Pilot Analysis: Exploratory Visualization

This document is intended to give an overview of the response distributions from our pilot.

Data

Load Worker Responses from Pilot

The data is already anonymous and in a tidy format at this stage in the analysis pipeline. We just need to read it in and do some preprocessing.

```
# read in data
full_df <- read_csv("pilot-anonymous.csv")
```

```
## Parsed with column specification:
## cols(
##   .default = col_double(),
##   workerId = col_character(),
##   batch = col_integer(),
##   condition = col_character(),
##   start_gain_frame = col_character(),
##   numeracy = col_integer(),
##   gender = col_character(),
##   age = col_character(),
##   education = col_character(),
##   chart_use = col_character(),
##   intervene = col_integer(),
##   outcome = col_character(),
##   pSup = col_integer(),
##   trial = col_character(),
##   trialIdx = col_character()
## )
```

```
## See spec(...) for full column specifications.
```

```
# preprocessing
responses_df <- full_df %>%
  rename( # rename to convert away from camel case
    worker_id = workerId,
    company_value = companyValue,
    ground_truth = groundTruth,
    p_contract_new = pContractNew,
    p_contract_old = pContractOld,
    p_superiority = pSup,
    start_time = startTime,
    resp_time = respTime,
    trial_dur = trialDur,
    trial_idx = trialIdx
  ) %>%
  mutate( # mutate to jitter probability of superiority away from boundaries
    p_superiority = ifelse(p_superiority == 0, 0.25, p_superiority),            # avo
id responses equal to zero
    p_superiority = ifelse(p_superiority == 100, 99.75, p_superiority)         # avoi
d responses equal to one-hundred
  ) %>%
  filter(trial_idx != "practice", trial_idx != "mock") # remove practice and mock tri
als from responses dataframe, leave in full version

head(responses_df)
```

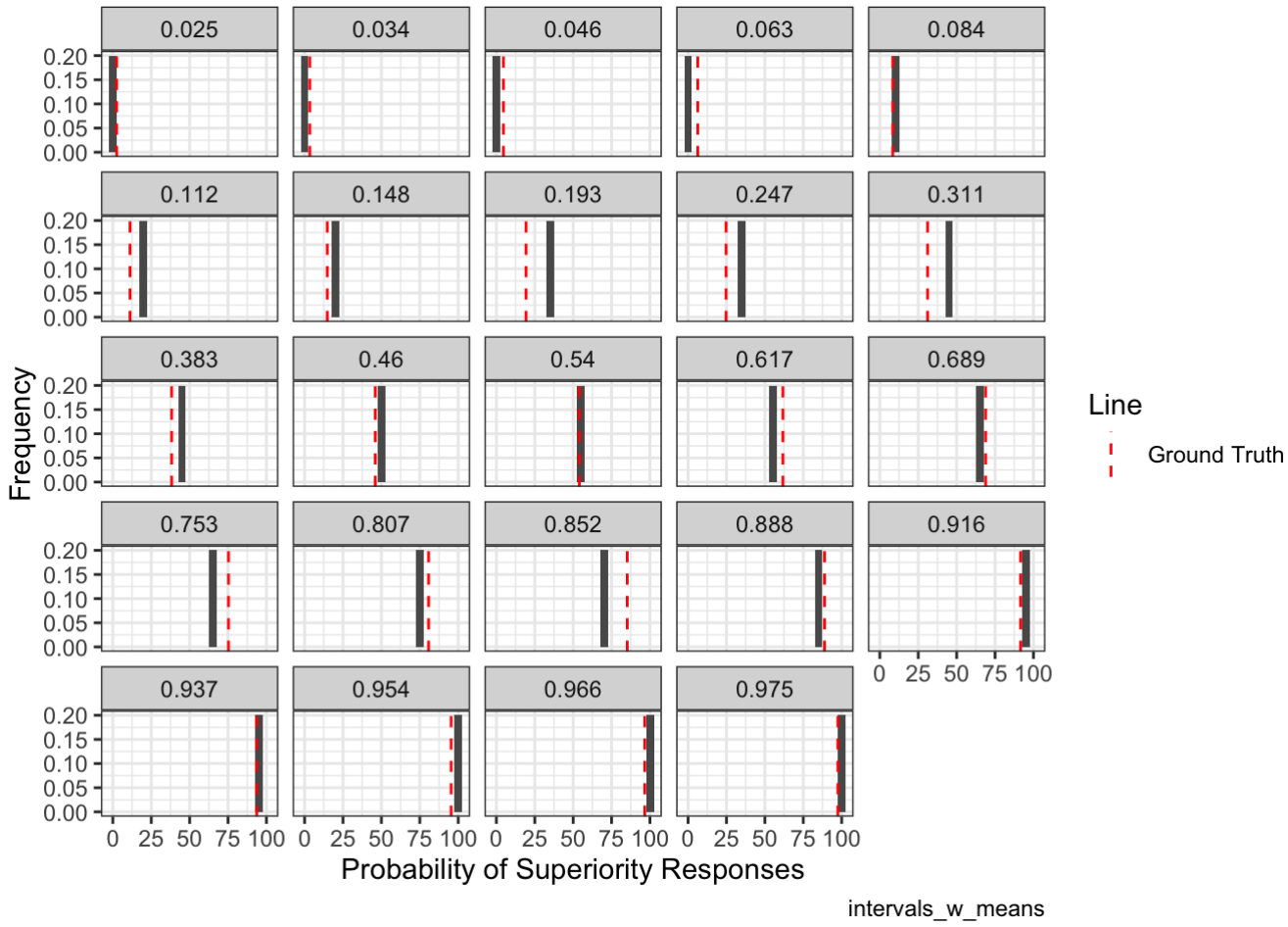
```
## # A tibble: 6 x 27
##   worker_id batch condition baseline contract_value exchange
##   <chr>     <int> <chr>        <dbl>          <dbl>    <dbl>
## 1 be209114      0 interval…      0.5           2.25   0.0480
## 2 be209114      0 interval…      0.5           2.25   0.0480
## 3 be209114      0 interval…      0.5           2.25   0.0480
## 4 be209114      0 interval…      0.5           2.25   0.0480
## 5 be209114      0 interval…      0.5           2.25   0.0480
## 6 be209114      0 interval…      0.5           2.25   0.0480
## # … with 21 more variables: start_gain_frame <chr>, total_bonus <dbl>,
## #   duration <dbl>, numeracy <int>, gender <chr>, age <chr>,
## #   education <chr>, chart_use <chr>, company_value <dbl>,
## #   ground_truth <dbl>, intervene <int>, outcome <chr>,
## #   p_contract_new <dbl>, p_contract_old <dbl>, p_superiority <int>,
## #   payoff <dbl>, resp_time <dbl>, start_time <dbl>, trial <chr>,
## #   trial_dur <dbl>, trial_idx <chr>
```

Response Distributions

Probability of Superiority Judgments

Let's plot histograms of probability of superiority judgments at each level of the ground truth probability of superiority. We show the ground truth in red. This will give us an overview of bias and precision in judgments. We do this separately for each visualization condition to limit the number of faceted subplots in a single view.

