced to achieve this balance, involving three phases of interaction between schools and students at different educational levels.

The search process in ECO continues until it meets the predetermined termination criterion. The full architecture of the algorithm is detailed through pseudo-code in Algorithm 1 and illustrated in Figure 4, providing a thorough walkthrough of the entire optimisation process, including its iterative stages and search tactics. ECO leverages the strengths of both exploration and exploitation phases, ensuring a thorough examination of the search space and efficient convergence to optimal solutions.

2.5. Computational complexity of ECO

In this section, we provide an overview of the overall computational complexity associated with the ECO approach. The computational burden of ECO

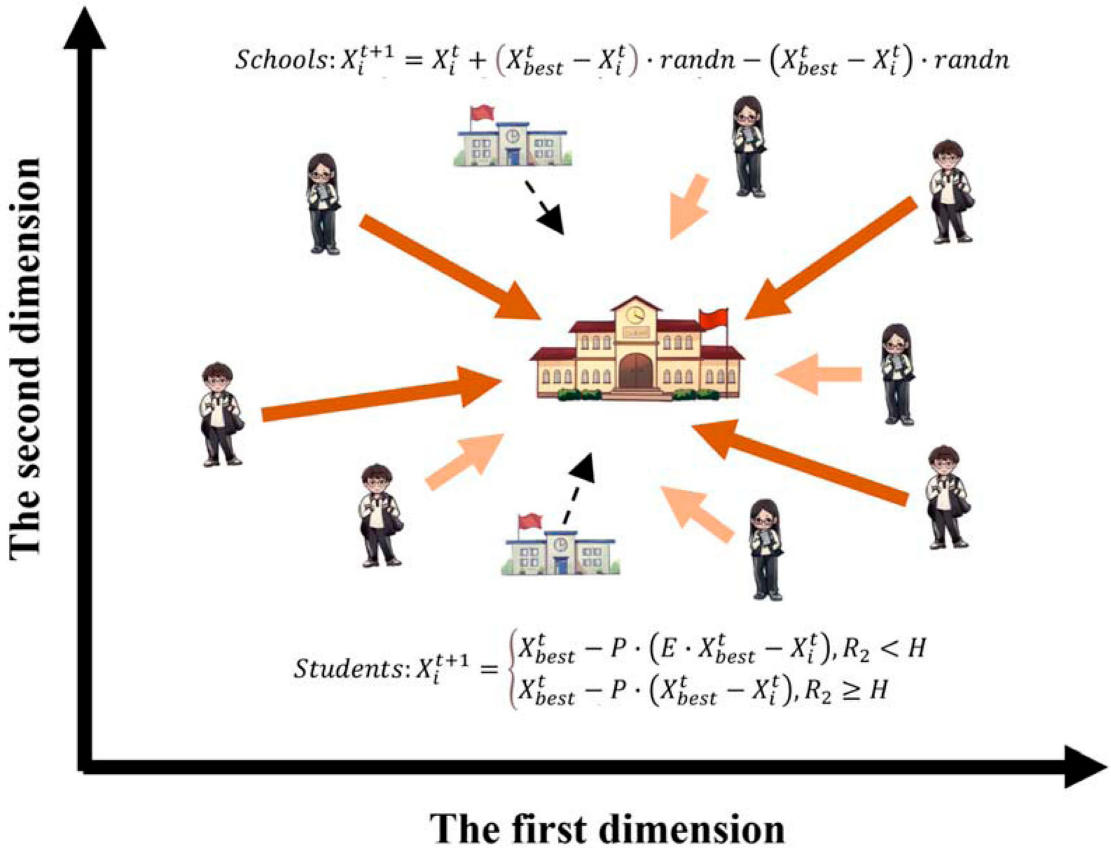

Figure 3. The behaviour at the high school stage.

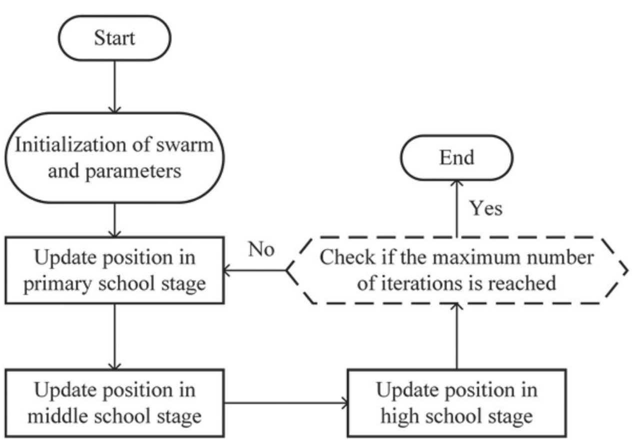

Figure 4. Flowchart of ECO algorithm.

primarily hinges on three key elements: the initialisation of solutions, the computation of fitness functions, and the solution update mechanism. Let us consider N as the count of solutions and $O(N)$ as the computational complexity associated with the initialisation of these solutions. The computational complexity of the updating processes is $O(T \times N) + O(T \times N \times dim) + O(T \times N \times logN)$, which includes searching for the optimal agents and revising the locations of all solutions, where the total number of iterations is called T and the dimension of the given case is called *dim*.

3. Results and discussion

We assess the efficacy of ECO algorithm by subjecting it to rigorous testing across 23 classical benchmark functions (Yao et al., 1999), 10 CEC2021 test functions (Mohamed et al., 2021), and 6 real-world engineering problems spanning various domains. Subsequently, we conduct a comprehensive comparative analysis by juxtaposing the performance results of ECO against those of nine established metaheuristic algorithms documented in the existing literature, including Ant Lion Optimizer (ALO) (Mirjalili, 2015), Grey Wolf Optimizer (GWO) (Faris et al., 2018), Whale Optimization Algorithm (WOA) (Mirjalili & Lewis, 2016), Salp Swarm Algorithm (SSA) (Abualigah et al., 2020), Arithmetic Optimization Algorithm (AOA) (Abualigah et al., 2021), Harris Hawks Optimization (HHO) (Heidari et al., 2019), Sine Cosine Algorithm (SCA)

Table 1. Parameter settings.

Algorithms	Name of parameters	Value of parameters
ALO	I ratio	10
	w	2–6
GWO	Convergence parameter a	Linear reduction from 2 to 0
WOA	α	Decreased from 2 to 0
	b	2
SSA	v_0	0
AOA	α	5
	μ	0.05
HHO	E_0	[−1,1]
SCA	-	-
MVO	WEP_{max}	1
	WEP_{min}	0.2
ROA	-	-

(Mirjalili, 2016), Multi-Verse Optimizer (MVO) (Mirjalili et al., 2016), Remora Optimization Algorithm (ROA) (Jia et al., 2021).

The algorithms utilised and their specific control parameters are outlined in Table 1. We executed these algorithms on both classical and CEC2021 test functions as well as engineering design problems, utilising MATLAB R2023a. For each algorithm, we conducted 30 independent runs. To assess the quality of the obtained solutions, we employed five performance indicators: best, worst, average, standard deviation (STD), and median values. These indicators were used to showcase the outcomes achieved by the ECO approach.

Researchers frequently utilise a set of 23 classical test functions to assess the performance and capabilities of optimisation algorithms. These test functions have been widely employed in numerous optimisation algorithm studies. The 23 classical test functions are categorised into three types: single-peak, multi-peak, and fixed-dimension multi-peak functions. In **Table A1** – **Table A3**, these functions are delineated alongside their specific details, including function types, search ranges, and theoretical optimal values.

To highlight the effectiveness of the ECO algorithm, we specifically chose to evaluate it using the CEC2021 test functions. These functions exhibit a wide array of characteristics, including unimodal, basic, hybrid, and composite functions. For a deeper understanding of these selected functions, detailed information is provided in **Table A4**.

Benchmark datasets serve as widely accepted instruments for assessing the performance of various technologies against established norms (Li & Lin, 2020; Liu et al., 2020). They facilitate the evaluation of distinct computational dimensions, thereby aiding in determining which technology surpasses the rest in multiple fields (Fan et al., 2021; Zhou et al., 2021). By assessing the algorithm's performance across these 33 test functions, we may acquire knowledge regarding how effective it is in tackling optimization challenges.

3.1. Parameter sensitivity analysis

Establishing the optimal population size is paramount for a developed algorithm. In our evaluation of optimisation algorithms, we employed the 23 classical test functions to gauge their performance and capabilities.

Tables 2–4 present a detailed analysis of the search results generated by the ECO algorithm following 500 iterations across different population sizes, namely N = 10, 20, 30, 40, 50, and 60. These tables provide comprehensive insights into the algorithm's performance, showcasing its efficacy in optimising solutions.

Figure 5 depicts the convergence curve displaying the fitness values attained by the ECO algorithm during its quest for optimal solutions spanning from F1 to F23 across a range of population sizes. This graphical representation aids in evaluating and comparing the algorithm's performance across different types of functions and population sizes.

Through our analysis, we observed a positive correlation between the search ability of the ECO algorithm and the population size. The results in Figure 5 can be categorised into four groups based on their impact on the outcomes:

(1) No effect on the results: Cases such as F11, F17 and F19 show minimal variation, indicating that parameter changes have little influence on the algorithm's performance.
(2) Slight effect on the results: F1-F4 demonstrate slight variations due to parameter changes, but the impact is relatively minor.
(3) Large effect on the results: F12 and F15 exhibit significant performance variations due to parameter changes.
(4) Larger populations lead to worse results: F4 and F23 show that increasing the population size can result in poorer algorithm performance.

Most cases fall into the first two categories, indicating the ECO model's strong robustness. It demonstrates stable and reliable performance across various scenarios and parameter settings. Notably, all the test

Table 2. Results of unimodal benchmark functions (different population).

Function	Item	N = 10	N = 20	N = 30	N = 40	N = 50	N = 60
F1	Best	2.10E-77	5.89E-80	1.38E-76	9.69E-92	4.68E-78	8.69E-89
	Median	4.43E-60	1.23E-64	4.76E-64	1.24E-62	1.49E-61	1.42E-67
	Mean	7.41E-44	1.48E-51	5.27E-46	3.77E-49	3.09E-45	1.41E-54
	Worst	2.22E-42	4.39E-50	1.58E-44	1.13E-47	9.28E-44	3.23E-53
	STD	1.58E-85	6.21E-101	8.06E-90	4.12E-96	2.77E-88	3.46E-107
F2	Best	1.22E-39	3.92E-40	1.02E-41	2.01E-45	6.58E-46	4.96E-42
	Median	5.64E-30	1.83E-31	9.71E-32	2.41E-33	2.29E-35	6.08E-34
	Mean	7.68E-24	8.54E-21	2.24E-27	1.33E-27	2.45E-24	2.51E-28
	Worst	1.55E-22	2.56E-19	5.72E-26	2.71E-26	7.34E-23	6.68E-27
	STD	8.79E-46	2.12E-39	1.05E-52	2.43E-53	1.74E-46	1.44E-54
F3	Best	2.53E-74	2.59E-80	1.63E-71	8.41E-83	1.50E-88	4.57E-84
	Median	1.31E-59	6.82E-61	1.30E-60	1.12E-63	4.40E-66	1.66E-66
	Mean	2.97E-44	1.70E-51	8.43E-46	1.72E-52	2.99E-48	5.19E-50
	Worst	7.92E-43	5.10E-50	2.53E-44	5.17E-51	8.76E-47	1.53E-48
	STD	2.04E-86	8.38E-101	2.06E-89	8.61E-103	2.47E-94	7.49E-98
F4	Best	3.08E-39	8.17E-38	1.31E-43	7.52E-45	1.88E-44	5.47E-43
	Median	3.87E-30	5.62E-30	1.32E-31	4.19E-33	5.66E-34	3.93E-34
	Mean	1.28E-26	7.13E-26	1.31E-27	2.90E-26	2.16E-28	1.53E-24
	Worst	2.86E-25	8.37E-25	3.82E-26	8.22E-25	6.04E-27	2.70E-23
	STD	2.65E-51	4.03E-50	4.68E-53	2.18E-50	1.18E-54	3.22E-47
F5	Best	2.73E + 01	2.71E + 01	2.67E + 01	2.65E + 01	2.56E + 01	2.58E + 01
	Median	2.84E + 01	2.77E + 01	2.73E + 01	2.69E + 01	2.67E + 01	2.66E + 01
	Mean	2.82E + 01	2.77E + 01	2.73E + 01	2.70E + 01	2.67E + 01	2.66E + 01
	Worst	2.87E + 01	2.84E + 01	2.80E + 01	2.77E + 01	2.74E + 01	2.76E + 01
	STD	1.41E-01	7.90E-02	1.28E-01	1.47E-01	1.27E-01	1.48E-01
F6	Best	1.24E-02	1.52E-03	4.72E-04	1.54E-04	3.94E-05	1.59E-05
	Median	6.71E-02	1.00E-02	2.75E-03	1.01E-03	3.53E-04	2.89E-04
	Mean	1.36E-01	1.77E-02	5.42E-03	2.69E-03	7.61E-04	5.27E-04
	Worst	5.91E-01	1.34E-01	5.62E-02	2.15E-02	3.30E-03	4.15E-03
	STD	2.56E-02	6.49E-04	1.03E-04	2.13E-05	7.11E-07	6.93E-07
F7	Best	2.08E-05	1.06E-05	1.06E-05	5.47E-06	4.30E-06	1.97E-05
	Median	5.80E-04	2.59E-04	2.73E-04	1.72E-04	1.16E-04	1.21E-04
	Mean	9.09E-04	3.95E-04	3.42E-04	2.01E-04	1.51E-04	1.50E-04
	Worst	2.97E-03	1.58E-03	9.83E-04	6.10E-04	5.59E-04	5.39E-04
	STD	6.97E-07	1.54E-07	7.17E-08	2.83E-08	1.78E-08	1.06E-08

Table 3. Results of multimodal benchmark functions (different population).

Function	Item	N = 10	N = 20	N = 30	N = 40	N = 50	N = 60
F8	Best	−1.21E + 04	−1.21E + 04	−1.23E + 04	−1.24E + 04	−1.24E + 04	−1.24E + 04
	Median	−1.08E + 04	−1.12E + 04	−1.14E + 04	−1.16E + 04	−1.17E + 04	−1.17E + 04
	Mean	−1.03E + 04	−1.11E + 04	−1.13E + 04	−1.14E + 04	−1.14E + 04	−1.15E + 04
	Worst	−7.51E + 03	−8.96E + 03	−9.25E + 03	−8.68E + 03	−8.49E + 03	−8.08E + 03
	STD	1.54E + 06	4.51E + 05	7.28E + 05	9.98E + 05	9.49E + 05	7.74E + 05
F9	Best	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
	Median	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
	Mean	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
	Worst	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
	STD	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
F10	Best	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16
	Median	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16
	Mean	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16
	Worst	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16
	STD	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
F11	Best	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
	Median	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
	Mean	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
	Worst	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
	STD	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00
F12	Best	2.44E-04	5.85E-05	4.32E-06	3.71E-06	1.54E-06	2.88E-07
	Median	2.63E-03	5.21E-04	8.88E-05	4.07E-05	1.59E-05	5.67E-06
	Mean	5.03E-03	7.11E-04	1.35E-04	6.02E-05	4.11E-05	1.87E-05
	Worst	3.11E-02	3.26E-03	6.94E-04	3.89E-04	3.03E-04	1.01E-04
	STD	4.02E-05	4.40E-07	2.12E-08	5.04E-09	4.09E-09	6.35E-10
F13	Best	4.14E-03	9.56E-04	1.36E-04	8.81E-05	4.14E-05	1.19E-05
	Median	2.45E-01	2.12E-02	1.26E-02	1.16E-02	1.12E-02	6.31E-04
	Mean	6.79E-01	2.48E-01	3.22E-01	1.10E-01	5.20E-02	1.06E-01
	Worst	2.97E + 00	2.97E + 00	2.97E + 00	2.66E + 00	1.22E + 00	2.97E + 00
	STD	1.07E + 00	5.36E-01	7.80E-01	2.29E-01	4.72E-02	2.82E-01

Table 4. Results of fixed-dimension multimodal benchmark functions (different population).

Function	Item	N = 10	N = 20	N = 30	N = 40	N = 50	N = 60
F14	Best	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	Median	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	Mean	2.61E + 00	1.49E + 00	1.36E + 00	1.03E + 00	1.78E + 00	1.45E + 00
	Worst	1.27E + 01	5.93E + 00	3.97E + 00	1.99E + 00	1.27E + 01	1.27E + 01
	STD	6.34E + 00	1.35E + 00	6.21E-01	3.18E-02	5.29E + 00	4.40E + 00
F15	Best	3.08E-04	3.07E-04	3.07E-04	3.07E-04	3.07E-04	3.07E-04
	Median	6.42E-04	7.80E-04	7.40E-04	7.62E-04	3.11E-04	3.08E-04
	Mean	2.63E-03	2.07E-03	1.97E-03	1.40E-03	1.29E-03	5.46E-04
	Worst	2.04E-02	2.04E-02	2.04E-02	2.04E-02	2.04E-02	1.22E-03
	STD	3.50E-05	2.40E-05	2.43E-05	1.25E-05	1.27E-05	1.16E-07
F16	Best	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00
	Median	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00
	Mean	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00
	Worst	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00
	STD	6.11E-14	1.76E-16	5.27E-18	2.71E-19	1.70E-24	2.60E-31
F17	Best	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Median	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Mean	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Worst	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	STD	1.93E-11	1.05E-13	3.25E-13	2.95E-17	1.03E-15	8.26E-18
F18	Best	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00
	Median	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00
	Mean	4.80E + 00	3.00E + 00	3.90E + 00	3.00E + 00	3.00E + 00	3.00E + 00
	Worst	3.00E + 01	3.00E + 00	3.00E + 01	3.00E + 00	3.00E + 00	3.00E + 00
	STD	4.54E + 01	7.15E-27	2.35E + 01	1.91E-29	5.36E-30	4.84E-30
F19	Best	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00
	Median	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00
	Mean	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00
	Worst	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00
	STD	1.82E-12	3.46E-21	2.26E-26	6.80E-30	6.25E-30	5.94E-30
F20	Best	−3.32E + 00	−3.32E + 00	−3.32E + 00	−3.32E + 00	−3.32E + 00	−3.32E + 00
	Median	−3.32E + 00	−3.20E + 00	−3.26E + 00	−3.32E + 00	−3.20E + 00	−3.20E + 00
	Mean	−3.27E + 00	−3.26E + 00	−3.26E + 00	−3.27E + 00	−3.24E + 00	−3.26E + 00
	Worst	−3.20E + 00	−3.20E + 00	−3.20E + 00	−3.20E + 00	−3.20E + 00	−3.20E + 00
	STD	3.42E-03	3.52E-03	3.53E-03	3.52E-03	2.97E-03	3.52E-03
F21	Best	−1.02E + 01	−1.02E + 01	−1.02E + 01	−1.02E + 01	−1.02E + 01	−1.02E + 01
	Median	−1.02E + 01	−1.02E + 01	−1.02E + 01	−1.02E + 01	−1.02E + 01	−1.02E + 01
	Mean	−9.30E + 00	−8.90E + 00	−9.57E + 00	−9.98E + 00	−9.39E + 00	−1.02E + 01
	Worst	−2.73E-01	−2.73E-01	−2.73E-01	−5.06E + 00	−2.63E + 00	−1.02E + 01
	STD	6.66E + 00	8.21E + 00	4.80E + 00	8.37E-01	3.91E + 00	8.41E-12
F22	Best	−1.04E + 01	−1.04E + 01	−1.04E + 01	−1.04E + 01	−1.04E + 01	−1.04E + 01
	Median	−1.04E + 01	−1.04E + 01	−1.04E + 01	−1.04E + 01	−1.04E + 01	−1.04E + 01
	Mean	−7.74E + 00	−8.95E + 00	−9.11E + 00	−9.38E + 00	−9.21E + 00	−9.54E + 00
	Worst	−2.94E-01	−2.77E + 00	−1.84E + 00	−2.77E + 00	−1.84E + 00	−2.77E + 00
	STD	1.53E + 01	7.12E + 00	8.56E + 00	6.74E + 00	7.34E + 00	5.03E + 00
F23	Best	−1.05E + 01	−1.05E + 01	−1.05E + 01	−1.05E + 01	−1.05E + 01	−1.05E + 01
	Median	−1.05E + 01	−1.05E + 01	−1.05E + 01	−1.05E + 01	−1.05E + 01	−1.05E + 01
	Mean	−8.76E + 00	−9.89E + 00	−9.43E + 00	−1.03E + 01	−9.91E + 00	−9.63E + 00
	Worst	−3.22E-01	−1.86E + 00	−1.68E + 00	−3.84E + 00	−2.42E + 00	−1.86E + 00
	STD	1.09E + 01	4.04E + 00	7.97E + 00	1.45E + 00	3.75E + 00	5.63E + 00

functions exhibit rapid convergence during the initial iteration phase, thanks to the alternating search dynamics of the three educational competition phases. This underscores ECO's remarkable ability to locate near-optimal solutions swiftly.

