Ray Casting

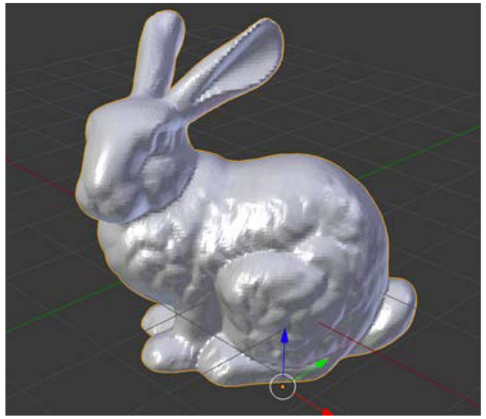
Some Slides/Images adapted from Marschner and Shirley

Topics

- The Ray Casting Algorithm
- Introduction to Rays
- The Camera
- Ray-Object Intersection
    - Ray-Plane Intersection
    - Ray-Sphere Intersection
    - Ray-Triangle Intersection

Photography

Input:



Objects

Lights

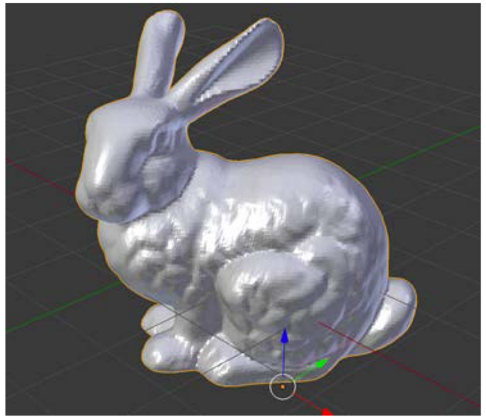
Camera

Output:

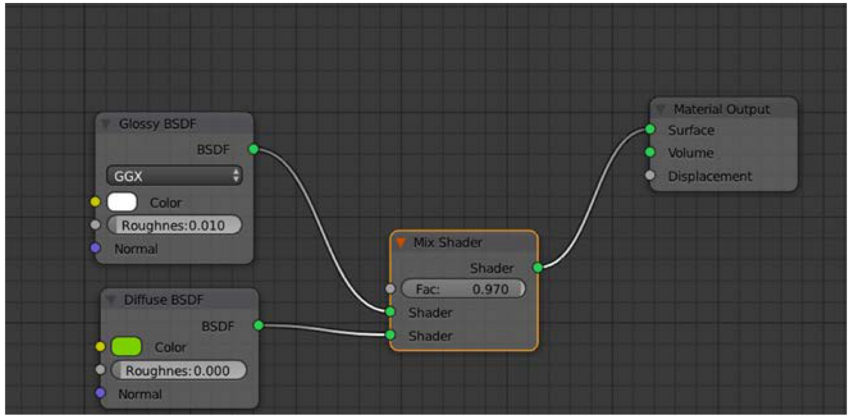
$$I\left(x,y\right)$$

Rendering

Input:



Objects



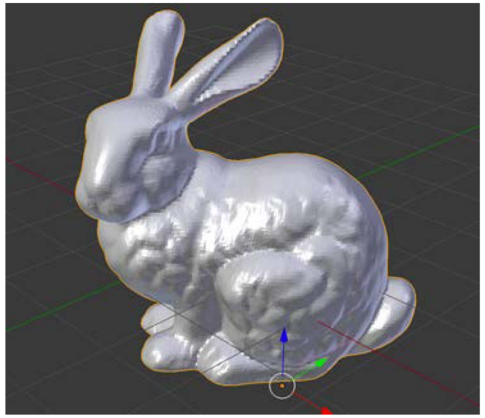
Materials

Output:

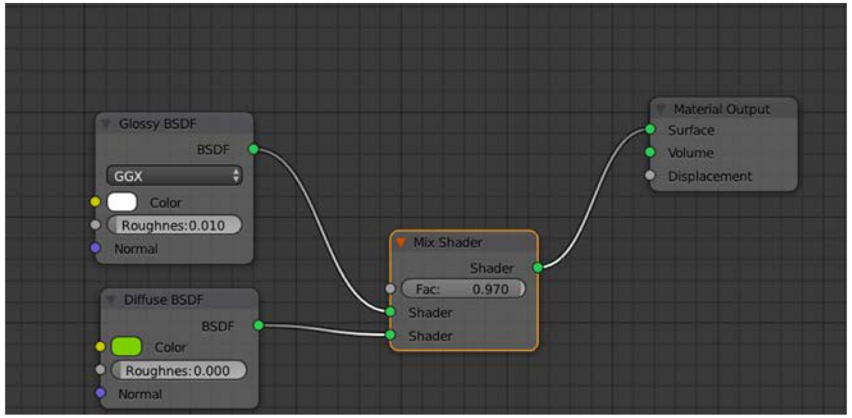
$I\left(x, y\right)$

Rendering

Input:



Objects

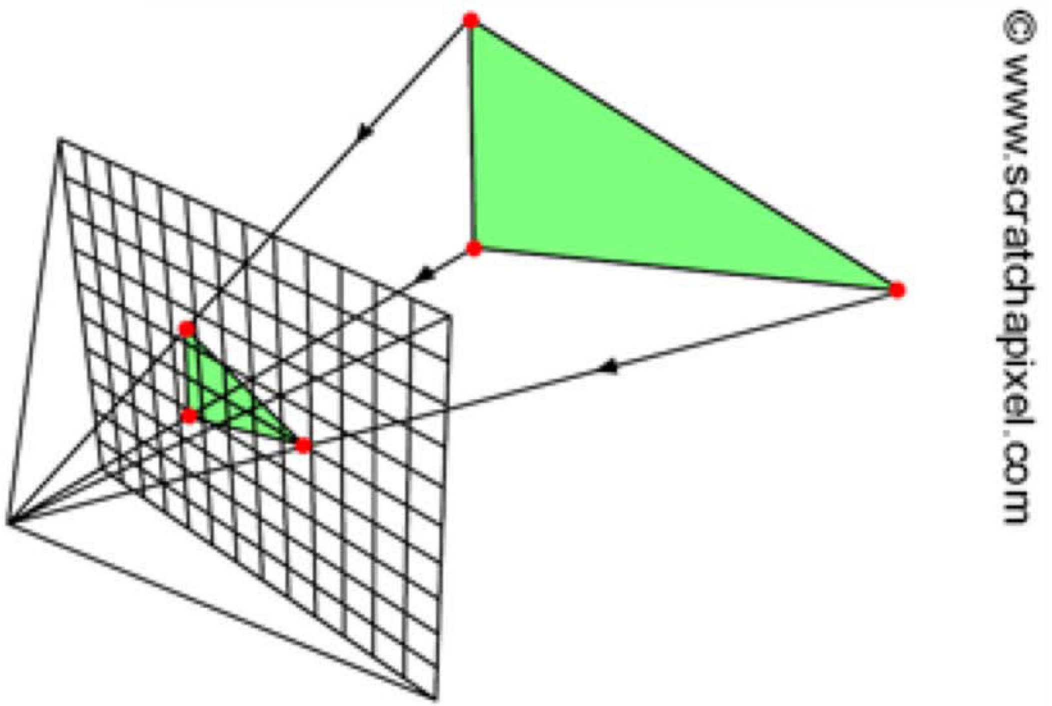


Materials

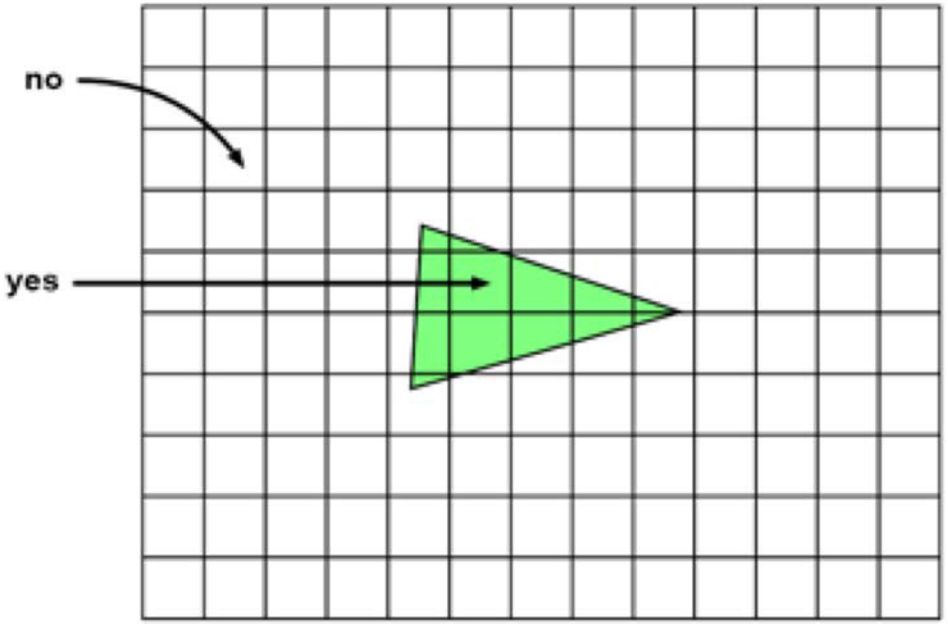
Output:

$$I\left(x, y\right)$$

Rasterization



1. Project Vertices to Image Plane



2. Turn on pixels inside triangle

https://www.scratchapixel.com/lessons/3d-basic-rendering/rasterization-practical-implementation

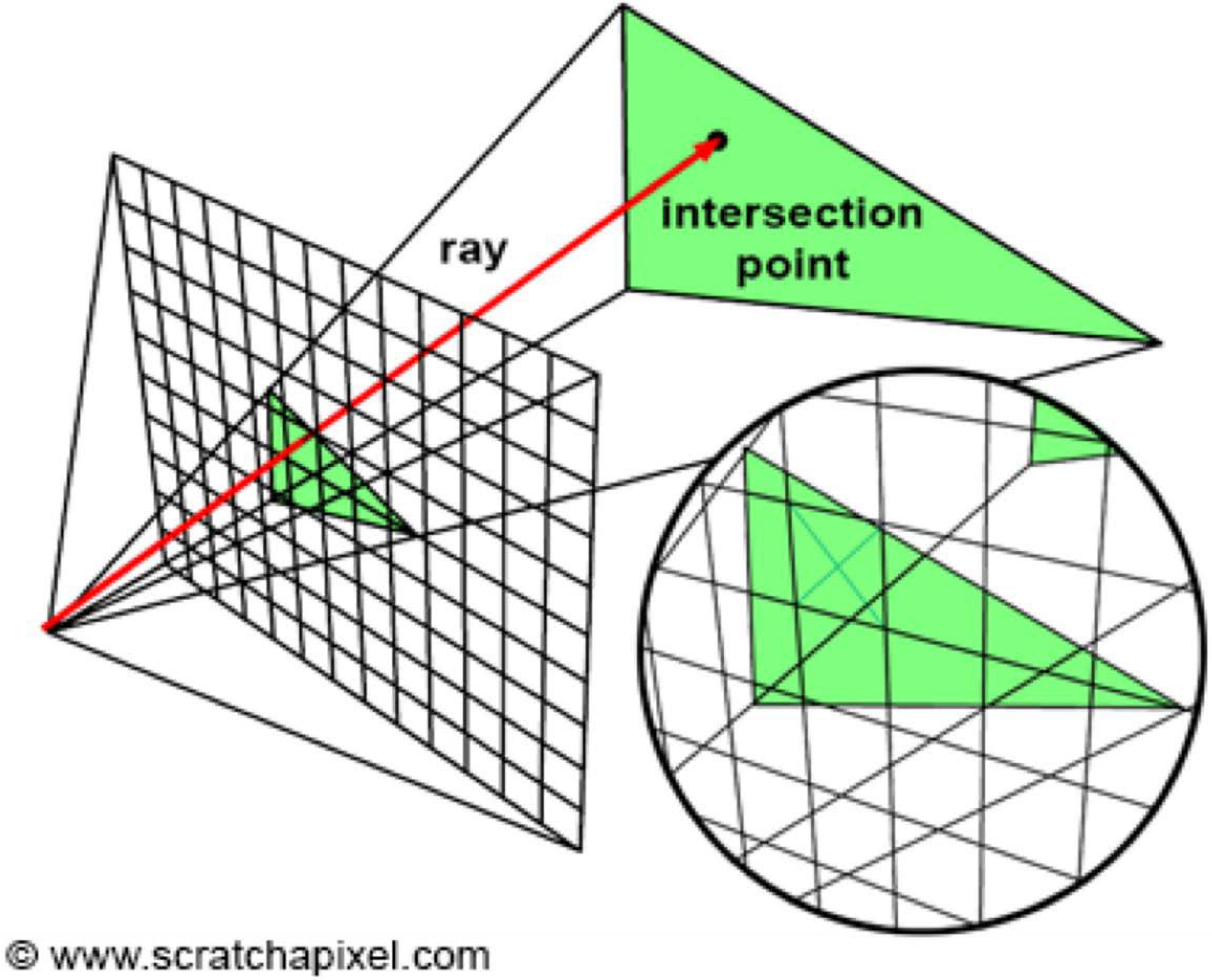
Fortnite | Epic Games

Rasterization

```
for each scene object {
    for each image pixel{
        if (object affects pixel) {
            do something
        }
    }
}
```

operations can be done quickly on the GPU!

Ray Casting



https://www.scratchapixel.com/lessons/3d-basic-rendering/rasterization-practical-implementation

Ray Casting

```
for each image pixel {
    generate a ray
    for each scene object {
        if (ray intersects object) {
            set pixel color
        }
    }
}
```

Ray Casting vs. Rasterization

Ray Casting

```
for each image pixel {
    for each scene object {
        ...
    }
}
```

Rasterization

```
for each scene object {
    for each image pixel {
        ...
    }
}
```