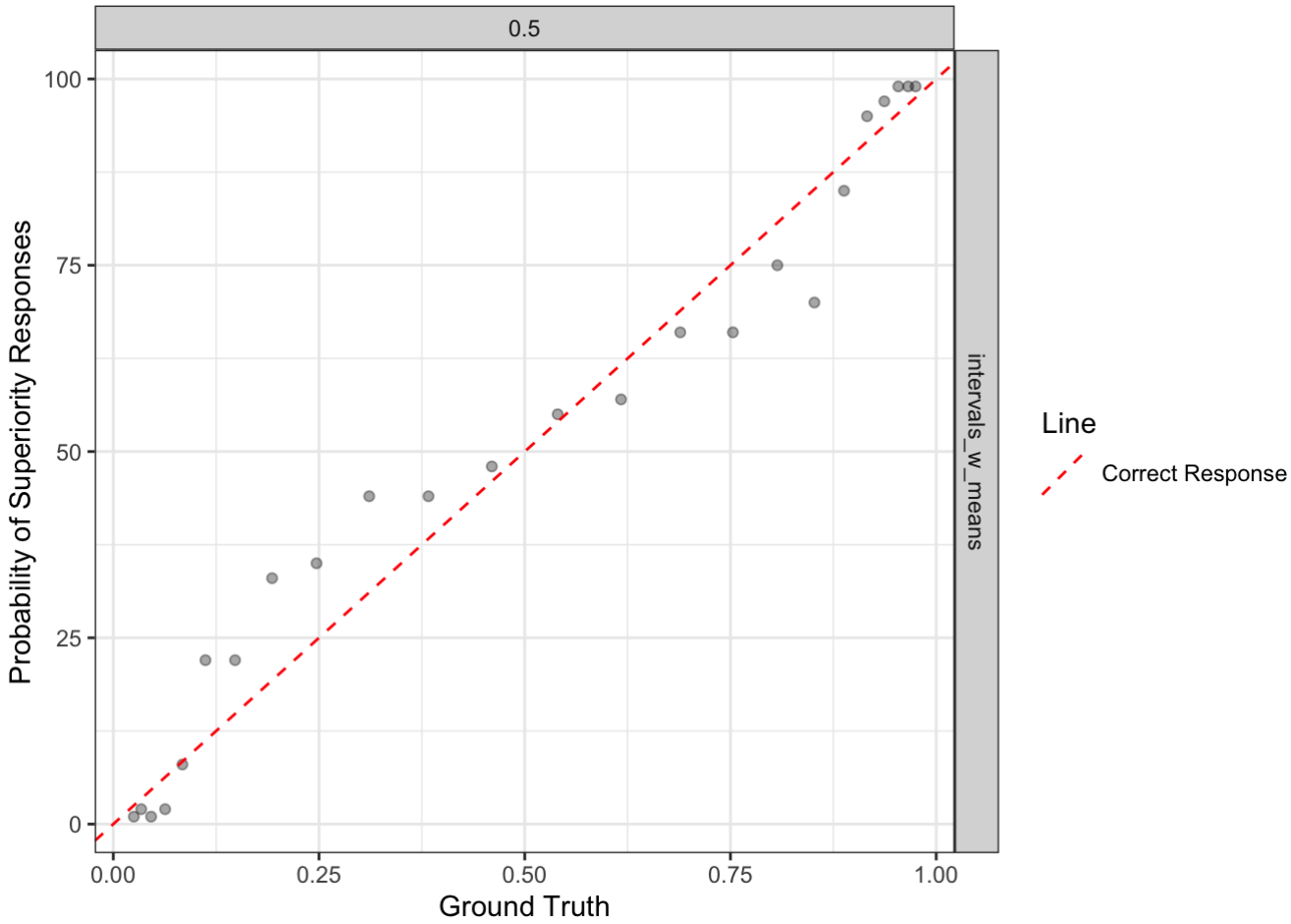
```
for (cond in unique(responses_df$condition)) {
  plt <- responses_df %>% filter(condition == cond) %>%
    ggplot(aes(x = p_superiority)) +
    geom_histogram(aes(y = ..density..), binwidth = 5) +
    geom_vline(aes(xintercept = ground_truth * 100, linetype = "Ground Truth"), color
= "red") +
    scale_linetype_manual(name = "Line", values = c(2,1), guide=guide_legend(overrid
e.aes = list(color = c("red")))) +
    theme_bw() +
    labs(
      caption=cond,
      x = "Probability of Superiority Responses",
      y = "Frequency"
    ) +
    facet_wrap( ~ ground_truth)
  print(plt)
}
```



As we would expect based on a linear log odds representation of probability, probability of superiority judgments tend to be biased toward 50% relative to the ground truth. We can also see that responses are highly variable, with some responses falling completely on the wrong side of 50%, suggesting that some participants may have been confused by this question.

Another more compact way of looking at the relationship between estimated probability of superiority and the ground truth is to just plot them against one another. Let's look at this even though its sort of a mess.

```
# plot estimated probability of superiority vs the ground truth
responses_df %>%
  ggplot(aes(x = ground_truth, y = p_superiority)) +
  geom_point(alpha = 0.35) +
  geom_abline(aes(intercept = 0, slope = 100, linetype = "Correct Response"), color =
"red") +
  scale_linetype_manual(name = "Line", values = c(2,1), guide=guide_legend(override.a
es = list(color = c("red")))) +
  theme_bw() +
  labs(
    x = "Ground Truth",
    y = "Probability of Superiority Responses"
  ) +
  facet_grid(condition ~ baseline)
```



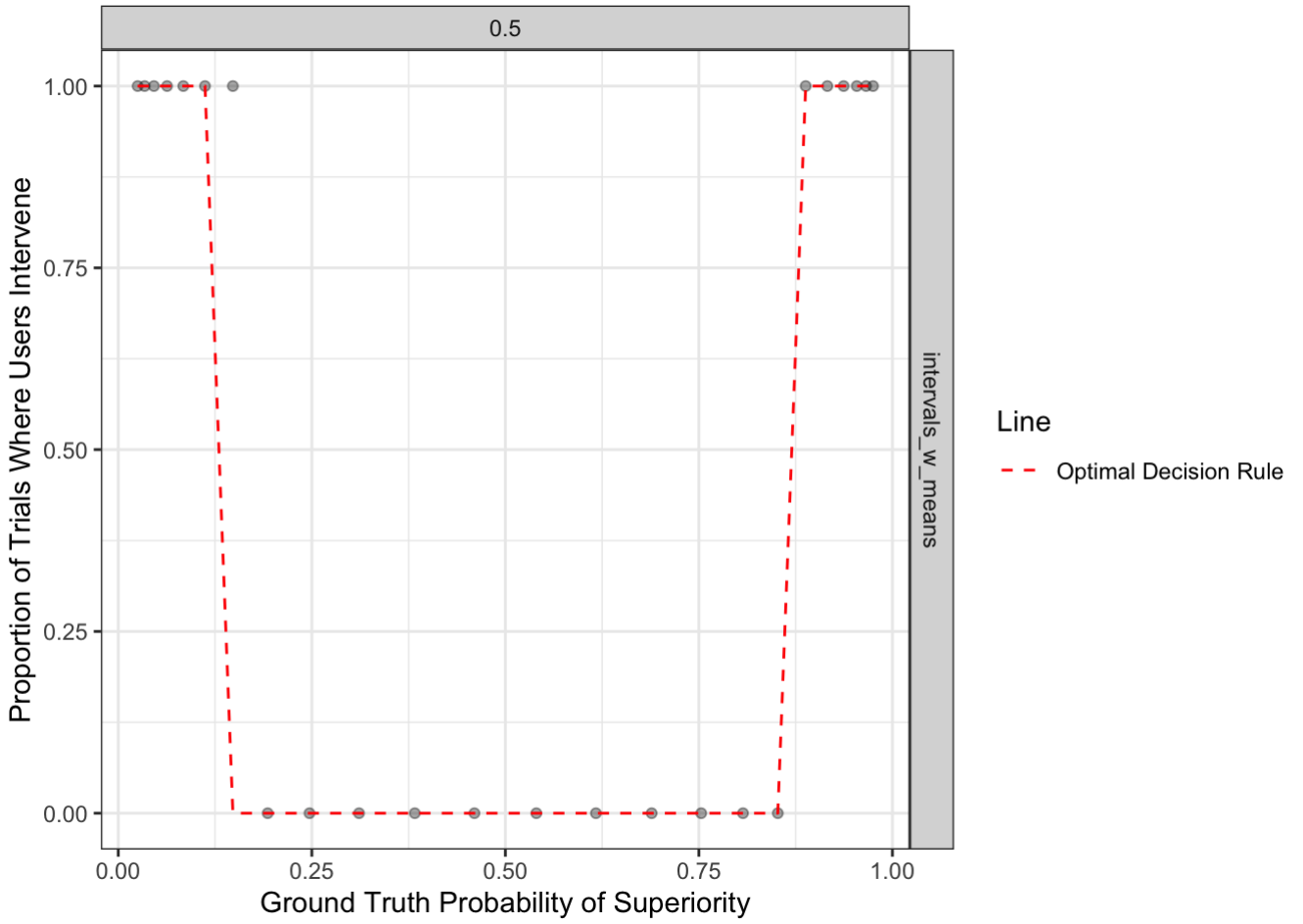
Decisions to Intervene

In order to see how people are doing on the decision task, we want to benchmark their performance against a utility optimal decision rule. The rule is different depending on whether the task is framed as a gain or a loss (i.e., whether the ground truth probability of superiority is greater than or less than 50%).

```
# determine whether or not intervention is utility optimal on each trial
responses_df <- responses_df %>%
  mutate(should_intervene = if_else(ground_truth > 0.5,
                                    (p_contract_old + 1 / contract_value) < p_contrac
t_new, # gain framing decision rule
                                    ((1 - p_contract_old) - 1 / contract_value) > (1
 - p_contract_new))) # loss framing decision rule
```