Intuitively, larger population sizes generally enhance the search scope and improve the likelihood of discovering an optimal solution. However, substantial populations may significantly prolong computational time. Notably, empirical findings from ECO studies reveal that the model's performance is relatively unaffected by population size variations. In certain test scenarios, like functions F13, F20, and F23, increasing population size actually diminishes the optimisation efficacy of the ECO model. To balance computational efficiency and optimisation performance, we opt for a population size parameter of N = 40 in the ECO framework.

To affirm the efficacy of the ECO algorithm, we executed experiments and scrutinised its convergence and trajectories. Figure 6 vividly presents these findings, offering a glimpse into the evolution of search points within the population and the concurrent fluctuations in the average fitness as the ECO algorithm pursues the optimal solution with a population size of N = 40. This visualisation in Figure 6 provides intricate insights into the dynamic interplay between schools and students throughout the optimisation process.

3.2. Comparison of different algorithms on classical test functions

To rigorously assess and contrast the search capabilities of the ECO algorithm, we meticulously selected nine state-of-the-art algorithms for a comparative analysis on a classical test function. To ensure impartiality in the comparison, all examined algorithms underwent execution with uniform parameters: 500 iterations and a population size of 40, aligning precisely with the settings of ECO.

This meticulous approach empowers us to gauge the relative performance and efficacy of ECO against the chosen algorithms under uniform experimental conditions. Through this meticulously crafted evaluation framework, we can thoroughly scrutinise and discern the search prowess of the algorithms on an equal footing.

Tables 5–7 offer a comprehensive comparison of the search results achieved by ECO and nine popular optimisation algorithms across F1 through F23, employing five evaluation metrics. The average convergence curves of the ten algorithms are depicted in Figure 7.

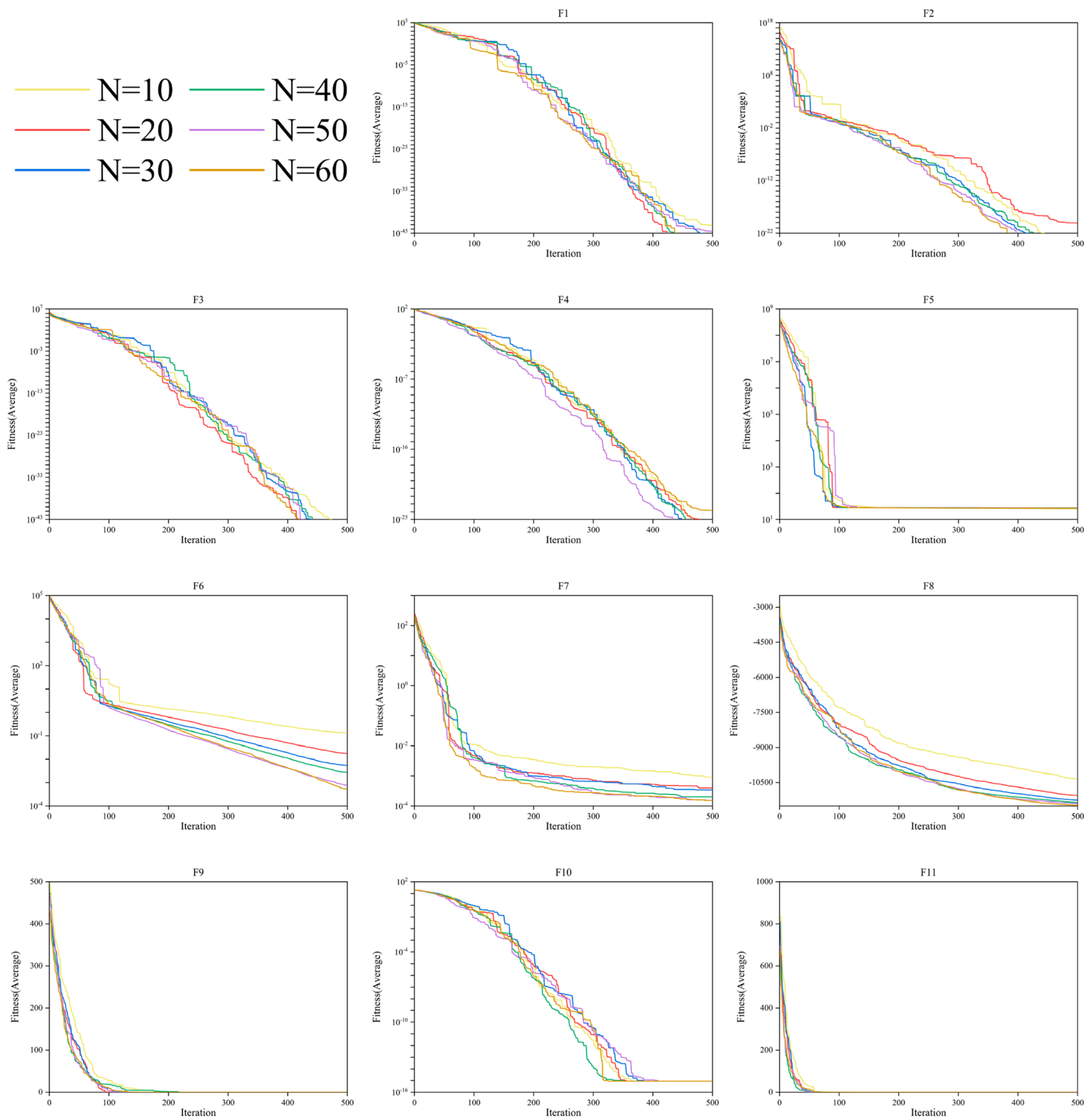

Figure 5. The influence of the population size.

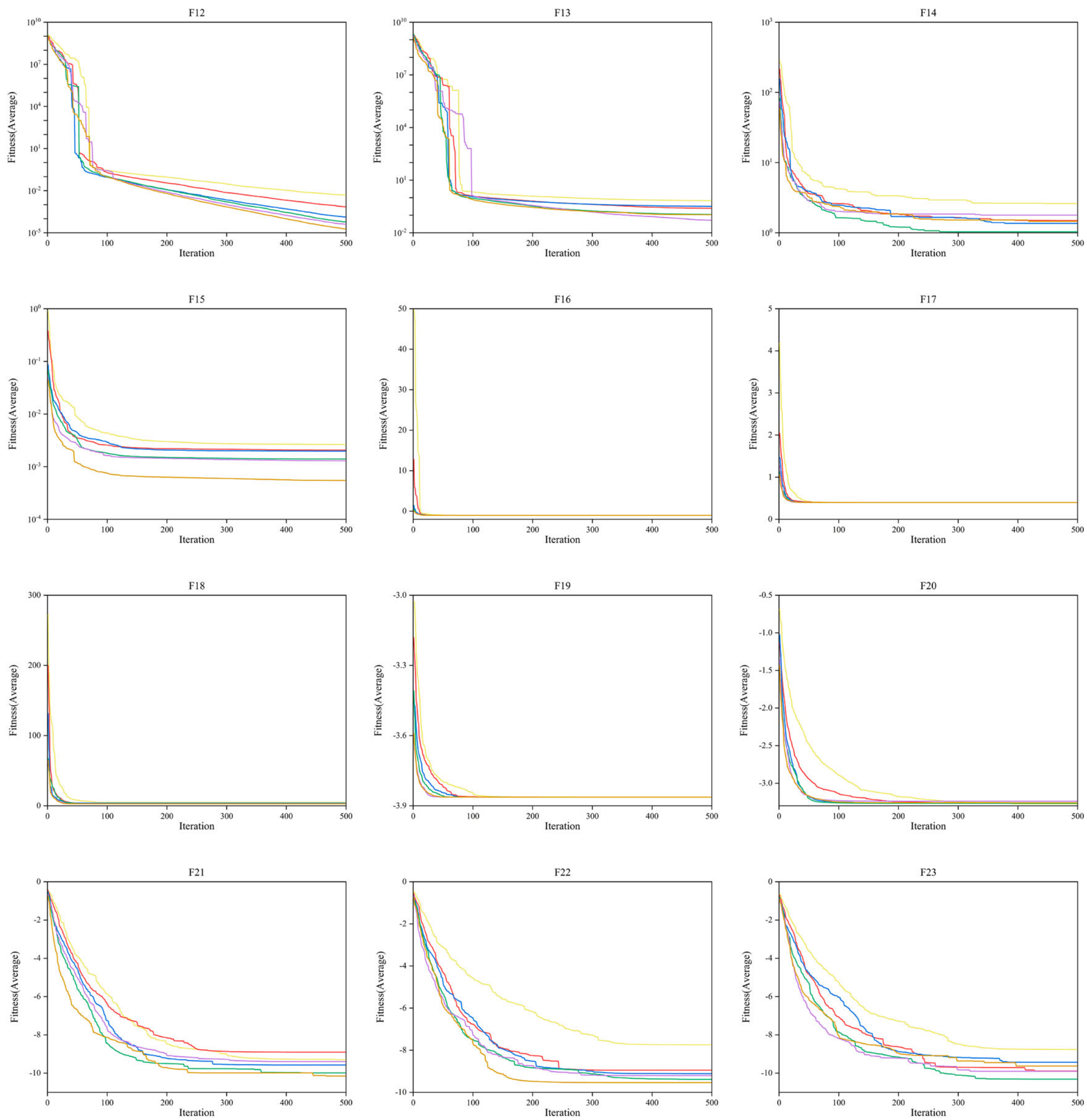

Figure 5. Continued.

The results reveal that ECO outperforms the other algorithms across most of the tested functions and consistently ranks highly across various test functions. Moreover, ECO demonstrates superior convergence abilities in the majority of the test functions. Therefore, ECO emerges as a comprehensive optimisation algorithm with robust search capabilities.

Notably, in functions F9 through F12, F16, F17, F18, F19, and F20, ECO showcases exceptional search capability and rapid convergence, enabling it to swiftly identify optimal or near-optimal solutions.

The performance evaluation of the ECO algorithm and nine existing frontier algorithms on F1 through F23 is presented in Table 8. It's worth noting that when ranking these ten algorithms, the criteria are prioritised in the order of mean and variance.

The results showcase that the ECO algorithm outperforms the other nine optimisation algorithms, securing a significant lead with an impressive average ranking of 2.39. This remarkable performance reaffirms the superiority of the ECO algorithm in solving the optimisation problems under consideration.

This thorough evaluation highlights the effectiveness and competitiveness of the ECO algorithm when compared to other peers. It confirms the robust performance of ECO across diverse function types and underscores its suitability for tackling complex optimisation problems.

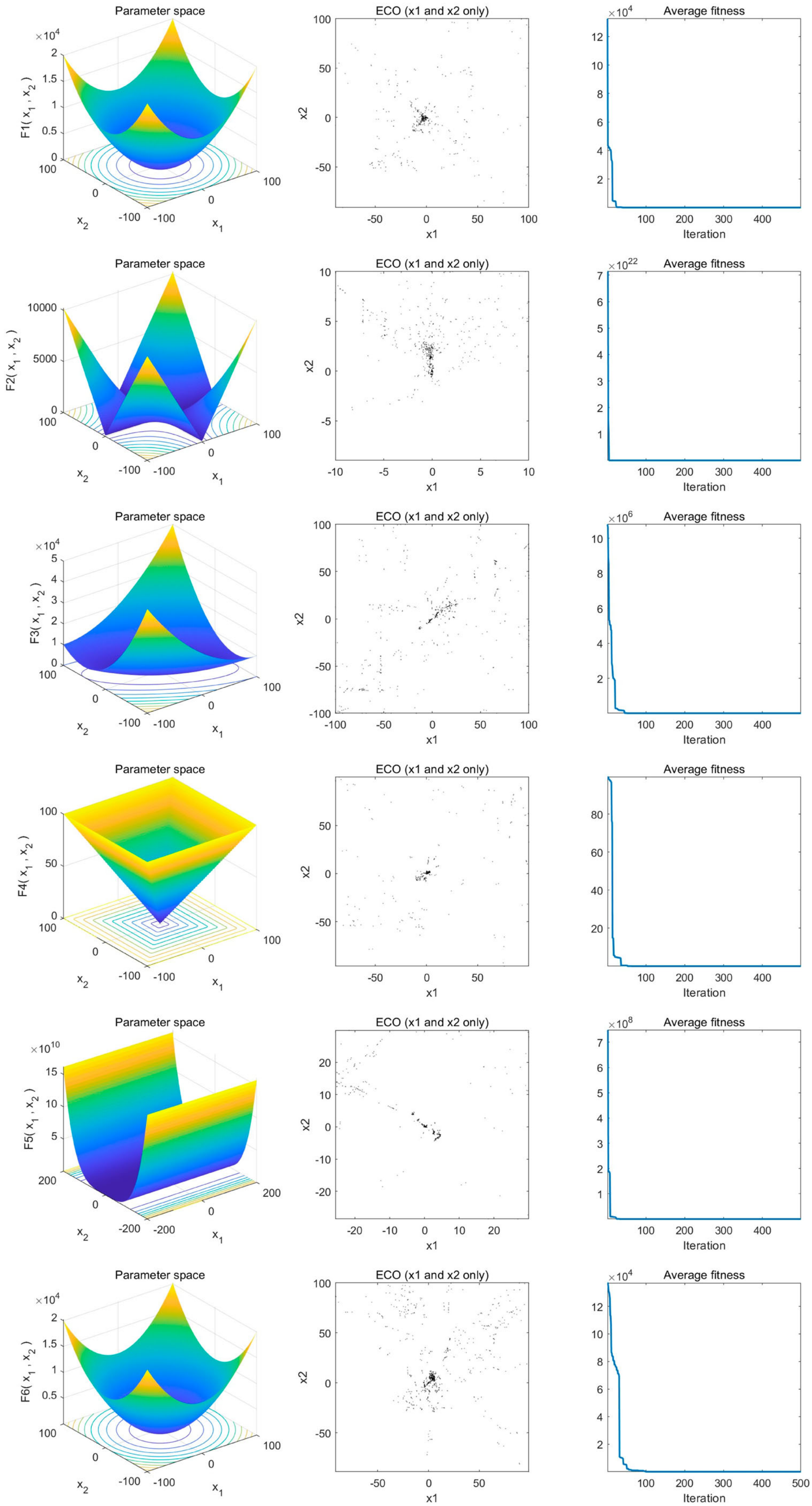

Figure 6. Qualitative results for the studied problems.

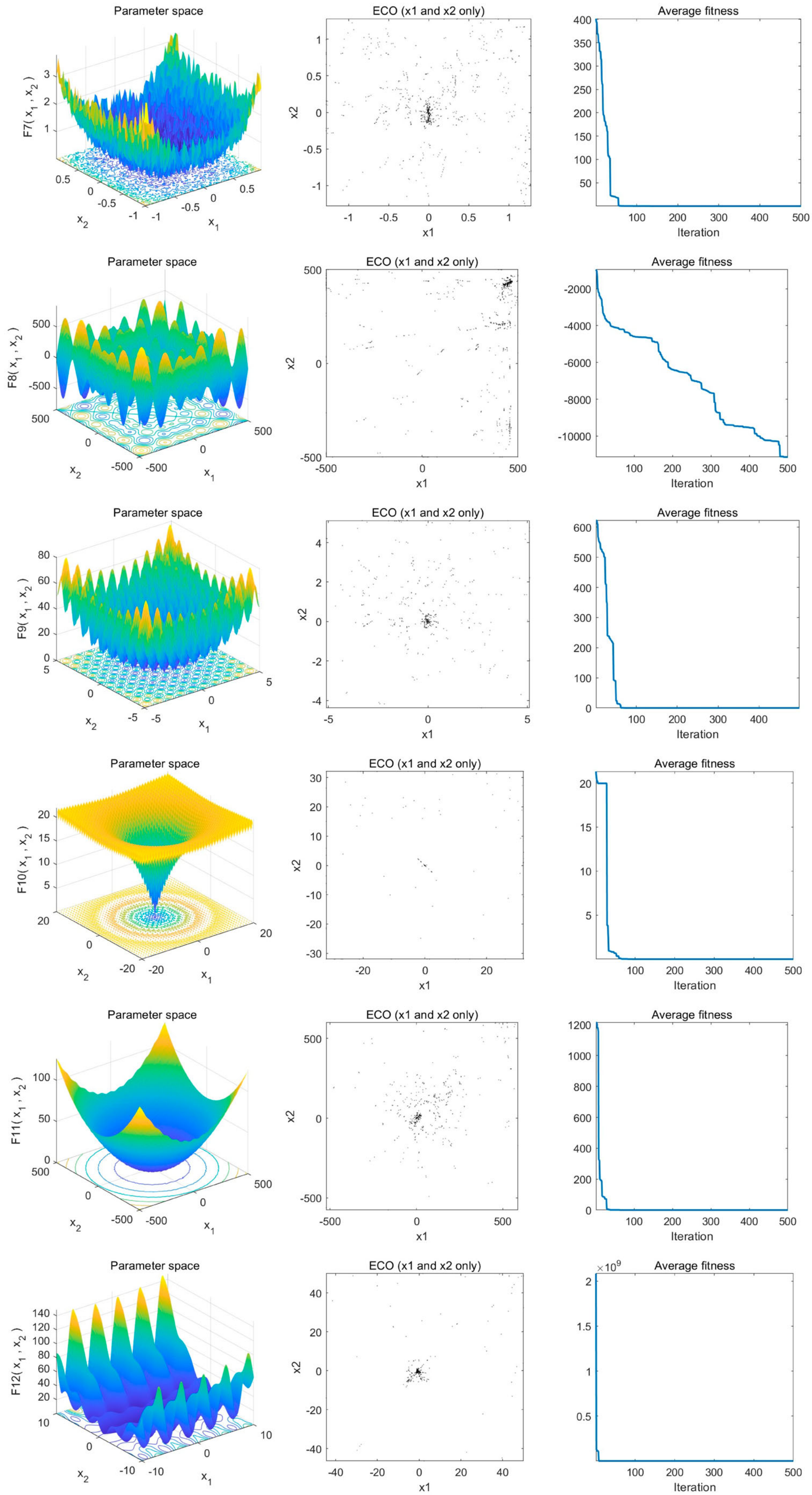

Figure 6. Continued.

