Assignment1 - Introduction and Basic Algorithms

Ji

Dec 2022

Contents

1 Principal Component Analysis

1.1 Method

The method of applying principle component analysis (PCA) to point cloud mainly has three steps:

1. Normalized by the center:

$$\tilde{X} = [\tilde{x_1}, ..., \tilde{x_m}], \tilde{x_i} = x_i - \bar{x}, i = 1, ..., m$$

2. Compute SVD or correlation:

$$H = \tilde{X}\tilde{X}^T = U_r \Sigma^2 {U_r}^T$$
$$H = \frac{1}{n} X X^T$$

3. The principle vectors are the columns of U_r. (Eigenvector of X = Eigenvector of H)

1.2 Results

In this section, I would like to visualize the results of an airplane. Figure 1 visualizes the original point cloud of an airplane.

Figure 1: Point cloud of an airplane.

As shown in Figure 2, the black and red lines are the first and second principal components (PCs) of the airplane, respectively. More specifically, the first PC is $[-0.9990599, -0.00894016, -0.04241916]$, and the second one is $[-0.04335102, 0.20626529, 0.97753533]$.

Furthremore, Figure 3 visualizes the projection of the point cloud onto the plane defined by its first and second principal components, which reduces the dimensionality from three to two.

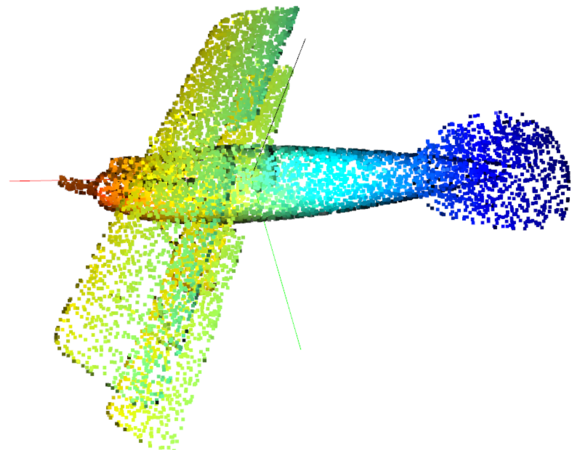


Figure 2: Three main principle components of an airplane.

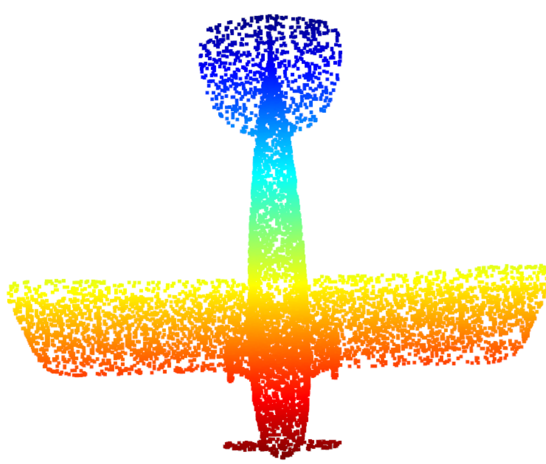


Figure 3: Projection of an airplane onto the plane defined by its first and second principle components.

2 Surface Normal Estimation

2.1 Method

The method of surface normal estimation on 3D point cloud mainly has three steps. For any point P as the anchor point:

1. find its k (an hyperparameter) nearest neighbors which define a surface.
2. apply PCA to those points.
3. the surface normal at P is the least significant vector of PCA.

2.2 Results

In this section, I would like to visualize the surface normal of a guitar. Figure 4 is the ground truth of the surface normal of a guitar provided by the dataset.

Figure 4: The surface normal of a guitar.

Figure 5 shows the estimation of the surface normal of the guitar. Obviously, there is difference between the ground truth and the estimation around the guitar neck and head.

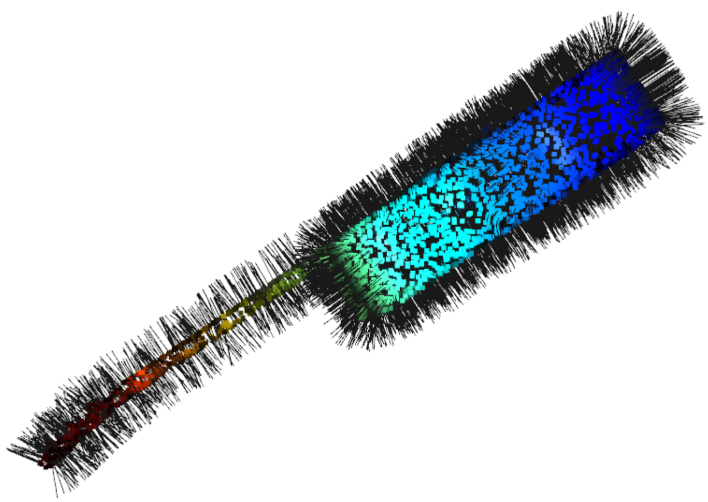


Figure 5: The estimation of the surface normal of a guitar.

