

# Mapper & De-Mapper System Document

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## **Mapper / De-Mapper**

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## 1. High Level System and Function Block

As seen in Figure 1, the high level modem system consists of a transmitter (Tx) and a receiver (Rx) section. The encoder and modulator in the transmitter receives data from the general purpose multiplexer (GPM) and creates the data frame for transmission through the DAC and analog radio frequency devices to the antenna. The encoder consists of two forward error correction codes: one is a Reed-Solomon (R-S) encoder and the other is a low density parity check (LDPC) encoder. GScom uses concatenated encoders of R-S and LDPC for high order modulation QAM signals and a single LDPC encoder for low order QAM signals. The encoder adds parity symbols and bits to inputted data symbols and bits, respectively, to make R-S encoded data and LDPC encoded data formats for forward error correction. The encoded data bits go to a Mapper to map those bits into symbols according to the selected modulation scheme. The symbols are then modulated into real signals (I) and imaginary signals (Q). The digital format of the airframe is converted to analog signals by a digital to analog converter (DAC) and is then sent for transmission.

At the receiver, the analog to digital converter (ADC) receives the analog signal, converts the received analog signal to a digital signal and then sends it to the demodulator block. The digital signals are processed by the demodulator, sent to the De-mapper for converting the demodulated symbols to bits according to the selected modulation scheme, and then sent to the decoder for error correction and for decoding. The decoder consists of two forward error correction codes: one is a Reed-Solomon (R-S) decoder and the other is a low density parity check (LDPC) decoder. GScom uses concatenated decoders of R-S and LDPC for high order modulation QAM signals and a single LDPC decoder for low order QAM signals.

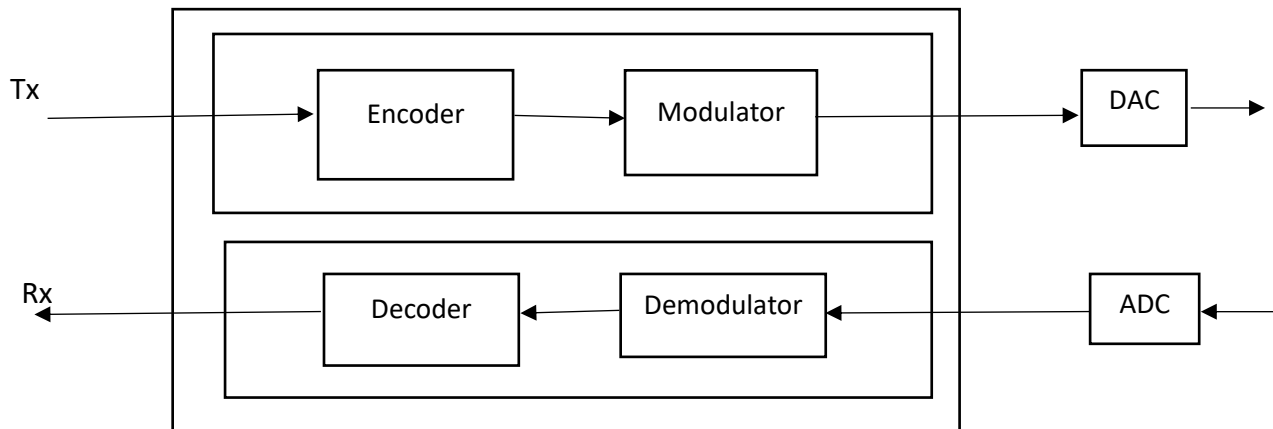


Figure 1, High level modem system

### 1.1. Mapper

The Mapper module takes a stream of coded bits, divides them into groups of  $m$  bits, maps each group of  $m$  bits in one of  $M (=2^m)$  signal constellations using Gray code mapping rule, and generates the stream of corresponding symbols as follows;

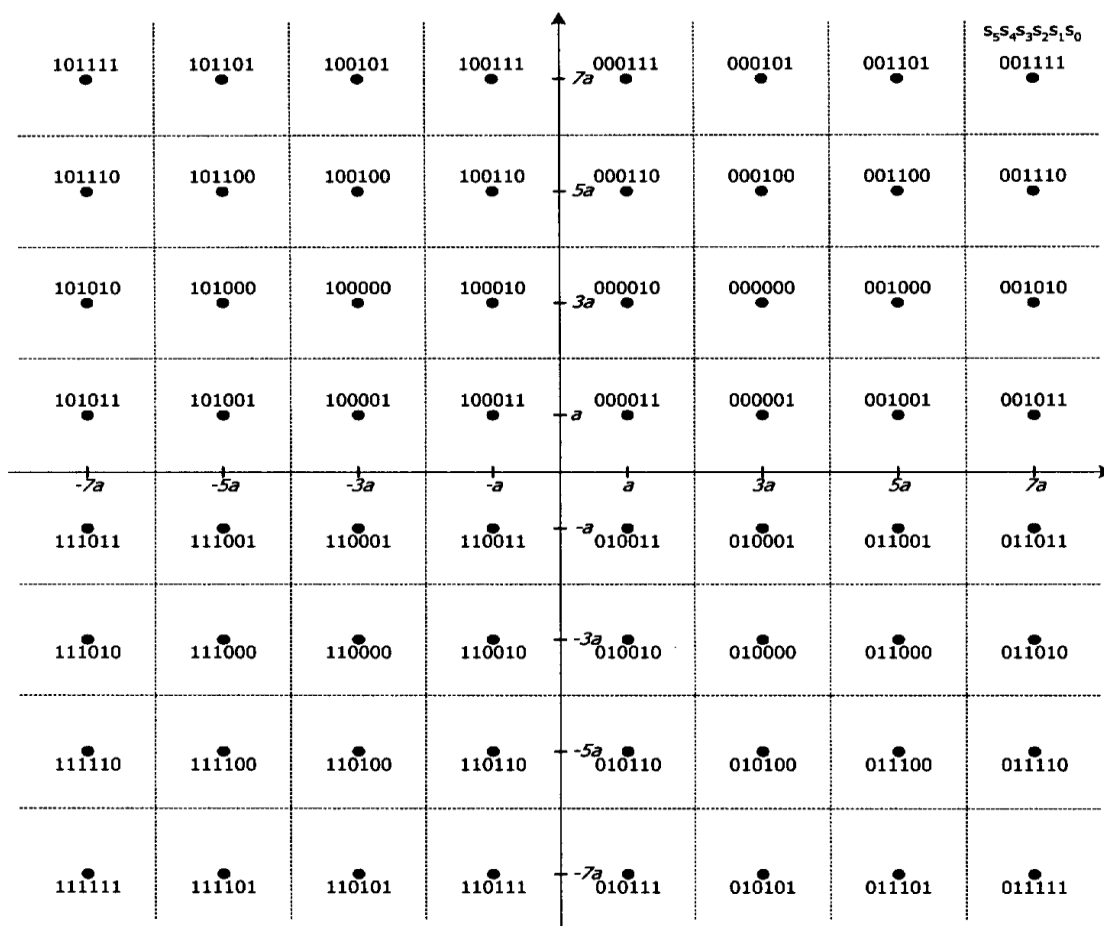
$$s_{k,m-1}s_{k,m-2}s_{k,m-3}\dots s_{k,0} \xrightarrow{\text{map}} I_k, Q_k$$