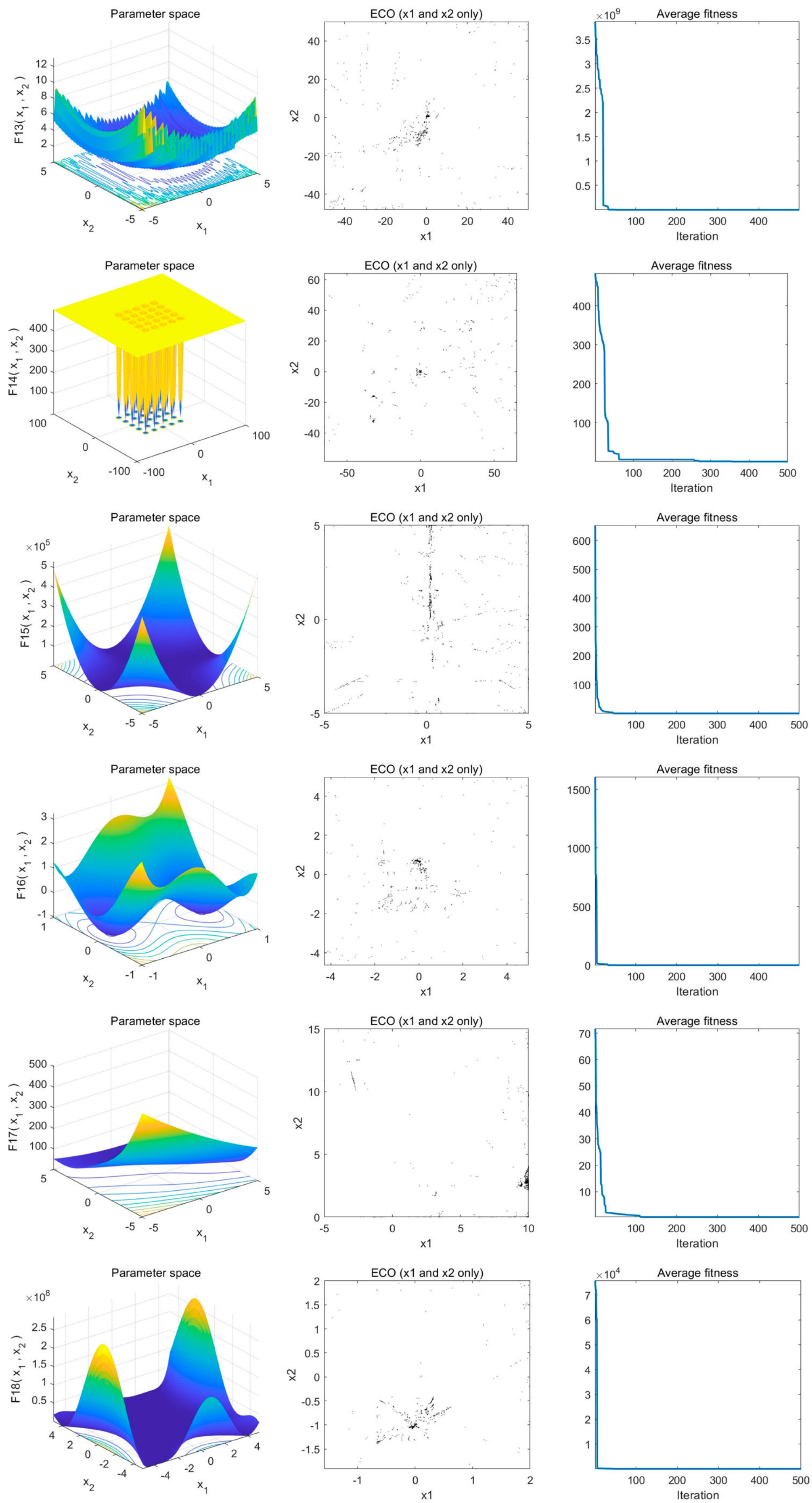

Figure 6. Continued.

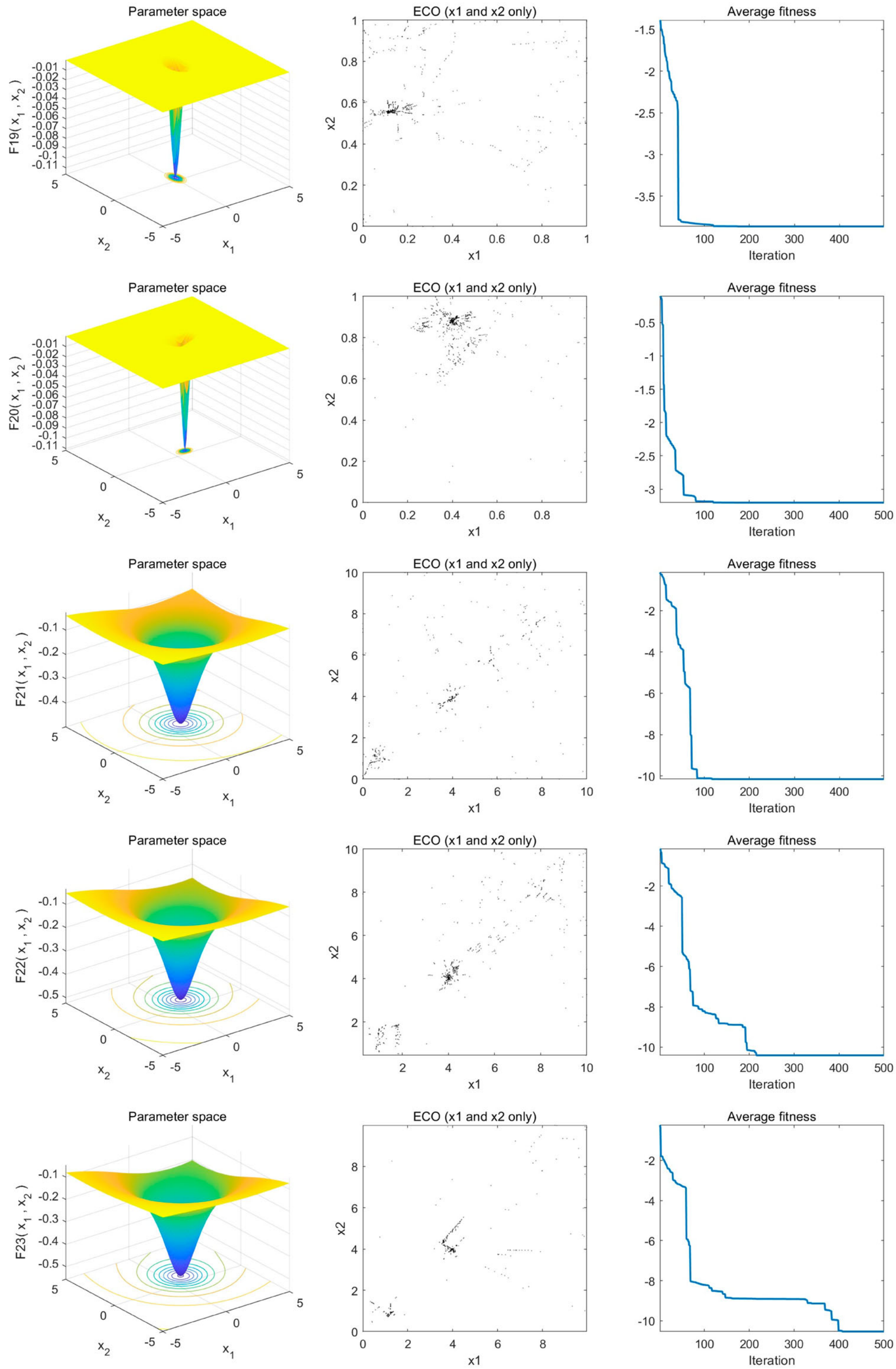

Figure 6. Continued.

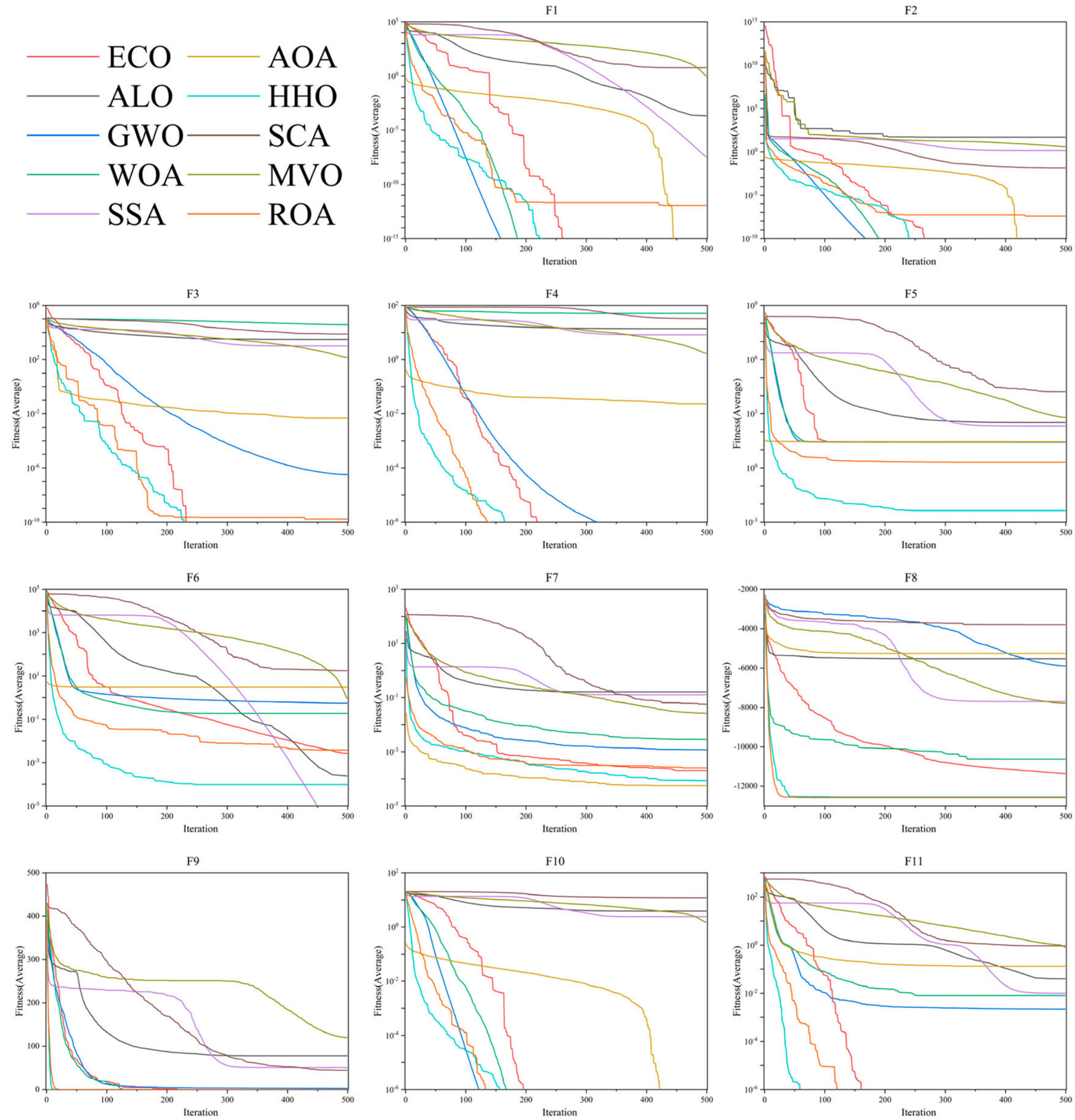

Figure 7. Comparison of convergence rates for different algorithms.

3.3. Comparison of different algorithms on CEC2021 test functions

To deepen our assessment of the effectiveness of the proposed ECO algorithm and scrutinise its ability to explore, exploit, and avoid local optima, we subjected it to one of the most rigorous benchmarks available: the CEC2021 test function suite. We compared the performance of ECO with the nine well-known optimisation algorithms mentioned above. All algorithms underwent independent runs 30 times with 500 iterations and a population size of 40.

Tables 9–14 document the search results as well as the ranking of the ten search algorithms for dimensions (dim) 2, 10, and 20, respectively. It's important to note that when ranking these ten algorithms, the criteria are prioritised in the order of mean and variance.

The results show that ECO outperforms the nine frontier algorithms being compared. More noteworthy is that ECO obtained the optimal solution for all ten test functions within 500 iterations whether in 2, 10, or 20 dimensions, which indicates that ECO is extremely powerful in searching and can deal with high local optimality of combinatorial functions. There is no doubt that ECO is the winner in the CEC2021.

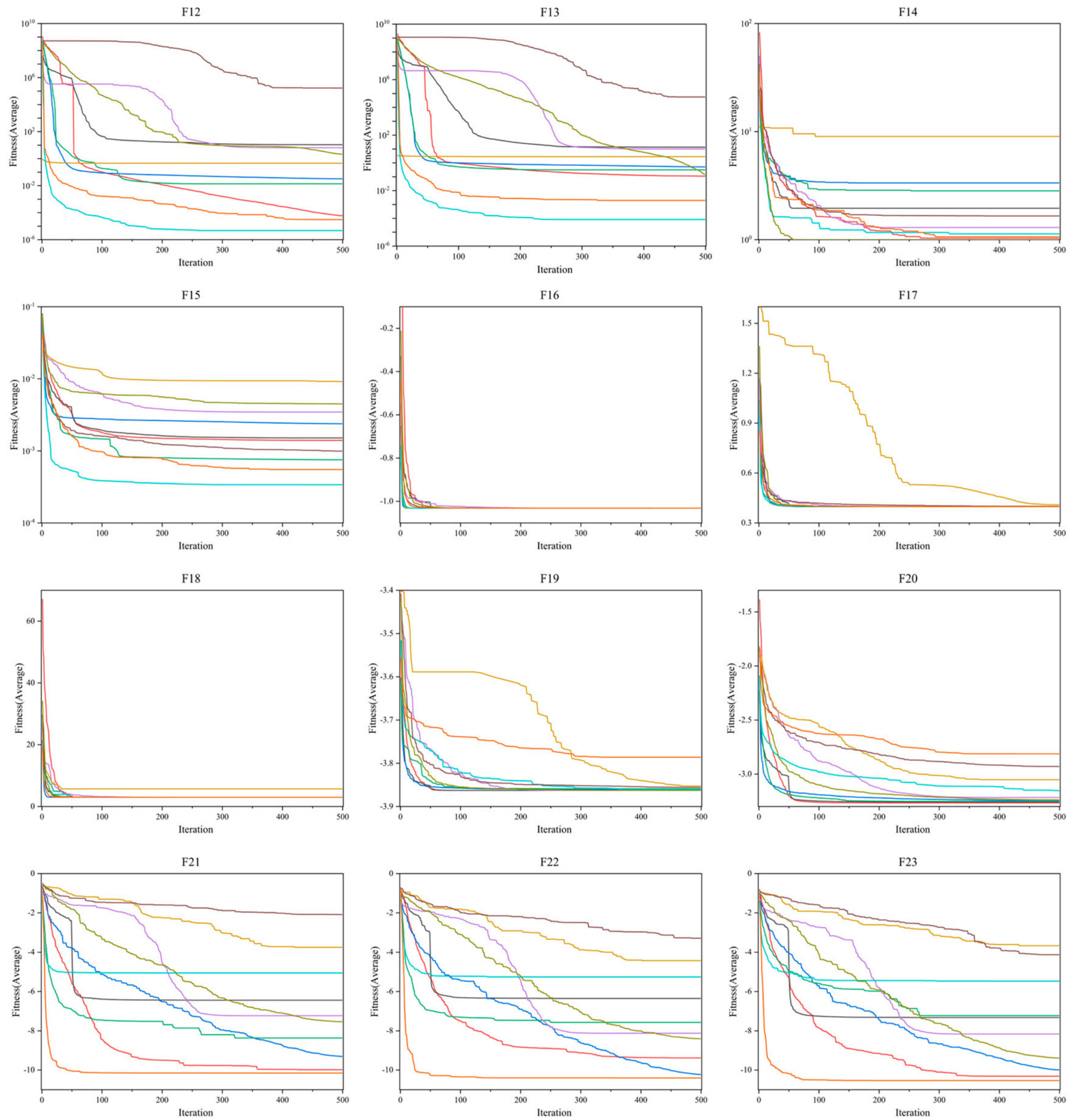

Figure 7. Continued.

The average convergence curves of the ten algorithms are illustrated in Figures 8–10 for the ten tested function species in 2, 10, and 20 dimensions, respectively.

3.4. Real-world applications

In this section, we leverage the recently introduced ECO algorithm to address six distinct engineering design problems (Kumar et al., 2020) and present the outcomes. These engineering optimisation problems have been formulated to seek optimal solutions while adhering to specific conditions and constraints.

Conventionally, metaheuristic method are not inherently designed to directly address constrained optimization problems (Liu, Li, Lu, Yin, & Zhou, 2024). Nonetheless, through the integration of constraint handling techniques (CHTs), these methods can adeptly deal with both the objective function and constraints (Wang & Zhang, 2023). In every iteration, the method evaluates the fitness of the candidate agents, taking both the objective function and restrictions into account. The subsequent batch of candidate swarms is then evaluated based on the estimated fitness values.

Table 5. Results of unimodal benchmark functions (different algorithms).

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	Best	9.69E-92	7.59E-05	2.15E-32	3.73E-91	1.19E-08	5.67E-243	6.67E-116	8.60E-03	4.52E-01	3.66E-34
	Median	1.24E-62	1.87E-04	3.66E-31	4.28E-85	3.05E-08	4.14E-132	3.48E-107	1.25E+00	9.47E-01	4.13E-21
	Mean	3.77E-49	2.12E-04	5.68E-31	4.10E-79	3.44E-08	7.77E-24	1.88E-95	5.98E+00	9.96E-01	1.11E-12
	Worst	1.13E-47	5.37E-04	2.84E-30	1.22E-77	8.11E-08	2.33E-22	5.60E-94	1.03E+02	1.95E+00	3.34E-11
	STD	4.12E-96	1.16E-08	4.70E-61	4.76E-156	2.52E-16	1.75E-45	1.01E-188	3.39E+02	1.46E-01	3.60E-23
F2	Best	2.01E-45	7.43E-01	3.19E-19	9.57E-58	9.09E-02	0.00E+00	3.63E-61	3.00E-05	3.96E-01	5.89E-16
	Median	2.41E-33	1.87E+01	1.07E-18	2.86E-55	1.32E+00	0.00E+00	2.06E-54	5.88E-03	7.13E-01	1.63E-10
	Mean	1.33E-27	5.00E+01	1.50E-18	4.20E-53	1.45E+00	0.00E+00	2.34E-50	1.38E-02	4.10E+00	3.77E-08
	Worst	2.71E-26	1.22E+02	4.11E-18	6.40E-52	3.71E+00	0.00E+00	6.81E-49	7.00E-02	9.90E+01	7.54E-07
	STD	2.43E-53	2.33E+03	1.16E-36	1.51E-104	8.92E-01	0.00E+00	1.49E-98	3.26E-04	3.11E+02	1.90E-14
F3	Best	8.41E-83	1.28E+03	5.66E-10	1.83E+04	9.13E+01	5.79E-153	1.12E-102	1.26E+03	4.14E+01	1.27E-23
	Median	1.12E-63	3.00E+03	4.54E-08	3.78E+04	7.56E+02	6.96E-43	6.05E-93	7.67E+03	1.30E+02	6.98E-15
	Mean	1.72E-52	3.11E+03	3.33E-07	3.78E+04	1.03E+03	4.76E-03	1.73E-84	7.82E+03	1.46E+02	1.69E-10
	Worst	5.17E-51	6.73E+03	2.74E-06	5.32E+04	3.60E+03	5.31E-02	3.50E-83	1.67E+04	3.23E+02	5.00E-09
	STD	8.61E-103	1.80E+06	4.63E-13	7.32E+07	7.65E+05	1.24E-04	4.25E-167	2.42E+07	4.03E+03	8.05E-19
F4	Best	7.52E-45	7.65E+00	8.86E-09	2.76E-01	3.80E+00	1.40E-84	3.40E-58	1.18E+01	6.48E-01	5.72E-17
	Median	4.19E-33	1.29E+01	6.90E-08	5.40E+01	8.43E+00	2.63E-02	4.15E-54	3.46E+01	1.62E+00	7.30E-11
	Mean	2.90E-26	1.38E+01	1.50E-07	5.13E+01	8.27E+00	2.32E-02	1.66E-50	3.22E+01	1.70E+00	6.56E-09
	Worst	8.22E-25	2.04E+01	9.64E-07	9.22E+01	1.31E+01	4.58E-02	4.25E-49	4.66E+01	3.18E+00	9.96E-08
	STD	2.18E-50	1.07E+01	5.09E-14	6.73E+02	6.86E+00	3.43E-04	5.83E-99	8.85E+01	3.79E-01	4.95E-16
F5	Best	2.65E+01	2.38E+01	2.56E+01	2.68E+01	2.62E+01	2.76E+01	1.93E-05	3.30E+01	3.07E+01	2.73E-06
	Median	2.69E+01	1.15E+02	2.66E+01	2.76E+01	7.89E+01	2.84E+01	2.46E-03	1.88E+03	2.39E+02	2.65E-02
	Mean	2.70E+01	3.34E+02	2.69E+01	2.76E+01	2.12E+02	2.84E+01	4.34E-03	1.67E+04	6.23E+02	2.04E+00
	Worst	2.77E+01	2.10E+03	2.86E+01	2.87E+01	1.28E+03	2.89E+01	1.96E-02	1.33E+05	2.85E+03	2.87E+01
	STD	1.47E-01	2.72E+05	6.98E-01	1.50E-01	9.32E+04	7.47E-02	2.63E-05	9.73E+08	7.41E+05	5.09E+01
F6	Best	1.54E-04	6.39E-05	4.80E-05	4.34E-02	1.42E-08	2.49E+00	1.19E-08	4.11E+00	4.60E-01	1.49E-06
	Median	1.01E-03	2.10E-04	5.02E-01	1.56E-01	3.07E-08	3.09E+00	3.30E-05	7.74E+00	8.44E-01	7.05E-04
	Mean	2.69E-03	2.42E-04	5.58E-01	1.88E-01	3.27E-08	3.05E+00	9.57E-05	1.77E+01	9.20E-01	3.79E-03
	Worst	2.15E-02	6.72E-04	1.00E+00	7.29E-01	8.01E-08	3.47E+00	4.35E-04	1.63E+02	1.38E+00	3.55E-02
	STD	2.13E-05	2.04E-08	8.56E-02	1.90E-02	1.78E-16	6.36E-02	1.50E-08	9.48E+02	6.43E-02	7.30E-05
F7	Best	5.47E-06	6.12E-02	3.29E-04	7.78E-05	4.94E-02	4.47E-07	7.19E-07	7.64E-03	7.17E-03	1.74E-05
	Median	1.72E-04	1.54E-01	1.05E-03	1.44E-03	1.10E-01	4.10E-05	6.17E-05	3.70E-02	2.41E-02	1.61E-04
	Mean	2.01E-04	1.60E-01	1.17E-03	2.86E-03	1.26E-01	5.58E-05	8.56E-05	5.78E-02	2.65E-02	2.53E-04
	Worst	6.10E-04	3.13E-01	2.73E-03	1.69E-02	3.38E-01	1.93E-04	5.14E-04	2.32E-01	5.89E-02	1.24E-03
	STD	2.83E-08	3.40E-03	3.32E-07	1.16E-05	3.27E-03	2.46E-09	9.68E-09	2.49E-03	1.31E-04	7.97E-08

Table 6. Results of multimodal benchmark functions (different algorithms).

