EE128 Feedback Control

Lecture 23, 11/28/2006

- State-Space Design (Ch 7)
  - Estimator Design (7.7)
    - Reduced-Order Estimators (7.7.2)
  - Compensator Design: Combined Control Law and Estimator (7.8)
  - Examples

Reduced-Order Estimators

- Reduce complexity of the estimator by using the state variables that are measured directly
  - If measurements are very noise it's better to use full-order estimation
- Reduces the order of the estimator by the number of sensed outputs

Directly measured

Remaining state variables

$$\begin{bmatrix} \dot{x}_a \\ \dot{\mathbf{x}}_b \end{bmatrix} = \begin{bmatrix} F_{aa} & \mathbf{F}_{ab} \\ \mathbf{F}_{ba} & \mathbf{F}_{bb} \end{bmatrix} \begin{bmatrix} x_a \\ \mathbf{x}_b \end{bmatrix} + \begin{bmatrix} G_a \\ \mathbf{G}_b \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_a \\ \mathbf{x}_b \end{bmatrix}$$

Known input

$$\dot{\mathbf{x}}_b = \mathbf{F}_{bb}\mathbf{x}_b + \mathbf{F}_{ba}x_a + \mathbf{G}_b u$$

$x_a = y \rightarrow$

$$\dot{x}_a = \dot{y} = F_{aa}y + \mathbf{F}_{ab}\mathbf{x}_b + G_a u$$

$$\dot{y} - F_{aa}y - G_a u = \mathbf{F}_{ab}\mathbf{x}_b$$

Known measurement

$$\begin{aligned} &\mathbf{x} \leftarrow \mathbf{x}_b \\ &\mathbf{F} \leftarrow \mathbf{F}_{bb} \\ &Gu \leftarrow \mathbf{F}_{ba}y + \mathbf{G}_b u \\ &y \leftarrow \dot{y} - F_{aa}y - G_a u \\ &\mathbf{H} \leftarrow \mathbf{F}_{ab} \end{aligned}$$

Substitutions in the original estimator eqs

Substitute reduced-order eqs into full-order estimator

$$\dot{\hat{\mathbf{x}}}_b = \mathbf{F}_{bb}\hat{\mathbf{x}}_b + \mathbf{F}_{ba}y + \mathbf{G}_b u + \mathbf{L}(\dot{y} - F_{aa}y - G_a u - \mathbf{F}_{ab}\mathbf{x}_b)$$

input measurement

$$\tilde{\mathbf{x}}_b \overset{\Delta}{=} \mathbf{x}_b - \hat{\mathbf{x}}_b$$

$$\dot{\tilde{\mathbf{x}}}_b = (\mathbf{F}_{bb} - \mathbf{L}\mathbf{F}_{ab})\tilde{\mathbf{x}}_b$$

$$\det[s\mathbf{I} - (\mathbf{F}_{bb} - \mathbf{L}\mathbf{F}_{ab})] = 0$$

Select $\mathbf{L}$

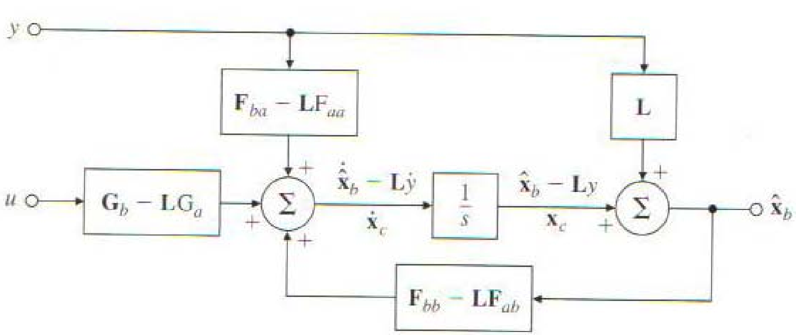
Derivative of the measurement!

$$\dot{\hat{\mathbf{x}}}_b = (\mathbf{F}_{bb} - \mathbf{L}\mathbf{F}_{ab})\hat{\mathbf{x}}_b + (\mathbf{F}_{ba} - \mathbf{L}F_{aa})y + (\mathbf{G}_b - \mathbf{L}G_a)u + \mathbf{L}\dot{y}$$

$$\mathbf{x}_c \overset{\Delta}{=} \hat{\mathbf{x}}_b - \mathbf{L}y$$

Removes noisy term

$$\dot{\mathbf{x}}_c = (\mathbf{F}_{bb} - \mathbf{L}\mathbf{F}_{ab})\hat{\mathbf{x}}_b + (\mathbf{F}_{ba} - \mathbf{L}F_{aa})y + (\mathbf{G}_b - \mathbf{L}G_a)u$$



Reduced-Order Estimators

- **Example 7.26** Design a reduced-order estimator for the pendulum that has the error pole at $-10\omega_0$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\omega_0^2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

(1) System equations

$$\begin{bmatrix}F_{aa} & \mathbf{F}_{ab}\\ \mathbf{F}_{ba} & \mathbf{F}_{bb}\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\omega_0^2 & 0\end{bmatrix}$$

$$\begin{bmatrix}G_a\\ \mathbf{G}_b\end{bmatrix}=\begin{bmatrix}0\\ 1\end{bmatrix}$$