Let's plot the proportion of users who intervene at each level of ground truth probability of superiority in each visualization * baseline condition. People should intervene more often at extreme probabilities. We show the utility optimal decision threshold in red. This should give us an overview of decision quality.

```
# summarise the data as the overall proportion of trials where users intervene vs wha
t they should do at each level of ground_truth * condition * baseline
responses_df %>%
  group_by(condition, baseline, ground_truth) %>%
  summarise(
    proportion_intervene = sum(intervene) / n(),
    optimal_decision = mean(should_intervene)
  ) %>%
  ggplot(aes(x = ground_truth, y = proportion_intervene)) +
  geom_point(alpha = 0.35) +
  geom_line(aes(y = optimal_decision, linetype="Optimal Decision Rule"), color="red")
+
  scale_linetype_manual(name="Line", values = c(2,1), guide=guide_legend(override.aes
=list(color=c("red")))) +
  theme_bw() +
  labs(
    x = "Ground Truth Probability of Superiority",
    y = "Proportion of Trials Where Users Intervene"
  ) +
  facet_grid(condition ~ baseline)
```

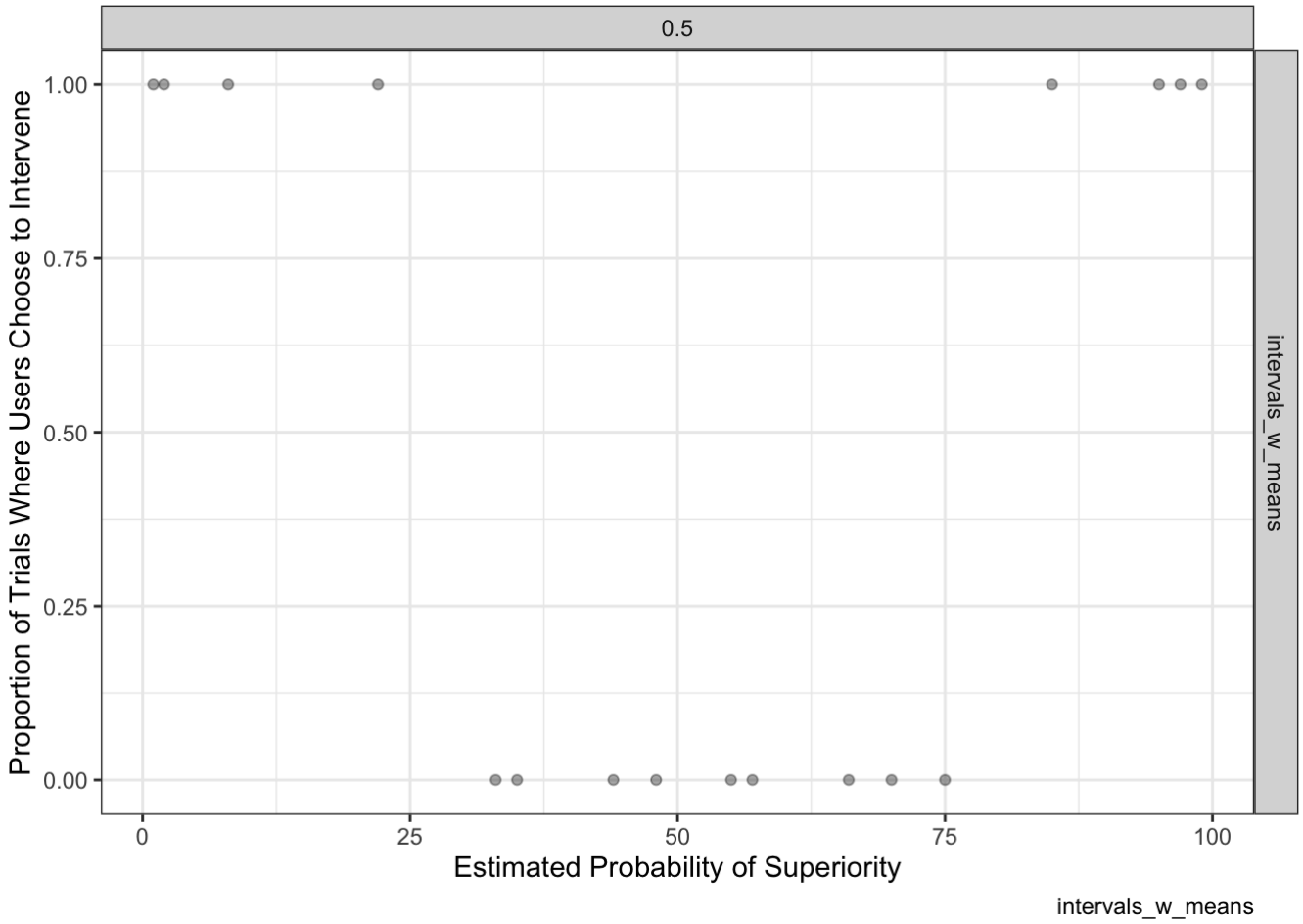


Unsurprisingly, users seem pretty bad at decision-making when they use only means to inform their reasoning. Interestingly, the baseline condition seems to have a large impact on decision quality (provisionally, people seem to make better decisions when entropy is lower or the baseline outcome is more certain).

Probability of Superiority Judgments vs Decisions to Intervene

It might also be interesting to see how decisions corresond to probability of superiority judgments. We omit the ground truth and optimal decision rule from this chart.

```
# summarise the data as the overall proportion of trials where users choose to interv
ene at each level of condition * baseline * p_superiority
responses_df %>%
  group_by(condition, baseline, p_superiority) %>%
  summarise(proportion_intervene = sum(intervene) / n()) %>%
  ggplot(aes(x = p_superiority, y = proportion_intervene)) +
  geom_point(alpha = 0.35) +
  theme_bw() +
  labs(
    caption=cond,
    x = "Estimated Probability of Superiority",
    y = "Proportion of Trials Where Users Choose to Intervene"
  ) +
  facet_grid(condition ~ baseline)
```