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F8	Best	−1.24E+04	−6.53E+03	−7.44E+03	−1.26E+04	−9.81E+03	−5.92E+03	−1.26E+04	−4.78E+03	−8.88E+03	−1.26E+04
	Median	−1.16E+04	−5.51E+03	−5.83E+03	−1.11E+04	−7.77E+03	−5.37E+03	−1.26E+04	−3.73E+03	−7.93E+03	−1.26E+04
	Mean	−1.14E+04	−5.52E+03	−5.89E+03	−1.06E+04	−7.70E+03	−5.26E+03	−1.25E+04	−3.80E+03	−7.78E+03	−1.26E+04
	Worst	−8.68E+03	−5.42E+03	−3.46E+03	−6.83E+03	−5.98E+03	−4.45E+03	−1.21E+04	−3.32E+03	−6.55E+03	−1.26E+04
	STD	9.98E+05	3.96E+04	8.11E+05	3.22E+06	6.12E+05	1.66E+05	7.75E+03	1.04E+05	4.35E+05	2.11E-05
F9	Best	0.00E+00	4.38E+01	0.00E+00	0.00E+00	2.39E+01	0.00E+00	0.00E+00	8.79E-03	7.02E+01	0.00E+00
	Median	0.00E+00	7.81E+01	5.02E-01	0.00E+00	4.78E+01	0.00E+00	0.00E+00	3.47E+01	1.14E+02	0.00E+00
	Mean	0.00E+00	7.77E+01	2.74E+00	0.00E+00	5.06E+01	0.00E+00	0.00E+00	4.42E+01	1.20E+02	3.73E-13
	Worst	0.00E+00	1.53E+02	1.61E+01	0.00E+00	9.25E+01	0.00E+00	0.00E+00	1.32E+02	1.99E+02	1.07E-11
	STD	0.00E+00	5.44E+02	1.45E+01	0.00E+00	3.07E+02	0.00E+00	0.00E+00	1.33E+03	8.67E+02	3.71E-24
F10	Best	8.88E-16	1.16E+00	3.95E-14	4.44E-16	1.16E+00	4.44E-16	4.44E-16	2.03E-02	5.25E-01	4.44E-16
	Median	8.88E-16	3.09E+00	5.55E-14	4.00E-15	2.50E+00	4.44E-16	4.44E-16	1.97E+01	1.50E+00	1.57E-11
	Mean	8.88E-16	3.92E+00	5.56E-14	4.47E-15	2.40E+00	4.44E-16	4.44E-16	1.21E+01	1.52E+00	1.13E-09
	Worst	8.88E-16	1.25E+01	6.44E-14	7.55E-15	3.98E+00	4.44E-16	4.44E-16	2.04E+01	2.64E+00	2.03E-08
	STD	0.00E+00	7.83E+00	4.10E-29	6.51E-30	6.19E-01	0.00E+00	0.00E+00	8.63E+01	2.52E-01	1.43E-17
F11	Best	0.00E+00	8.45E-03	0.00E+00	0.00E+00	4.53E-05	7.42E-03	0.00E+00	4.74E-01	6.55E-01	0.00E+00
	Median	0.00E+00	3.53E-02	0.00E+00	0.00E+00	8.41E-03	1.18E-01	0.00E+00	9.07E-01	8.44E-01	0.00E+00
	Mean	0.00E+00	3.98E-02	2.17E-03	8.09E-03	1.00E-02	1.31E-01	0.00E+00	9.40E-01	8.11E-01	1.19E-13
	Worst	0.00E+00	8.93E-02	1.71E-02	1.57E-01	3.72E-02	3.93E-01	0.00E+00	2.50E+00	9.44E-01	2.14E-12
	STD	0.00E+00	5.08E-04	2.48E-05	1.00E-03	9.32E-05	8.69E-03	0.00E+00	1.50E-01	5.27E-03	1.74E-25
F12	Best	3.71E-06	6.39E+00	6.68E-03	2.57E-03	1.75E+00	3.48E-01	5.78E-08	1.20E+00	3.88E-01	1.59E-09
	Median	4.07E-05	9.52E+00	2.68E-02	8.30E-03	6.29E+00	4.63E-01	2.03E-06	1.12E+01	1.99E+00	1.71E-05
	Mean	6.02E-05	1.09E+01	3.23E-02	1.38E-02	6.42E+00	4.58E-01	4.73E-06	1.70E+05	2.15E+00	3.04E-05
	Worst	3.89E-04	2.29E+01	9.12E-02	9.77E-02	1.38E+01	5.58E-01	2.12E-05	4.85E+06	5.30E+00	1.83E-04
	STD	5.04E-09	2.03E+01	2.83E-04	3.10E-04	6.38E+00	2.14E-03	3.66E-11	7.58E+11	1.61E+00	1.68E-09
F13	Best	8.81E-05	1.14E-02	9.83E-02	5.18E-02	6.91E-05	2.57E+00	1.92E-07	2.45E+00	3.86E-02	4.30E-06
	Median	1.16E-02	5.09E+00	5.14E-01	2.78E-01	1.11E+00	2.83E+00	5.37E-05	9.98E+01	1.23E-01	7.23E-04
	Mean	1.10E-01	1.35E+01	5.14E-01	3.15E-01	1.00E+01	2.82E+00	8.22E-05	5.61E+04	1.44E-01	1.97E-03
	Worst	2.66E+00	6.22E+01	9.17E-01	7.31E-01	4.38E+01	2.99E+00	5.33E-04	6.92E+05	3.31E-01	1.29E-02
	STD	2.29E-01	2.62E+02	3.88E-02	3.09E-02	1.48E+02	1.02E-02	1.13E-08	2.79E+10	4.83E-03	1.11E-05

Table 7. Results of fixed-dimension multimodal benchmark functions (different algorithms).

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F14	Best	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	Median	9.98E-01	1.99E + 00	2.98E + 00	1.50E + 00	9.98E-01	1.12E + 01	9.98E-01	9.99E-01	9.98E-01	9.98E-01
	Mean	1.03E + 00	1.96E + 00	3.35E + 00	2.83E + 00	1.30E + 00	9.03E + 00	1.13E + 00	1.66E + 00	9.98E-01	1.06E + 00
	Worst	1.99E + 00	6.90E + 00	1.27E + 01	1.08E + 01	3.97E + 00	1.27E + 01	1.99E + 00	2.98E + 00	9.98E-01	2.98E + 00
	STD	3.18E-02	1.85E + 00	1.17E + 01	8.71E + 00	5.34E-01	1.63E + 01	1.14E-01	8.74E-01	1.52E-21	1.27E-01
F15	Best	3.07E-04	3.08E-04	3.08E-04	3.15E-04	4.21E-04	3.56E-04	3.08E-04	3.46E-04	5.68E-04	3.08E-04
	Median	7.62E-04	8.24E-04	3.29E-04	6.47E-04	7.85E-04	2.82E-03	3.31E-04	7.87E-04	7.69E-04	3.45E-04
	Mean	1.40E-03	1.51E-03	2.38E-03	7.52E-04	3.46E-03	9.13E-03	3.38E-04	9.91E-04	4.48E-03	5.52E-04
	Worst	2.04E-02	2.04E-02	2.04E-02	1.86E-03	2.04E-02	3.56E-02	4.01E-04	1.84E-03	2.04E-02	1.69E-03
	STD	1.26E-05	1.23E-05	3.60E-05	2.17E-07	4.40E-05	1.19E-04	7.82E-10	1.48E-07	5.32E-05	1.59E-07
F16	Best	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00
	Median	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00
	Mean	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00
	Worst	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00	−1.03E + 00
	STD	2.71E-19	2.90E-26	2.21E-16	1.26E-19	8.64E-28	1.30E-14	5.07E-21	1.33E-09	6.63E-14	9.84E-10
F17	Best	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Median	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	4.07E-01	3.98E-01	3.99E-01	3.98E-01	3.98E-01
	Mean	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	4.09E-01	3.98E-01	4.00E-01	3.98E-01	3.98E-01
	Worst	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	4.29E-01	3.98E-01	4.09E-01	3.98E-01	4.01E-01
	STD	2.95E-17	6.05E-27	1.62E-11	9.36E-12	5.14E-28	7.57E-05	4.80E-11	7.45E-06	2.28E-13	4.78E-07
F18	Best	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00
	Median	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00
	Mean	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	5.70E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00
	Worst	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 01	3.00E + 00	3.00E + 00	3.00E + 00	3.00E + 00
	STD	1.91E-29	1.49E-25	3.71E-10	5.29E-10	7.70E-26	6.56E + 01	6.63E-13	9.63E-09	5.22E-12	9.44E-08
F19	Best	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00
	Median	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.85E + 00	−3.86E + 00	−3.85E + 00	−3.86E + 00	−3.84E + 00
	Mean	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.85E + 00	−3.86E + 00	−3.86E + 00	−3.86E + 00	−3.79E + 00
	Worst	−3.86E + 00	−3.86E + 00	−3.85E + 00	−3.83E + 00	−3.86E + 00	−3.84E + 00	−3.85E + 00	−3.85E + 00	−3.86E + 00	−3.61E + 00
	STD	6.80E-30	1.01E-26	5.77E-06	3.33E-05	2.47E-25	1.06E-05	1.24E-05	8.33E-06	1.15E-12	8.39E-03
F20	Best	−3.32E + 00	−3.32E + 00	−3.32E + 00	−3.32E + 00	−3.32E + 00	−3.19E + 00	−3.32E + 00	−3.17E + 00	−3.32E + 00	−3.25E + 00
	Median	−3.32E + 00	−3.20E + 00	−3.26E + 00	−3.32E + 00	−3.20E + 00	−3.08E + 00	−3.17E + 00	−3.01E + 00	−3.20E + 00	−2.91E + 00
	Mean	−3.27E + 00	−3.26E + 00	−3.25E + 00	−3.25E + 00	−3.22E + 00	−3.05E + 00	−3.15E + 00	−2.93E + 00	−3.24E + 00	−2.81E + 00
	Worst	−3.20E + 00	−3.20E + 00	−3.09E + 00	−3.05E + 00	−3.18E + 00	−2.83E + 00	−2.88E + 00	−1.45E + 00	−3.20E + 00	−1.17E + 00
	STD	3.52E-03	3.53E-03	6.26E-03	8.35E-03	2.30E-03	8.34E-03	1.13E-02	1.20E-01	3.03E-03	1.50E-01
F21	Best	−1.02E + 01	−1.02E + 01	−1.02E + 01	−1.02E + 01	−1.02E + 01	−6.78E + 00	−5.06E + 00	−4.88E + 00	−1.02E + 01	−1.02E + 01
	Median	−1.02E + 01	−5.06E + 00	−1.02E + 01	−1.01E + 01	−1.02E + 01	−3.60E + 00	−5.05E + 00	−8.81E-01	−7.63E + 00	−1.02E + 01
	Mean	−9.98E + 00	−6.44E + 00	−9.31E + 00	−8.37E + 00	−7.23E + 00	−3.75E + 00	−5.05E + 00	−2.08E + 00	−7.54E + 00	−1.02E + 01
	Worst	−5.06E + 00	−2.63E + 00	−5.06E + 00	−2.63E + 00	−2.63E + 00	−2.14E + 00	−5.05E + 00	−4.97E-01	−2.63E + 00	−1.01E + 01
	STD	8.37E-01	7.50E + 00	3.57E + 00	6.50E + 00	1.05E + 01	1.30E + 00	4.88E-06	2.96E + 00	7.03E + 00	5.09E-06
F22	Best	−1.04E + 01	−1.04E + 01	−1.04E + 01	−1.04E + 01	−1.04E + 01	−7.24E + 00	−1.03E + 01	−8.29E + 00	−1.04E + 01	−1.04E + 01
	Median	−1.04E + 01	−5.11E + 00	−1.04E + 01	−7.73E + 00	−1.04E + 01	−4.04E + 00	−5.09E + 00	−3.24E + 00	−1.04E + 01	−1.04E + 01
	Mean	−9.38E + 00	−6.35E + 00	−1.02E + 01	−7.57E + 00	−8.12E + 00	−4.43E + 00	−5.26E + 00	−3.29E + 00	−8.41E + 00	−1.04E + 01
	Worst	−2.77E + 00	−2.77E + 00	−5.13E + 00	−2.77E + 00	−2.75E + 00	−1.85E + 00	−5.08E + 00	−9.06E-01	−2.77E + 00	−1.04E + 01
	STD	6.74E + 00	1.02E + 01	8.96E-01	8.22E + 00	1.08E + 01	1.93E + 00	8.81E-01	3.83E + 00	8.36E + 00	9.33E-06
F23	Best	−1.05E + 01	−1.05E + 01	−1.05E + 01	−1.05E + 01	−1.05E + 01	−7.27E + 00	−1.05E + 01	−7.86E + 00	−1.05E + 01	−1.05E + 01
	Median	−1.05E + 01	−1.05E + 01	−1.05E + 01	−7.45E + 00	−1.05E + 01	−3.67E + 00	−5.13E + 00	−4.35E + 00	−1.05E + 01	−1.05E + 01
	Mean	−1.03E + 01	−7.32E + 00	−9.99E + 00	−7.23E + 00	−8.16E + 00	−3.67E + 00	−5.47E + 00	−4.13E + 00	−9.39E + 00	−1.05E + 01
	Worst	−3.84E + 00	−2.43E + 00	−2.42E + 00	−2.42E + 00	−2.42E + 00	−1.77E + 00	−5.11E + 00	−9.41E-01	−2.43E + 00	−1.05E + 01
	STD	1.45E + 00	1.23E + 01	4.10E + 00	1.11E + 01	1.16E + 01	1.25E + 00	1.67E + 00	3.51E + 00	6.80E + 00	5.85E-05

Using the ECO method in these engineering design challenges allows for the joint analysis of the objective function and restriction. This simplifies finding of optimal agents that meet the design objectives and restrictions. The incorporation of CHTs demonstrates the ECO algorithm's efficacy in dealing with difficult engineering optimization challenges. Utilising the ECO algorithm in engineering design problems enables the simultaneous consideration of both the objective function and constraints, thereby facilitating the pursuit of optimal solutions that meet design specifications and constraints. The effectiveness of the ECO algorithm in addressing intricate engineering optimisation challenges is showcased through the incorporation of CHTs.

3.4.1. The tension/compression spring design (TSD)

The primary aim of this challenge is to minimise the weight of the tension/compression spring, emphasising efficiency in material usage and design optimisation (Tzanetos & Blondin, 2023), as showed in Figure 11. The problem involves three decision variables: wire diameter ($d = x_1$), mean coil diameter ($D = x_2$), and the number of active coils ($N = x_3$). Eq. (14) is formulated to address this optimisation challenge, and the outcomes are presented in Table 15, alongside the results obtained from other competing algorithms.

Minimize:

$$f(\vec{l}) = (l_3 + 2)l_2 l_1^2 \tag{14}$$

Table 8. Rank of classical benchmark functions.

	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	3	8	4	2	7	5	1	10	9	6
F2	4	10	5	2	8	1	3	7	9	6
F3	2	8	4	10	7	5	1	9	6	3
F4	2	8	4	10	7	5	1	9	6	3
F5	4	8	3	5	7	6	1	10	9	2
F6	4	3	7	6	1	9	2	10	8	5
F7	3	10	5	6	9	1	2	8	7	4
F8	3	8	7	4	6	9	2	10	5	1
F9	1	9	6	1	8	1	1	7	10	5
F10	3	9	5	4	8	1	1	10	7	6
F11	3	9	5	4	8	6	1	10	7	2
F12	1	7	4	5	6	8	1	10	9	3
F13	3	9	6	5	8	7	1	10	4	2
F14	2	7	9	8	5	10	4	6	1	3
F15	5	6	7	3	8	10	1	4	9	2
F16	1	1	1	1	1	1	1	1	1	1
F17	1	1	1	1	1	10	1	9	1	8
F18	1	1	6	6	1	10	1	8	1	9
F19	1	1	5	7	1	9	6	8	1	10
F20	1	2	4	3	6	8	7	9	5	10
F21	2	7	3	4	6	9	8	10	5	1
F22	3	7	2	6	5	9	8	10	4	1
F23	2	6	3	7	5	10	8	9	4	1
Average Rank	2.39	6.30	4.61	4.78	5.61	6.52	2.74	8.43	5.57	4.09
Final Ranking	1	8	4	5	6	9	2	10	7	3

subject to:

$$g_1(\vec{l}) = 1 - \frac{4l_2^2 - l_2 l_3}{717851^4} \leq 0,$$

$$g_2(\vec{l}) = \frac{4l_2^2 - l_1 l_2}{12566(l_3 l_1^3 - l_1^4)} + \frac{1}{5108 l_1^2} \leq 0,$$

$$g_3(\vec{l}) = 1 - \frac{150.45 l_1}{l_2^2 l_3} \leq 0,$$

$$g_4(\vec{l}) = \frac{l_1 + l_2}{1.5} - 1 \leq 0,$$

with bounds:

$$0.05 \leq l_1 \leq 2.00, 0.25 \leq l_2 \leq 1.30, 2.00 \leq l_3 \leq 15.0$$

3.4.2. Gear train design (GTD)

This subsection verifies the ability of the ECO algorithm to tackle the design problem of gear trains. Figure 12 displays a visual illustration of this problem. The goal is to find the number of teeth for each one of the four wheels: A ($= x_1$), B ($= x_2$), C ($= x_3$), and D ($= x_4$) of the gear to minimise the gear ratio. Eq. (15) is formulated to address this optimisation challenge, and the outcomes are presented in Table 16, alongside the results obtained from other competing algorithms.

Minimise:

$$f(x) = \left(\frac{1}{6.931} - \frac{x_2 x_3}{x_1 x_4}\right)^2 \tag{15}$$

with bounds:

$$12 \leq x_1, x_2, x_3, x_4 \leq 60.$$

3.4.3. The optimal design of an industrial refrigeration system (ODIS)

The mathematical model of this problem is described in (Kumar et al., 2020). This problem can be formulated as a nonlinear inequality constrained optimisation problem, Eq. (16) is formulated. and the outcomes are presented in Table 17, alongside the results obtained from other competing algorithms.