(2) Partitioned matrices

(3) Characteristic equation

$$s-(0-L)=0$$

(4) Desired equation

$$\alpha_e(s)=s+10\omega_0=0$$

$$L=10\omega_0$$

$$\dot{\mathbf{x}}_c=(\mathbf{F}_{bb}-\mathbf{L}\mathbf{F}_{ab})\hat{\mathbf{x}}_b+(\mathbf{F}_{ba}-\mathbf{L}F_{aa})y+(\mathbf{G}_b-\mathbf{L}G_a)u$$

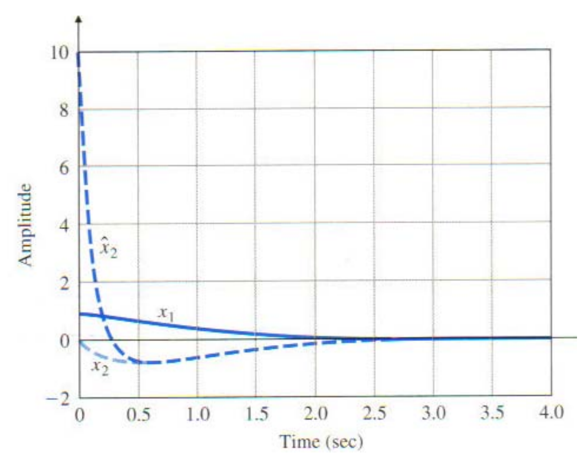
$$\dot{x}_c=-10\omega_0\hat{x}_2-\omega_0^2 y+u$$

(5) Estimator equation

(6) State estimate

$$\mathbf{x}_c=\hat{\mathbf{x}}_b-\mathbf{L}y\rightarrow\hat{x}_2=\hat{\mathbf{x}}_b$$

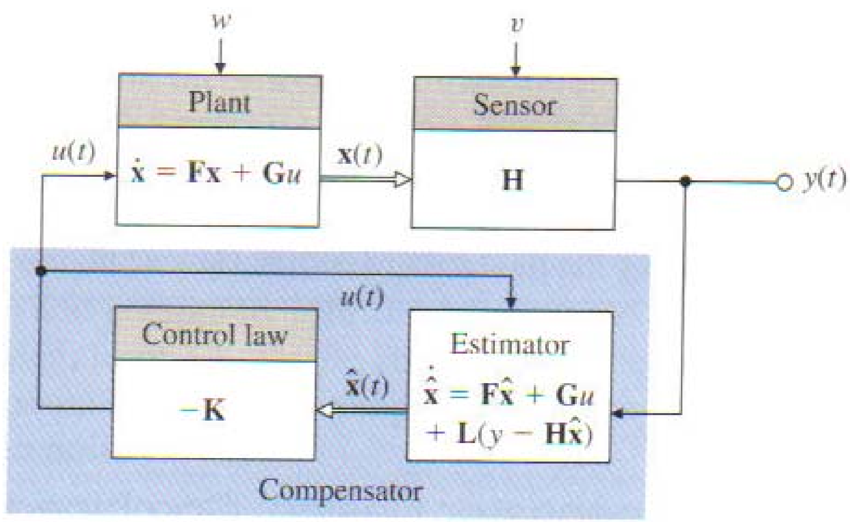
$$\hat{x}_2=x_c+10\omega_0 y$$



The condition for the existence of a full or reduced-order estimator is the **observability of (F,H)**

Compensator Design
Combined Control Law and Estimator

- **Regulator:** Combination of control law + estimator
  - can reject disturbances but has no reference input to be tracked



$$\left.\begin{aligned}\dot{\mathbf{x}} &= \mathbf{F}\mathbf{x} - \mathbf{G}\mathbf{K}\hat{\mathbf{x}} \\ \dot{\mathbf{x}} &= \mathbf{F}\mathbf{x} - \mathbf{G}\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}})\end{aligned}\right\} \text{Plant equation with feedback}$$