In the above equation,  $s_{k,i}$  ( $i = 0,1,2, \dots, m-1$ ) represents the  $i^{th}$  bit in the  $k^{th}$  symbol to be coded by the Gray code mapping, and  $I_k$  and  $Q_k$  represents the in-phase and quadrature components of the  $k^{th}$  symbol, respectively. The Gray mapping for mapping for coded binary bits into symbols can be done with the following recursive formula.

$$\text{gray}(k) = 2^{n-1} + \text{gray}(2^n - 1 - k), \quad 2^{n-1} \leq k < 2^n$$

with  $\text{gray}(0) = 0$ ,  $\text{gray}(1) = 1$ , as initial values.

The Mapper is programmable to support different signal constellations and different modulations up to 4096QAM. The Mapper supports six standard gray mapped constellations: QPSK, 16QAM, 64QAM, 256QAM, 1024QAM, and 4096QAM. In case of 64QAM, as an example,  $m = 6$  and its corresponding signal constellation is as follows;



## 2. Demodulator Functional Block

The demodulator in the receiver receives a stream of symbol data from ADC (analog to digital converter). The demodulator performs frequency and timing (symbol and frame) synchronization process with keeping AGC (automatic gain control) process on the received symbols. After obtaining the synchronization, the demodulator performs an equalization process to mitigate the channel effect on the received symbols. Then the output of the Equalizer goes to De-mapper (or Slicer) and then goes to the LDPC decoder for data correction.

The following Figure 2 shows the functional block diagram of demodulator.