Basic Components of Ray Casting

- Ray

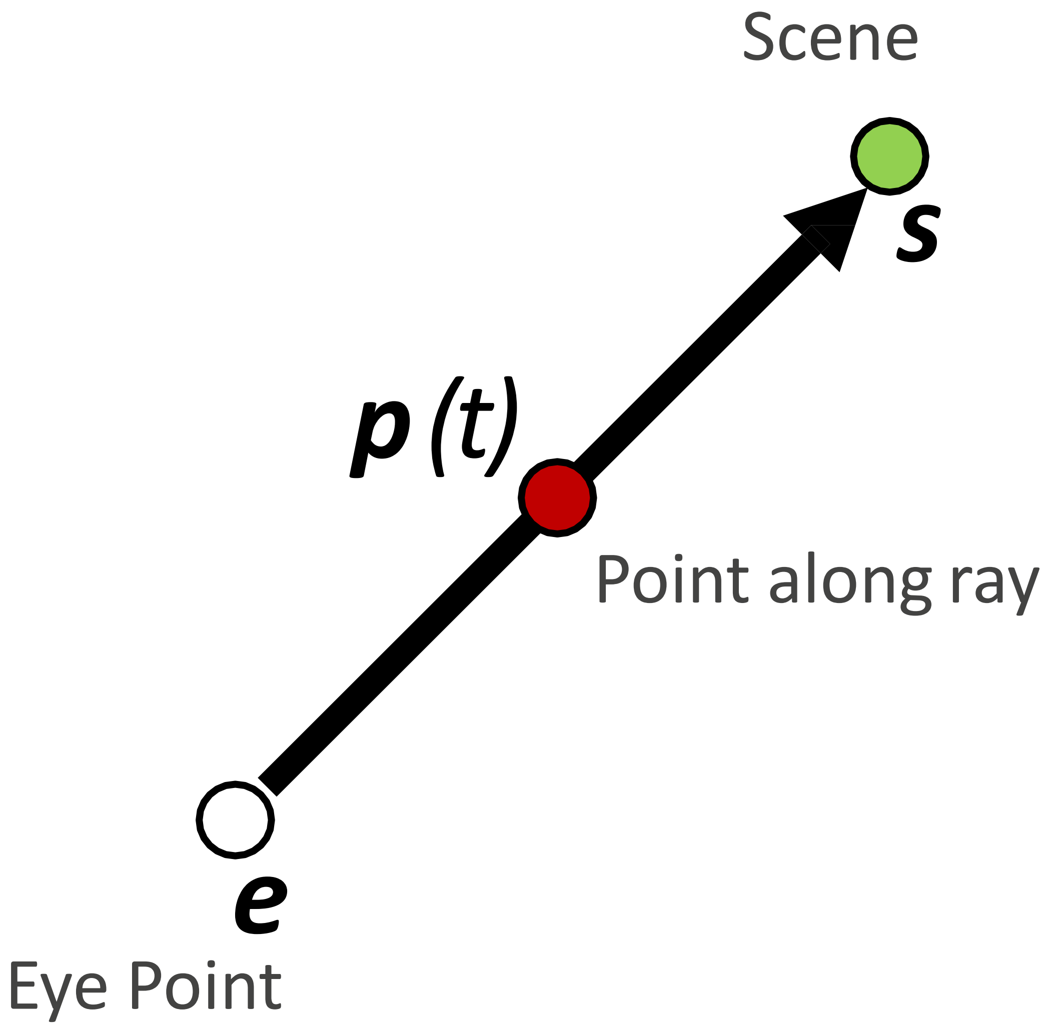
- Camera

- Intersection Tests

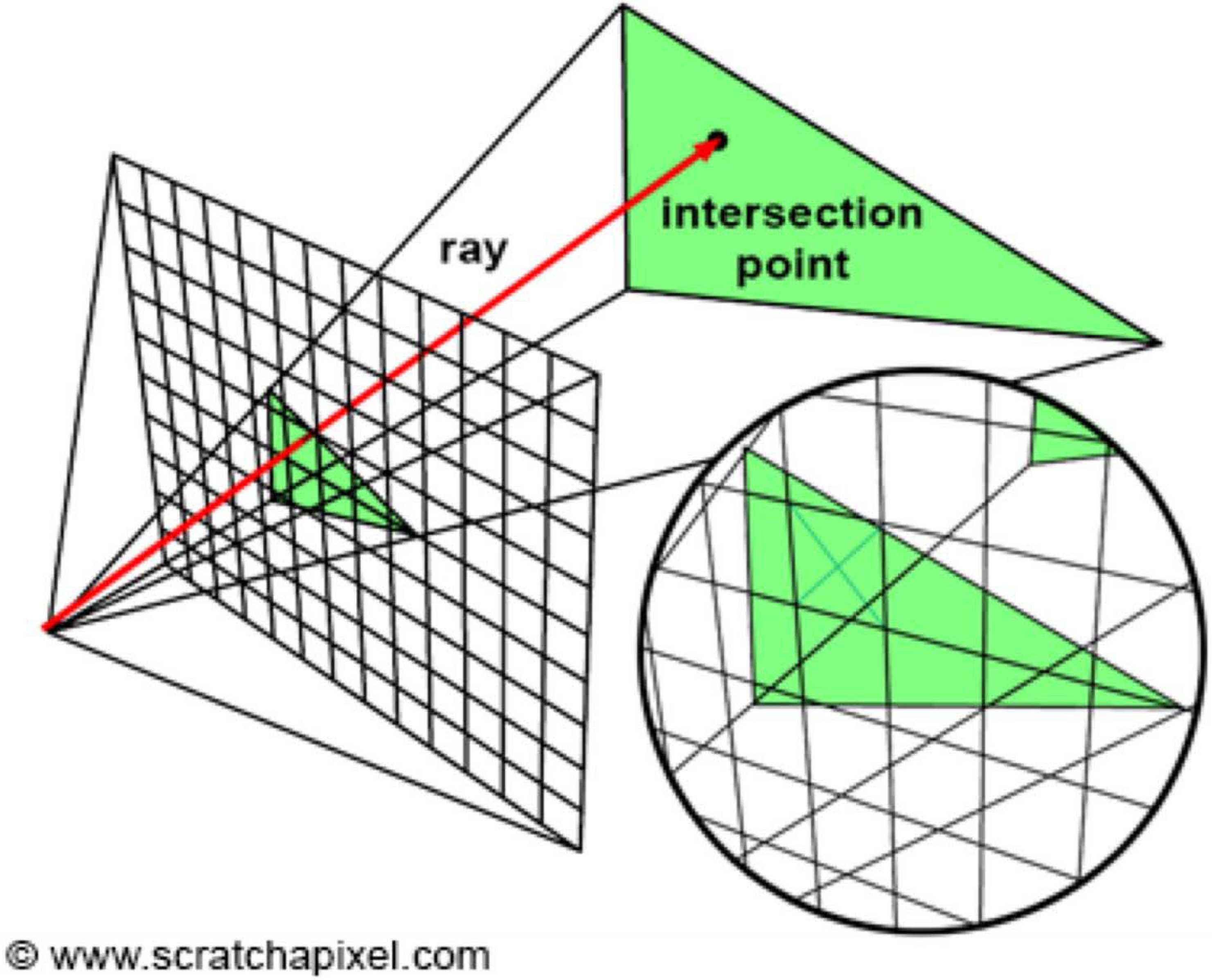
Ray

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

$$\mathbf{p}(0) = \mathbf{e}$$
$$\mathbf{p}(1) = \mathbf{s}$$

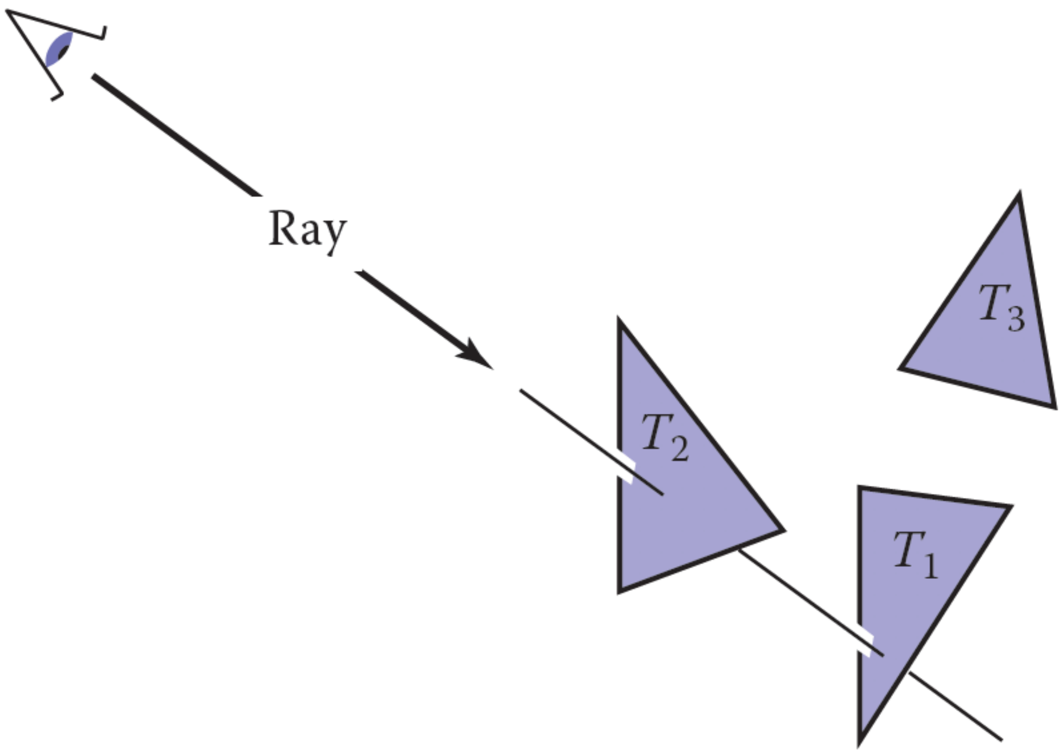


Ray Casting



https://www.scratchapixel.com/lessons/3d-basic-rendering/rasterization-practical-implementation