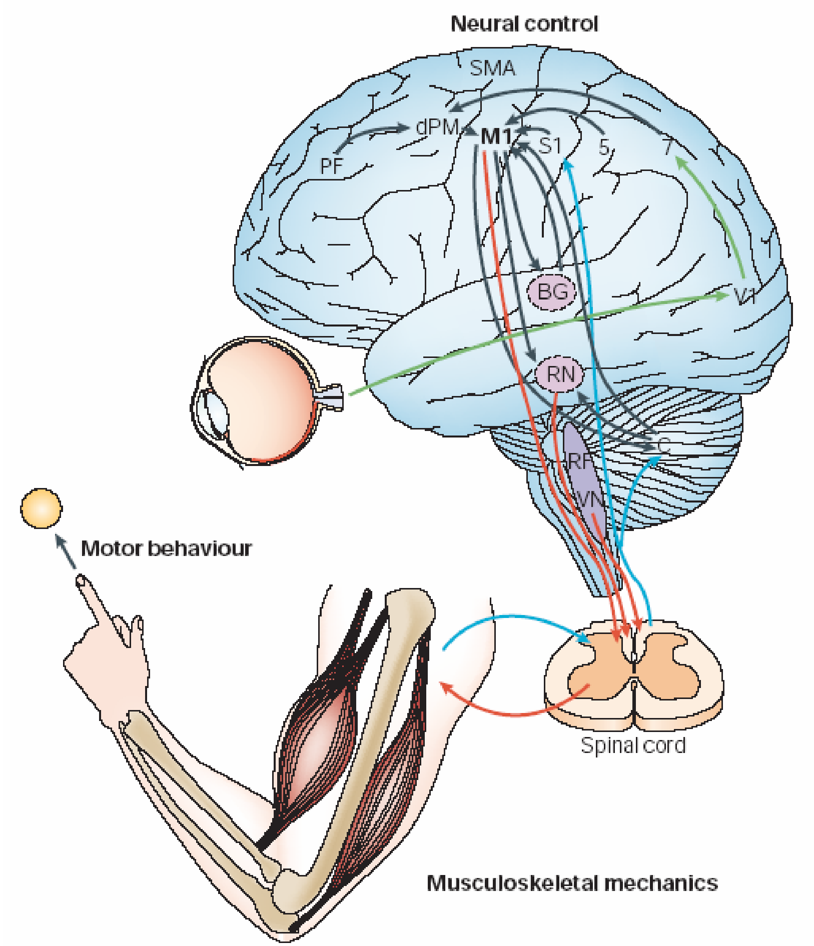
$$\left.\begin{bmatrix}\dot{\mathbf{x}} \\ \dot{\tilde{\mathbf{x}}}\end{bmatrix} = \begin{bmatrix}\mathbf{F} - \mathbf{G}\mathbf{K} & \mathbf{G}\mathbf{K} \\ 0 & \mathbf{F} - \mathbf{L}\mathbf{H}\end{bmatrix}\begin{bmatrix}\mathbf{x} \\ \tilde{\mathbf{x}}\end{bmatrix}\right\} \text{System dynamics}$$

$$\det\begin{bmatrix}s\mathbf{I} - \mathbf{F} + \mathbf{G}\mathbf{K} & -\mathbf{G}\mathbf{K} \\ 0 & s\mathbf{I} - \mathbf{F} + \mathbf{L}\mathbf{H}\end{bmatrix} = 0$$

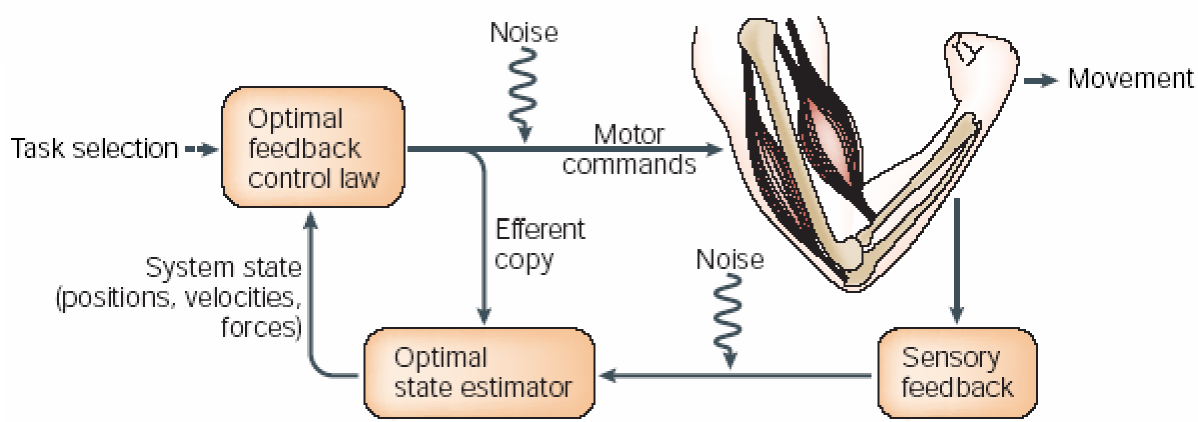
$$\det(s\mathbf{I} - \mathbf{F} + \mathbf{G}\mathbf{K}) \cdot \det(s\mathbf{I} - \mathbf{F} + \mathbf{L}\mathbf{H}) = \underbrace{\alpha_c(s)\alpha_e(s)}_{} = 0$$

The designs of control law and the estimator can be carried out independently!