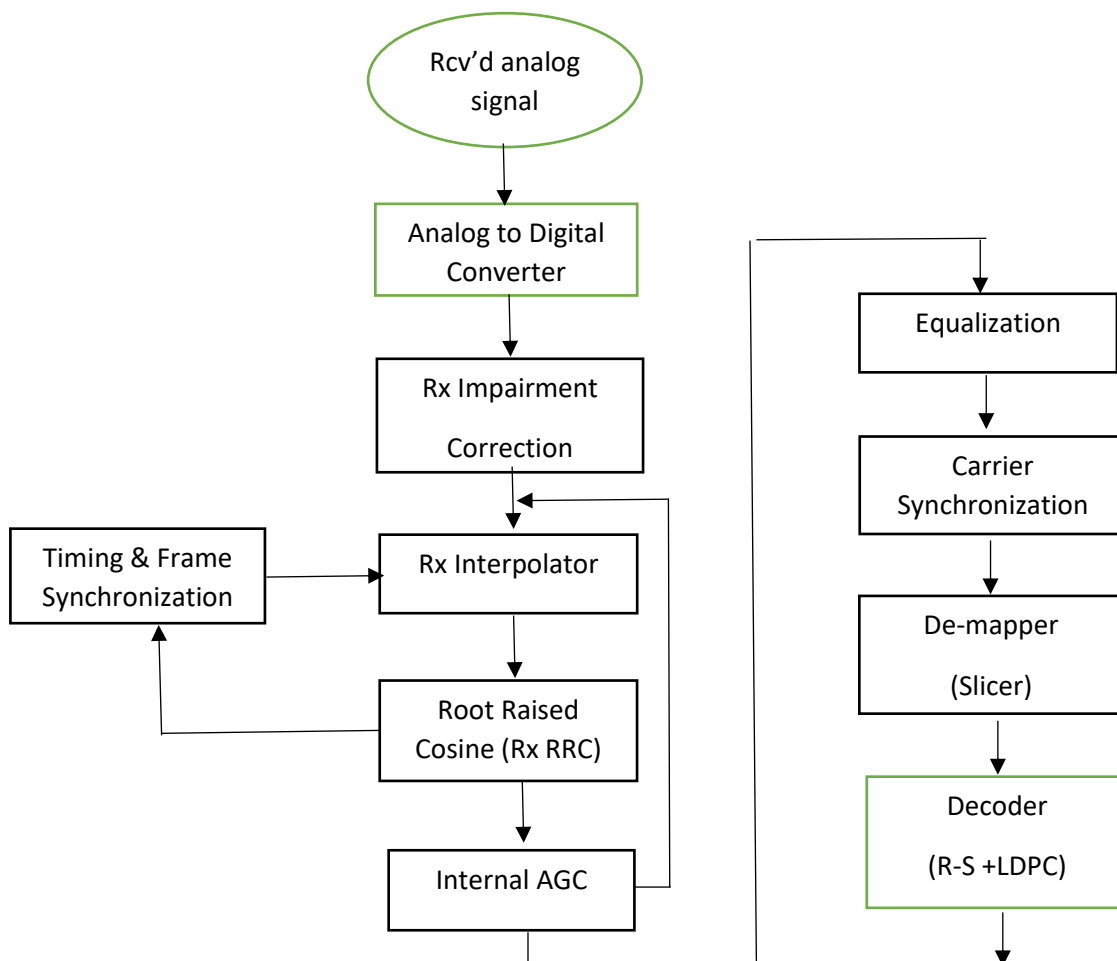


Figure 2: Demodulator Functional Block Diagram

## 2.1 Decoder Block

The Rx decoder consists of a De-Mapper (or Slicer), an LDPC decoder, and a Reed-Solomon decoder. The Rx decoder receives demodulated symbols from demodulator, converts the demodulated symbols to bit stream through a de-mapping process, and then performs the error correction on the received bit streams of data.

### 2.1.1. De-Mapper

The Mapper module takes the coded bits and maps the bit stream into symbol streams. The Mapper is programmable to support different signal constellations and different modulations up to 4096QAM. The Mapper supports six standard gray mapped constellations: QPSK, 16QAM, 64QAM, 256QAM, 1024QAM, and 4096QAM.

The modulator in the transmitter receives a stream of symbol data from Mapper. The symbols are then modulated into real signals (I) and imaginary signals (Q). The digital format of the airframe is converted to analog signals by digital to analog converter (DAC) and is sent for transmission.

The receiver receives the transmitted signal through an analog to digital converter (ADC) and demodulates the received signal symbols. The De-Mapper receives the demodulated symbols from demodulator, converts the received symbols into the soft values of the corresponding bit stream through a soft decision decoding process. The De-Mapper output signals are the soft decision value of the bit stream of the corresponding symbols. It is well established that LDPC provides the best performance when the soft value of bit is used as its input.

We can represent the demodulated received signal,  $R_k$ , as follows;

$$R_k = X_k + jY_k = g_k(I_k + jQ_k) + (n_k^I + jn_k^Q)$$

where  $X_k$  and  $Y_k$  are the in-phase and quadrature components of the modulated signal, respectively,  $g_k$  is the transmission path gain of the signal,  $n_k^I$  and  $n_k^Q$  are independently distributed zero mean white Gaussian noise.

The estimation of bit,  $S_{k,i}$  ( $i = 0, 1, 2 \dots m$ ), in symbol  $S_k$  is obtained by taking the log likelihood ratio (LLR) of the conditional probability of the bit being correct, as follows;

$$\Lambda(S_{k,i}) = \log \frac{\Pr\{S_{k,i}=0 | X_k, Y_k\}}{\Pr\{S_{k,i}=1 | X_k, Y_k\}}, \quad i = 0, 1, 2, \dots, m-1$$

The above LLR can be approximated by dual minimum metric approximation as follows;

[Equation 1]

$$\begin{aligned} \Lambda(S_{k,i}) &= \log \frac{\sum_{z_i} \exp\{-1/\sigma_n^2 [R_k - z_k(S_{k,i}=0)]^2\}}{\sum_{z_i} \exp\{-1/\sigma_n^2 [R_k - z_k(S_{k,i}=1)]^2\}} \\ &\approx \{ \min [R_k - z_k(S_{k,i}=1)]^2 - \min [R_k - z_k(S_{k,i}=0)]^2 \} \\ &= (2n_{k,i} - 1) \{ [R_k - z_k(S_{k,i}=n_{k,i})]^2 - \min [R_k - z_k(S_{k,i}=\bar{n}_{k,i})]^2 \} \end{aligned}$$

Where  $z_k(S_{k,i}=0)$  and  $z_k(S_{k,i}=1)$  are the values of  $I_k + jQ_k$  when  $S_{k,i}=0$  and  $S_{k,i}=1$ , respectively. In the above equation,  $n_{k,i}$  represent the value of the  $i^{th}$  bit of the symbol which is the closest to the received symbol,  $R_k$ . The  $\bar{n}_{k,i}$  represent the negative value of  $n_{k,i}$ .

As seen from the above equation, the LLR can be calculated by finding the values of  $z_k(S_{k,i} = 1)$  and  $z_k(S_{k,i} = 0)$  which minimize the values of  $[R_k - z_k(S_{k,i} = 1)]^2$  and  $[R_k - z_k(S_{k,i} = 0)]^2$ , respectively. The values of  $z_k(S_{k,i} = 1)$  and  $z_k(S_{k,i} = 0)$  which minimize the values of  $[R_k - z_k(S_{k,i} = 1)]^2$  and  $[R_k - z_k(S_{k,i} = 0)]^2$  can be determined by the range of the values of the in-phase and quadrature components of the received symbol,  $R_k$ .

The first term in the third line in [Equation 1] can be represented using the in-phase and quadrature components of the received symbol (or signal) as follows;

[Equation 2]

$$[R_k - z_k(S_{k,i} = n_{k,i})]^2 = (X_k - U_k)^2 + (Y_k - V_k)^2$$

The  $U_k$  and  $V_k$  are the in-phase and quadrature components of the received symbol on a signal constellation, respectively.

