Subobj

3. Sum of subspaces definition 12 is a subspace of V.

Let V be a vector space over field F and let $W_1, \dots, W_k$ be subspaces of V.

Show $W_1 + \dots + W_k = w_1 + \dots + w_k$, $w_i \in W_i$ is a subspace of V.

1) Not empty because $W_1, \dots, W_k$ are not empty.

6 2) Let $a, b \in W_1 + \dots + W_k$, a, b are vectors. Then $a+b \in W$.

show.

8 So this is a subspace by def 12.

Also check: multiplication by scalars.

5. Let $W_i, i \in I$ be a collection of subspaces of vector space V over field F. Let $W_i, i\in I$ be a collection of subspaces. For I finite this yields the

$$\sum_{i \in I} W_i = \mathrm{Span}_F\left(\bigcup_{i \in I} W_i\right)$$

Verify for I finite this yields the same as def. 12.

6 Suppose $w \in \mathrm{Span}_F(\bigcup_{i \in I} W_i)$. $w = w_1 + w_2 \dots + w_i$. Each w_i is in $\bigcup_{i \in I} W_i$, so clearly $w \in \sum_{i \in I} W_i$. Now suppose $w \in \sum_{i \in I} W_i$, then $w = w_1 + w_2 + \dots + w_i \in \bigcup_{i \in I} W_i$ X

This sum is not in the union.

So $w \in \mathrm{Span}(\bigcup_i W_i)$.

w_i need not be in W_i (You may have to group terms)

6. Let V be a vector space over field F. Assume W is a subspace of V and $S, S_i, i \in I$ are arbitrary subsets.

1) $\mathrm{Span}_F(W) = W$ If $W = \emptyset$, $\mathrm{Span}_F(W) = \{0\}$ and we are done.

If W is nonempty, then $\mathrm{Span}_F(W)$ is the intersection of subspaces of V containing W, by definition.

7 Let vector $v \in \mathrm{Span}_F(W)$, then $v = \sum_{i=1}^{n} a_i w_i$, $a_i \in F$, $w_i \in W$.

By definition of subspace, $v \in W$ and $v \in \mathrm{Span}_F(W)$. So $\mathrm{Span}\, W = W$.

Fields 26 Let F be a field and $Q \subseteq F$. F is a vector space over Q with dimension 2, show $\exists\, a \in F$ $a \notin Q$ and satisfies $a^2 - n = 0$ $n \in \mathbb{Z}$ $n \neq 0$.

Conclude F is isomorphic to $Q[\sqrt{n}]$ and n is square free.

~~Let $b \in \{F - Q\}$. Then $\exists\, 1, b, b^2$ such that $x \cdot 1 + y \cdot b + z \cdot b^2$, $x, y, z \in Q$.~~

~~Since $b \notin Q$, $z \neq 0$.~~

F is isomorphic to $Q[\sqrt{n}]$ as vector spaces. $Q[\sqrt{n}]$ is also of dimension 2.

$F = \{a + bk \mid a, b \in Q\}$ is a closed set. Let $k^2 = a + bk$, then $(k - \frac{1}{2}b)^2 = k^2 - bk + \frac{1}{4}b^2 = a + bk - bk + \frac{1}{4}b^2 = a + \frac{1}{4}b^2$. Then $(2k - b)^2 = 4a + b^2$

$\underset{0}{\shortparallel}$

So F is isomorphic to $Q[\sqrt{4a + b^2}]$.

10

Suppose $Q[\sqrt{n}]$ is not square free. Then $a + b\sqrt{n} = a + bc\sqrt{n^*}$, where $n = c^2 n^*$. $Q[\sqrt{n}]$ is isomorphic to square free $Q[\sqrt{n}]$, written as $Q[\sqrt{n^*}]$. So $Q[\sqrt{n}]$ must be square free as well.

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