Current views on the **neural control of movement**

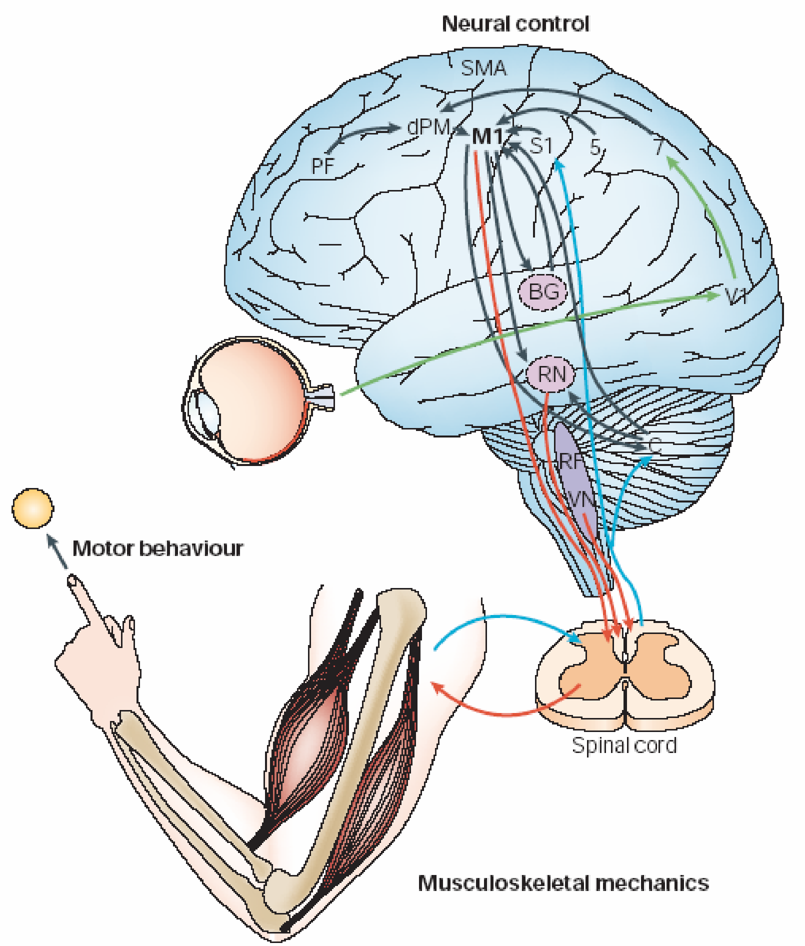


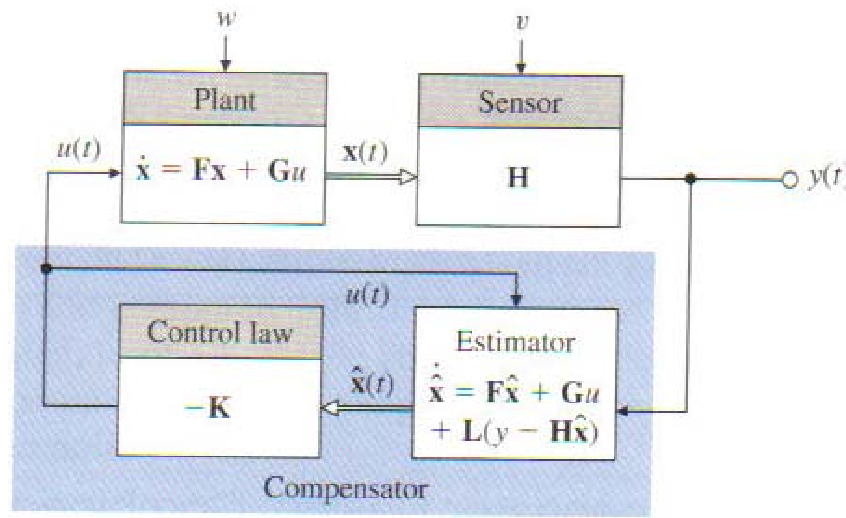
Afferent
Efferent



Scott, Nat Rev Neurosc, 2004

Current views on the
neural control of movement





Afferent

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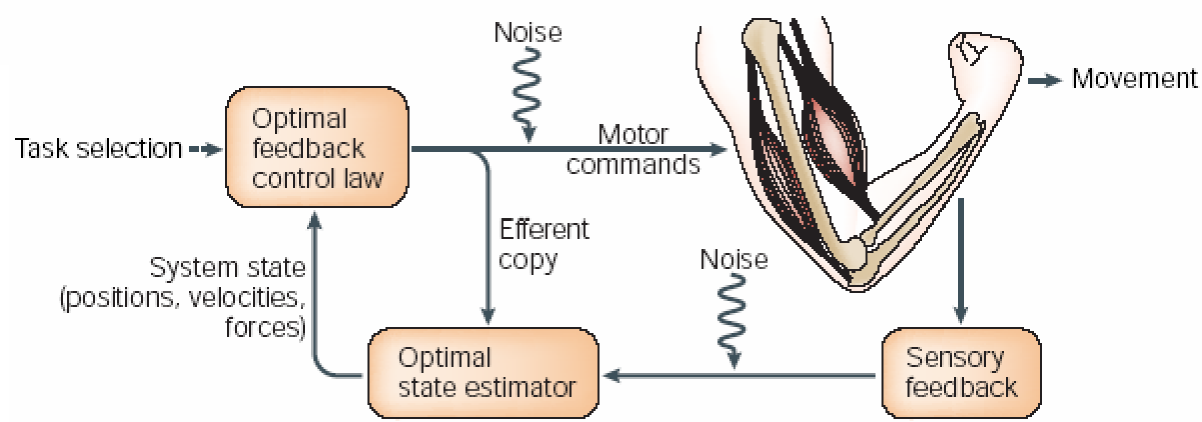