The second term in the third line in [Equation 1] can be represented using the in-phase and quadrature components of the received symbol (or signal) as follows;

[Equation 3]

$$\min [R_k - z_k(S_{k,i} = \bar{n}_{k,i})]^2 = (X_k - U_{k,i})^2 + (Y_k - V_{k,i})^2$$

The  $U_{k,i}$  and  $V_{k,i}$  are the in-phase and quadrature components of the received symbol on a signal constellation, respectively, which minimize  $[R_k - z_k(S_{k,i} = \bar{n}_{k,i})]^2$ .

Using the [Equation 2] and [Equation 3] in [Equation 1], the LLR can be obtained as follows;

[Equation 4]

$$\begin{aligned} \Lambda(S_{k,i}) &= (2n_{k,i}-1) [ \{ (X_k - U_k)^2 + (Y_k - V_k)^2 \} - \{ (X_k - U_{k,i})^2 + (Y_k - V_{k,i})^2 \} ] \\ &= (2n_{k,i}-1) [ (U_k + U_{k,i} - 2X_k)(U_k - U_{k,i}) + (V_k + V_{k,i} - 2Y_k)(V_k - V_{k,i}) ] \end{aligned}$$

We can use [Equation 4] to derive the soft decision value of the LLR of any M-ary QAM for the input of the decoder. As we support from 4QAM up to 4096QAM in the modem, we need to derive six sets of those soft decision value of the LLRs for those of 4QAM, 16QAM, 64QAM, 256QAM, 1024QAM, and 4096QAM.

As seen in [Equation 4], there is need to find the values of  $U_k$ ,  $U_{k,i}$ ,  $V_k$ , and  $V_{k,i}$  for the corresponding received symbol  $S_{k,i}$  ( $= X_k + jY_k$ ) as a function of bit position of  $i$  ( $= 0, 1, \dots, m-1$ ) to obtain the soft decision value of the LLR,  $\Lambda(S_{k,i})$ , of the received symbol.



## LLR Derivation For 256QAM

GScom illustrates the derivation methodology for the soft decision values of the LLR for 256QAM as an example. For the 256QAM, the following two tables (Table 1 & Table 2) show the range of the quadrature component  $Y_k$  and in-phase component  $X_k$  of the received symbol  $R_k$ , their corresponding bit values, and their corresponding representative values of the  $V_k$  and  $U_k$  in those ranges.

[Table 1]

| $Y_k$               | $(n_{k,7} \ n_{k,6} \ n_{k,5} \ n_{k,4})$ | $V_k$ |
|---------------------|---|-------|
| $Y_k > 14a$         | (0000)                                    | 15a   |
| $12a < Y_k < 14a$   | (0001)                                    | 13a   |
| $10a < Y_k < 12a$   | (0011)                                    | 11a   |
| $8a < Y_k < 10a$    | (0010)                                    | 9a    |
| $6a < Y_k < 8a$     | (0110)                                    | 7a    |
| $4a < Y_k < 6a$     | (0111)                                    | 5a    |
| $2a < Y_k < 4a$     | (0101)                                    | 3a    |
| $0 < Y_k < 2a$      | (0100)                                    | a     |
| $-2a < Y_k < 0$     | (1100)                                    | -a    |
| $-4a < Y_k < -2a$   | (1101)                                    | -3a   |
| $-6a < Y_k < -4a$   | (1111)                                    | -5a   |
| $-8a < Y_k < -6a$   | (1110)                                    | -7a   |
| $-10a < Y_k < -8a$  | (1010)                                    | -9a   |
| $-12a < Y_k < -10a$ | (1011)                                    | -11a  |
| $-14a < Y_k < -12a$ | (1001)                                    | -13a  |
| $Y_k < -14a$        | (1000)                                    | -15a  |

[Table 2]

| $X_k$               | $(n_{k,3} \ n_{k,2} \ n_{k,1} \ n_{k,0})$ | $U_k$ |
|---------------------|---|-------|
| $X_k > 14a$         | (0000)                                    | 15a   |
| $12a < X_k < 14a$   | (0001)                                    | 13a   |
| $10a < X_k < 12a$   | (0011)                                    | 11a   |
| $8a < X_k < 10a$    | (0010)                                    | 9a    |
| $6a < X_k < 8a$     | (0110)                                    | 7a    |
| $4a < X_k < 6a$     | (0111)                                    | 5a    |
| $2a < X_k < 4a$     | (0101)                                    | 3a    |
| $0 < X_k < 2a$      | (0100)                                    | a     |
| $-2a < X_k < 0$     | (1100)                                    | -a    |
| $-4a < X_k < -2a$   | (1101)                                    | -3a   |
| $-6a < X_k < -4a$   | (1111)                                    | -5a   |
| $-8a < X_k < -6a$   | (1110)                                    | -7a   |
| $-10a < X_k < -8a$  | (1010)                                    | -9a   |
| $-12a < X_k < -10a$ | (1011)                                    | -11a  |
| $-14a < X_k < -12a$ | (1001)                                    | -13a  |
| $X_k < -14a$        | (1000)                                    | -15a  |

The following Table 3 shows the bit sequence  $\{m_{k,7}, m_{k,6}, \dots, m_{k,2}, m_{k,1}, m_{k,0}\}$  in the symbol which minimize  $[R_k - z_k(S_{k,i} = \bar{n}_{k,i})]^2$  for each bit position of  $i$  ( $i=0,1,2,\dots,7$ ), their representation in terms of  $\{n_{k,7}, n_{k,6}, \dots, n_{k,2}, n_{k,1}, n_{k,0}\}$ , their corresponding in-phase component  $U_{k,i}$ , and the quadrature component  $V_{k,i}$  of the symbol  $z_k(S_{k,i} = \bar{n}_{k,i})$ .

