Stat 310B: Problem Set 1

Due Tuesday, January 24

1. On the unit interval with Lebesgue measure, let $X(\omega) = \omega$ and for an arbitrary positive integer n let $Y(\omega) = n\omega - [n\omega]$, where $[x]$ denotes the largest integer $\leq x$. What is $E(X|Y)$? Explain your answer.

2. Suppose $E(X_i)$ exists for $i = 1, 2$ and $E(X_1; A) \leq E(X_2; A)$ for all $A \in \mathcal{F}$. Show that $P\{X_1 \leq X_2\} = 1$.

3. Suppose X and Y are square integrable. Show that for every sub-sigma-algebra $\mathcal{G} \subset \mathcal{F}$, $E[XE(Y|\mathcal{G})] = E[E(X|\mathcal{G})Y]$.

4. Let $Var(X|\mathcal{F}) = E(X^2|\mathcal{F}) - E(X|\mathcal{F})^2$. Prove that $Var(X) = E(Var(X|\mathcal{F})) + Var(E(X|\mathcal{F}))$.