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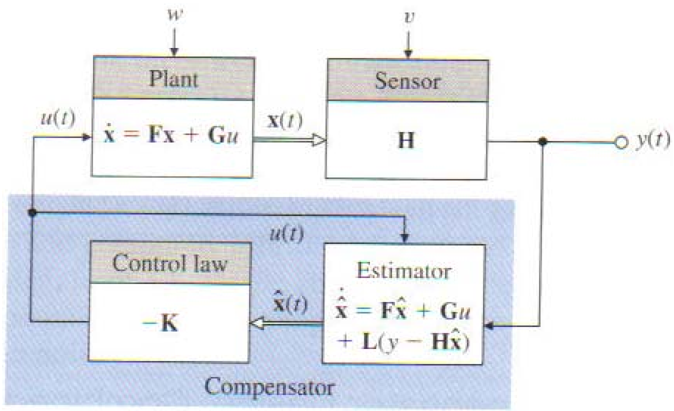
Compensator Design
Combined Control Law and Estimator



$$\dot{\hat{\mathbf{x}}} = (\mathbf{F} - \mathbf{GK} - \mathbf{LH})\hat{\mathbf{x}} + \mathbf{L}y$$

$$u = -\mathbf{K}\hat{\mathbf{x}}$$

$$\left.\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Fx} + \mathbf{Gu} \\ \det(s\mathbf{I} - \mathbf{F}) &= 0 \end{aligned}\right\} \text{Comparing with the general form…}$$

$$\det(s\mathbf{I} - \mathbf{F} + \mathbf{GK} + \mathbf{LH}) = 0$$

$$\left\{\begin{aligned} G(s) &= \frac{Y(s)}{U(s)} = \mathbf{H}(s\mathbf{I} - \mathbf{F})^{-1}\mathbf{G} + J \\ D_c(s) &= \frac{U(s)}{Y(s)} = \underbrace{-\mathbf{K}(s\mathbf{I} - \mathbf{F} + \mathbf{GK} + \mathbf{LH})^{-1}\mathbf{L}}_{\text{Compensator transfer function}} \end{aligned}\right.$$

$$u = -\begin{bmatrix} K_a & \mathbf{K}_b \end{bmatrix} \begin{bmatrix} x_a \\ \hat{\mathbf{x}}_b \end{bmatrix} = -K_a y - \mathbf{K}_b \hat{\mathbf{x}}_b$$

$$\dot{\mathbf{x}}_c = \mathbf{A}_r \mathbf{x}_c + \mathbf{B}_r y$$

$$u = \mathbf{C}_r \mathbf{x} + \mathbf{D}_r y$$

$$\mathbf{A}_r = \mathbf{F}_{bb} - \mathbf{L}\mathbf{F}_{ab} - (\mathbf{G}_b - \mathbf{L}G_a)\mathbf{K}_b$$

$$\mathbf{B}_r = \mathbf{A}_r \mathbf{L} + \mathbf{F}_{ba} - \mathbf{L}F_{aa} - (\mathbf{G}_b - \mathbf{L}G_a)K_a$$

$$\mathbf{C}_r = -\mathbf{K}_b$$

$$D_r = -K_a - \mathbf{K}_b \mathbf{L}$$

$$D_{cr}(s) = \frac{U(s)}{Y(s)} = \underbrace{\mathbf{C}_r (s\mathbf{I} - \mathbf{A}_r)^{-1} \mathbf{B}_r + D_r}_{\text{Reduced-order compensator transfer function}}$$

- **Full-Order** Compensator Design for Satellite Attitude Control

Design a compensator using pole placement for the satellite plant $1/s^2$. Place the control poles at $s=-0.707\pm0.707j$ (ω_n=1rad/sec, ζ=0.707) and place the estimator poles at ω_n=5rad/sec, ζ=0.5.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