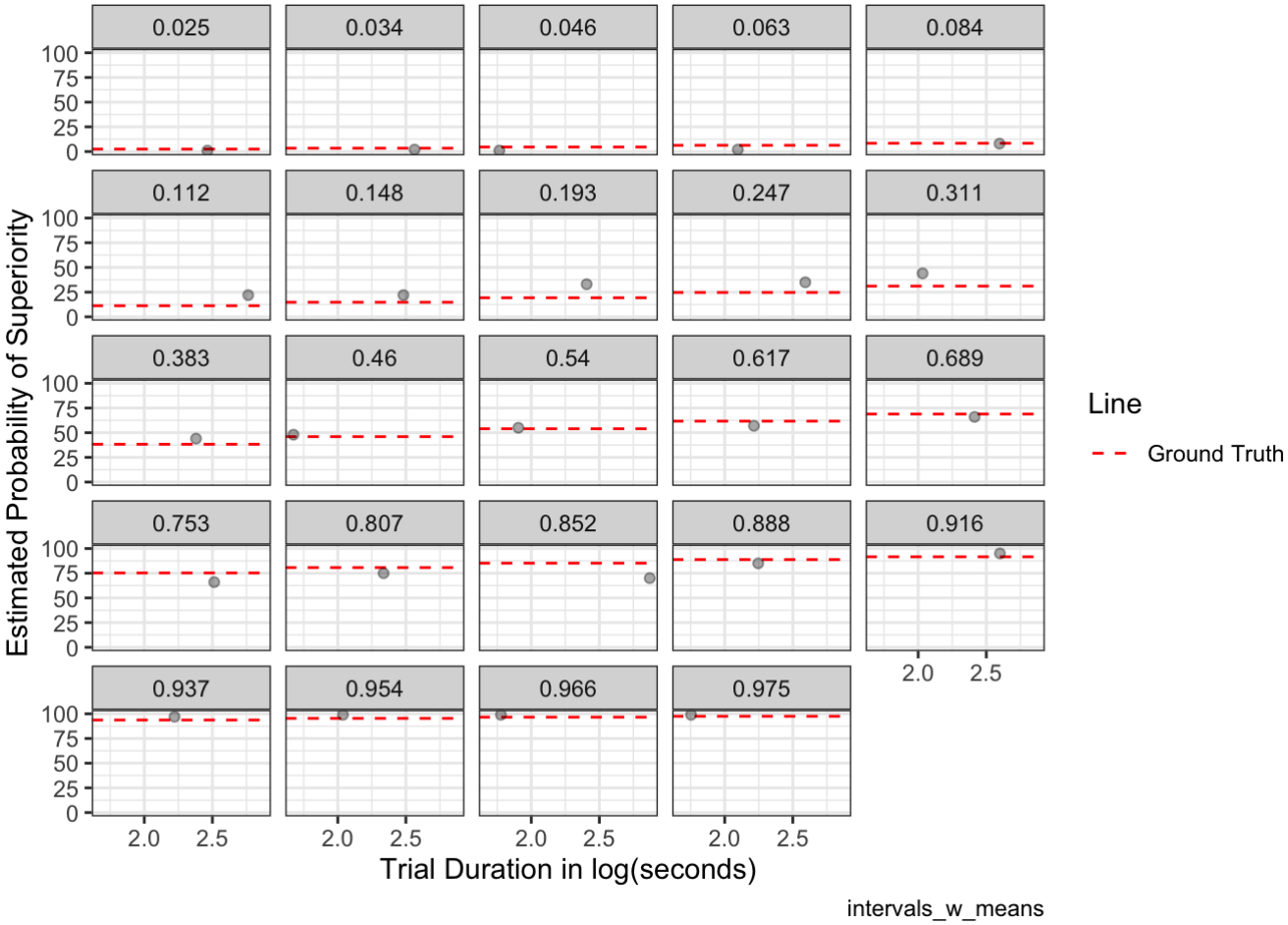
Relationships with Trial Duration

We want to know when, if at all, spending more time on a response results in improved performance.

Trial Duration vs Probability of Superiority Judgments

Let's look at probability of superiority estimates as a function of trial duration. As before, we show the ground truth in red and separate visualization conditions into different views to limit the number of faceted subplots in a single view.

```
for (cond in unique(responses_df$condition)) {
  plt <- responses_df %>% filter(condition == cond) %>%
    ggplot(aes(x = log(trial_dur), y = p_superiority)) +
    geom_hline(aes(yintercept = ground_truth * 100, linetype = "Ground Truth"), color
= "red") +
    scale_linetype_manual(name = "Line", values = c(2,1), guide=guide_legend(overrid
e.aes = list(color = c("red")))) +
    geom_point(alpha = 0.35) +
    theme_bw() +
    labs(
        caption=cond,
        x = "Trial Duration in log(seconds)",
        y = "Estimated Probability of Superiority"
    ) +
    facet_wrap( ~ ground_truth)
  print(plt)
}
```



Trial duration seems mostly unrelated to probability of superiority judgments except for in the case of HOPs, where responses seem to cluster closer to the ground truth on longer trial durations (with some exceptions). We should expect that people will have more accurate perceptions of probability the longer they watch HOPs.

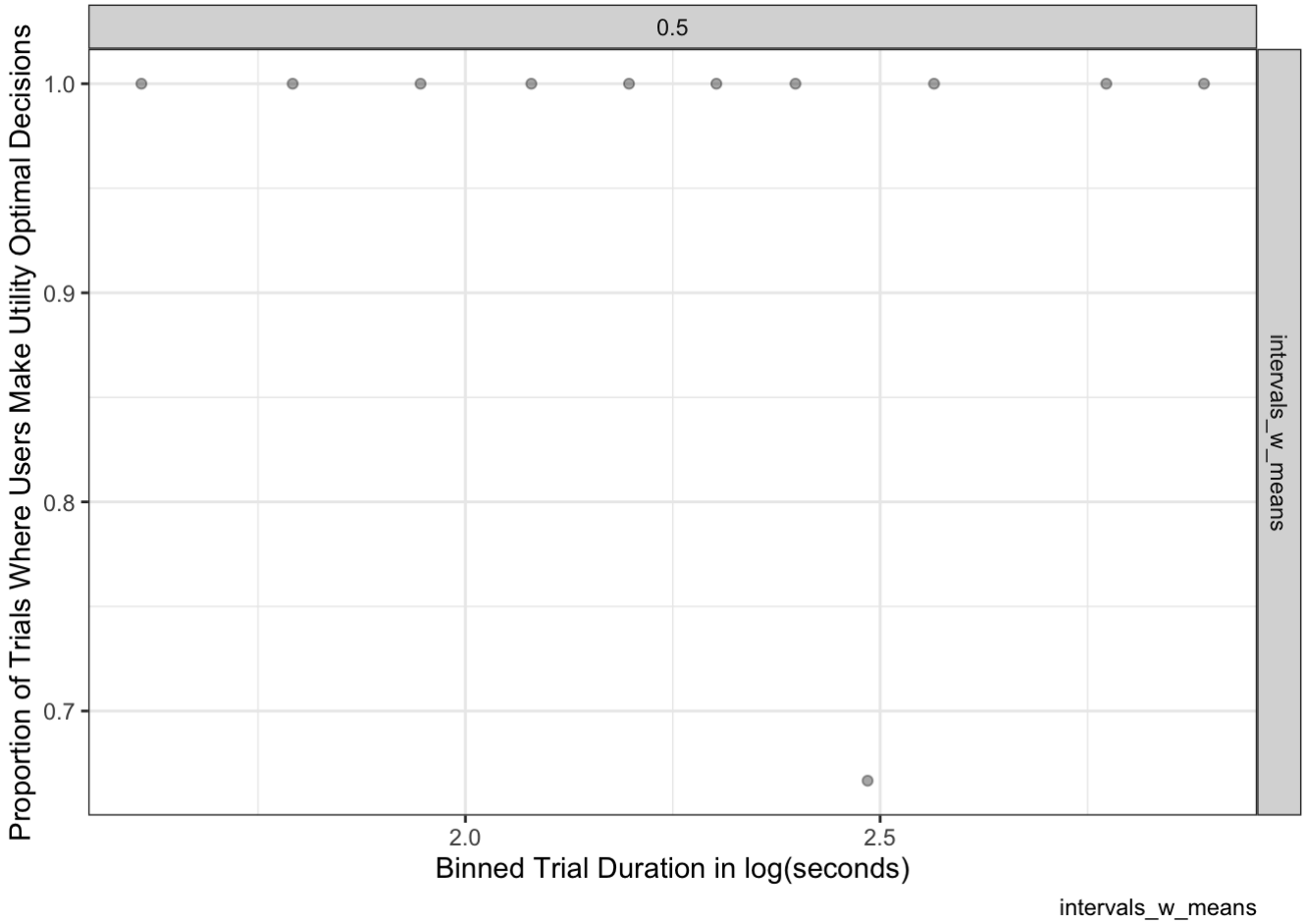
Trial Duration vs Decision Quality

A nice matric for decision quality is whether users responded "correctly" or in line with the normative utility optimal decision rule. We calculate whether the user was "correct"" or not on each trial.

```
# determine whether response on each trial is utility optimal
responses_df <- responses_df %>%
  mutate(correct = intervene == should_intervene)
```

Let's look at the proportion correct as a function of trial duration, faceting baseline and visualization conditions as above.

```
# summarise the data as the overall proportion of trials where users make utility opt
imal decisions at each level of condition * baseline * trial_dur
responses_df %>%
  mutate(trial_dur_binned = round(trial_dur)) %>%
  group_by(condition, baseline, trial_dur_binned) %>%
  summarise(proportion_correct = sum(correct) / n()) %>%
  ggplot(aes(x = log(trial_dur_binned), y = proportion_correct)) +
  geom_point(alpha = 0.35) +
  theme_bw() +
  labs(
    caption=cond,
    x = "Binned Trial Duration in log(seconds)",
    y = "Proportion of Trials Where Users Make Utility Optimal Decisions"
  ) +
  facet_grid(condition ~ baseline)
```



Trial duration seems to have little to do with decision quality.