[Table 3]

| $i$ | $\{m_{k,7} m_{k,6} m_{k,5} m_{k,4} m_{k,3} m_{k,2} m_{k,1} m_{k,0}\}$       | $V_{k,i}$ | $U_{k,i}$ |
|-----|---|-----------|-----------|
| 7   | $\{\bar{n}_{k,7} 1 0 0 n_{k,3} n_{k,2} n_{k,1} n_{k,0}\}$                   | $V_{k,7}$ | $U_k$     |
| 6   | $\{n_{k,7} \bar{n}_{k,6} 1 0 n_{k,3} n_{k,2} n_{k,1} n_{k,0}\}$             | $V_{k,6}$ | $U_k$     |
| 5   | $\{n_{k,7} n_{k,6} \bar{n}_{k,5} 1 n_{k,3} n_{k,2} n_{k,1} n_{k,0}\}$       | $V_{k,5}$ | $U_k$     |
| 4   | $\{n_{k,7} n_{k,6} n_{k,5} \bar{n}_{k,4} n_{k,3} n_{k,2} n_{k,1} n_{k,0}\}$ | $V_{k,4}$ | $U_k$     |
| 3   | $\{n_{k,7} n_{k,6} n_{k,5} n_{k,4} \bar{n}_{k,3} 1 0 0\}$                   | $V_k$     | $U_{k,3}$ |
| 2   | $\{n_{k,7} n_{k,6} n_{k,5} n_{k,4} n_{k,3} \bar{n}_{k,2} 1 0\}$             | $V_k$     | $U_{k,2}$ |
| 1   | $\{n_{k,7} n_{k,6} n_{k,5} n_{k,4} n_{k,3} n_{k,2} \bar{n}_{k,1} 1\}$       | $V_k$     | $U_{k,1}$ |
| 0   | $\{n_{k,7} n_{k,6} n_{k,5} n_{k,4} n_{k,3} n_{k,2} n_{k,1} \bar{n}_{k,0}\}$ | $V_k$     | $U_{k,0}$ |

The values of  $V_{k,i}$  and  $U_{k,i}$  obtained by using the values of  $(m_{k,7}, m_{k,6}, m_{k,5}, m_{k,4})$  and  $(m_{k,3}, m_{k,2}, m_{k,1}, m_{k,0})$ , respectively, as a function of bit position  $i$  in the above table are shown in the next two tables (Table 4 & Table 5) as a function of the value of bits,  $n_{k,i}$  ( $i = 0, 1, \dots, m-1$ ).

[Table 4]

| $(n_{k,7} n_{k,6} n_{k,5} n_{k,4})$ | $V_{k,7}$ | $V_{k,6}$ | $V_{k,5}$ | $V_{k,4}$ |
|-------------------------------------|-----------|-----------|-----------|-----------|
| (0000)                              | -a        | 7a        | 11a       | 13a       |
| (0001)                              | -a        | 7a        | 11a       | 15a       |
| (0011)                              | -a        | 7a        | 13a       | 9a        |
| (0010)                              | -a        | 7a        | 13a       | 11a       |
| (0110)                              | -a        | 9a        | 3a        | 5a        |
| (0111)                              | -a        | 9a        | 3a        | 7a        |
| (0101)                              | -a        | 9a        | 5a        | a         |
| (0100)                              | -a        | 9a        | 5a        | 3a        |
| (1100)                              | a         | -9a       | -5a       | -3a       |
| (1101)                              | a         | -9a       | -5a       | -a        |
| (1111)                              | a         | -9a       | -3a       | -7a       |
| (1110)                              | a         | -9a       | -3a       | -5a       |
| (1010)                              | a         | -7a       | -13a      | -11a      |
| (1011)                              | a         | -7a       | -13a      | -9a       |
| (1001)                              | a         | -7a       | -11a      | -15a      |
| (1000)                              | a         | -7a       | -11a      | -13a      |

[Table 5]

| $(n_{k,3} \ n_{k,2} \ n_{k,1} \ n_{k,0})$ | $U_{k,3}$ | $U_{k,2}$ | $U_{k,1}$ | $U_{k,0}$ |
|---|-----------|-----------|-----------|-----------|
| (0000)                                    | -a        | 7a        | 11a       | 13a       |
| (0001)                                    | -a        | 7a        | 11a       | 15a       |
| (0011)                                    | -a        | 7a        | 13a       | 9a        |
| (0010)                                    | -a        | 7a        | 13a       | 11a       |
| (0110)                                    | -a        | 9a        | 3a        | 5a        |
| (0111)                                    | -a        | 9a        | 3a        | 7a        |
| (0101)                                    | -a        | 9a        | 5a        | a         |
| (0100)                                    | -a        | 9a        | 5a        | 3a        |
| (1100)                                    | a         | -9a       | -5a       | -3a       |
| (1101)                                    | a         | -9a       | -5a       | -a        |
| (1111)                                    | a         | -9a       | -3a       | -7a       |
| (1110)                                    | a         | -9a       | -3a       | -5a       |
| (1010)                                    | a         | -7a       | -13a      | -11a      |
| (1011)                                    | a         | -7a       | -13a      | -9a       |
| (1001)                                    | a         | -7a       | -11a      | -15a      |
| (1000)                                    | a         | -7a       | -11a      | -13a      |

The values of the  $\Lambda(S_{k,i})$  of the received symbol obtained by using the values of the  $U_k$ ,  $U_{k,i}$ ,  $V_k$ , and  $V_{k,i}$  from the above tables ( [Table 1], [Table 2], [Table 4], [Table 5]) in [Equation 4] are shown in the following two tables ( [Table 6] & [Table 7] ) as a function of the value of bits,  $n_{k,i}$  (  $i = 0, 1, \dots, m - 1$  ).