$$s = -0.707 \pm 0.707\,j \;\; (\omega_n = 1\,\text{rad/sec},\;\; \zeta = 0.7)$$

$$\alpha_c(s) = s^2 + s\sqrt{2} + 1$$

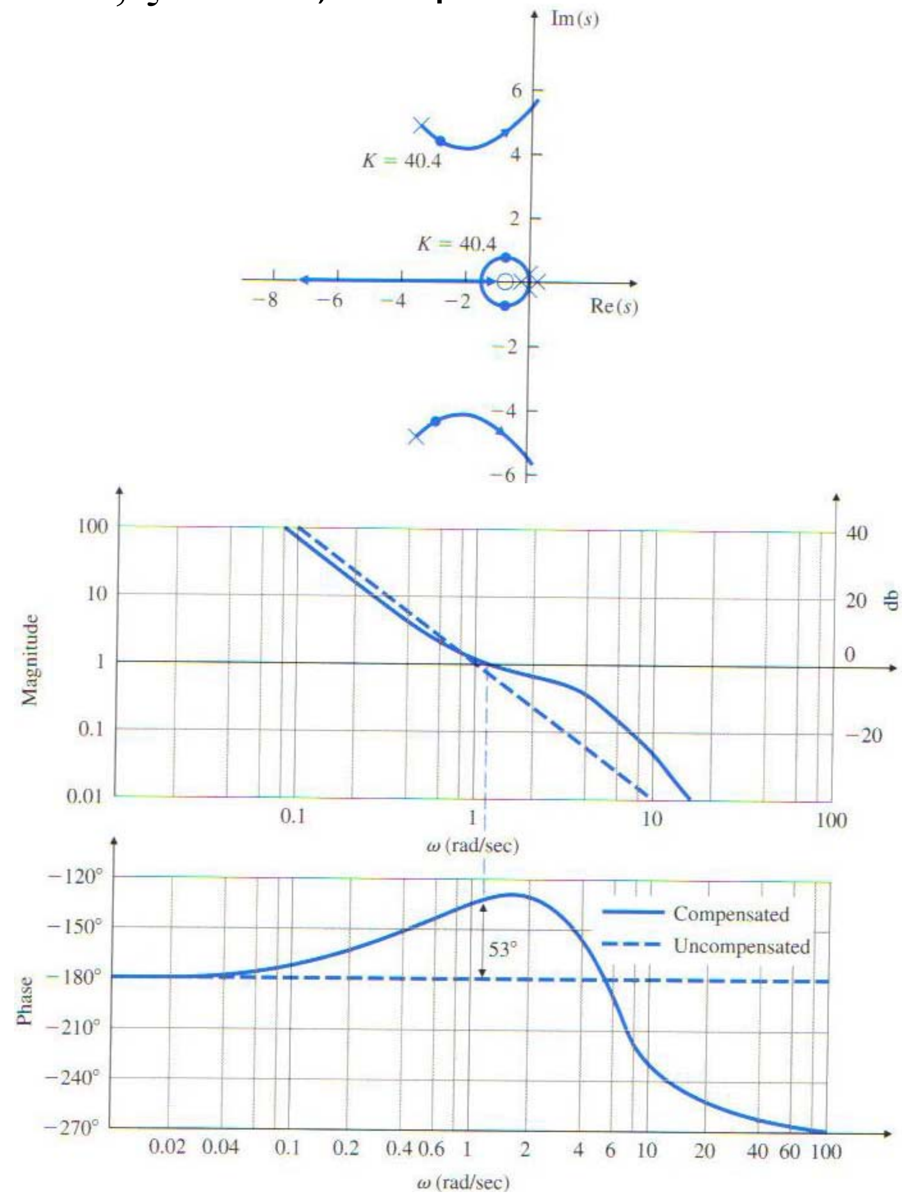
$$\mathbf{K} = \begin{bmatrix} 1 & \sqrt{2} \end{bmatrix}$$

$$\alpha_e(s) = s^2 + 5s + 25 = s + 2.5 \pm 4.3\,j$$

$$\mathbf{L} = \begin{bmatrix} 5 \\ 25 \end{bmatrix}$$

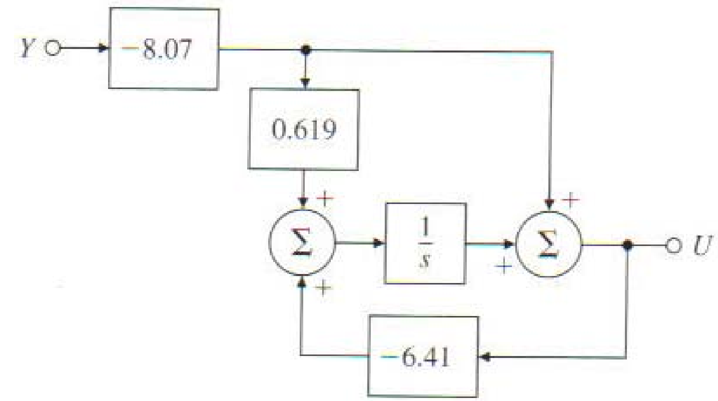
$$D_c(s) = -40.4\frac{s+0.619}{s+3.21\pm4.77\,j}$$

$$1 + K\frac{s+0.619}{(s+3.21\pm4.77\,j)s^2} = 0$$



• **Reduced-Order** Compensator Design for Satellite Attitude Control

Repeat the design for the $1/s^2$ plant, but use a reduced order estimator. Place the one estimator pole at -5rad/sec