Error Analysis

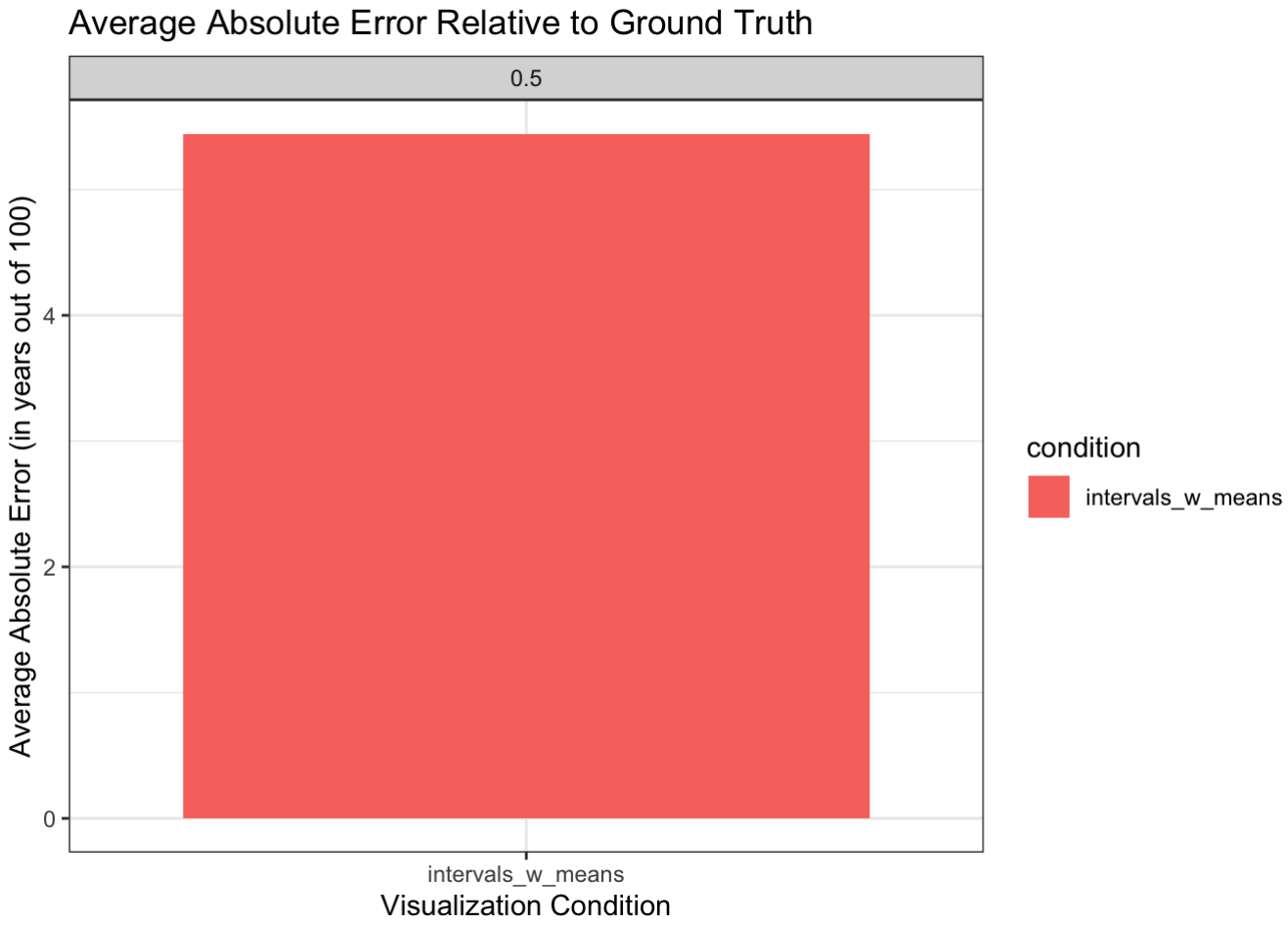
In this section, we look for patterns of interest in response errors. We'll start by adding error and absolute error in probability of superiority judgments to the dataframe. We already have a metric for correctness of decisions.

```
# add error and absolute error to df
responses_df <- responses_df %>%
  mutate(
    err_p_sup = ground_truth * 100 - p_superiority,
    abs_err_p_sup = abs(err_p_sup)
  )
```

Mean Absolute Error

Let's look at the average absolute error in probability of superiority judgments in each condition, regardless of the ground truth.

```
# avg absolute error per condition
responses_df %>%
  group_by(condition, baseline) %>%
  summarise(avg_abs_err_p_sup = mean(abs_err_p_sup)) %>%
  ggplot(aes(x = condition, y = avg_abs_err_p_sup, fill = condition)) +
    geom_bar(stat = "identity") +
    theme_bw() +
    labs(title = "Average Absolute Error Relative to Ground Truth",
        x = "Visualization Condition",
        y = "Average Absolute Error (in years out of 100)"
    ) +
  facet_grid(~ baseline)
```

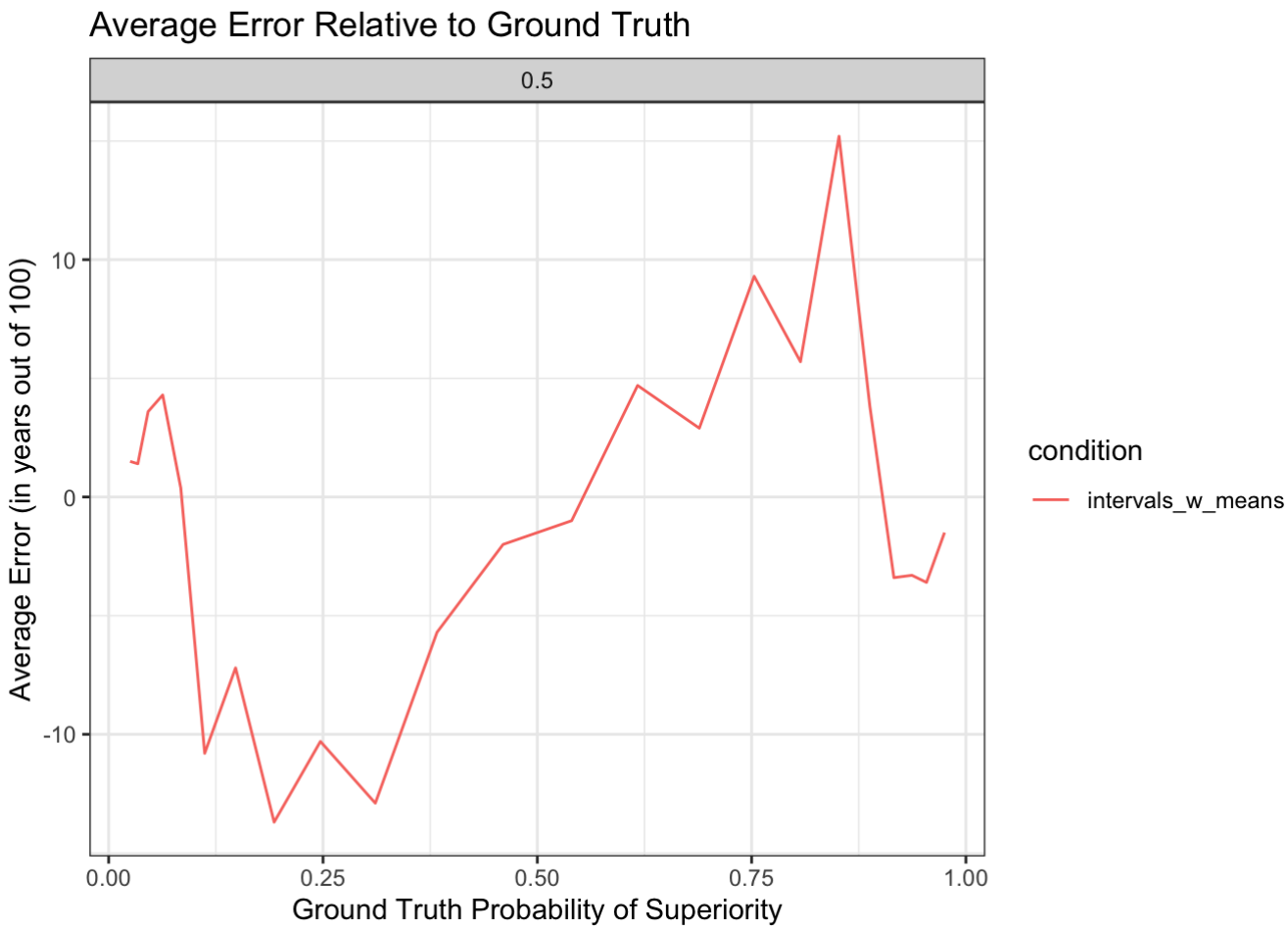


On average, errors in probability of superiority judgments are high across the board, with the average error equal to about a quarter the range of possible responses.