Ray Casting



Basic Components of Ray Casting

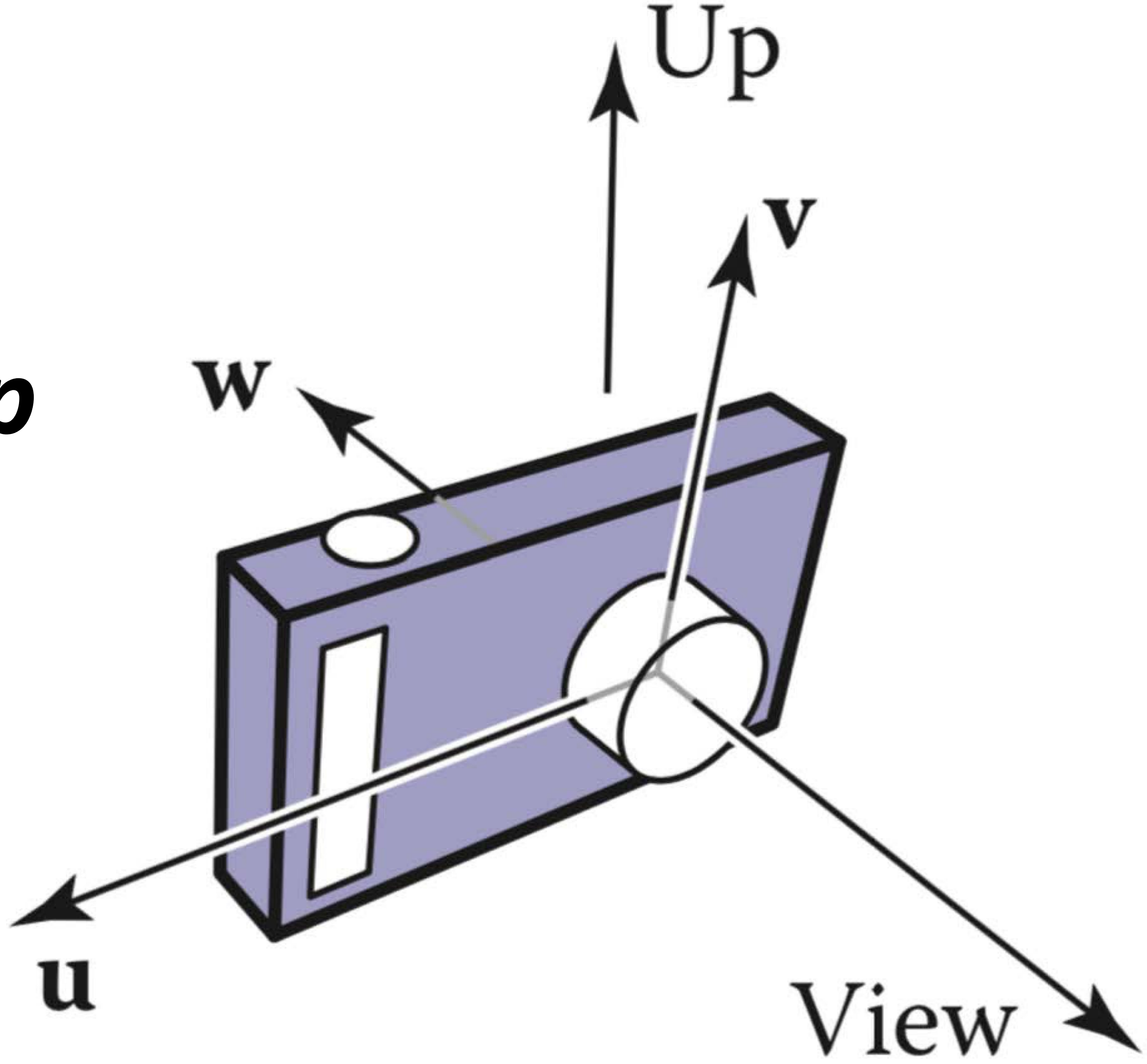
- Ray
- Camera
- Intersection Tests

Camera

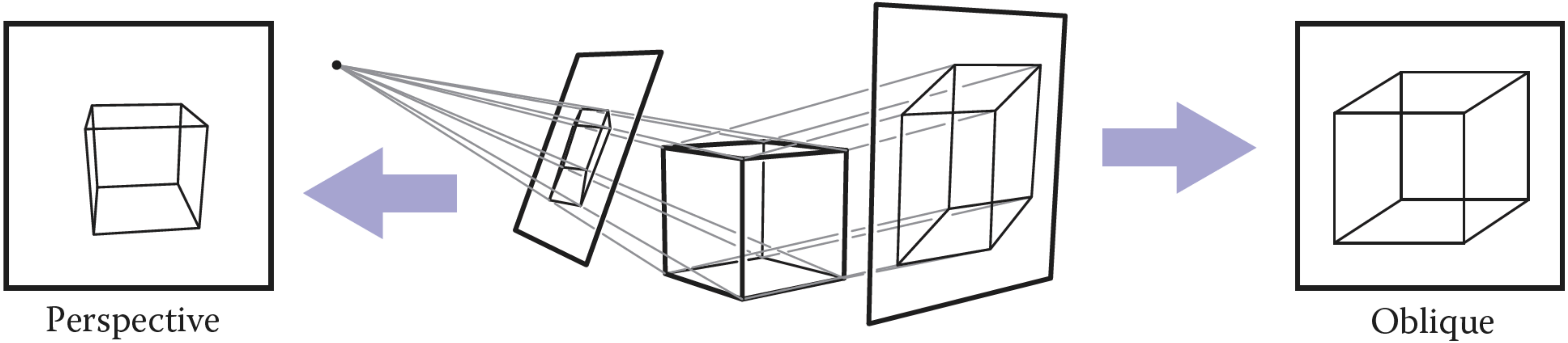