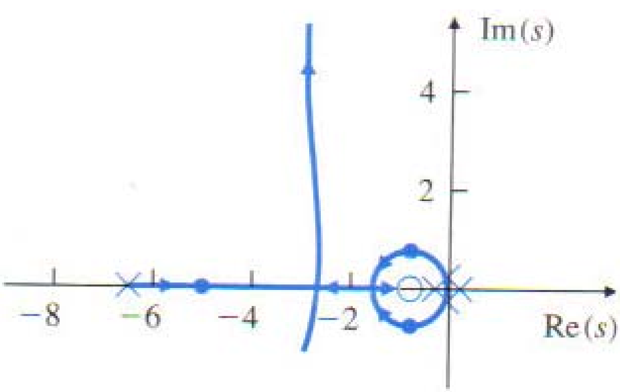
$$L = 5$$

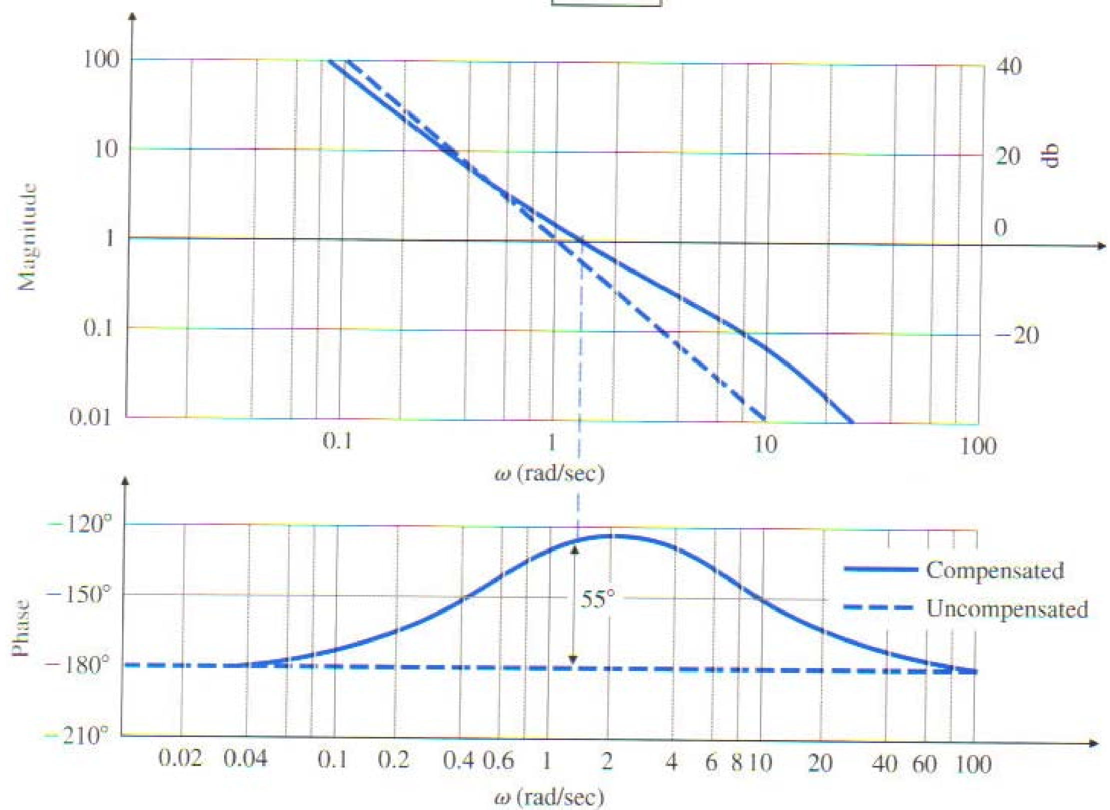
$$\dot{x}_c = -6.41x_c - 33.1y$$

$$u = -1.41x_c - 8.07y$$

$$x_c = \hat{x}_2 - 5y$$

$$D_{cr}(s) = -\frac{8.07(s + 0.619)}{s + 0.641}$$





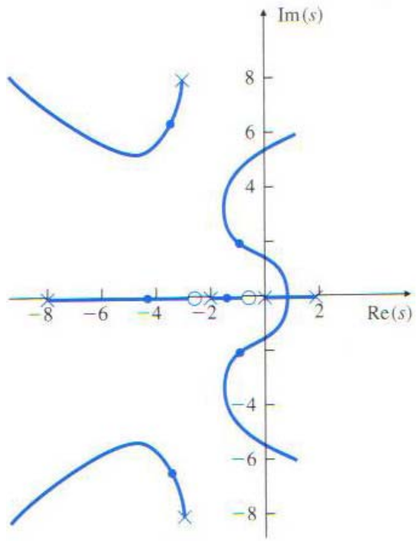
- **Full-Order** Compensator Design for DC Servo

Use the state-space pole placement method to design a compensator for the DC servo system with the given transfer function. Using a state-space description in observer canonical form, place the control poles at *pc*=[-1.42; -1.04±2.14*j*] locations and the full order estimator poles at *pe*=[-4.25; -3.13±6.41*j*].

$$G(s) = \frac{10}{s(s+2)(s+8)}$$

$$\mathbf{F} = \begin{bmatrix} -10 & 1 & 0 \\ -16 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad J = 0$$