Mean Error vs Ground Truth

Let's look at the average signed error in probability of superiority judgments. This time we'll plot error in each condition in relation to ground truth.

```
# error by ground truth, per condition
responses_df %>%
  group_by(ground_truth, condition, baseline) %>%
  summarise(avg_err_p_sup = mean(err_p_sup)) %>%
  ggplot(aes(x = ground_truth, y = avg_err_p_sup, color = condition)) +
    geom_line() +
    theme_bw() +
    labs(title = "Average Error Relative to Ground Truth",
        x = "Ground Truth Probability of Superiority",
        y = "Average Error (in years out of 100)"
    ) +
    facet_grid(~ baseline)
```



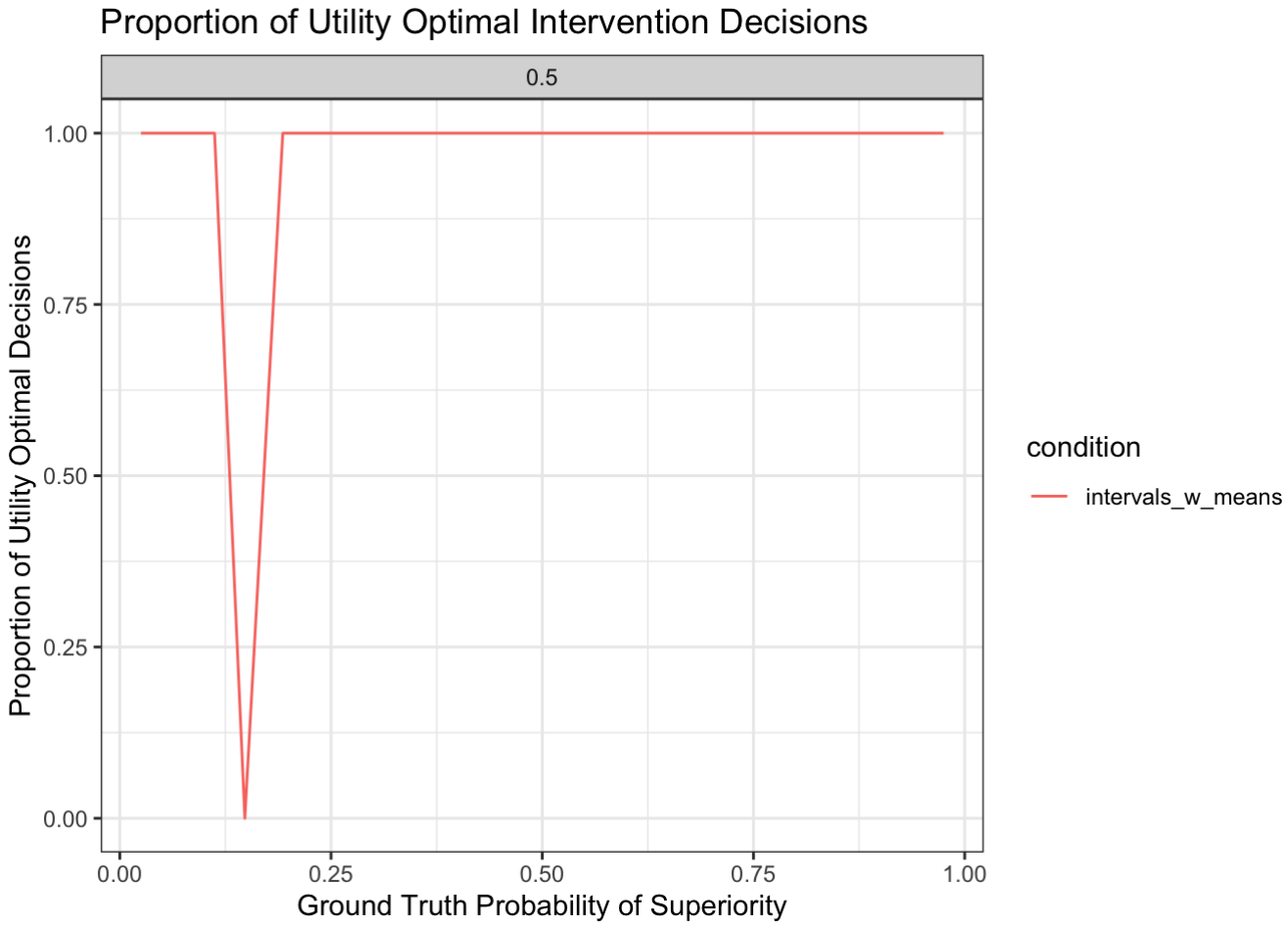
Again, we can see that errors are large on average, especially at the extreme ends of the probability scale. Higher errors at the extremes of the probability scale are expected based on *the cental tendency of judgments*.

We can also see that errors are larger in the HOPs condition when the baseline probability of gaining the contract is high. This seems to only be the case for ground truth values greater than 50%. Maybe this asymmetry can be explained by outliers?

Proportion of Utility Optimal Decisions

Let's take a similar approach to visualizing decisions by looking at the proportion of utility optimal decisions as a function of ground truth probability of superiority and condition.

```
# error by ground truth, per condition
responses_df %>%
  group_by(ground_truth, condition, baseline) %>%
  summarise(proportion_correct = sum(correct) / n()) %>%
  ggplot(aes(x = ground_truth, y = proportion_correct, color = condition)) +
    geom_line() +
    theme_bw() +
    labs(title = "Proportion of Utility Optimal Intervention Decisions",
        x = "Ground Truth Probability of Superiority",
        y = "Proportion of Utility Optimal Decisions"
    ) +
    facet_grid(~ baseline)
```



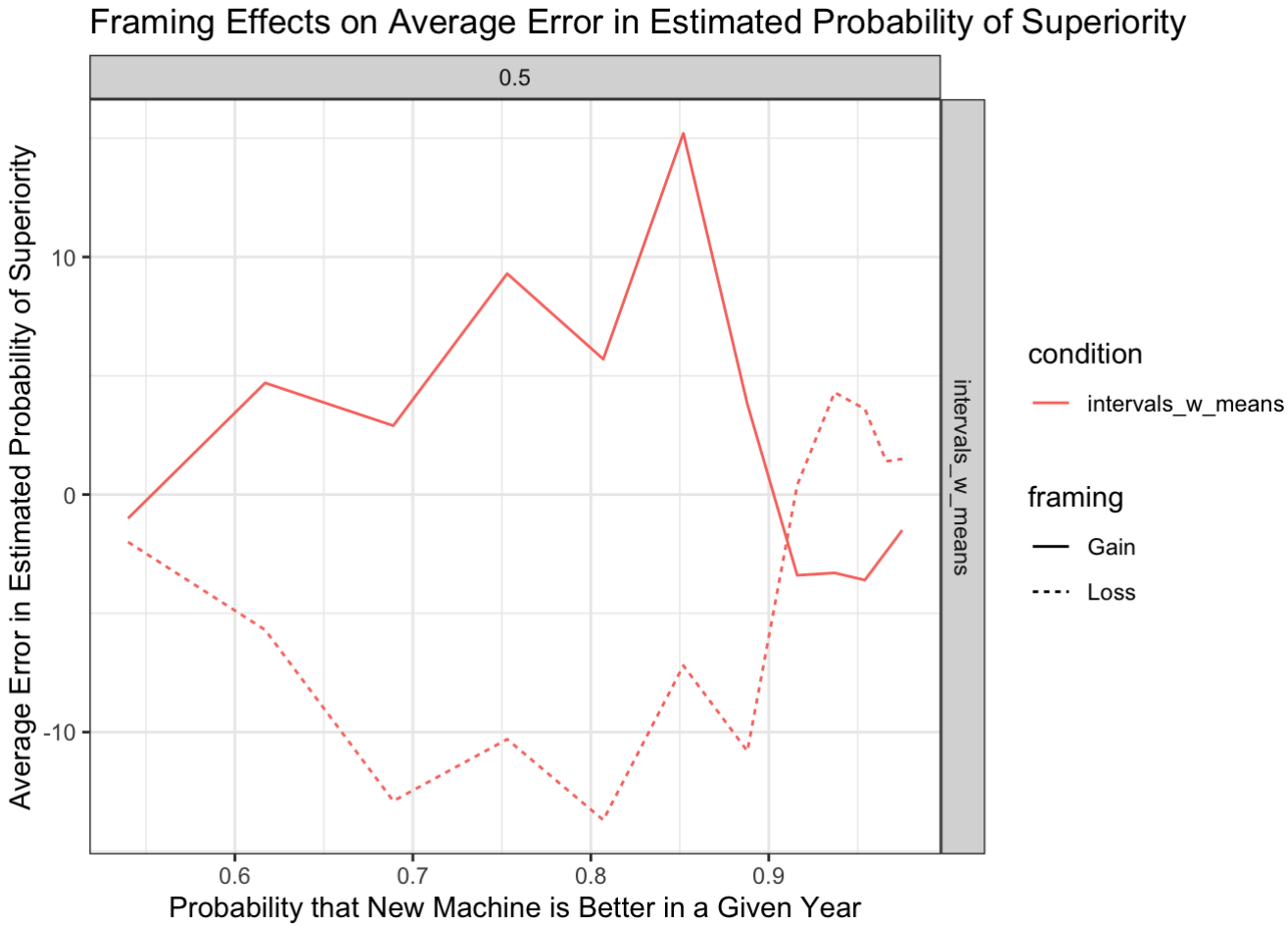
We can see that there are dips in performance near 25% and 75% probability of superiority. This is expected considering that these are the charts for which the intervention condition is most ambiguous. Interestingly, when the ground truth is near 50% users of charts that shows only means perform much worse than the users of other charts *when the baseline probability of gaining/keeping the contract is 50%*. This difference seems to disappear when the baseline is increased to 85%. Are means particularly ill suited for decision-making under conditions of maximum ambiguity but less harmful otherwise? Is this trend just noise?