w = -***view***

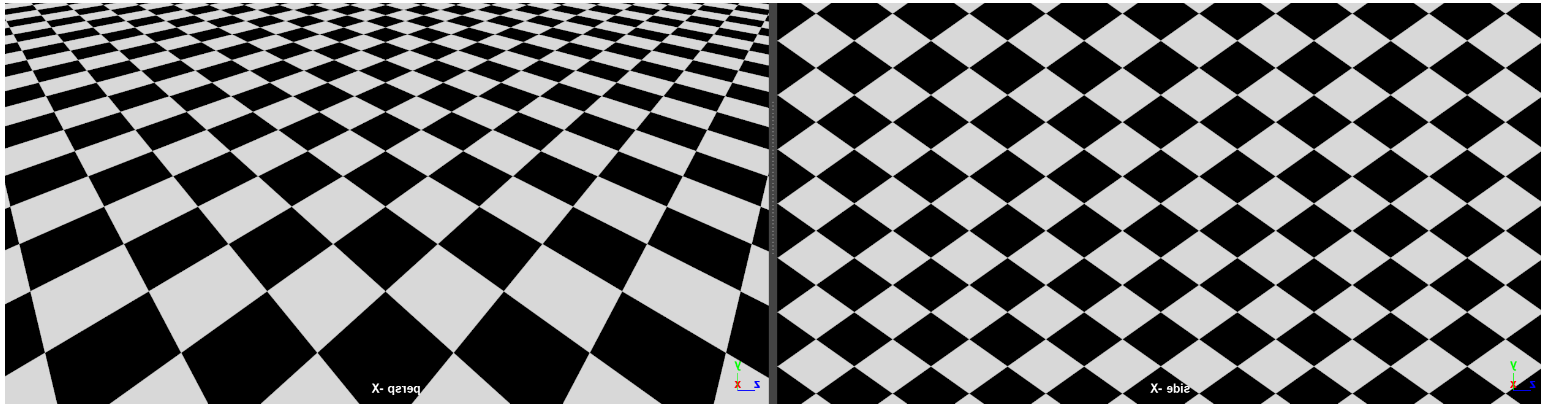
u = ***view*** x ***up***

v = ***w*** x ***u***

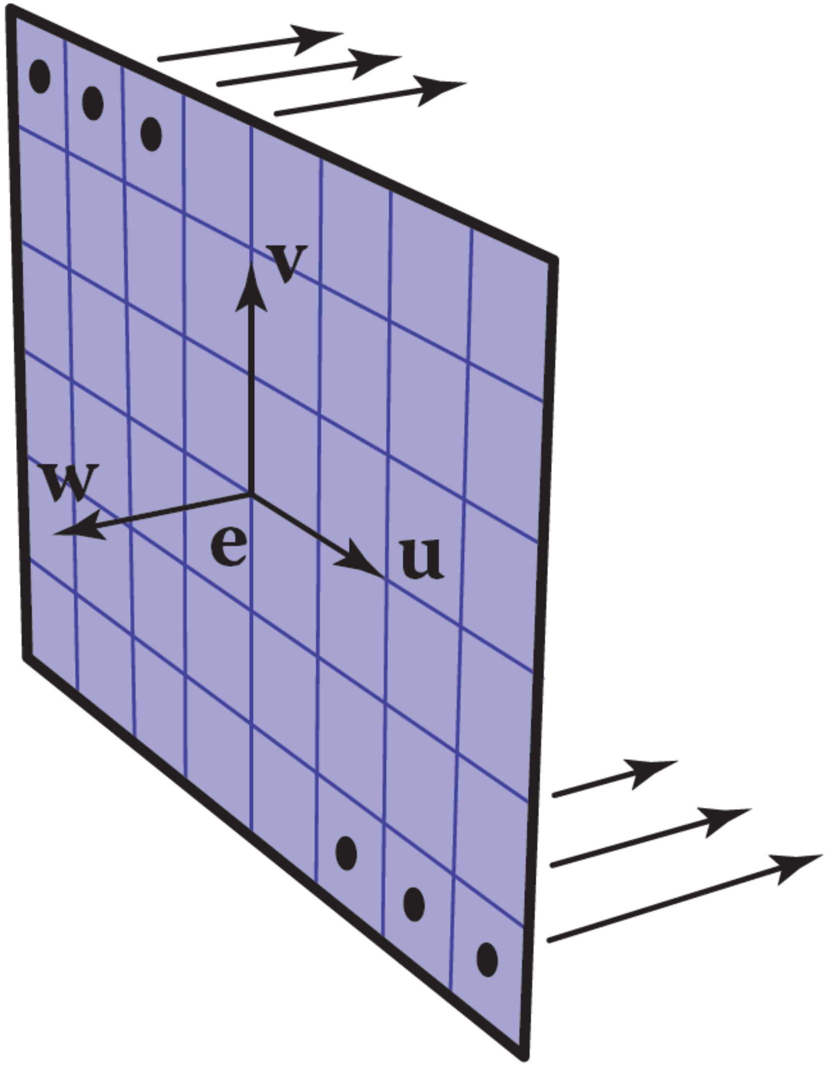