[Table 6]

| $Y_k$ Range         | $\Lambda(S_{k,7})$ | $\Lambda(S_{k,6})$ | $\Lambda(S_{k,5})$ | $\Lambda(S_{k,4})$ |
|---------------------|--------------------|--------------------|--------------------|--------------------|
| $Y_k > 14a$         | $8Y_k - 56a$       | $4Y_k - 44a$       | $2Y_k - 26a$       | $Y_k - 14a$        |
| $12a < Y_k < 14a$   | $7Y_k - 42a$       | $3Y_k - 30a$       | $Y_k - 12a$        | $Y_k - 14a$        |
| $10a < Y_k < 12a$   | $6Y_k - 30a$       | $2Y_k - 18a$       | $Y_k - 12a$        | $-Y_k + 10a$       |
| $8a < Y_k < 10a$    | $5Y_k - 20a$       | $Y_k - 8a$         | $2Y_k - 22a$       | $-Y_k + 10a$       |
| $6a < Y_k < 8a$     | $4Y_k - 12a$       | $Y_k - 8a$         | $-2Y_k + 10a$      | $Y_k - 6a$         |
| $4a < Y_k < 6a$     | $3Y_k - 6a$        | $2Y_k - 14a$       | $-Y_k + 4a$        | $Y_k - 6a$         |
| $2a < Y_k < 4a$     | $2Y_k - 2a$        | $3Y_k - 18a$       | $-Y_k + 4a$        | $-Y_k + 2a$        |
| $0 < Y_k < 2a$      | $Y_k$              | $4Y_k - 20a$       | $-2Y_k + 6a$       | $-Y_k + 2a$        |
| $-2a < Y_k < 0$     | $Y_k$              | $-4Y_k - 20a$      | $2Y_k + 6a$        | $Y_k + 2a$         |
| $-4a < Y_k < -2a$   | $2Y_k + 2a$        | $-3Y_k - 18a$      | $Y_k + 4a$         | $Y_k + 2a$         |
| $-6a < Y_k < -4a$   | $3Y_k + 6a$        | $-2Y_k - 14a$      | $Y_k + 4a$         | $-Y_k - 6a$        |
| $-8a < Y_k < -6a$   | $4Y_k + 12a$       | $-Y_k - 8a$        | $2Y_k + 10a$       | $-Y_k - 6a$        |
| $-10a < Y_k < -8a$  | $5Y_k + 20a$       | $-Y_k - 8a$        | $-2Y_k - 22a$      | $Y_k + 10a$        |
| $-12a < Y_k < -10a$ | $6Y_k + 30a$       | $-2Y_k - 18a$      | $-Y_k - 12a$       | $Y_k + 10a$        |
| $-14a < Y_k < -12a$ | $7Y_k + 42a$       | $-3Y_k - 30a$      | $-Y_k - 12a$       | $-Y_k - 14a$       |
| $Y_k < -14a$        | $8Y_k + 56a$       | $-4Y_k - 44a$      | $-2Y_k - 26a$      | $-Y_k - 14a$       |

[Table 7]

| $X_k$ Range         | $\Lambda(S_{k,3})$ | $\Lambda(S_{k,2})$ | $\Lambda(S_{k,1})$ | $\Lambda(S_{k,0})$ |
|---------------------|--------------------|--------------------|--------------------|--------------------|
| $X_k > 14a$         | $8X_k - 56a$       | $4X_k - 44a$       | $2X_k - 26a$       | $X_k - 14a$        |
| $12a < X_k < 14a$   | $7X_k - 42a$       | $3X_k - 30a$       | $X_k - 12a$        | $X_k - 14a$        |
| $10a < X_k < 12a$   | $6X_k - 30a$       | $2X_k - 18a$       | $X_k - 12a$        | $-X_k + 10a$       |
| $8a < X_k < 10a$    | $5X_k - 20a$       | $X_k - 8a$         | $2X_k - 22a$       | $-X_k + 10a$       |
| $6a < X_k < 8a$     | $4X_k - 12a$       | $X_k - 8a$         | $-2X_k + 10a$      | $X_k - 6a$         |
| $4a < X_k < 6a$     | $3X_k - 6a$        | $X_k - 14a$        | $-X_k + 4a$        | $X_k - 6a$         |
| $2a < X_k < 4a$     | $2X_k - 2a$        | $3Y_k - 18a$       | $-X_k + 4a$        | $-X_k + 2a$        |
| $0 < X_k < 2a$      | $X_k$              | $4X_k - 20a$       | $-2X_k + 6a$       | $-X_k + 2a$        |
| $-2a < X_k < 0$     | $X_k$              | $-4X_k - 20a$      | $2X_k + 6a$        | $X_k + 2a$         |
| $-4a < X_k < -2a$   | $2X_k + 2a$        | $-3Y_k - 18a$      | $X_k + 4a$         | $X_k + 2a$         |
| $-6a < X_k < -4a$   | $3X_k + 6a$        | $-2Y_k - 14a$      | $X_k + 4a$         | $-X_k - 6a$        |
| $-8a < X_k < -6a$   | $4X_k + 12a$       | $-X_k - 8a$        | $2X_k + 10a$       | $-X_k - 6a$        |
| $-10a < X_k < -8a$  | $5X_k + 20a$       | $-X_k - 8a$        | $-2X_k - 22a$      | $X_k + 10a$        |
| $-12a < X_k < -10a$ | $6X_k + 30a$       | $-2X_k - 18a$      | $-X_k - 12a$       | $X_k + 10a$        |
| $-14a < X_k < -12a$ | $7X_k + 42a$       | $-3X_k - 30a$      | $-X_k - 12a$       | $-X_k - 14a$       |
| $X_k < -14a$        | $8X_k + 56a$       | $-4X_k - 44a$      | $-2X_k - 26a$      | $-X_k - 14a$       |