3 Voxel Grid Downsampling

3.1 Method

The method of voxel grid downsampling on 3D point cloud mainly has six steps:

1. Compute the min or max of the point set $\{p_1, p_2, ..., p_N\}$:

$$\begin{aligned} x_{\max} &= \max(x_1, x_2, ..., x_N) \\ x_{\min} &= \min(x_1, x_2, ..., x_N) \\ &y_{\max}, y_{\min} \\ &z_{\max}, z_{\min} \end{aligned}$$

2. Deterine the voxel grid size r.

3. Compute the diemnsion of the voxel grid:

$$\begin{aligned} D_x &= \lceil (x_{\max} - x_{\min})/r \rceil \\ D_y &= \lceil (y_{\max} - y_{\min})/r \rceil \\ D_z &= \lceil (z_{\max} - z_{\min})/r \rceil \end{aligned}$$

4. Compute the voxel index for each point:

$$\begin{aligned} h_x &= \lfloor (x - x_{\min})/r \rfloor \\ h_y &= \lfloor (y - y_{\min})/r \rfloor \\ h_z &= \lfloor (z - z_{\min})/r \rfloor \\ h &= h_x + h_y \times D_x + h_z \times D_x \times D_y \end{aligned}$$

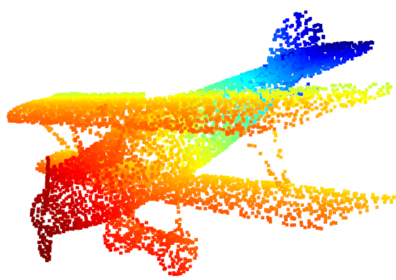
5. Use a hash function to map each point to a container G_i in $\{G_1, G_2, ..., G_M\}$.

6. Iterate $\{G_1, G_2, ..., G_M\}$ and get M points.

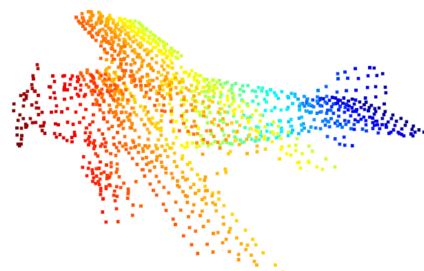
3.2 Results

In voxel grid downsampling, the algorithm creates a 3D voxel grid over the input point cloud data. Then, in each voxel, all the points present can be approximated by two methods. The first method is to compute the centroid of those points, while the other is to randomly select one point from them. Figure 6 visualizes the downsampled results of the first method, and Figure 7 visualizes that of the second method. Both of them show that a larger value of r has generated a sparser point cloud.

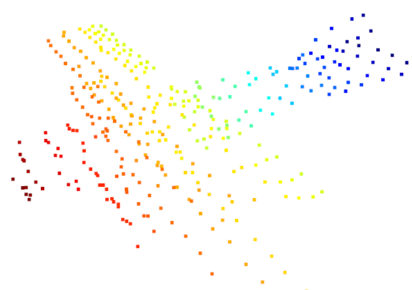
Furthermore, Figure 8 compares two downsampled point clouds of a chair. In this case, the point cloud obtained by the centroid approximation, Figure 8 (a), better reserves the chair's surface information.



(a) r=0.01

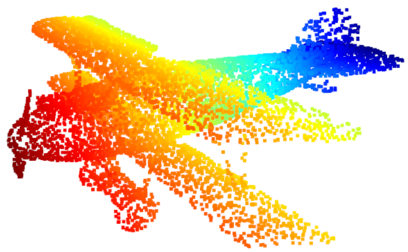


(b) r=0.05

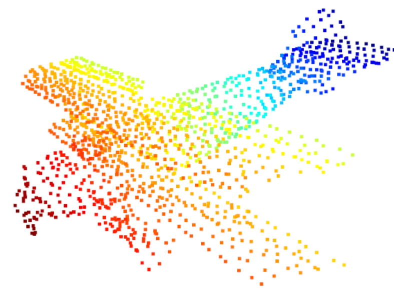


(c) r=0.1

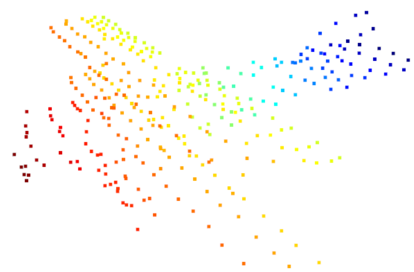
Figure 6: Three point clouds with a different density of an airplane. In each voxel, all the points present are approximated with their centroid.



(a) r=0.01

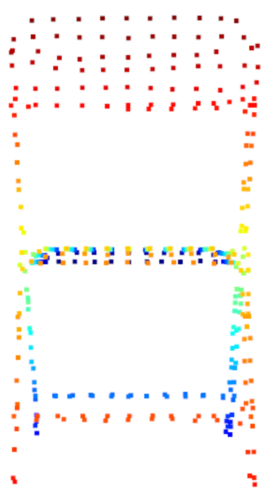


(b) r=0.05

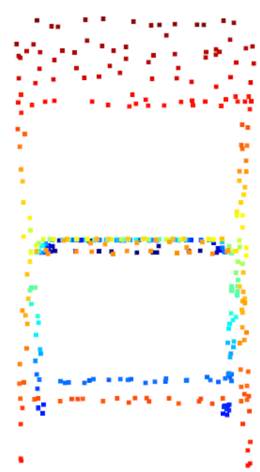


(c) r=0.1

Figure 7: Three point clouds with a different density of an airplane. In each voxel, all the pints present are approximated with one randomly selected point from them.



(a) centroid approximation



(b) random approximation

Figure 8: A comparison between two downsampled point clouds of a chair by two approximation methods.