Orthographic v Perspective Projection

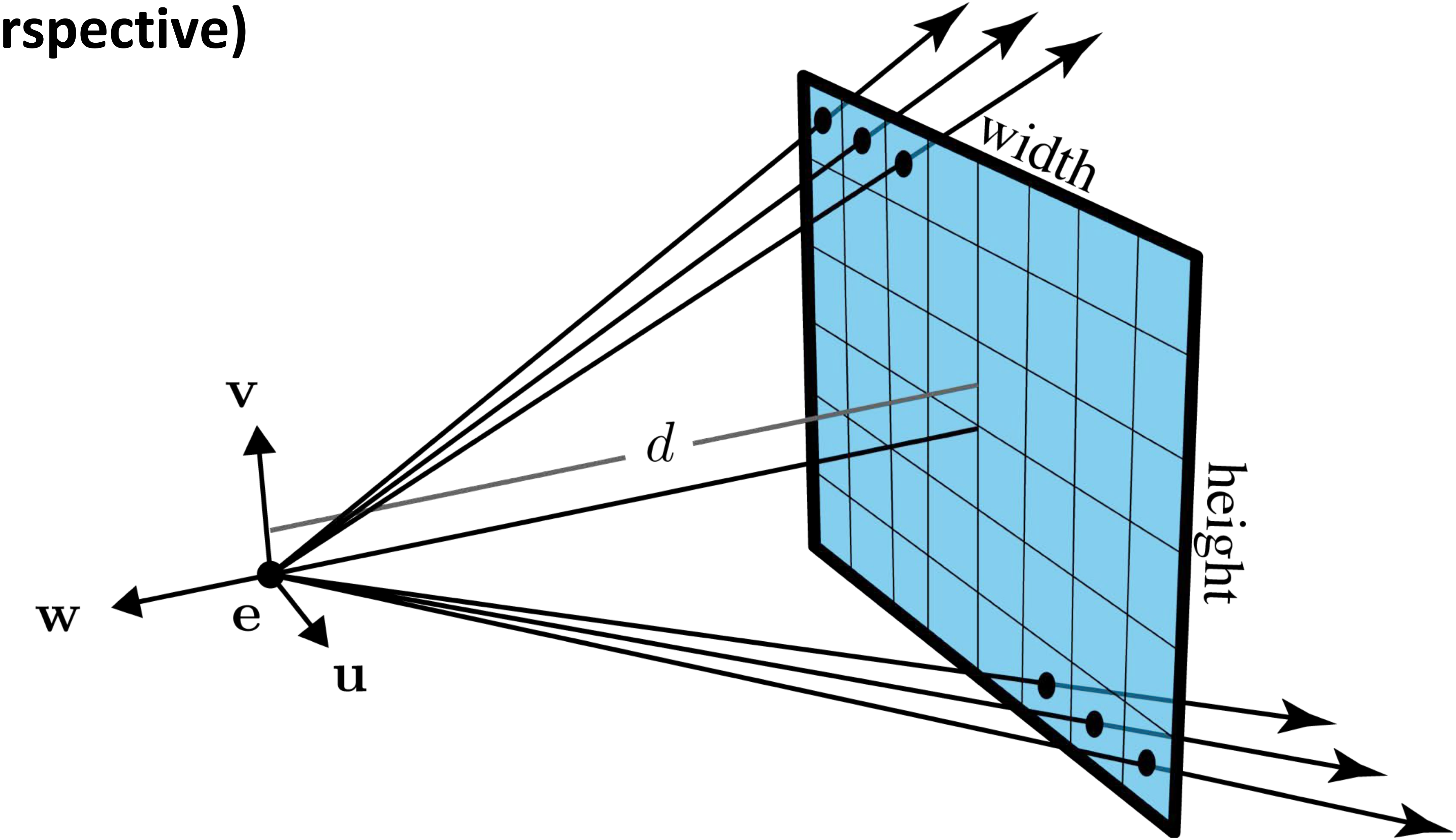




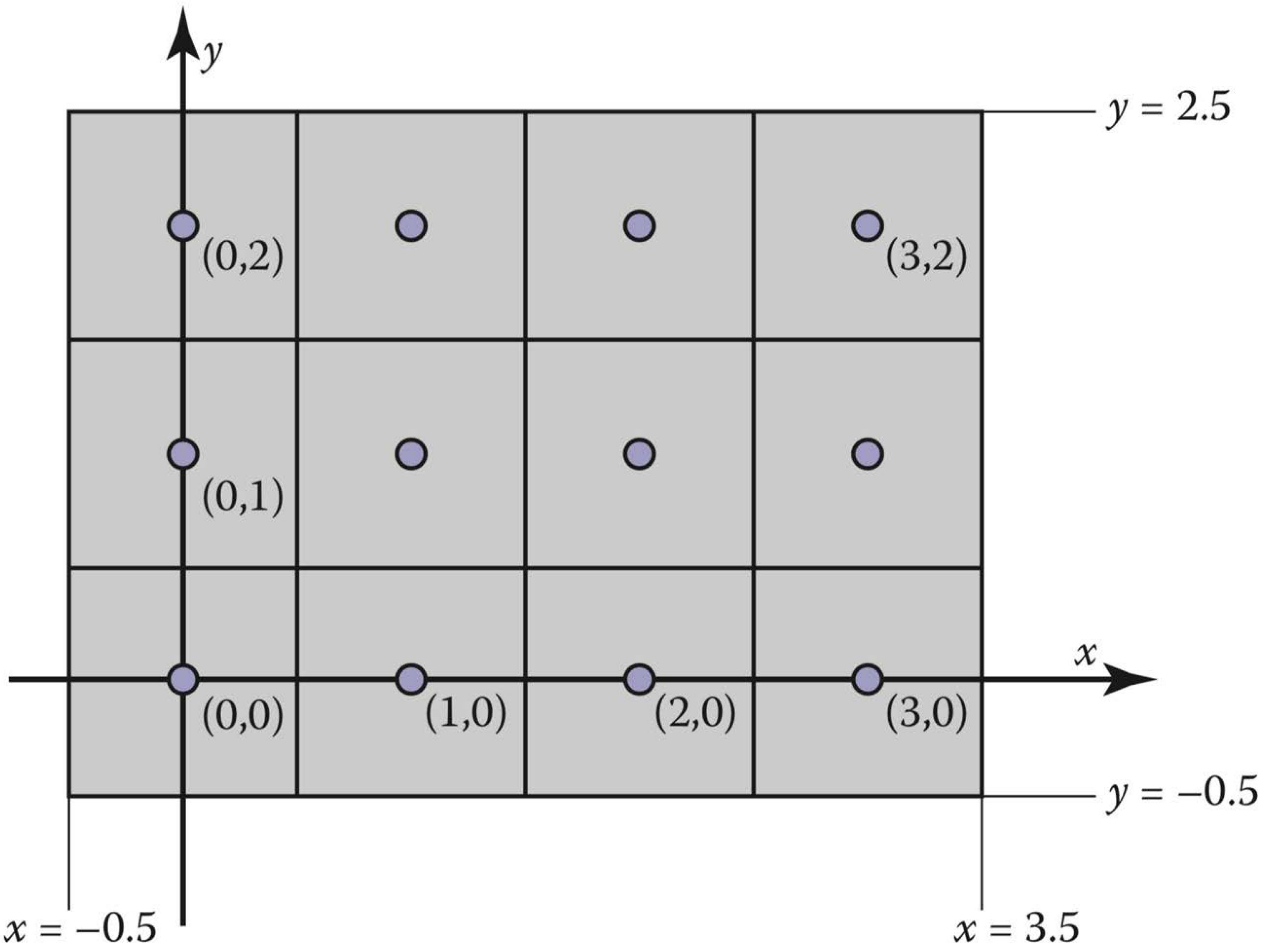
Generating Rays (Orthographic)