Minimise:

$$\begin{aligned} f(\bar{x}) = {} & 63098.88x_2x_4x_{12} + 5441.5x_2^2x_{12} \\ & + 115055.5x_2^{1.664}x_6 + 6172.27x_2^2x_6 \\ & + 63098.88x_1x_3x_{11} + 5441.5x_1^2x_{11} \\ & + 115055.5x_1^{1.664}x_5 + 6172.27x_1^2x_5 \\ & + 140.53x_1x_{11} + 281.29x_3x_{11} + 70.26x_1^2 \\ & + 281.29x_1x_3 + 281.29x_3^2 \\ & + 14437x_8^{1.8812}x_{12}^{0.3424}x_{10}x_{14}^{-1}x_1^2x_7x_9^{-1} \\ & + 20470.2x_7^{2.893}x_{11}^{0.316}x_1^2 \end{aligned} \tag{16}$$

subject to:

$$g_1(\bar{x}) = 1.524x_7^{-1} \leq 0,$$

$$g_2(\bar{x}) = 1.524x_8^{-1} \leq 1,$$

Table 9. Results of CEC2021 benchmark functions (dim = 2).

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	Best	0.00E + 00	5.07E-09	2.44E-289	2.94E-150	3.18E-02	0.00E + 00	3.45E-128	2.58E-88	1.39E-04	3.02E-32
	Median	0.00E + 00	7.58E-06	5.87E-253	4.29E-137	3.76E + 01	0.00E + 00	3.09E-113	6.95E-73	4.86E-02	4.97E-15
	Mean	0.00E + 00	2.69E + 02	2.57E-236	1.20E-121	2.91E + 02	0.00E + 00	1.86E-104	6.15E-70	9.63E-02	7.56E-11
	Worst	0.00E + 00	5.55E + 03	7.48E-235	3.60E-120	2.11E + 03	0.00E + 00	5.57E-103	1.48E-68	5.52E-01	1.87E-09
	STD	0.00E + 00	1.02E + 06	0.00E + 00	4.18E-241	2.73E + 05	0.00E + 00	9.99E-207	7.13E-138	1.43E-02	1.13E-19
F2	Best	0.00E + 00	4.55E-12	0.00E + 00	0.00E + 00	1.93E-12	0.00E + 00	0.00E + 00	0.00E + 00	4.25E-07	0.00E + 00
	Median	0.00E + 00	3.12E-01	0.00E + 00	0.00E + 00	3.12E-01	0.00E + 00	0.00E + 00	0.00E + 00	3.44E-04	0.00E + 00
	Mean	0.00E + 00	3.64E + 00	8.33E-02	0.00E + 00	7.67E-01	0.00E + 00	0.00E + 00	0.00E + 00	1.07E + 01	6.94E-12
	Worst	0.00E + 00	1.71E + 01	6.24E-01	0.00E + 00	1.71E + 01	0.00E + 00	0.00E + 00	0.00E + 00	1.18E + 02	1.58E-10
	STD	0.00E + 00	4.44E + 01	4.51E-02	0.00E + 00	9.21E + 00	0.00E + 00	0.00E + 00	0.00E + 00	8.68E + 02	8.49E-22
F3	Best	0.00E + 00	8.32E-13	0.00E + 00	0.00E + 00	4.49E-13	0.00E + 00	0.00E + 00	0.00E + 00	1.33E-05	0.00E + 00
	Median	0.00E + 00	2.04E + 00	2.04E + 00	0.00E + 00	2.28E-11	0.00E + 00	0.00E + 00	0.00E + 00	2.04E + 00	1.20E-23
	Mean	0.00E + 00	1.18E + 00	1.42E + 00	6.57E-33	8.66E-01	0.00E + 00	0.00E + 00	1.93E-01	1.56E + 00	3.69E-14
	Worst	0.00E + 00	2.34E + 00	2.13E + 00	1.97E-31	2.34E + 00	0.00E + 00	0.00E + 00	2.50E + 00	2.04E + 00	5.49E-13
	STD	0.00E + 00	1.07E + 00	8.76E-01	1.25E-63	1.13E + 00	0.00E + 00	0.00E + 00	3.89E-01	7.43E-01	1.87E-26
F4	Best	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	1.13E-14	0.00E + 00
	Median	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	1.70E-10	0.00E + 00
	Mean	0.00E + 00	6.58E-03	2.30E-03	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	1.32E-03	0.00E + 00
	Worst	0.00E + 00	4.93E-02	1.97E-02	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	1.98E-02	0.00E + 00
	STD	0.00E + 00	1.84E-04	3.69E-05	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	2.44E-05	0.00E + 00
F5	Best	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Median	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Mean	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Worst	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	STD	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
F6	Best	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	0.00E + 00	1.53E-10	0.00E + 00
	Median	0.00E + 00	6.69E-15	0.00E + 00	0.00E + 00	2.22E-16	0.00E + 00	0.00E + 00	0.00E + 00	8.92E-09	0.00E + 00
	Mean	0.00E + 00	3.89E-14	0.00E + 00	0.00E + 00	6.03E-16	0.00E + 00	0.00E + 00	0.00E + 00	4.97E-08	4.11E-15
	Worst	0.00E + 00	2.23E-13	0.00E + 00	0.00E + 00	2.78E-15	0.00E + 00	0.00E + 00	0.00E + 00	5.37E-07	1.23E-13
	STD	0.00E + 00	4.40E-27	0.00E + 00	0.00E + 00	5.01E-31	0.00E + 00	0.00E + 00	0.00E + 00	1.25E-14	4.84E-28
F7	Best	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Median	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Mean	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Worst	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	STD	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
F8	Best	0.00E + 00	1.41E-11	0.00E + 00	0.00E + 00	2.37E-12	0.00E + 00	0.00E + 00	0.00E + 00	3.76E-05	0.00E + 00
	Median	0.00E + 00	2.71E-10	0.00E + 00	0.00E + 00	3.95E-11	0.00E + 00	0.00E + 00	0.00E + 00	6.92E-04	7.40E-15
	Mean	0.00E + 00	7.50E-01	0.00E + 00	1.23E-17	7.17E-11	0.00E + 00	0.00E + 00	0.00E + 00	7.93E-04	3.00E-11
	Worst	0.00E + 00	1.12E + 01	0.00E + 00	3.70E-16	5.59E-10	0.00E + 00	0.00E + 00	0.00E + 00	2.39E-03	8.98E-10
	STD	0.00E + 00	7.87E + 00	0.00E + 00	4.41E-33	1.13E-20	0.00E + 00	0.00E + 00	0.00E + 00	3.19E-07	2.60E-20
F9	Best	0.00E + 00	2.12E-06	1.01E-286	5.07E-154	3.15E-07	0.00E + 00	4.17E-139	1.37E-86	4.00E-03	8.88E-15
	Median	0.00E + 00	1.27E-05	2.43E-255	6.46E-132	7.70E-06	0.00E + 00	1.90E-119	4.66E-80	2.86E-02	5.40E-10
	Mean	0.00E + 00	1.40E-05	1.55E-236	3.26E-15	7.29E-06	0.00E + 00	1.17E-115	1.75E-75	2.89E-02	8.25E-06
	Worst	0.00E + 00	2.98E-05	4.65E-235	8.88E-15	1.44E-05	0.00E + 00	2.84E-114	3.83E-74	6.08E-02	2.43E-04
	STD	0.00E + 00	4.74E-11	0.00E + 00	1.83E-29	1.79E-11	0.00E + 00	2.68E-229	4.98E-149	2.18E-04	1.90E-09
F10	Best	0.00E + 00	1.33E-03	1.26E-04	2.44E-04	7.62E-04	0.00E + 00	5.66E-128	4.55E-05	1.99E-02	3.09E-40
	Median	0.00E + 00	2.74E-03	4.80E-04	3.61E-03	2.20E-03	0.00E + 00	2.34E-07	4.52E-04	5.50E-02	3.30E-04
	Mean	0.00E + 00	3.07E-03	5.74E-04	5.36E-03	2.22E-03	0.00E + 00	4.78E-05	6.89E-04	5.48E-02	5.31E-04
	Worst	0.00E + 00	5.90E-03	1.35E-03	1.93E-02	4.63E-03	0.00E + 00	3.49E-04	2.92E-03	1.09E-01	2.11E-03
	STD	0.00E + 00	1.61E-06	1.12E-07	2.72E-05	6.88E-07	0.00E + 00	8.74E-09	4.11E-07	2.85E-04	3.18E-07

Table 10. Rank of CEC2021 benchmark functions (dim = 2).

	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	1	9	3	4	10	1	5	6	8	7
F2	1	9	7	1	8	1	1	1	10	6
F3	1	8	9	4	7	1	1	6	10	5
F4	1	10	9	1	1	1	1	1	8	1
F5	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
F6	1	9	3	4	10	1	5	6	8	7
F7	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
F8	1	10	1	6	8	1	1	1	9	7
F9	1	9	3	6	7	1	4	5	10	8
F10	1	8	5	9	7	1	3	6	10	4
Average Rank	1	9	5	4.375	7.25	1	2.625	4	9.125	5.625
Final Ranking	1	9	6	5	8	1	3	4	10	7

Table 11. Results of CEC2021 benchmark functions (dim = 10).

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	Best	0.00E+00	2.46E-03	1.21E-62	6.14E-87	4.49E+00	0.00E+00	4.40E-114	2.13E-12	4.12E+03	1.69E-20
	Median	0.00E+00	9.09E+02	5.87E-60	2.48E-82	5.64E+02	0.00E+00	1.59E-101	1.14E-09	1.25E+04	2.79E-16
	Mean	0.00E+00	1.70E+03	1.08E-57	4.87E-78	1.49E+03	0.00E+00	3.25E-92	2.16E-08	1.37E+04	1.68E-09
	Worst	0.00E+00	9.83E+03	2.08E-56	3.97E-77	5.97E+03	0.00E+00	9.58E-91	4.74E-07	3.17E+04	2.99E-08
	STD	0.00E+00	5.31E+06	1.42E-113	1.27E-154	2.85E+06	0.00E+00	2.95E-182	7.28E-15	4.76E+07	3.69E-17
F2	Best	0.00E+00	4.45E+02	0.00E+00	0.00E+00	7.02E+00	0.00E+00	0.00E+00	0.00E+00	3.94E+00	0.00E+00
	Median	0.00E+00	9.38E+02	7.30E-02	0.00E+00	5.34E+02	0.00E+00	0.00E+00	3.96E-09	5.63E+02	0.00E+00
	Mean	0.00E+00	1.01E+03	5.58E+00	1.08E+02	5.70E+02	0.00E+00	0.00E+00	4.37E+01	5.17E+02	1.50E-11
	Worst	0.00E+00	1.69E+03	1.03E+02	1.48E+03	1.40E+03	0.00E+00	0.00E+00	5.51E+02	9.81E+02	2.82E-10
	STD	0.00E+00	1.04E+05	3.46E+02	1.17E+05	1.08E+05	0.00E+00	0.00E+00	1.72E+04	5.10E+04	2.88E-21
F3	Best	0.00E+00	1.69E+01	0.00E+00	0.00E+00	9.95E+00	0.00E+00	0.00E+00	3.57E-14	9.15E+00	1.77E-30
	Median	0.00E+00	3.42E+01	3.53E+01	0.00E+00	3.08E+01	0.00E+00	0.00E+00	2.35E-07	2.60E+01	3.18E-22
	Mean	0.00E+00	3.79E+01	3.35E+01	6.57E-33	3.25E+01	0.00E+00	0.00E+00	1.04E+01	2.60E+01	3.57E-15
	Worst	0.00E+00	9.55E+01	5.93E+01	1.97E-31	7.86E+01	0.00E+00	0.00E+00	9.68E+01	4.18E+01	8.92E-14
	STD	0.00E+00	2.82E+02	3.09E+02	1.25E-63	2.81E+02	0.00E+00	0.00E+00	6.85E+02	7.23E+01	2.63E-28
F4	Best	0.00E+00	4.14E-01	0.00E+00	0.00E+00	3.46E-01	0.00E+00	0.00E+00	0.00E+00	6.08E-01	0.00E+00
	Median	0.00E+00	1.30E+00	2.15E-01	0.00E+00	1.27E+00	0.00E+00	0.00E+00	1.00E-08	1.39E+00	0.00E+00
	Mean	0.00E+00	1.47E+00	4.66E-01	1.22E-01	1.54E+00	0.00E+00	0.00E+00	1.01E+00	1.43E+00	0.00E+00
	Worst	0.00E+00	4.40E+00	1.87E+00	1.59E+00	3.37E+00	0.00E+00	0.00E+00	6.60E+00	2.42E+00	0.00E+00
	STD	0.00E+00	5.98E-01	3.21E-01	1.15E-01	7.11E-01	0.00E+00	0.00E+00	3.07E+00	2.68E-01	0.00E+00
F5	Best	0.00E+00	8.79E+02	4.35E-25	8.14E-81	7.17E+02	0.00E+00	3.75E-110	2.09E-16	2.59E+02	1.79E-18
	Median	0.00E+00	7.68E+03	1.27E-22	5.28E-23	6.49E+03	0.00E+00	2.15E-89	4.40E-12	8.79E+02	2.33E-13
	Mean	0.00E+00	1.65E+04	2.91E-01	1.42E-16	8.29E+03	0.00E+00	3.02E-74	2.22E-01	9.10E+02	1.57E-10
	Worst	0.00E+00	8.99E+04	2.64E+00	2.82E-15	2.95E+04	0.00E+00	9.07E-73	6.49E+00	1.92E+03	4.41E-09
	STD	0.00E+00	4.05E+08	5.21E-01	2.80E-31	5.68E+07	0.00E+00	2.65E-146	1.36E+00	8.47E+04	6.26E-19
F6	Best	0.00E+00	5.72E+00	2.03E-02	1.36E-09	2.56E+00	0.00E+00	0.00E+00	7.89E-04	2.32E+00	2.98E-08
	Median	0.00E+00	2.52E+01	1.20E-01	7.31E-02	1.58E+01	0.00E+00	1.49E-12	1.05E-01	1.61E+01	2.67E-05
	Mean	0.00E+00	3.94E+01	1.03E+00	1.63E+01	2.98E+01	0.00E+00	8.49E-06	2.84E+00	3.76E+01	4.95E-05
	Worst	0.00E+00	1.59E+02	1.79E+01	2.93E+02	2.43E+02	0.00E+00	1.39E-04	6.62E+01	1.45E+02	2.35E-04
	STD	0.00E+00	1.77E+03	1.02E+01	3.73E+03	2.17E+03	0.00E+00	6.59E-10	1.43E+02	2.37E+03	3.91E-09
F7	Best	0.00E+00	8.34E+02	5.67E-03	5.24E-04	6.73E+02	0.00E+00	8.91E-114	6.78E-04	2.72E+01	4.22E-08
	Median	0.00E+00	3.19E+03	3.34E-02	1.19E-02	2.85E+03	0.00E+00	1.93E-13	3.05E-02	2.55E+02	4.02E-06
	Mean	0.00E+00	4.01E+03	2.92E-01	4.22E-02	2.96E+03	0.00E+00	7.78E-07	1.52E-01	2.48E+02	3.70E-05
	Worst	0.00E+00	1.18E+04	2.56E+00	2.84E-01	1.02E+04	0.00E+00	1.60E-05	2.76E+00	5.23E+02	4.34E-04
	STD	0.00E+00	8.85E+06	3.63E-01	4.12E-03	4.34E+06	0.00E+00	8.80E-12	2.42E-01	2.13E+04	6.82E-09
F8	Best	0.00E+00	4.33E+01	0.00E+00	0.00E+00	4.21E+01	0.00E+00	0.00E+00	1.48E-14	2.17E+01	0.00E+00
	Median	0.00E+00	4.12E+02	0.00E+00	0.00E+00	1.97E+02	0.00E+00	0.00E+00	4.42E-12	9.09E+01	5.55E-16
	Mean	0.00E+00	4.24E+02	0.00E+00	0.00E+00	2.35E+02	0.00E+00	0.00E+00	2.37E-08	2.31E+02	2.04E-11
	Worst	0.00E+00	9.62E+02	0.00E+00	0.00E+00	6.34E+02	0.00E+00	0.00E+00	5.22E-07	1.07E+03	5.28E-10
	STD	0.00E+00	6.05E+04	0.00E+00	0.00E+00	2.24E+04	0.00E+00	0.00E+00	9.11E-15	7.87E+04	8.99E-21
F9	Best	0.00E+00	1.41E-04	8.88E-15	2.42E-95	5.36E-05	0.00E+00	1.67E-122	2.96E-10	3.17E-01	1.78E-14
	Median	0.00E+00	4.67E+00	8.88E-15	8.88E-15	8.77E-05	0.00E+00	1.41E-111	5.23E-08	7.13E-01	1.70E-09
	Mean	0.00E+00	1.25E+01	1.15E-14	9.47E-15	1.48E+00	0.00E+00	2.98E-103	3.26E-07	1.73E+00	1.24E-06
	Worst	0.00E+00	6.79E+01	1.78E-14	1.78E-14	6.95E+00	0.00E+00	8.37E-102	4.13E-06	6.08E+00	1.78E-05
	STD	0.00E+00	5.08E+02	1.66E-29	2.59E-29	6.44E+00	0.00E+00	2.26E-204	6.00E-13	3.13E+00	1.39E-11
F10	Best	0.00E+00	4.82E+01	4.03E-03	1.76E-02	4.85E+01	0.00E+00	6.58E-114	4.73E-03	4.84E+01	1.58E-04
	Median	0.00E+00	4.97E+01	4.96E+01	5.59E-02	5.07E+01	0.00E+00	1.52E-06	6.10E+01	4.91E+01	9.45E-04
	Mean	0.00E+00	5.24E+01	5.10E+01	6.33E-02	5.91E+01	0.00E+00	2.28E-04	4.65E+01	5.21E+01	1.29E-03
	Worst	0.00E+00	8.32E+01	7.82E+01	1.42E-01	1.09E+02	0.00E+00	3.55E-03	8.54E+01	8.00E+01	3.52E-03
	STD	0.00E+00	7.02E+01	1.56E+02	1.15E-03	2.46E+02	0.00E+00	5.55E-07	1.14E+03	8.51E+01	9.73E-07

Table 12. Rank of CEC2021 benchmark functions (dim = 10).

	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	1	9	5	4	8	1	3	7	10	6
F2	1	10	5	7	9	1	1	6	8	4
F3	1	10	9	4	8	1	1	6	7	5
F4	1	9	6	5	10	1	1	7	8	1
F5	1	10	7	4	9	1	3	6	8	5
F6	1	9	5	4	8	1	3	7	10	6
F7	1	10	7	5	9	1	3	6	8	4
F8	1	10	1	1	9	1	1	7	8	6
F9	1	10	5	4	8	1	3	6	9	7
F10	1	9	7	5	10	1	3	6	8	4
Average Rank	1	9.6	5.7	4.3	8.8	1	2.2	6.4	8.4	4.8
Final Ranking	1	10	6	4	9	1	3	7	8	5

Table 13. Results of CEC2021 benchmark functions (dim = 20).

