Midterm Exam (Stat 583) - March '04

Problem 1. Let $X_1, \ldots, X_n$ be i.i.d. random variables with density function

$$f(x \mid \theta) = (\theta + 1)x^{\theta}, \quad 0 \le x \le 1$$

(a) Find the method of moments estimate of θ.
(b) Find the mle of θ.
(c) Find the asymptotic variance of the mle.

Problem 2. Suppose that the number of minutes a person must wait for a bus each morning has a uniform distribution on the interval $[0, \theta]$, where the value of the endpoint θ is unknown. Suppose also that the prior p.d.f. of θ is as follows:

$$\xi(\theta) = \begin{cases} \frac{192}{\theta^4} & \text{for } \theta \ge 4, \\ 0 & \text{otherwise.} \end{cases}$$

If the observed waiting times on three successive mornings are 5, 3, and 8 minutes:

(a) What is the posterior p.d.f. of θ?
(b) What is the Bayes estimator if squared loss is used?

Problem 3. (a) Suppose that a sample of size 15 from a normal distribution gives $\bar{X} = 10$ and $s^2 = 25$. Find 90% confidence intervals for μ and σ^2.

(b) Suppose that the five random variables $X_1, \ldots, X_8$ are i.i.d., and each has a standard normal distribution. Determine a constant c such that the random variable

$$\frac{c(X_1 + X_2 + X_3)}{(X_4^2 + X_5^2 + X_6^2 + X_7^2 + X_8^2)^{\frac{1}{2}}}$$

will have a t distribution.

Problem 4. Suppose that $X_1, \ldots, X_n$ form a random sample from a Poisson distribution with unknown mean θ, and let $Y = \sum_{i=1}^{n} X_i$.

(a) Determine the value of a constant c such that the estimator e^{-cY} is an unbiased estimator of $e^{-\theta}$.
(b) Use the information inequality to obtain a lower bound for the variance of the unbiased estimator found in part (a).