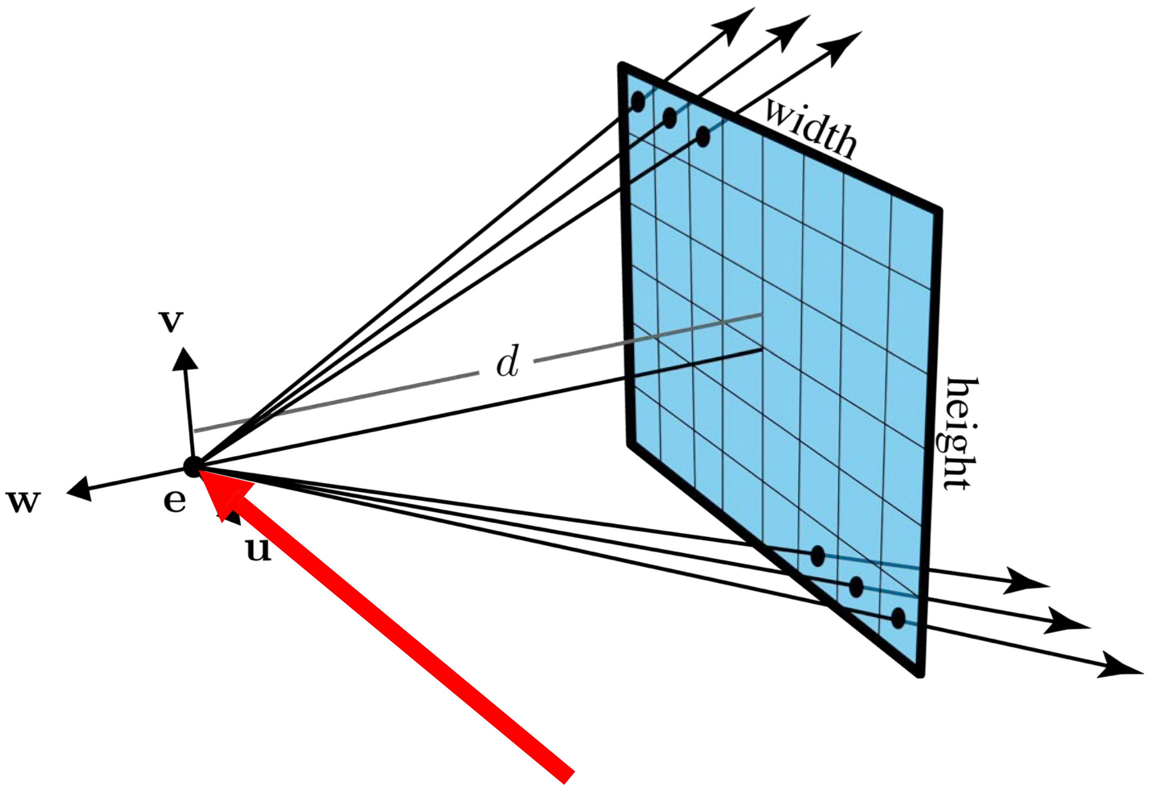


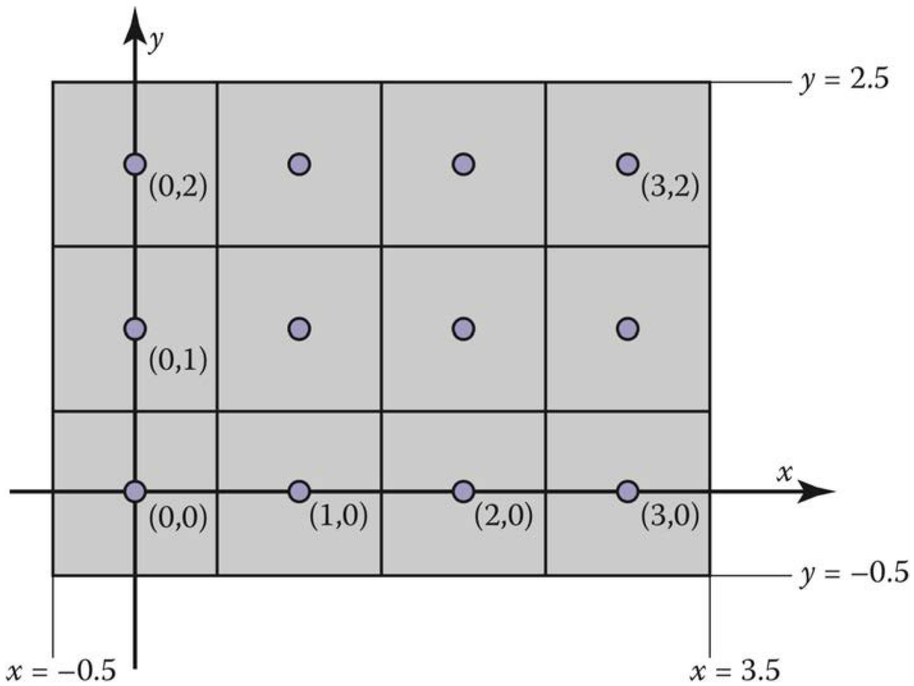
Generating Rays (Perspective)



Recall: Standard Pixel Coordinate System

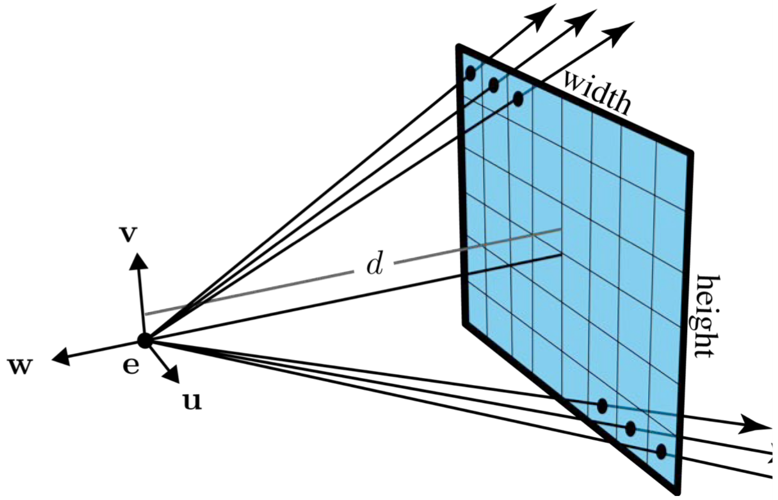




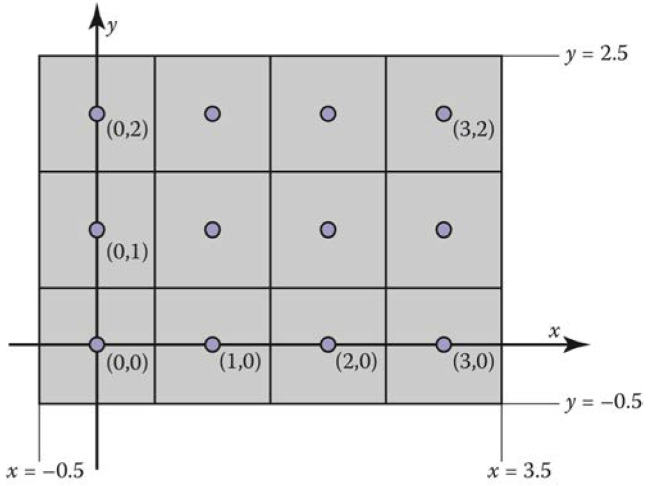


Origin of camera frame (the eye)

What are the coordinates for pixel (i, j) in the camera frame?



Camera space



Pixel space

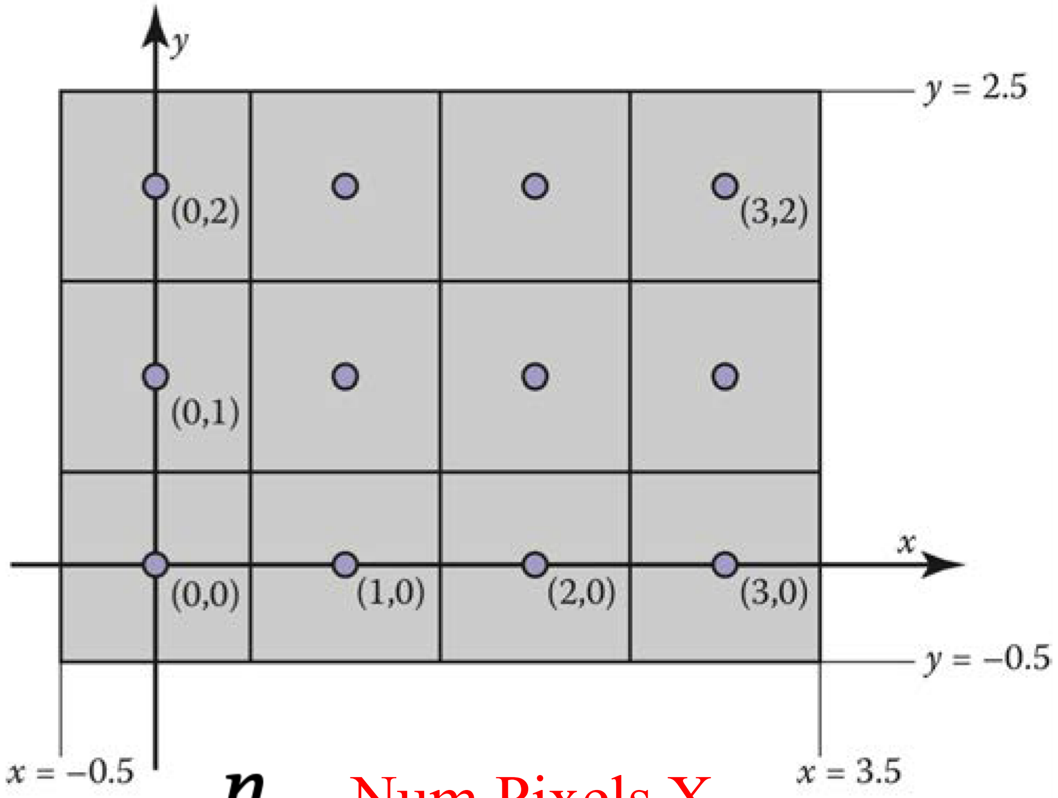
Bottom Left Corner (i, j) : ?