The LLR of the  $\Lambda(S_{k,i})$  can be obtained by applying the following transform variable to [Table 6] and [Table 7] which does not require a lookup table or memory for the calculation of the  $\Lambda(S_{k,i})$ . The transform variable used are as follows;

$$Z_{1k} = |Y_k| - 8a, Z_{2k} = |Z_{1k}| - 4a, Z_{3k} = |Z_{2k}| - 2a,$$

$$Z'_{1k} = |X_k| - 8a, Z'_{2k} = |Z'_{1k}| - 4a, Z'_{3k} = |Z'_{2k}| - 2a,$$

Under the assumption of the possibility of using sign bit to represent the sign of those value of transform variables ( $X_k, Y_k, Z_{1k}, Z_{2k}, Z_{3k}, Z'_{1k}, Z'_{2k}, Z'_{3k}$ ), the LLR tables for the calculation of  $\Lambda(S_{k,i})$  of the received symbol obtained can be found by representing the values of LLR in terms of the transform variables in the [Table 6] and [Table 7]. The resulting tables are shown in [Table 8] and [Table 9], respectively.

[Table 8]

| MSB( $Y_k$ ) | MSB( $Z_{1k}$ ) | MSB( $Z_{2k}$ ) | MSB( $Z_{3k}$ ) | $\Lambda(S_{k,7})$       | $\Lambda(S_{k,6})$         | $\Lambda(S_{k,5})$ | $\Lambda(S_{k,4})$ |
|--------------|-----------------|-----------------|-----------------|--------------------------|----------------------------|--------------------|--------------------|
| 0            | 0               | 0               | 0               | $Y_k + 7Z_{1k}$          | $Y_k - 2Z_{1k} + 5Z_{2k}$  | $Z_{2k} + Z_{3k}$  | $Z_{3k}$           |
|              |                 |                 | 1               | $Y_k + 7Z_{1k} - Z_{3k}$ | $Y_k - Z_{2k} + 3Z_{3k}$   | $Z_{2k}$           | $Z_{3k}$           |
|              |                 | 1               | 0               | $3Y_k - 2Z_{3k}$         | $Z_{1k}$                   | $Z_{2k} - Z_{3k}$  | $Z_{3k}$           |
|              |                 |                 | 1               | $3Y_k - 3Z_{3k}$         | $Z_{1k} - Z_{3k}$          | $Z_{2k}$           | $Z_{3k}$           |
|              | 1               | 0               | 0               | $Y_k$                    | $2Z_{1k} - 2Z_{3k}$        | $Z_{2k} + Z_{3k}$  | $Z_{3k}$           |
|              |                 |                 | 1               | $Y_k - Z_{3k}$           | $2Z_{1k} - Z_{3k}$         | $Z_{2k}$           | $Z_{3k}$           |
|              |                 | 1               | 0               | $2Y_k + 2Z_{3k}$         | $Z_{1k}$                   | $Z_{2k} - Z_{3k}$  | $Z_{3k}$           |
|              |                 |                 | 1               | $2Y_k + Z_{3k}$          | $Z_{1k} + Z_{3k}$          | $Z_{2k}$           | $Z_{3k}$           |
| 1            | 0               | 0               | 0               | $Y_k - 7Z_{1k}$          | $-Y_k - 2Z_{1k} + 5Z_{2k}$ | $Z_{2k} + Z_{3k}$  | $Z_{3k}$           |
|              |                 |                 | 1               | $Y_k - 7Z_{1k} + Z_{3k}$ | $-Y_k - Z_{2k} + 3Z_{3k}$  | $Z_{2k}$           | $Z_{3k}$           |
|              |                 | 1               | 0               | $3Y_k + 2Z_{3k}$         | $Z_{1k}$                   | $Z_{2k} - Z_{3k}$  | $Z_{3k}$           |
|              |                 |                 | 1               | $3Y_k + 3Z_{3k}$         | $Z_{1k} - Z_{3k}$          | $Z_{2k}$           | $Z_{3k}$           |
|              | 1               | 0               | 0               | $Y_k$                    | $2Z_{1k} - 2Z_{3k}$        | $Z_{2k} + Z_{3k}$  | $Z_{3k}$           |
|              |                 |                 | 1               | $Y_k + Z_{3k}$           | $2Z_{1k} - Z_{3k}$         | $Z_{2k}$           | $Z_{3k}$           |
|              |                 | 1               | 0               | $2Y_k - 2Z_{3k}$         | $Z_{1k}$                   | $Z_{2k} - Z_{3k}$  | $Z_{3k}$           |
|              |                 |                 | 1               | $2Y_k - Z_{3k}$          | $Z_{1k} + Z_{3k}$          | $Z_{2k}$           | $Z_{3k}$           |