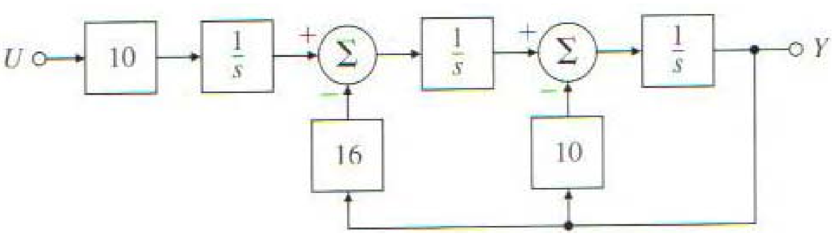


$$\text{pc} = \begin{bmatrix} -1.42; & -1.04+2.14j; & -1.04-2.14j \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} -46.4 & 5.76 & -0.65 \end{bmatrix}$$

$$\text{pe} = \begin{bmatrix} -4.25; & -3.13+6.41j; & -3.13-6.41j; \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 0.5 \\ 61.4 \\ 216 \end{bmatrix}$$



$$D_c(s) = -190 \frac{(s+0.432)(s+2.10)}{(s-1.88)(s+2.94 \pm 8.32j)}$$

• **Reduced-Order** Compensator Design for Satellite Attitude Control

Design a compensator for the DC servo system by using the same control poles but with a reduced order estimator. Place the estimator poles at -4.24±4.24j positions with ω_n=6 and ζ=0.707.

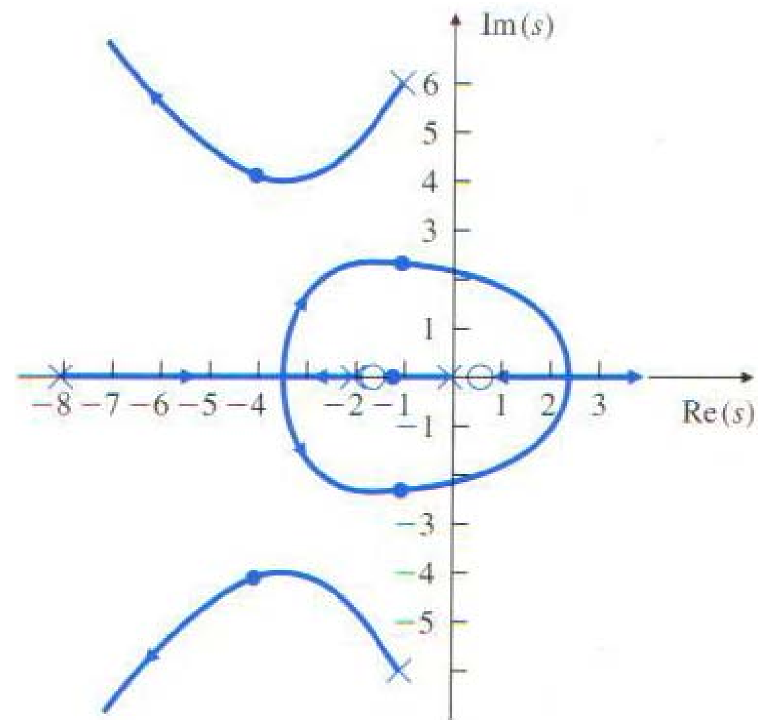
$$\mathrm{pc} = [-4.24+4.24j;\ -4.24-4.24j]$$

$$\begin{bmatrix} F_{aa} & \mathbf{F}_{ab} \\ \mathbf{F}_{ba} & \mathbf{F}_{bb} \end{bmatrix} = \begin{bmatrix} -10 & 1 & 0 \\ -16 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} G_a \\ \mathbf{G}_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\det(s\mathbf{I} - \mathbf{F}_{bb} + \mathbf{L}\mathbf{F}_{ab}) = \alpha_e(s)$$

$$\mathbf{L} = \begin{bmatrix} 8.5 \\ 36 \end{bmatrix}$$

$$D_{cr}(s) = 20.93\frac{(s-0.735)(s+1.871)}{s+0.990 \pm 6.120j}$$



• Problem 7.51

51. A simplified model for the control of a flexible robotic arm is shown in Fig. 7.97, where

$$\begin{aligned} k/M &= 900 \text{ rad/ sec}^2, \\ y &= \text{output, the mass position,} \\ u &= \text{input, the position of the end of the spring.} \end{aligned}$$

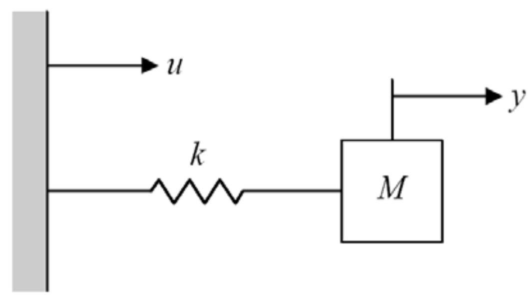


Figure 7.97: [Text Fig. 7.97] Simple robotic arm for Problem 7.51.

a) Write the equations of motion in state-space form.
b) Design an estimator with roots as $s = -100 \pm 100j$.
c) Could both state variables of the system be estimated if only a measurement of $\dot{y}$ was available?
d) Design a full-state feedback controller with roots at $s = -20 \pm 20j$.
e) Would it be reasonable to design a control law for the system with roots at $s = -200 \pm 200j$? State your reasons.
f) Write equations for the compensator, including a command input for y. Draw a Bode plot for the closed-loop system, and give the gain and phase margins for the design.