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	Best	0.00E+00	5.94E+00	2.99E-37	1.94E-86	2.09E-01	0.00E+00	5.75E-108	3.51E-01	7.58E+04	1.03E-27
	Median	0.00E+00	1.04E+03	6.71E-35	1.78E-79	3.46E+02	4.93E-260	1.11E-101	1.50E+02	1.37E+05	3.43E-14
	Mean	0.00E+00	1.44E+03	2.40E-34	6.79E-73	8.26E+02	1.76E-83	1.54E-94	2.91E+03	1.51E+05	1.62E-07
	Worst	0.00E+00	4.87E+03	2.57E-33	2.03E-71	4.80E+03	5.27E-82	4.58E-93	7.25E+04	2.62E+05	4.84E-06
	STD	0.00E+00	1.84E+06	2.86E-67	1.33E-143	1.22E+06	8.96E-165	6.77E-187	1.68E+08	2.27E+09	7.56E-13
F2	Best	0.00E+00	1.37E+03	0.00E+00	0.00E+00	2.18E+02	0.00E+00	0.00E+00	4.41E-02	1.34E+03	0.00E+00
	Median	0.00E+00	2.36E+03	5.46E+00	0.00E+00	1.73E+03	0.00E+00	0.00E+00	1.25E+02	1.86E+03	0.00E+00
	Mean	0.00E+00	2.29E+03	9.10E+00	1.60E+02	1.72E+03	0.00E+00	0.00E+00	4.78E+02	2.05E+03	3.50E-11
	Worst	0.00E+00	3.46E+03	3.04E+01	2.30E+03	2.95E+03	0.00E+00	0.00E+00	2.87E+03	3.07E+03	7.77E-10
	STD	0.00E+00	3.67E+05	1.04E+02	2.55E+05	4.24E+05	0.00E+00	0.00E+00	5.51E+05	2.39E+05	2.11E-20
F3	Best	0.00E+00	6.19E+01	1.77E-30	0.00E+00	4.78E+01	0.00E+00	0.00E+00	1.04E-03	3.31E+01	0.00E+00
	Median	0.00E+00	1.36E+02	7.01E+01	0.00E+00	9.55E+01	0.00E+00	0.00E+00	1.61E+01	7.56E+01	1.39E-21
	Mean	0.00E+00	1.41E+02	7.30E+01	6.57E-33	9.79E+01	6.57E-33	0.00E+00	3.01E+01	7.80E+01	2.39E-15
	Worst	0.00E+00	2.24E+02	1.82E+02	1.97E-31	1.70E+02	1.97E-31	0.00E+00	1.49E+02	1.29E+02	7.15E-14
	STD	0.00E+00	1.70E+03	3.47E+03	1.25E-63	9.66E+02	1.25E-63	0.00E+00	1.37E+03	4.32E+02	1.65E-28
F4	Best	0.00E+00	1.66E+00	0.00E+00	0.00E+00	2.15E+00	0.00E+00	0.00E+00	7.93E-08	3.61E+00	0.00E+00
	Median	0.00E+00	6.93E+00	4.99E-02	0.00E+00	5.86E+00	0.00E+00	0.00E+00	6.60E+00	6.44E+00	0.00E+00
	Mean	0.00E+00	6.25E+00	3.87E-01	2.18E-01	6.07E+00	0.00E+00	0.00E+00	6.22E+00	6.42E+00	0.00E+00
	Worst	0.00E+00	9.44E+00	3.57E+00	3.55E+00	1.09E+01	0.00E+00	0.00E+00	1.42E+01	1.04E+01	0.00E+00
	STD	0.00E+00	4.21E+00	5.31E-01	5.12E-01	4.54E+00	0.00E+00	0.00E+00	2.00E+01	2.75E+00	0.00E+00
F5	Best	0.00E+00	3.97E+04	1.30E-21	1.05E-83	6.72E+03	0.00E+00	2.64E-109	2.35E-05	5.14E+03	1.54E-19
	Median	0.00E+00	1.64E+05	7.24E-14	2.70E-25	1.20E+05	0.00E+00	6.40E-94	2.25E-01	1.86E+04	1.29E-13
	Mean	0.00E+00	2.47E+05	2.96E+00	4.65E-16	2.35E+05	0.00E+00	1.22E-76	1.16E+01	2.57E+04	7.63E-09
	Worst	0.00E+00	7.62E+05	1.94E+01	1.39E-14	1.13E+06	0.00E+00	3.67E-75	1.14E+02	7.38E+04	2.29E-07
	STD	0.00E+00	3.75E+10	2.23E+01	6.27E-30	7.97E+10	0.00E+00	4.33E-151	6.44E+02	3.47E+08	1.69E-15
F6	Best	−2.22E-16	6.60E+01	2.65E-02	−2.22E-16	1.09E+01	−1.11E-16	−1.11E-16	2.55E-01	6.40E+01	3.44E-06
	Median	−1.11E-16	5.35E+02	1.23E+00	1.41E-01	3.19E+02	0.00E+00	1.71E-08	2.10E+00	2.23E+02	6.49E-05
	Mean	−1.11E-16	5.42E+02	2.62E+00	7.31E+01	3.27E+02	−7.40E-18	2.83E-05	9.27E+00	2.51E+02	2.54E-04
	Worst	0.00E+00	1.17E+03	2.48E+01	7.20E+02	7.91E+02	0.00E+00	3.09E-04	1.61E+02	5.38E+02	1.85E-03
	STD	1.23E-32	8.95E+04	2.34E+01	3.11E+04	4.52E+04	7.67E-34	4.84E-09	8.25E+02	1.71E+04	2.17E-07
F7	Best	−2.22E-16	6.81E+03	3.45E-02	5.38E-03	7.28E+03	0.00E+00	−2.22E-16	1.34E-01	1.72E+03	1.19E-07
	Median	0.00E+00	7.44E+04	3.19E-01	6.79E-02	4.51E+04	0.00E+00	3.28E-14	5.80E-01	5.01E+03	1.77E-05
	Mean	−5.92E-17	9.24E+04	2.13E+00	6.96E-02	6.19E+04	1.60E-270	5.78E-07	2.32E+00	5.75E+03	3.25E-05
	Worst	0.00E+00	2.40E+05	2.11E+01	1.92E-01	2.84E+05	4.80E-269	1.15E-05	2.72E+01	2.13E+04	2.32E-04
	STD	9.64E-33	3.98E+09	2.03E+01	2.42E-03	3.85E+09	0.00E+00	5.04E-12	2.90E+01	1.34E+07	2.28E-09
F8	Best	0.00E+00	5.78E+02	0.00E+00	0.00E+00	3.66E+02	0.00E+00	0.00E+00	3.37E-05	4.05E+02	0.00E+00
	Median	0.00E+00	1.47E+03	0.00E+00	0.00E+00	7.36E+02	0.00E+00	0.00E+00	3.95E-03	1.75E+03	7.10E-14
	Mean	0.00E+00	1.44E+03	0.00E+00	0.00E+00	8.20E+02	0.00E+00	0.00E+00	1.86E+00	1.64E+03	5.98E-11
	Worst	0.00E+00	2.67E+03	0.00E+00	0.00E+00	1.45E+03	0.00E+00	0.00E+00	3.63E+01	3.12E+03	1.02E-09
	STD	0.00E+00	2.34E+05	0.00E+00	0.00E+00	8.88E+04	0.00E+00	0.00E+00	4.77E+01	5.33E+05	3.74E-20
F9	Best	0.00E+00	3.15E+00	1.78E-14	8.96E-96	1.32E-04	0.00E+00	1.34E-115	1.76E-03	2.35E+00	7.11E-14
	Median	0.00E+00	5.82E+01	3.55E-14	8.88E-15	1.79E-04	5.96E-250	1.75E-110	2.06E-02	5.79E+00	2.74E-09
	Mean	0.00E+00	4.94E+01	3.29E-14	9.18E-15	1.19E+01	6.50E-83	1.49E-99	3.49E-02	5.52E+00	1.37E-07
	Worst	0.00E+00	1.14E+02	3.55E-14	1.78E-14	9.33E+01	1.95E-81	3.07E-98	2.57E-01	8.62E+00	1.74E-06
	STD	0.00E+00	1.42E+03	3.76E-29	3.41E-29	6.12E+02	1.23E-163	3.57E-197	2.65E-03	3.73E+00	1.23E-13
F10	Best	0.00E+00	4.98E+01	5.19E+01	3.64E-02	4.97E+01	0.00E+00	4.21E-120	7.79E-02	4.99E+01	2.68E-04
	Median	0.00E+00	5.13E+01	8.13E+01	9.98E-02	7.01E+01	3.12E-252	1.66E-06	9.72E+01	5.08E+01	9.50E-04
	Mean	0.00E+00	5.23E+01	7.55E+01	9.69E-02	7.68E+01	1.54E-60	1.02E-04	8.40E+01	5.35E+01	1.39E-03
	Worst	0.00E+00	8.05E+01	9.09E+01	1.83E-01	1.80E+02	4.62E-59	1.73E-03	1.33E+02	7.91E+01	5.04E-03
	STD	0.00E+00	2.84E+01	1.69E+02	1.90E-03	9.18E+02	6.88E-119	9.79E-08	1.54E+03	6.71E+01	1.48E-06

Table 14. Rank of CEC2021 benchmark functions (dim = 20).

	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	1	8	5	4	7	3	2	9	10	6
F2	1	10	5	6	8	1	1	7	9	4
F3	1	10	7	3	9	3	1	6	8	5
F4	1	9	6	5	7	1	1	8	10	1
F5	1	10	6	4	9	1	3	7	8	5
F6	1	8	5	4	7	3	2	9	10	6
F7	1	10	6	5	9	2	3	7	8	4
F8	1	9	1	1	8	1	1	7	10	6
F9	1	10	5	4	9	3	2	7	8	6
F10	1	6	8	5	9	2	3	10	7	4
Average Rank	1	9	5.4	4.1	8.2	2	1.9	7.7	8.8	4.7
Final Ranking	1	10	6	4	8	3	2	7	9	5

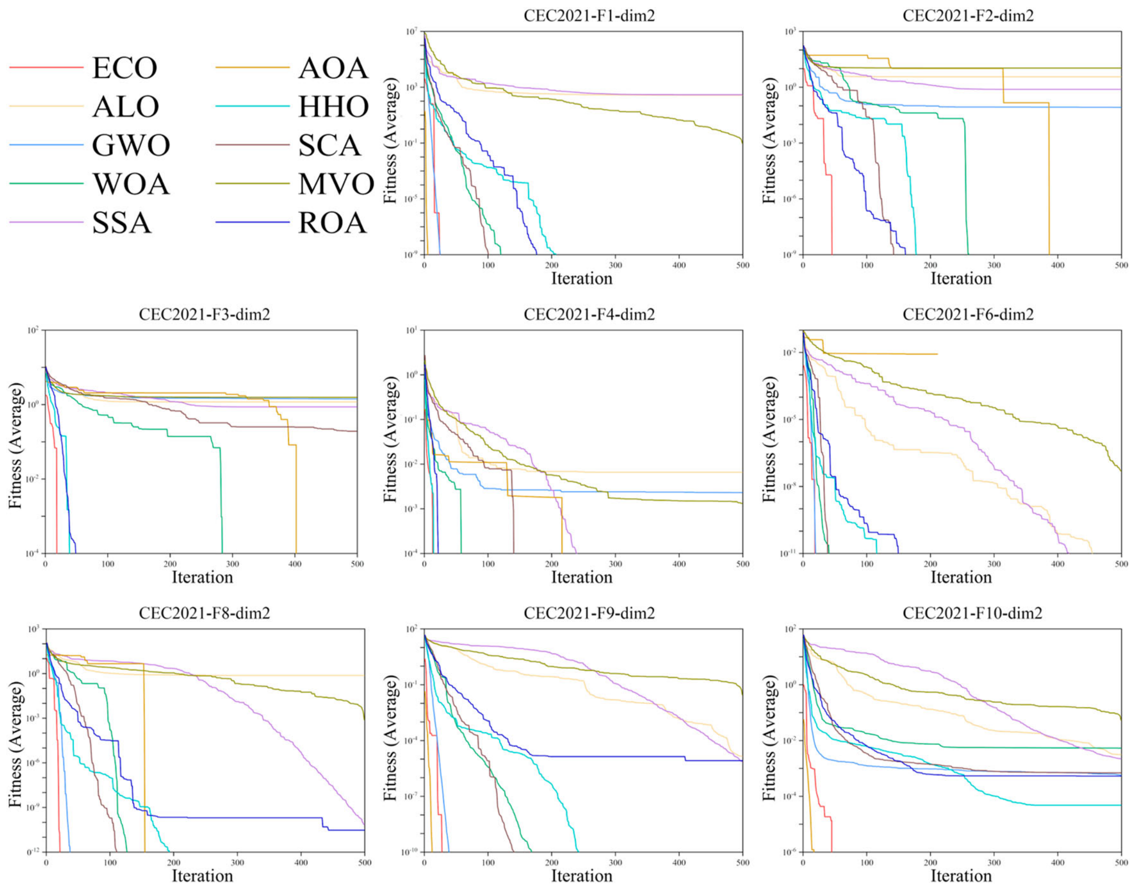

Figure 8. Comparison of CEC2021 benchmark functions (dim = 2).

Table 15. Results of TSD problem.

	x1	x2	x3	x4	x5	Best	Worst	Average	STD	Median
ECO	5.19E-02	3.62E-01	1.14E + 01	5.09E + 02	2.00E + 00	1.27E-02	1.32E-02	1.31E-02	3.80E-08	1.32E-02
ALO	5.19E-02	3.62E-01	1.06E + 01	4.89E + 02	2.00E + 00	1.27E-02	1.42E-02	1.32E-02	2.41E-07	1.31E-02
GWO	5.19E-02	3.62E-01	1.07E + 01	1.60E + 02	2.00E + 00	1.27E-02	1.36E-02	1.28E-02	2.74E-08	1.27E-02
WOA	5.27E-02	3.81E-01	9.70E + 00	1.00E + 03	2.00E + 00	1.27E-02	1.78E-02	1.38E-02	1.20E-06	1.36E-02
SSA	6.92E-02	9.41E-01	1.82E + 00	8.00E + 01	6.00E + 01	1.80E-02	3.03E-02	2.28E-02	1.05E-05	2.26E-02
AOA	5.22E-02	3.66E-01	1.09E + 01	1.00E + 03	2.00E + 00	1.30E-02	3.18E-02	1.51E-02	2.99E-05	1.32E-02
HHO	5.27E-02	3.81E-01	9.73E + 00	2.33E + 02	2.00E + 00	1.27E-02	1.67E-02	1.39E-02	9.51E-07	1.36E-02
SCA	5.07E-02	3.32E-01	1.35E + 01	1.00E + 03	2.00E + 00	1.28E-02	1.33E-02	1.32E-02	1.33E-08	1.32E-02
MVO	5.00E-02	3.10E-01	1.50E + 01	9.51E + 02	2.00E + 00	1.32E-02	1.86E-02	1.71E-02	2.91E-06	1.79E-02
ROA	5.37E-02	4.08E-01	8.70E + 00	1.00E + 03	2.00E + 00	1.30E-02	1.63E-02	1.44E-02	8.84E-07	1.43E-02

Table 16. Results of GTD problem.

	x1	x2	x3	x4	Best	Worst	Average	STD	Median
ECO	4.33E + 01	1.64E + 01	1.87E + 01	4.91E + 01	2.70E-12	9.92E-10	5.35E-10	1.73E-19	7.74E-10
ALO	5.65E + 01	3.08E + 01	1.33E + 01	4.94E + 01	9.94E-11	2.73E-08	7.76E-09	8.18E-17	4.50E-09
GWO	4.86E + 01	1.62E + 01	1.90E + 01	4.29E + 01	2.70E-12	2.36E-09	3.83E-10	3.25E-19	2.31E-11
WOA	4.90E + 01	1.94E + 01	1.57E + 01	4.34E + 01	2.70E-12	2.73E-08	2.96E-09	3.25E-17	1.26E-09
SSA	4.91E + 01	1.64E + 01	1.89E + 01	4.31E + 01	2.70E-12	1.12E-08	2.12E-09	7.09E-18	9.92E-10
AOA	5.35E + 01	2.56E + 01	1.47E + 01	5.07E + 01	2.31E-11	2.73E-08	1.13E-08	1.20E-16	6.35E-09
HHO	4.87E + 01	1.93E + 01	1.58E + 01	4.27E + 01	2.70E-12	2.73E-08	2.07E-09	2.38E-17	1.09E-09
SCA	5.33E + 01	1.53E + 01	2.57E + 01	5.12E + 01	2.31E-11	1.62E-08	2.49E-09	1.03E-17	1.36E-09
MVO	5.30E + 01	1.27E + 01	3.02E + 01	5.06E + 01	2.31E-11	2.36E-09	8.69E-10	5.22E-19	8.89E-10
ROA	3.66E + 01	2.33E + 01	1.26E + 01	5.64E + 01	6.60E-10	3.10E-03	1.04E-04	3.11E-07	2.04E-08

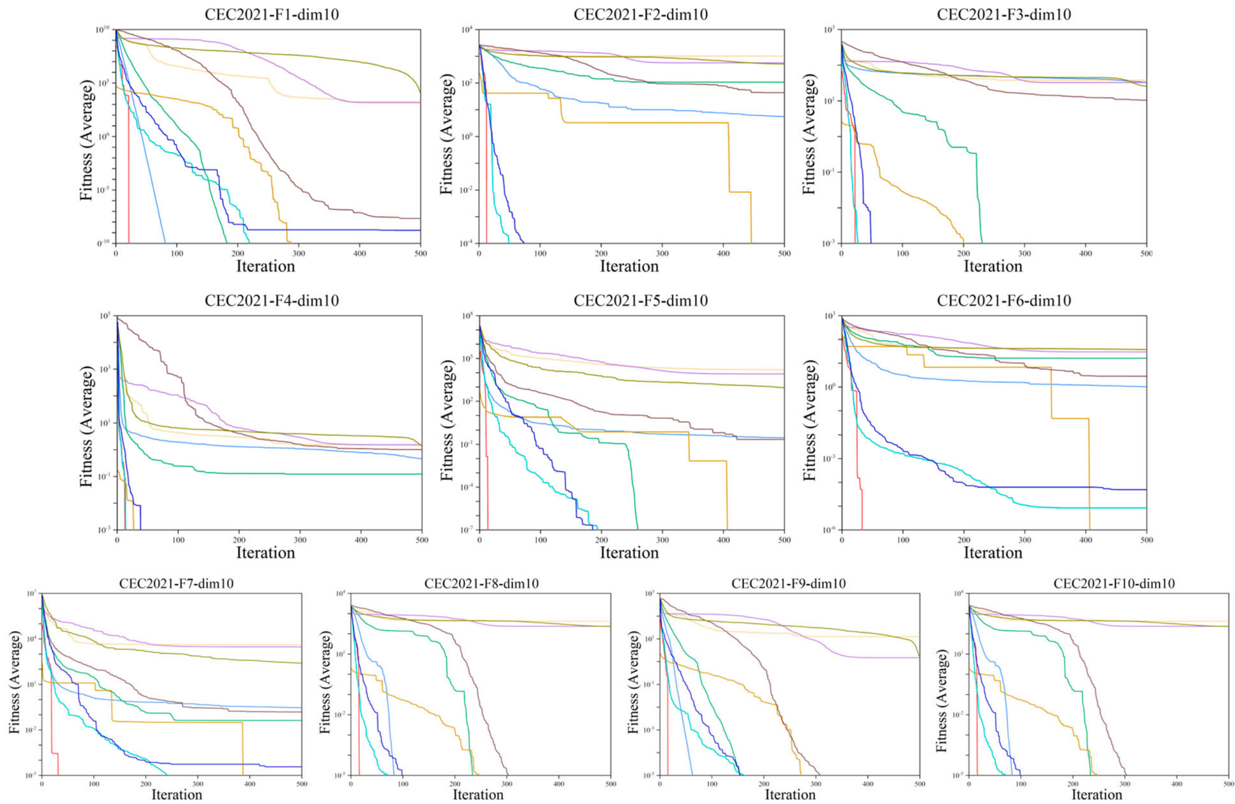

Figure 9. Comparison of CEC2021 benchmark functions (dim = 10).