[Table 9]

| MSB( $X_k$ ) | MSB( $Z'_{1k}$ ) | MSB( $Z'_{2k}$ ) | MSB( $Z'_{3k}$ ) | $\Lambda(S_{k,3})$         | $\Lambda(S_{k,2})$           | $\Lambda(S_{k,1})$  | $\Lambda(S_{k,0})$ |
|--------------|------------------|------------------|------------------|----------------------------|------------------------------|---------------------|--------------------|
| 0            | 0                | 0                | 0                | $X_k + 7Z'_{1k}$           | $X_k - 2Z'_{1k} + 5Z'_{2k}$  | $Z'_{2k} + Z'_{3k}$ | $Z'_{3k}$          |
|              |                  |                  | 1                | $X_k + 7Z'_{1k} - Z'_{3k}$ | $X_k - Z'_{2k} + 3Z'_{3k}$   | $Z'_{2k}$           | $Z'_{3k}$          |
|              |                  | 1                | 0                | $3X_k - 2Z'_{3k}$          | $Z'_{1k}$                    | $Z'_{2k} - Z'_{3k}$ | $Z'_{3k}$          |
|              |                  |                  | 1                | $3X_k - 3Z'_{3k}$          | $Z'_{1k} - Z'_{3k}$          | $Z'_{2k}$           | $Z'_{3k}$          |
|              | 1                | 0                | 0                | $X_k$                      | $2Z'_{1k} - 2Z'_{3k}$        | $Z'_{2k} + Z'_{3k}$ | $Z'_{3k}$          |
|              |                  |                  | 1                | $X_k - Z'_{3k}$            | $2Z'_{1k} - Z'_{3k}$         | $Z'_{2k}$           | $Z'_{3k}$          |
|              |                  | 1                | 0                | $2X_k + 2Z'_{3k}$          | $Z'_{1k}$                    | $Z'_{2k} - Z'_{3k}$ | $Z'_{3k}$          |
|              |                  |                  | 1                | $2X_k + Z'_{3k}$           | $Z'_{1k} + Z'_{3k}$          | $Z'_{2k}$           | $Z'_{3k}$          |
| 1            | 0                | 0                | 0                | $X_k - 7Z'_{1k}$           | $-X_k - 2Z'_{1k} + 5Z'_{2k}$ | $Z'_{2k} + Z'_{3k}$ | $Z'_{3k}$          |
|              |                  |                  | 1                | $X_k - 7Z'_{1k} + Z'_{3k}$ | $-X_k - Z'_{2k} + 3Z'_{3k}$  | $Z'_{2k}$           | $Z'_{3k}$          |
|              |                  | 1                | 0                | $3X_k + 2Z'_{3k}$          | $Z'_{1k}$                    | $Z'_{2k} - Z'_{3k}$ | $Z'_{3k}$          |
|              |                  |                  | 1                | $3X_k + 3Z'_{3k}$          | $Z'_{1k} - Z'_{3k}$          | $Z'_{2k}$           | $Z'_{3k}$          |

|  |   |   |   |                   |                       |                     |           |
|--|---|---|---|-------------------|-----------------------|---------------------|-----------|
|  | 1 | 0 | 0 | $Y_k$             | $2Z'_{1k} - 2Z'_{3k}$ | $Z'_{2k} + Z'_{3k}$ | $Z'_{3k}$ |
|  |   |   | 1 | $X_k + Z'_{3k}$   | $2Z'_{1k} - Z'_{3k}$  | $Z'_{2k}$           | $Z'_{3k}$ |
|  |   | 1 | 0 | $2X_k - 2Z'_{3k}$ | $Z'_{1k}$             | $Z'_{2k} - Z'_{3k}$ | $Z'_{3k}$ |
|  |   |   | 1 | $2X_k - Z'_{3k}$  | $Z'_{1k} + Z'_{3k}$   | $Z'_{2k}$           | $Z'_{3k}$ |

The soft decision values of LLR for the upper 4 bits (  $\Lambda(S_{k,7})$ ,  $\Lambda(S_{k,6})$ ,  $\Lambda(S_{k,5})$ ,  $\Lambda(S_{k,4})$  ) can be obtained by exploiting the information in [Table 8] as follows;

$$\Lambda(S_{k,7}) = \alpha Y_k + \delta (\beta Z'_{1k} + \gamma Z'_{3k})$$

$$\Lambda(S_{k,6}) = a Y_k + b Z'_{1k} + c Z'_{2k} + d Z'_{3k}$$

$$\Lambda(S_{k,5}) = Z'_{2k} + f Z'_{3k}$$

$$\Lambda(S_{k,4}) = Z'_{3k}$$

Where the values of  $\alpha$ ,  $\delta$ ,  $\beta$ ,  $\gamma$ ,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $f$  are coefficient constants and can be obtained by exploiting the information in [Table 8].

The soft decision values of LLR for the lower 4 bits (  $\Lambda(S_{k,3})$ ,  $\Lambda(S_{k,2})$ ,  $\Lambda(S_{k,1})$ ,  $\Lambda(S_{k,0})$  ) can be obtained by exploiting the information in [Table 9] as follows;

$$\Lambda(S_{k,3}) = \alpha' X_k + \delta' (\beta' Z'_{1k} + \gamma' Z'_{3k})$$

$$\Lambda(S_{k,2}) = a' X_k + b' Z'_{1k} + c' Z'_{2k} + d' Z'_{3k}$$

$$\Lambda(S_{k,1}) = Z'_{2k} + f' Z'_{3k}$$

$$\Lambda(S_{k,0}) = Z'_{3k}$$

Where the values of  $\alpha'$ ,  $\delta'$ ,  $\beta'$ ,  $\gamma'$ ,  $a'$ ,  $b'$ ,  $c'$ ,  $d'$ , and  $f'$  are coefficient constants and can be obtained by exploiting the information in [Table 9].

### 2.1.2. Implementation of De-Mapper into Hardware

As seen from the above LLR equations, the soft decision values of LLR of one symbol for 256QAM can be obtained by using six bit clocks. One symbol in 256-QAM consists of eight bits. The de-mapping process developed here is a real time process. GScom uses the same principle to derive real time mapping & de-mapping processors for QPSK, 16QAM, 64QAM, 1024QAM, and 4096QAM.