Checking for Bias Due to Framing

We should check whether responses are biased by the framing of the problem either as a potential *gain* when probability of superiority is greater than 50% or as a potential *loss* when probability of superiority is less than 50%.

Let's start by looking at signed errors in probability of superiority estimates. We facet out visualization conditions in this view to make it easier to detect asymmetries between framing conditions.

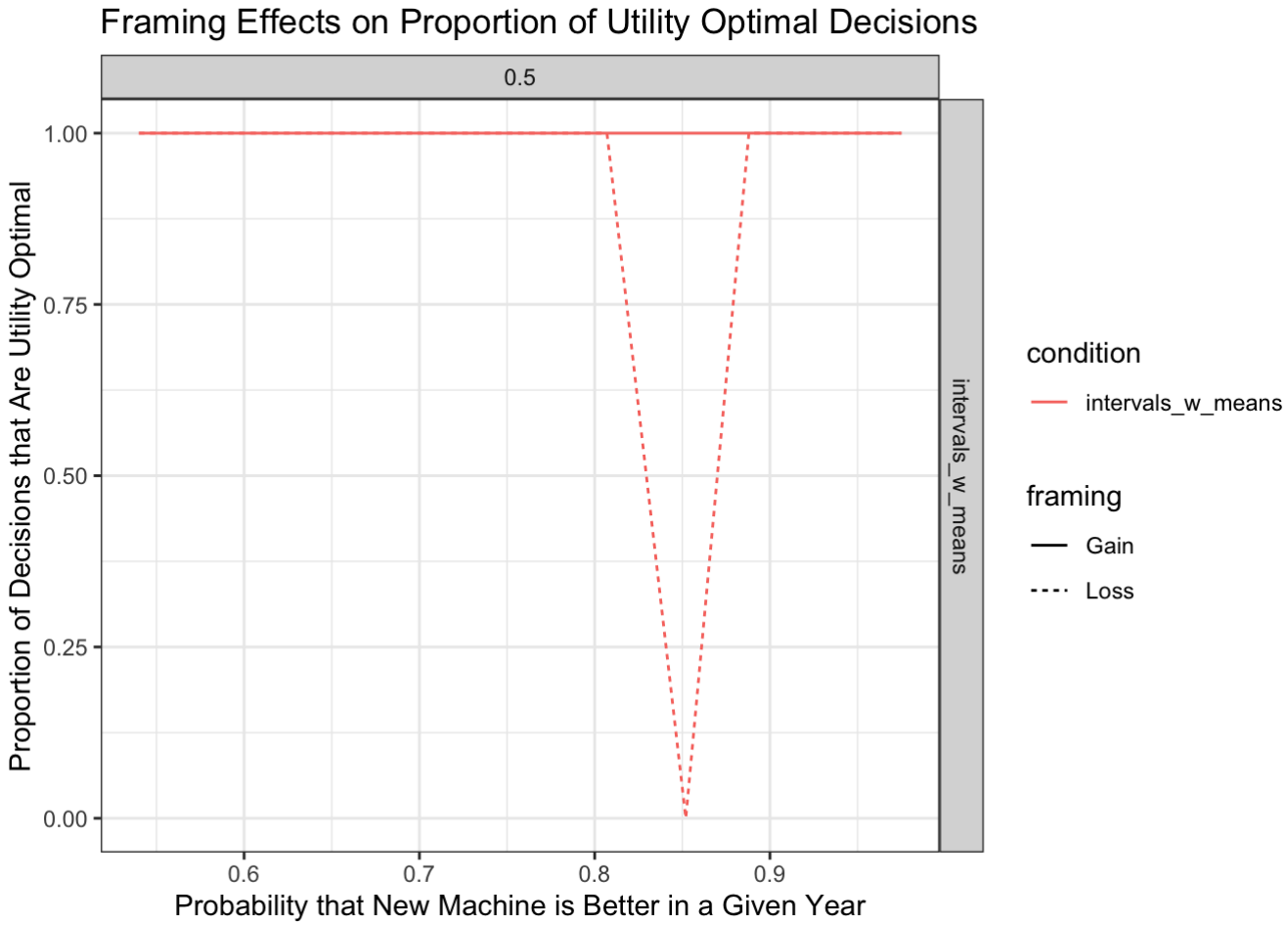
```
# reflect error where probability of superiority < 50% onto range between 0.5 and 1
responses_df %>%
  mutate(
    ground_truth_50_100 = ifelse(ground_truth < 0.5, 1 - ground_truth, ground_truth),
    framing = ifelse(ground_truth > 0.5, "Gain", "Loss")
  ) %>%
  group_by(ground_truth_50_100, condition, baseline, framing) %>%
  summarise(avg_err_p_sup = mean(err_p_sup)) %>%
  ggplot(aes(x = ground_truth_50_100, y = avg_err_p_sup, color = condition)) +
    geom_line(aes(linetype = framing)) +
    theme_bw() +
    labs(title = "Framing Effects on Average Error in Estimated Probability of Superi
ority",
         x = "Probability that New Machine is Better in a Given Year",
         y = "Average Error in Estimated Probability of Superiority"
    ) +
    facet_grid(condition ~ baseline)
```



The means only condition looks pretty strange at a baseline of 85%. It's hard to say why the lines would cross like that. With the amount of data collected, it is difficult to differentiate any apparent asymmetry from noise. We'll need to tease this out using formal statistical analysis.

Next, let's use a similar visualization to investigate framing bias in the proportion of utility optimal decisions.

```
# reflect error where Pr(A > B) < 0.5 onto range between 0.5 and 1
responses_df %>%
  mutate(
    ground_truth_50_100 = ifelse(ground_truth < 0.5, 1 - ground_truth, ground_truth),
    framing = ifelse(ground_truth > 0.5, "Gain", "Loss")
  ) %>%
  group_by(ground_truth_50_100, condition, baseline, framing) %>%
  summarise(proportion_correct = sum(correct) / n()) %>%
  ggplot(aes(x = ground_truth_50_100, y = proportion_correct, color = condition)) +
    geom_line(aes(linetype = framing)) +
    theme_bw() +
    labs(title = "Framing Effects on Proportion of Utility Optimal Decisions",
        x = "Probability that New Machine is Better in a Given Year",
        y = "Proportion of Decisions that Are Utility Optimal"
    ) +
    facet_grid(condition ~ baseline)
```



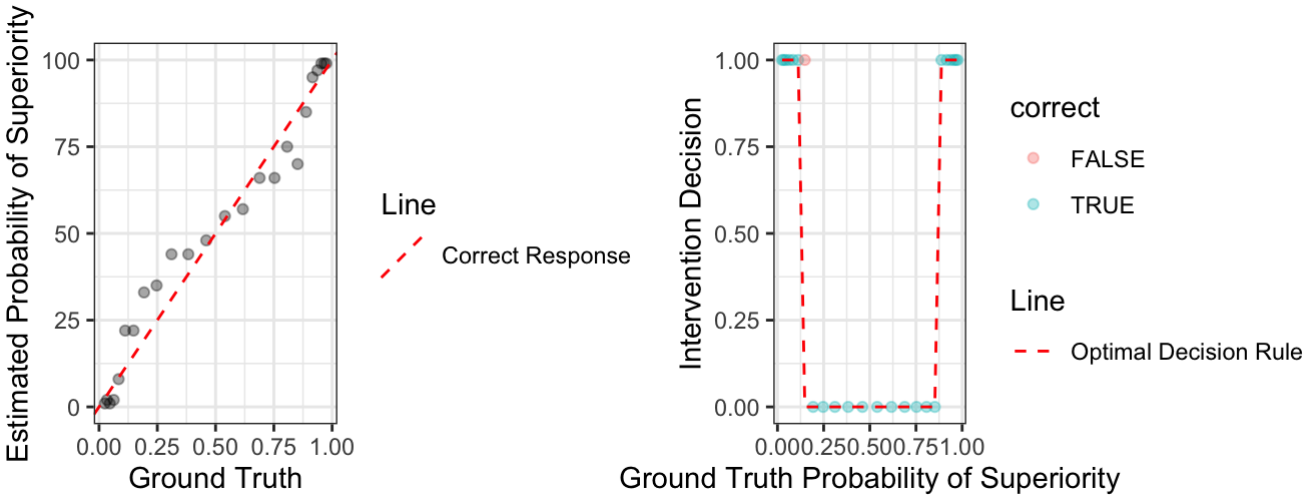
Again, the means only condition looks strange at a baseline of 85%. It makes me wonder if there is an outlier in that condition (e.g., a partipant who was speeding or didn't get the task). Otherwise, there doesn't seem to be much of a framing effect on decision performance.