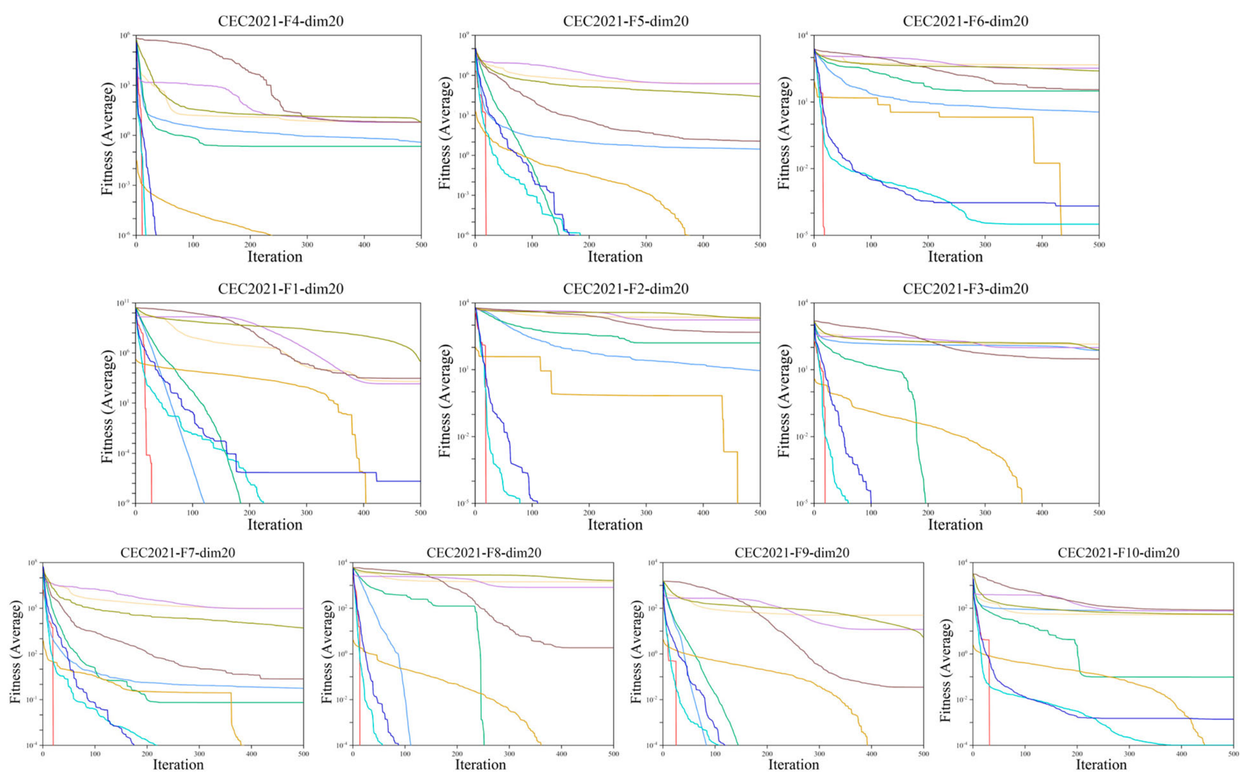

Figure 10. Comparison of CEC2021 benchmark functions (dim = 20).

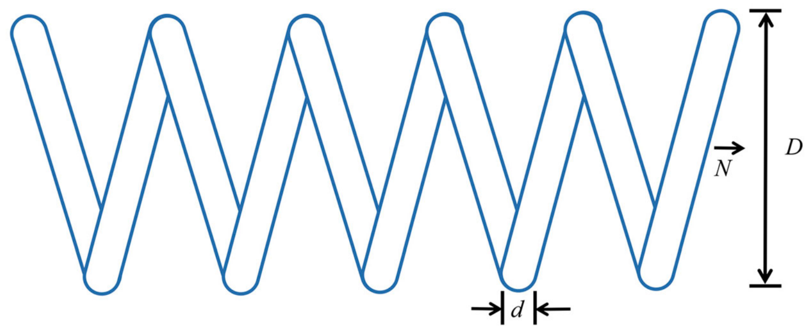

Figure 11. Schematic illustration of TSD.

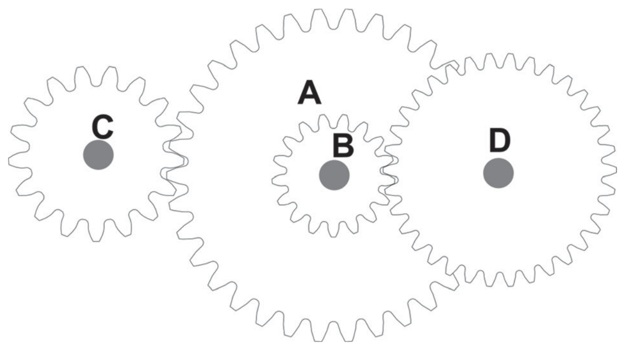

Figure 12. Schematic illustration of GTD.

$$
\begin{aligned}
g_3(\bar{x}) &= 0.07789x_1 - 2x_7^{-1}x_9 - 1 \leq 0,\\
g_4(\bar{x}) &= 7.05305x_9^{-1}x_1^2x_{10}x_8^{-1}x_2^{-1}x_{14}^{-1} - 1 \leq 0,\\
g_5(\bar{x}) &= 0.0833x_{13}^{-1}x_{14} - 1 \leq 0,\\
g_6(\bar{x}) &= 47.136x_2^{0.333}x_{10}^{-1}x_{12} - 1.333x_8x_{13}^{2.1195}\\
&\quad + 62.08x_{13}^{2.1195}x_{12}^{-1}x_8^{0.2}x_{10}^{-1} - 1 \leq 0,\\
g_7(\bar{x}) &= 0.04771x_{10}x_8^{1.8812}x_{12}^{0.3424} - 1 \leq 0,\\
g_8(\bar{x}) &= 0.0488x_9x_7^{1.893}x_{11}^{0.316} - 1 \leq 0,\\
g_9(\bar{x}) &= 0.0099x_1x_3^{-1} - 1 \leq 0,\\
g_{10}(\bar{x}) &= 0.0193x_2x_4^{-1} - 1 \leq 0,\\
g_{11}(\bar{x}) &= 0.0298x_1x_5^{-1} - 1 \leq 0,\\
g_{12}(\bar{x}) &= 0.056x_2x_6^{-1} - 1 \leq 0,\\
g_{13}(\bar{x}) &= 2x_9^{-1} - 1 \leq 0,\\
g_{14}(\bar{x}) &= 2x_{10}^{-1} - 1 \leq 0,\\
g_{15}(\bar{x}) &= x_{12}x_{11}^{-1} - 1 \leq 0
\end{aligned}
$$

with bounds:

$$0.001 \leq x_i \leq 5,\ i = 1, \ldots, 14.$$

3.4.4. Multiple disk clutch brake design (MDCBD)

The multiple disk clutch brake design problem aims to minimise the mass of the multiple disk clutch brake, as defined by Eq. (13). This problem is characterised by nine nonlinear constraints and involves five discrete design variables, namely the inner radius (x_1), outer radius (x_2), disk thickness (x_3), actuator force (x_4), and number of frictional surfaces (x_5). The schematic representation of this problem is depicted in Figure 13. The results of optimisation algorithms applied to this problem are tabulated in Table 18, revealing the superior performance of the ECO algorithm in comparison to other algorithms under consideration.

Minimise:

$$f(\bar{x}) = \pi(x_2^2 - x_1^2)x_3(x_5 + 1)\rho \tag{17}$$

subject to:

$$
\begin{aligned}
g_1(\bar{x}) &= -p_{max} + p_{rz} \leq 0,\\
g_2(\bar{x}) &= p_{rz}V_{sr} - V_{sr,max}p_{max} \leq 0,\\
g_3(\bar{x}) &= \Delta R + x_1 - x_2 \leq 0,\\
g_4(\bar{x}) &= -L_{max} + (x_5 + 1)(x_3 + \delta) \leq 0,\\
g_5(\bar{x}) &= sM_s - M_h \leq 0,\\
g_6(\bar{x}) &= T \geq 0,\\
g_7(\bar{x}) &= -V_{sr,max} + V_{sr} \leq 0,\\
g_7(\bar{x}) &= T - T_{max} \leq 0,
\end{aligned}
$$

where,

$$
\begin{aligned}
M_h &= \frac{2}{3}\mu x_4x_5\frac{x_2^3 - x_1^3}{x_2^2 - x_1^2}N.mm,\\
\omega &= \frac{\pi n}{30}rad/s,\\
A &= \pi(x_2^2 - x_1^2)mm^2,\\
p_{rz} &= \frac{x_4}{A}N/mm^2,
\end{aligned}
$$

Table 17. Results of ODIS problem.

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	Best	Worst	Average	STD	Median
ECO	1.00E-03	1.00E-03	1.00E-03	1.00E-03	1.00E-03	1.00E-03	1.55E + 00	1.52E + 00	4.97E + 00	4.97E + 00	2.86E-01	1.16E-01	3.74E-02	2.79E-01	2.85E-01	2.12E + 01	5.73E + 00	3.75E + 01	2.91E + 00
ALO	1.00E-03	1.00E-03	1.00E-03	8.07E-01	1.00E-03	1.00E-03	1.62E + 00	3.58E + 00	4.05E + 00	2.09E + 00	1.00E-03	1.00E-03	6.60E-03	7.92E-02	2.21E-01	8.78E + 20	1.18E + 20	3.67E + 40	9.64E + 19
GWO	1.00E-03	1.01E-03	1.07E-03	2.87E-03	1.60E-03	2.82E-03	1.53E + 00	1.53E + 00	4.99E + 00	2.00E + 00	2.86E-03	2.75E-03	1.17E-02	1.39E-01	3.76E-02	9.87E + 19	1.27E + 19	1.04E + 39	5.58E-02
WOA	1.00E-03	1.00E-03	1.00E-03	1.46E + 00	1.10E-03	2.20E-03	1.52E + 00	1.53E + 00	5.00E + 00	2.22E + 00	1.57E-03	1.37E-03	2.02E-03	1.57E-02	2.85E-01	3.57E + 21	6.32E + 20	1.41E + 42	4.91E + 06
SSA	1.00E-03	1.00E-03	1.00E-03	1.01E + 00	9.75E-01	2.21E-01	2.62E + 00	4.02E + 00	4.64E + 00	2.05E + 00	9.72E-02	5.30E-03	1.47E-02	1.77E-01	2.17E + 00	1.01E + 20	3.35E + 18	3.26E + 38	2.69E + 01
AOA	1.00E-03	1.00E-03	1.00E-03	2.70E + 00	1.80E + 00	4.64E-01	1.79E + 00	2.73E + 00	5.00E + 00	5.00E + 00	3.37E-03	1.00E-03	5.16E-03	1.32E-02	4.07E + 00	2.40E + 20	7.06E + 19	4.19E + 39	9.31E + 19
HHO	1.00E-03	1.00E-03	1.00E-03	1.98E + 00	1.00E-03	1.00E-03	1.98E + 00	1.53E + 00	2.01E + 00	2.00E + 00	1.00E-03	1.00E-03	6.35E-03	7.28E-02	2.27E-01	7.48E + 20	4.40E + 19	1.89E + 40	1.86E + 03
SCA	1.01E-03	1.00E-03	1.63E-03	5.21E-03	1.47E-03	1.25E-03	1.99E + 00	1.55E + 00	5.00E + 00	2.48E + 00	9.78E-03	3.19E-03	1.27E-02	1.02E-01	9.29E-02	3.49E-01	2.01E-01	4.21E-03	1.90E-01
MVO	1.00E-03	1.00E-03	1.00E-03	1.00E-03	1.00E-03	1.00E-03	2.08E + 00	2.14E + 00	2.53E + 00	4.55E + 00	2.14E-01	2.62E-02	4.80E-02	5.64E-01	3.30E-01	9.38E + 19	2.50E + 19	1.72E + 39	5.46E + 00
ROA	5.63E-02	3.09E-02	1.61E-01	4.95E + 00	2.24E + 00	1.32E-01	8.13E-01	2.68E + 00	4.95E + 00	4.95E + 00	2.77E + 00	4.95E + 00	4.95E + 00	4.95E + 00	3.96E + 20	6.39E + 21	3.88E + 21	1.60E + 42	4.34E + 21

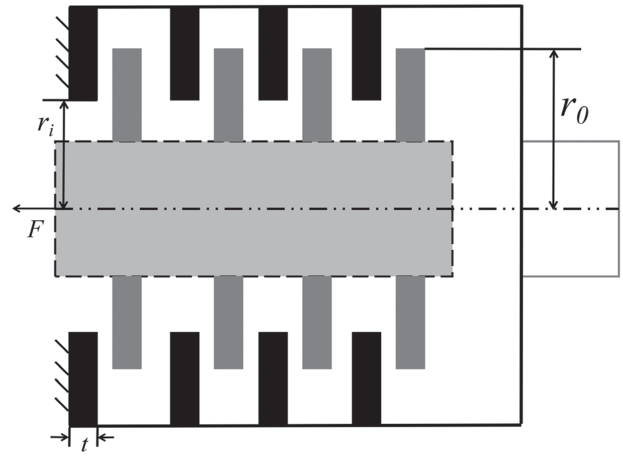

Figure 13. Schematic illustration of MDCBD.

$$V_{sr} = \frac{\pi R_{sr} n}{30} mm/s,$$
$$R_{sr} = \frac{2}{3}\frac{x_2^3 - x_1^3}{x_2^2 x_1^2} mm,$$
$$T = \frac{I_z \omega}{M_h + M_f},$$
$$\Delta R = 20mm,\ L_{max} = 30mm,\ L_{max} = 30mm,$$
$$L_{max} = 30mm,\ \mu = 0.6,$$
$$V_{sr,max} = 10m/s,\ \delta = 0.5mm,\ s = 1.5,$$
$$T_{max} = 15s,\ n = 250rpm,\ I_z = 55Kg \cdot m^2,$$
$$M_s = 40Nm,\ M_f = 3Nm,\ \text{and}\ p_{max} = 1,$$

with bounds:

$$60 \leq x_1 \leq 80,\ 90 \leq x_2 \leq 110,\ 1 \leq x_3 \leq 3$$
$$0 \leq x_4 \leq 1000,\ 2 \leq x_5 \leq 9.$$

3.4.5. Speed reducer design (SRD)

The primary aim of the SRD problem, classified as a discrete challenge, is to identify the optimal weight for the speed reducer while adhering to four essential design constraints. These constraints encompass the bending stress of the gear teeth, covering stress, transverse deflections of the shafts, and stresses within the shafts, all depicted in Figure 14. Consequently, the problem involves managing one discrete variable and six continuous variables. Specifically, x_1 signifies the face width, x_2 represents the module of teeth, and x_3 denotes a discrete design variable pertaining to the arrangement of teeth in the pinion. Correspondingly,

Table 18. Results of MDCBD problem.

	x1	x2	x3	x4	x5	Best	Worst	Average	STD	Median
ECO	7.00E + 01	9.00E + 01	1.00E + 00	2.72E + 02	2.00E + 00	2.35E-01	2.35E-01	2.35E-01	3.30E-14	2.35E-01
ALO	7.00E + 01	9.00E + 01	1.00E + 00	4.89E + 02	2.00E + 00	2.35E-01	2.35E-01	2.35E-01	8.66E-17	2.35E-01
GWO	7.00E + 01	9.00E + 01	1.00E + 00	1.60E + 02	2.00E + 00	2.35E-01	2.36E-01	2.35E-01	5.00E-09	2.35E-01
WOA	7.00E + 01	9.00E + 01	1.00E + 00	1.00E + 03	2.00E + 00	2.35E-01	2.35E-01	2.35E-01	1.95E-14	2.35E-01
SSA	6.00E + 01	8.00E + 01	6.00E + 01	8.00E + 01	6.00E + 01	1.34E + 27	1.34E + 27	1.34E + 27	0.00E + 00	1.34E + 27
AOA	7.00E + 01	9.00E + 01	1.00E + 00	1.00E + 03	2.00E + 00	2.35E-01	2.66E-01	2.40E-01	6.78E-05	2.36E-01
HHO	7.00E + 01	9.00E + 01	1.00E + 00	2.33E + 02	2.00E + 00	2.35E-01	2.35E-01	2.35E-01	2.59E-17	2.35E-01
SCA	7.00E + 01	9.00E + 01	1.00E + 00	1.00E + 03	2.00E + 00	2.35E-01	2.40E-01	2.37E-01	1.48E-06	2.37E-01
MVO	7.00E + 01	9.00E + 01	1.00E + 00	9.51E + 02	2.00E + 00	2.35E-01	2.53E-01	2.36E-01	1.04E-05	2.35E-01
ROA	7.00E + 01	9.00E + 01	1.00E + 00	1.00E + 03	2.00E + 00	2.35E-01	3.31E-01	3.04E-01	9.43E-04	3.15E-01

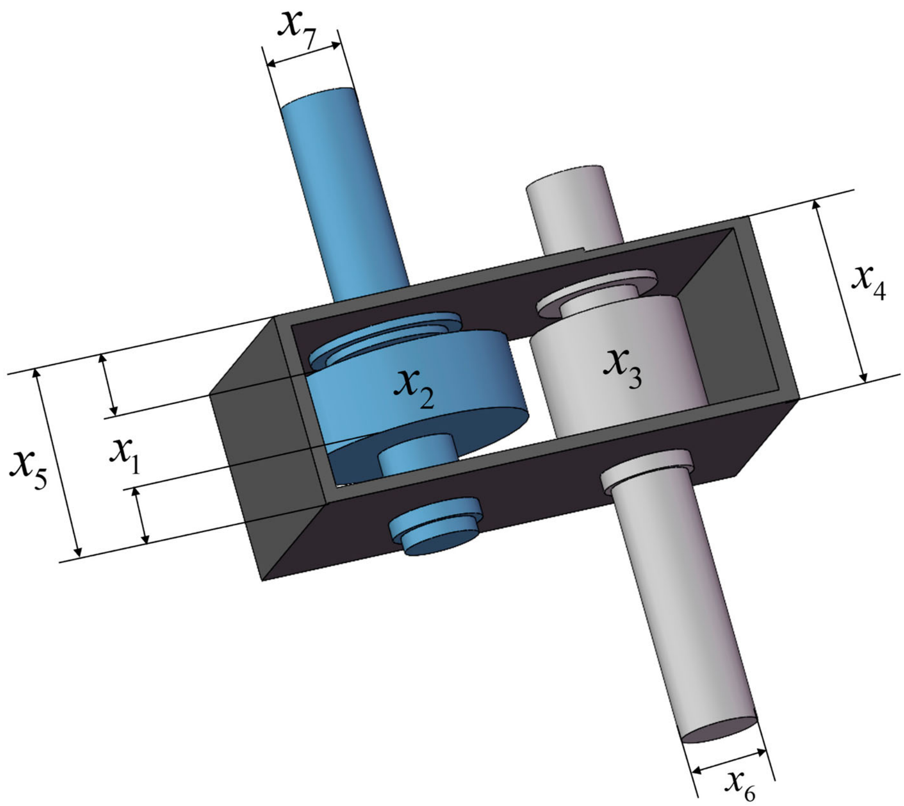

Figure 14. Schematic illustration of SRD.

x_4 signifies the length of the first shaft between bearings, while x_5 pertains to the length of the second shaft between bearings. The sixth and seventh design variables (x_6 and x_7) correspond to the diameters of the first and second shaft, respectively. The mathematical formulation of this task is as follows. Table 19 reports the results obtained by optimisation algorithms for this problem, in which ECO algorithms outperform other comparative algorithms.

Minimise:

$$f(x) = 0.7854x_2^2x_1(14.9334x_3 - 43.0934 + 3.3333x_3^2) + 0.7854(x_5x_7^2 + x_4x_6^2) - 1.508x_1(x_7^2 + x_6^2) + 7.477(x_7^2 + x_6^2) \quad (18)$$

subject to:

$$g_1(x) = -x_1x_2^2x_3 + 27 \le 0,$$

$$g_2(x) = -x_1x_2^2x_3 + 397.5 \le 0,$$

$$g_3(x) = -x_2x_6^4x_3x_4^{-3} + 1.93 \le 0,$$

$$g_4(x) = -x_2x_7^4x_3x_5^{-3} + 1.93 \le 0,$$

$$g_5(x) = 10x_6^{-3}\sqrt{16.91 \times 10^6 + (745x_4x_2^{-1}x_3^{-1})^2} - 1100 \le 0,$$

$$g_6(x) = 10x_7^{-3}\sqrt{157.5 \times 10^6 + (745x_5x_2^{-1}x_3^{-1})^2} - 850 \le 0,$$

$$g_7(x) = x_2x_3 - 40 \le 0,$$

Table 19. Results of SRD problem.

	x1	x2	x3	x4	x5	x6	x7	Best	Worst	Average	STD	Median
ECO	3.50E+00	7.00E-01	1.70E+01	7.33E+00	7.72E+00	3.35E+00	5.29E+00	2.99E+03	3.04E+03	3.01E+03	1.43E+02	3.01E+03
ALO	3.50E+00	7.00E-01	1.70E+01	7.40E+00	7.72E+00	3.35E+00	5.29E+00	3.00E+03	3.02E+03	3.00E+03	1.76E+01	3.01E+03
GWO	3.50E+00	7.00E-01	1.70E+01	7.37E+00	7.84E+00	3.35E+00	5.29E+00	3.00E+03	3.03E+03	3.01E+03	2.80E+01	3.01E+03
WOA	3.50E+00	7.00E-01	1.70E+01	7.98E+00	8.03E+00	3.38E+00	5.29E+00	3.01E+03	5.32E+03	3.26E+03	1.97E+05	3.13E+03
SSA	3.60E+00	3.60E+00	3.60E+00	3.60E+00	2.60E+00	3.35E+00	3.60E+00	3.39E+26	3.39E+26	3.39E+26	3.43E+39	3.39E+26
AOA	3.60E+00	7.00E-01	1.70E+01	8.30E+00	8.30E+00	3.36E+00	5.30E+00	3.07E+03	3.23E+03	3.16E+03	1.96E+03	3.17E+03
HHO	3.53E+00	7.00E-01	1.70E+01	8.04E+00	8.07E+00	3.35E+00	5.30E+00	3.03E+03	5.37E+03	3.66E+03	4.08E+05	3.49E+03
SCA	3.57E+00	7.00E-01	1.70E+01	7.60E+00	8.23E+00	3.39E+00	5.32E+00	3.07E+03	3.20E+03	3.14E+03	1.73E+03	3.14E+03
MVO	3.50E+00	7.00E-01	1.70E+01	7.90E+00	8.27E+00	3.36E+00	5.29E+00	3.02E+03	3.08E+03	3.05E+03	1.84E+02	3.05E+03
ROA	3.52E+00	7.00E-01	1.70E+01	8.12E+00	8.12E+00	3.49E+00	5.29E+00	3.06E+03	2.50E+19	2.17E+18	3.62E+37	3.26E+03

$$g_8(x) = -x_1x_2^{-1} + 5 \leq 0,$$
$$g_9(x) = x_1x_2^{-1} - 12 \leq 0,$$
$$g_{10}(x) = 1.5x_6 - x_4 + 1.9 \leq 0,$$
$$g_{11}(x) = 1.1x_7 - x_5 + 1.9 \leq 0,$$

where,

$$2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28,$$

$$x_4 \leq 8.3, 7.3 \leq x_5, 2.9 \leq x_6 \leq 3.9, 5 \leq x_7 \leq 5.5,$$

3.4.6. Rolling element bearing design (REBD)

This engineering problem involves 10 geometric variables and considers nine assembly constraints along with geometric-based limitations. Our objective in addressing this scenario is to optimise (maximise) the dynamic load-carrying capacity. The formulation of this test case is outlined below. The schematic of this problem is shown in Figure 15. Table 20 reports the obtained results of optimisation algorithms for this problem in which ECO algorithms outperform other comparative algorithms.

Maximise:

$$f(\bar{x}) = \begin{cases} f_c Z^{\frac{2}{3}} D_b^{1.8} & \text{if } D_b \leq 25.4\text{mm} \\ 3.647 f_c Z^{\frac{2}{3}} D_b^{1.4}, & \text{otherwise} \end{cases} \quad (19)$$

where:

$$g_1(\bar{x}) = Z - \frac{\phi_0}{2\sin^{-1}\left(\frac{D_b}{D_m}\right)} - 1 \leq 0,$$
$$g_2(\bar{x}) = K_{Dmin}(D-d) - 2D_b \leq 0,$$
$$g_3(\bar{x}) = 2D_b - K_{Dmax}(D-d) \leq 0,$$
$$g_4(\bar{x}) = D_b - w \leq 0,$$
$$g_5(\bar{x}) = 0.5(D+d) - D_m \leq 0,$$
$$g_6(\bar{x}) = D_m - (0.5+e)(D+d) \leq 0,$$
$$g_7(\bar{x}) = \xi D_b - 0.5(D - D_m - D_b) \leq 0,$$
$$g_8(\bar{x}) = 0.515 - f_i \leq 0,$$
$$g_9(\bar{x}) = 0.515 - f_0 \leq 0,$$

where,

$$f_c = 37.91\left\{1 + \left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72} \times \left(\frac{f_i(2f_0-1)}{f_0(2f_i-1)}\right)^{0.41}\right\}^{\frac{10}{3}}\right\}^{-0.3},$$

$$\gamma = \frac{D_b\cos(\alpha)}{D_m}, f_i = \frac{r_i}{D_b}, f_0 = \frac{r_0}{D_b}$$

$$\phi_0 = 2\pi - 2\times\cos^{-1}\left(\frac{\left\{\frac{(D-d)}{2-3\left(\frac{T}{4}\right)}\right\}^2 + \left\{\frac{D}{2} - \left(\frac{T}{4}\right) - D_b\right\}^2 - \left\{\frac{d}{2+\left(\frac{T}{4}\right)}\right\}^2}{2\left\{\frac{(D-d)}{2} - 3\left(\frac{T}{4}\right)\right\}\left\{\frac{D}{2} - \left(\frac{T}{4}\right) - D_b\right\}}\right)$$

$$T = D = d - 2D_b, D = 160, d = 90, B_w = 30.$$

with bounds:

$$0.5(D+d) \leq D_m \leq 0.6(D+d),$$
$$0.15(D-d) \leq D_b \leq 0.45(D-d),$$
$$4 \leq Z \leq 50,$$
$$0.515 \leq f_i \leq 0.6,$$
$$0.515 \leq f_0 \leq 0.6,$$
$$0.4 \leq K_{Dmin} \leq 0.5,$$
$$0.6 \leq K_{Dmax} \leq 0.7,$$
$$0.3 \leq \xi \leq 0.4,$$

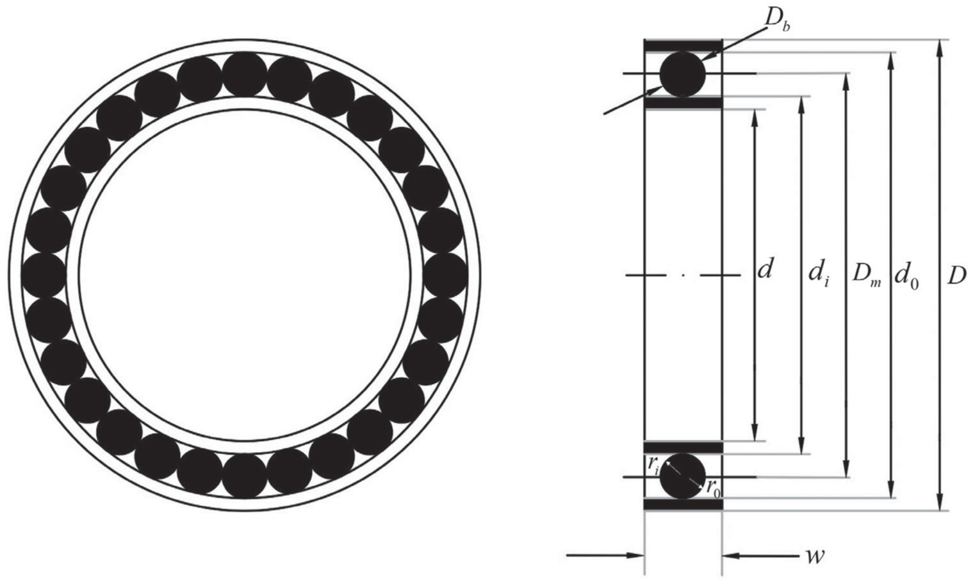

Figure 15. Schematic illustration of REBD.

$$0.02 \leq e \leq 0.1,$$

$$0.6 \leq \zeta \leq 0.85,$$

3.4.7. The ECO algorithm is good at solving real-world problems

Table 21 presents the performance evaluation of the ECO algorithm and nine existing frontier algorithms on six real-world engineering problems. The outcomes unequivocally indicate that the ECO algorithm outperforms the other nine optimisation algorithms and exhibits superior stability. These findings demonstrate the compelling competitiveness of the ECO algorithm in regard to solving optimisation problems with realistic constraints. The algorithm's consistently delivering excellent results further underscores its potential as an effective and robust optimisation tool for real-world applications.

3.5. Strengths and limitations of the ECO

As highlighted in the paper, the proposed ECO algorithm demonstrates significant theoretical potential in solving a wide range of optimisation problems and surpasses nine state-of-the-art algorithms. ECO adopts a simulation of intense competition in education by categorising populations into schools and students. It performs exceptionally well in handling single-peak, multipeak, and hybrid functions and in solving practical real-world problems.

It must be remembered to keep in mind that, like many other techniques, ECO cannot ensure the calculation of the best possible outcome. Especially when the original answer is already near the ideal option (e.g. cases F21-F23), the ECO algorithm may exhibit a lower convergence rate, necessitating more iterations to obtain a better solution. Future study should emphasize further assessing ECO's performance on real-world situations, which will provide helpful information that will contribute to its continuing improvement.

Moreover, the experimental findings suggest that ECO may occasionally converge to local optimal solutions within a limited set of functions, as exemplified by F13 among the 23 classical test functions. Addressing this issue warrants further investigation in our upcoming research endeavours.

4. Conclusions and future works

In conclusion, the Educational Competition Optimizer (ECO) presented in this paper has demonstrated its effectiveness as a meta-heuristic algorithm inspired by educational competition. ECO models the progressive competitiveness of students through three stages: elementary, middle, and high school, effectively

Table 20. Results of REBD problem.

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	Best	Worst	Average	STD	Median
ECO	1.26E + 02	2.14E + 01	1.06E + 01	5.15E-01	5.89E-01	4.98E-01	6.98E-01	3.02E-01	9.03E-02	6.35E-01	−8.52E + 04	−8.35E + 04	−8.45E + 04	5.72E + 04	−8.45E + 04
ALO	1.26E + 02	2.14E + 01	1.14E + 01	5.15E-01	5.15E-01	4.00E-01	6.19E-01	3.00E-01	9.96E-02	6.00E-01	−8.55E + 04	−8.46E + 04	−8.54E + 04	5.54E + 04	−8.55E + 04
GWO	1.26E + 02	2.14E + 01	1.10E + 01	5.15E-01	5.64E-01	4.86E-01	6.95E-01	3.00E-01	2.26E-02	6.18E-01	−8.55E + 04	−8.47E + 04	−8.53E + 04	2.84E + 04	−8.54E + 04
WOA	1.25E + 02	2.14E + 01	1.09E + 01	5.15E-01	5.25E-01	4.10E-01	6.11E-01	3.08E-01	3.15E-02	6.44E-01	−8.51E + 04	−4.52E + 04	−7.67E + 04	1.05E + 08	−8.17E + 04
SSA	1.25E + 02	1.25E + 02	1.50E + 02	1.50E + 02	1.25E + 02	1.50E + 02	1.25E + 02	1.50E + 02	1.25E + 02	1.25E + 02	6.55E + 04	6.55E + 04	6.55E + 04	0.00E + 00	6.55E + 04
AOA	1.25E + 02	2.19E + 01	9.61E + 00	5.15E-01	5.61E-01	4.00E-01	6.45E-01	3.00E-01	2.42E-02	6.00E-01	−8.32E + 04	−6.87E + 04	−7.59E + 04	1.66E + 07	−7.68E + 04
HHO	1.26E + 02	2.14E + 01	1.13E + 01	5.15E-01	5.15E-01	4.38E-01	6.25E-01	3.00E-01	5.63E-02	6.66E-01	−8.55E + 04	−4.23E + 04	−6.80E + 04	2.25E + 08	−6.98E + 04
SCA	1.25E + 02	2.12E + 01	1.12E + 01	5.15E-01	5.15E-01	4.00E-01	7.00E-01	3.00E-01	2.00E-02	6.00E-01	−8.36E + 04	−7.12E + 04	−7.90E + 04	7.82E + 06	−7.91E + 04
MVO	1.26E + 02	2.14E + 01	1.08E + 01	5.15E-01	5.20E-01	4.46E-01	6.75E-01	3.00E-01	9.29E-02	6.78E-01	−8.55E + 04	−7.85E + 04	−8.48E + 04	2.82E + 06	−8.54E + 04
ROA	1.25E + 02	2.10E + 01	1.12E + 01	5.15E-01	5.15E-01	4.00E-01	6.00E-01	3.00E-01	5.42E-02	6.00E-01	−8.25E + 04	−7.08E + 04	−8.07E + 04	9.96E + 06	−8.21E + 04

mirroring the process of acquiring a better education. The algorithm classifies students into two categories based on learning patience and employs an alternating stage strategy to achieve rapid convergence, demonstrating its potential for solving optimisation problems, particularly in constrained engineering design scenarios.

In the experiments we conducted, we initially performed a comprehensive parameter sensitivity analysis to optimise ECO's performance, revealing that a population size of 40 offers the highest performance. Interestingly, the population size does not correlate linearly with convergence ability. Subsequently, ECO was rigorously compared with nine state-of-the-art algorithms across a diverse set of functions encompassing single-peak, multimodal, hybrid, and combined functions. The results showcased ECO's superior ability to balance exploration and exploitation, positioning it as a robust contender among its peers. Moreover, ECO exhibited its prowess by successfully addressing five engineering optimisation problems. In comparative evaluations against nine renowned algorithms, ECO consistently outperformed or held its ground in five selected performance metrics. This robust and efficient performance can be theoretically attributed to three key factors: 1) A constant alternation of exploration and exploitation through a strategy involving three educational competition phases for efficient convergence. 2) A multi-strategy search methodology ensuring algorithmic stochasticity and population diversity. 3) Drawing inspiration from merit-based educational competition to eliminate low-quality solutions from the search population, effectively reducing performance degradation.

While ECO has demonstrated its potential, areas warranting further investigation in future research remain. Striking the optimal balance between the three phases of ECO presents a significant challenge but can potentially enhance algorithm efficiency and performance. Besides, the application of ECO can also be extended to other domains such as energy (Chen et al., 2023), image segmentation (Shi et al., 2023), the Internet of Things (Lakhan et al., 2022), or scheduling problems (Hussain et al., 2022), thereby enhancing its universality. Moreover, delving into hybrid methodologies that fuse ECO with other well-established metaheuristics shows great potential. These efforts could yield more robust and adaptable optimisation techniques, unlocking novel avenues and

Table 21. Rank of real-world problem.

	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
TSD	2	4	1	5	10	8	6	3	9	7
GTD	2	8	1	7	5	9	4	6	3	10
ODIS	2	8	4	9	3	7	6	1	5	10
MDCBD	1	3	5	4	10	8	2	7	6	9
SRD	2	1	3	7	10	6	8	5	4	9
REBD	4	1	2	7	10	8	9	6	3	5
Average Rank	2.17	4.17	2.67	6.50	8.00	7.67	5.83	4.67	5.00	8.33
Final Ranking	1	3	2	7	9	8	6	4	5	10

potentially propelling the field of optimisation forward significantly.

CRediT authorship contribution statement

Junbo Lian: Conceptualisation, Investigation, Methodology, Resources, Software, Formal analysis, Data curation, Visualisation, Writing – Original Draft, Writing – Review & Editing.

Ting Zhu: Data curation, Validation, Writing – Original Draft, Writing – Review & Editing, Formal analysis.

Ling Ma: Data curation, Validation, Writing – Original Draft, Writing – Review & Editing, Formal analysis.

Xincan Wu: Visualisation, Writing – Original Draft, Writing – Review & Editing, Formal analysis.

Ali Asghar Heidari: Formal analysis, Resources, Investigation, Writing – Review & Editing, Writing – Original Draft, Software.

Yi Chen: Investigation, Writing – Review & Editing, Writing – Original Draft, Visualisation, Formal analysis.

Huiling Chen: Supervision, Formal analysis, Resources, Investigation, Writing – Original Draft, Writing – Review & Editing, Formal analysis, Software.

Guohua Hui: Supervision, Funding acquisition, Project administration, Writing – Original Draft, Writing – Review & Editing, Formal analysis, Software.

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Data availability statement

Data will be made available on request.

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Table A1. Unimodal benchmark functions.

Function	Dim	Range	Shift position	f_{min}
$F_1(x) = \sum_{i=1}^{n} x_i^2$	30	$[-100, 100]$	$[-30, -30, \cdots, -30]$	0
$F_2(x) = \sum_{i=1}^{n} \lvert x_i\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$	30	$[-10, 10]$	$[-3, -3, \cdots, -3]$	0
$F_3(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{n} x_j\right)^2$	30	$[-100, 100]$	$[-30, -30, \cdots, -30]$	0
$F_4(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j\right)^2$	30	$[-100, 100]$	$[-30, -30, \cdots, -30]$	0
$F_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30	$[-30, 30]$	$[-15, -15, \cdots, -15]$	0
$F_6(x) = \sum_{i=1}^{n} (\lvert x_i + 0.5\rvert)^2$	30	$[-100, 100]$	$[-750, -750, \cdots, -750]$	0
$F_7(x) = \sum_{i=1}^{n} ix^4 + random[0, 1)$	30	$[-1.28, 1.28]$	$[-0.25, -0.25, \cdots, -0.25]$	0

Table A2. Multimodal benchmark functions.

Function	Dim	Range	Shift position	f_{min}
$F_8(x) = \sum_{i=1}^{n} -x_i sin\left(\sqrt{\lvert x_i\rvert}\right)$	30	$[-500, 500]$	$[-300, \cdots, -300]$	-12569.5
$F_9(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$	30	$[-5.12, 5.12]$	$[-2, -2, \cdots, -2]$	0
$F_{10}(x) = -20exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	30	$[-32, 32]$		0
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{(i)}}\right) + 1$	30	$[-600, 600]$	$[-400, \cdots, -400]$	0
$F_{12}(x) = \frac{\pi}{n}\left\{10sin(\pi y_i) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10sin^2(\pi y_{i+1})] + (y_n - 1)^2\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{x_i + 1}{4}$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m x_i > a \\ 0 - a < x_i < a \\ k(-x_i - a)^m x_i < -a \end{cases}$	30	$[-50, 50]$	$[-30, -30, \cdots, -30]$	0
$F_{13}(x) = 0.1\left\{sin^2(3\pi x_1) + \sum_{i=1}^{n1}(x_i - 1)^2[1 + sin^2(3\pi x_i + 1)] + (x_n - 1)^2[1 + sin^2(2\pi x_n)]\right\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	30	$[-50, 50]$	$[-100, \cdots, -100]$	0

Table A3. Fixed-dimension multimodal benchmark functions.

Function	Dim	Range	Shift position	f_{min}
$F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2} (x_i - a_{ij})^6}\right)^{-1}$	2	$[-65, 65]$	$[-2, -2, \cdots, -2]$	1
$F_{15}(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$	4	$[-5, 5]$	$[-2, -2, \cdots, -2]$	0.0003075
$F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	$[-5, 5]$	$[-2, -2, \cdots, -2]$	−1.0316285
$F_{17}(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right) + 10\left(1 - \frac{1}{8\pi}\right)\cos x_1 + 10$	2	$[-5, 5]$	$[-2, -2, \cdots, -2]$	0.398
$F_{18}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 16x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	2	$[-2, 2]$	$[-2, -2, \cdots, -2]$	3
$F_{19}(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2\right)$	3	$[1, 3]$	$[-2, -2, \cdots, -2]$	−3.86
$F_{20}(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2\right)$	6	$[0, 1]$	$[-2, -2, \cdots, -2]$	−3.32
$F_{21}(x) = -\sum_{i=1}^{5} [(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	$[0, 10]$	$[-2, -2, \cdots, -2]$	−10.1532
$F_{22}(x) = -\sum_{i=1}^{7} [(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	$[0, 10]$	$[-2, -2, \cdots, -2]$	−10.4028
$F_{23}(x) = -\sum_{i=1}^{1} [(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	$[0, 10]$	$[-2, -2, \cdots, -2]$	−10.536

Table A4. CEC2021 benchmark functions.

	No.	Functions	F_i^*
Unimodal Function	1	Shifted and Rotated Bent Cigar Function	100
Basic Functions	2	Shifted and Rotated Schwefel's Function	1100
	3	Shifted and Rotated Lunacek bi-Rastrigin Function	700
	4	Expanded Rosenbrock's plus Griewangk's Function	1900
Hybrid Functions	5	Hybrid Function 1 (N = 3)	1700
	6	Hybrid Function 2 (N = 4)	1600
	7	Hybrid Function 3 (N = 5)	2100
Composition Functions	8	Composition Function 1(N = 3)	2200
	9	Composition Function 2 (N = 4)	2400
	10	Composition Function 3(N = 5)	2500
Search range: $[-100,100]^{dim}$			