Individual Patterns of Behavior

As is often the case with judgments from visualizations, the data seem highly heterogenious. We try to get a sense of this by looking at individual patterns of responses in conjunction with individual characteristics such as gender, age, education, chart use, and numeracy. Below we create an overview of performance and individual characteristics for each participant separately.

```
for (worker in unique(responses_df$worker_id)) {
  # get a df for just this worker
  worker_df <- responses_df %>% filter(worker_id == worker)
  # plot probability of superiority judgments vs ground truth
  p_sup_plt <- worker_df %>%
    ggplot(aes(x = ground_truth, y = p_superiority)) +
    geom_point(alpha = 0.35) +
    geom_abline(aes(intercept = 0, slope = 100, linetype = "Correct Response"), color
= "red") +
    scale_linetype_manual(name = "Line", values = c(2,1), guide=guide_legend(overrid
e.aes = list(color = c("red")))) +
    theme_bw() +
    ylim(0, 100) +
    labs(
      x = "Ground Truth",
      y = "Estimated Probability of Superiority"
    )
  # plot intervention decisions vs ground truth, noting which are in line with the ut
ility optimal decision rule
  decision_plt <- worker_df %>%
    ggplot(aes(x = ground_truth, y = intervene, color = correct)) +
    geom_point(alpha = 0.35) +
    geom_line(aes(y = as.numeric(should_intervene), linetype="Optimal Decision Rule"
), color="red") +
    scale_linetype_manual(name="Line", values = c(2,1), guide=guide_legend(override.a
es=list(color=c("red")))) +
    theme_bw() +
    labs(
      x = "Ground Truth Probability of Superiority",
      y = "Intervention Decision"
    )
  # create a table summarizing this worker
  summary_table <- worker_df %>%
    group_by(worker_id) %>%
    summarise(
      condition = unique(condition),
      baseline = unique(baseline),
      gender = unique(gender),
      age = unique(age),
      education = unique(education),
      chart_use = unique(chart_use),
      numeracy = unique(numeracy)
    ) %>%
    select(-worker_id) %>%
    ggtexttable(rows = NULL, theme = ttheme("blank"))
  # stitch together these three views
  charts <- ggarrange(p_sup_plt, decision_plt, ncol = 2, nrow = 1)
  figure <- ggarrange(summary_table, charts, ncol = 1, nrow = 2)
  print(figure)
}
```

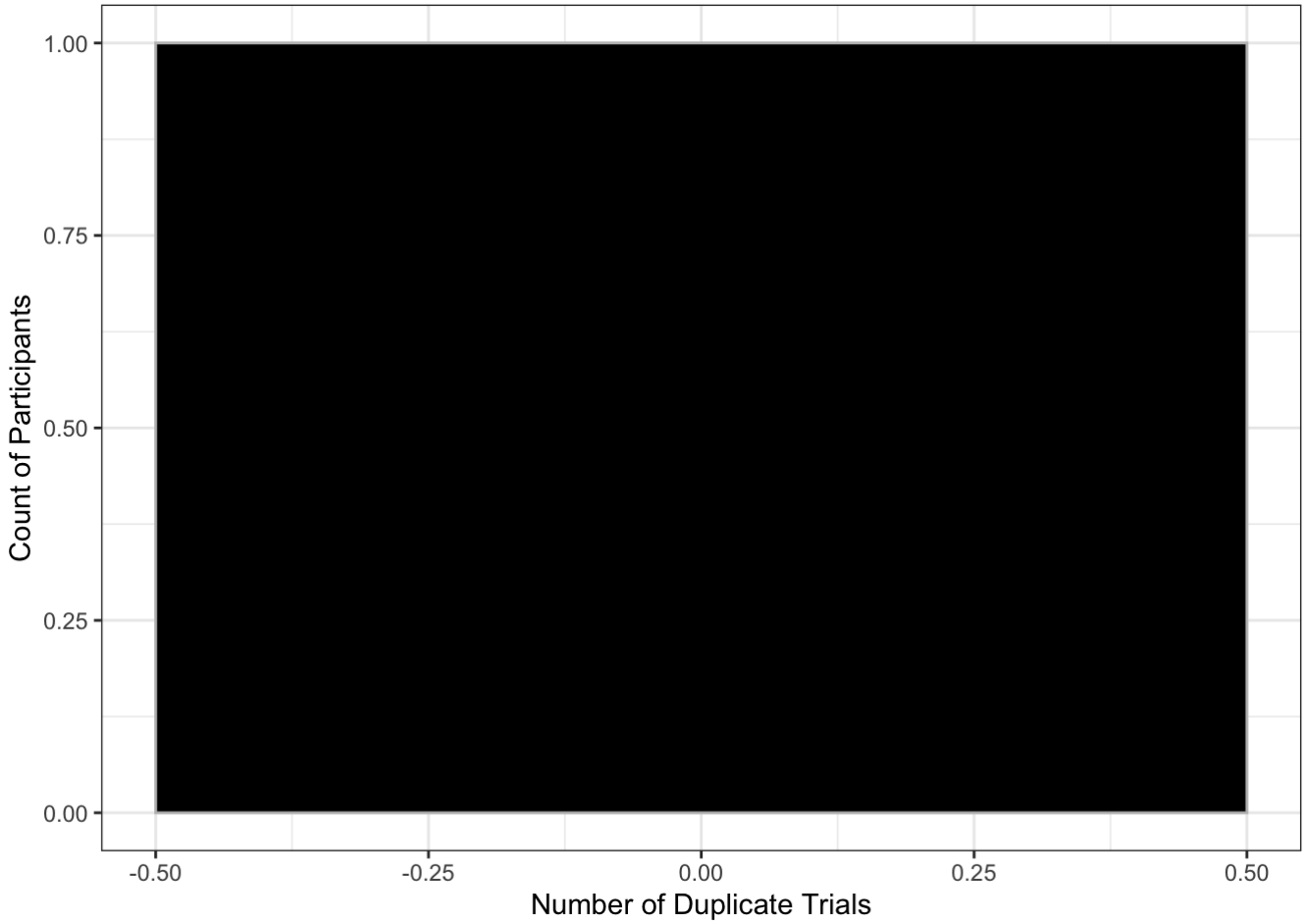
condition	baseline	gender	age	education	chart_use	numeracy
intervals_w_means	0.5	M	25-34	Bachelor's degree	Daily	11



Looking at these plots for individual participants, we can see that not everyone was shown all 24 levels of ground truth as intended. This is due to a bug in the interface code that reshuffled the trial set throughout the experiment for some participants. This was fixed after HIT assignment batch 7. Let's see what the damage looks like. How many participants were shown various numbers of duplicate trials?

```
# create a grid of worker ids * trial indices, every trial that should exist
trials_should_exist_df <- data_grid(responses_df, worker_id = unique(worker_id), tria
l = unique(trial_idx))

# check the number of times each worker was shown each trial, and plot the number of
 workers shown various numbers of duplicates
responses_df %>% select(worker_id, trial_idx) %>%
  right_join(trials_should_exist_df, by = "worker_id") %>%
  group_by(worker_id, trial) %>%
  summarise(n_times_shown_trial = sum(trial == trial_idx)) %>%
  group_by(worker_id) %>%
  summarise(n_duplicates = sum(n_times_shown_trial > 1)) %>%
  ggplot(aes(x = n_duplicates)) +
  geom_histogram(aes(y = ..count..), binwidth = 1, fill="black", col="grey") +
  theme_bw() +
  labs(
    x = "Number of Duplicate Trials",
    y = "Count of Participants"
  )
```


1.00
0.75
0.50
0.25
0.00
Count of Participants
-0.50
-0.25
0.00
0.25
0.50
Number of Duplicate Trials