Top Right Corner (i, j): ?

Bottom Left Corner (u, v) : ?

Top Right Corner (u, v) : ?

n_y Num Pixels Y



n_x Num Pixels X

Bottom Left Corner (i, j) : $(-\frac{1}{2}, -\frac{1}{2})$

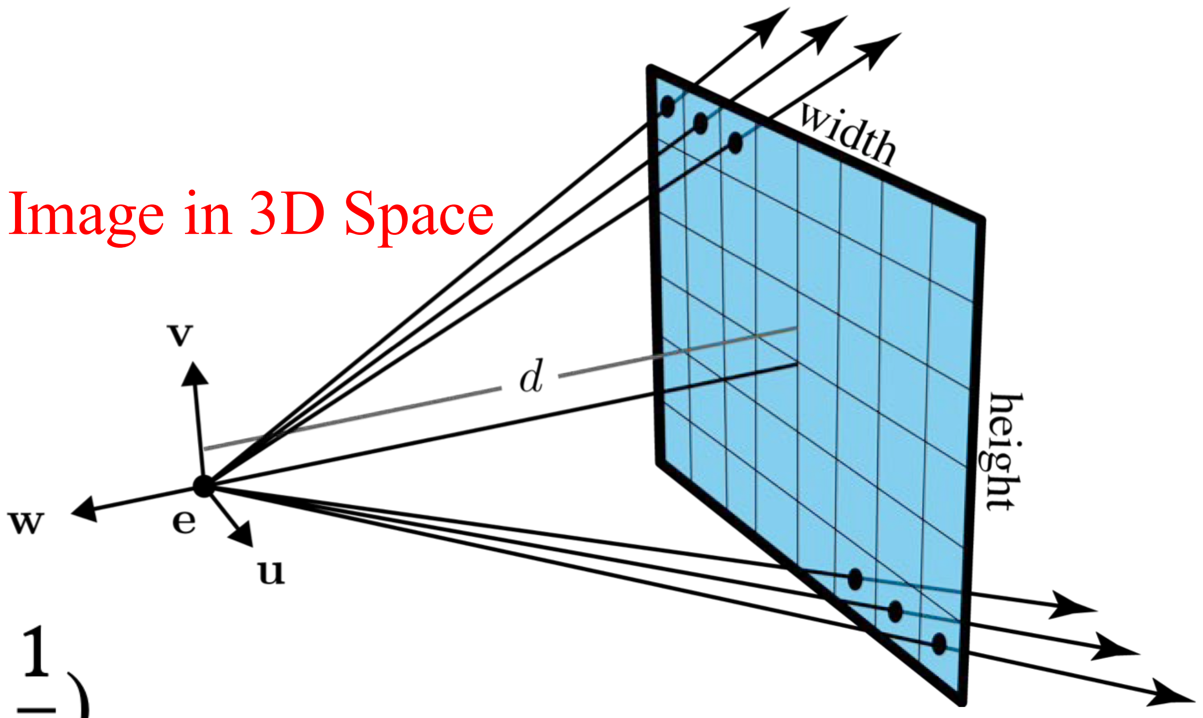
Top Right Corner (i, j) : $(n_x - \frac{1}{2}, n_y - \frac{1}{2})$

Bottom Left Corner (u, v) :

Top Right Corner (u, v) :

Physical Width of Image in 3D Space

Physical Height of Image in 3D Space



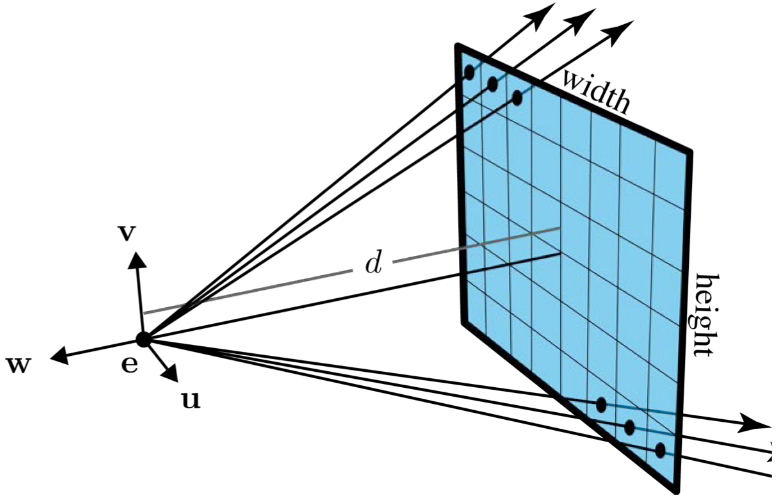
Bottom Left Corner (i, j) : $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

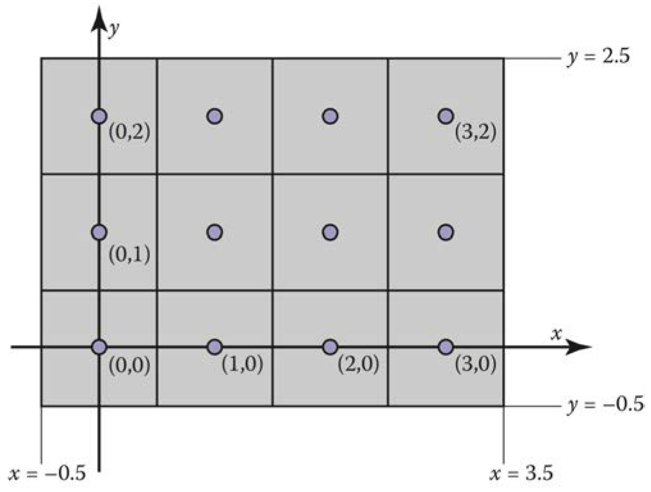
Top Right Corner (i, j) : $\left(n_x - \frac{1}{2}, n_y - \frac{1}{2}\right)$

Bottom Left Corner (u, v) : $\left(-\frac{\text{width}}{2}, -\frac{\text{height}}{2}\right)$

Top Right Corner (u, v) : $\left(\frac{\text{width}}{2}, \frac{\text{height}}{2}\right)$

pixel at position (i, j) in the raster image has the position:





Physical Width of Image in 3D Space

$$u = \frac{\text{width}}{n_x} \cdot \left(i + \frac{1}{2}\right) - \frac{\text{width}}{2}$$

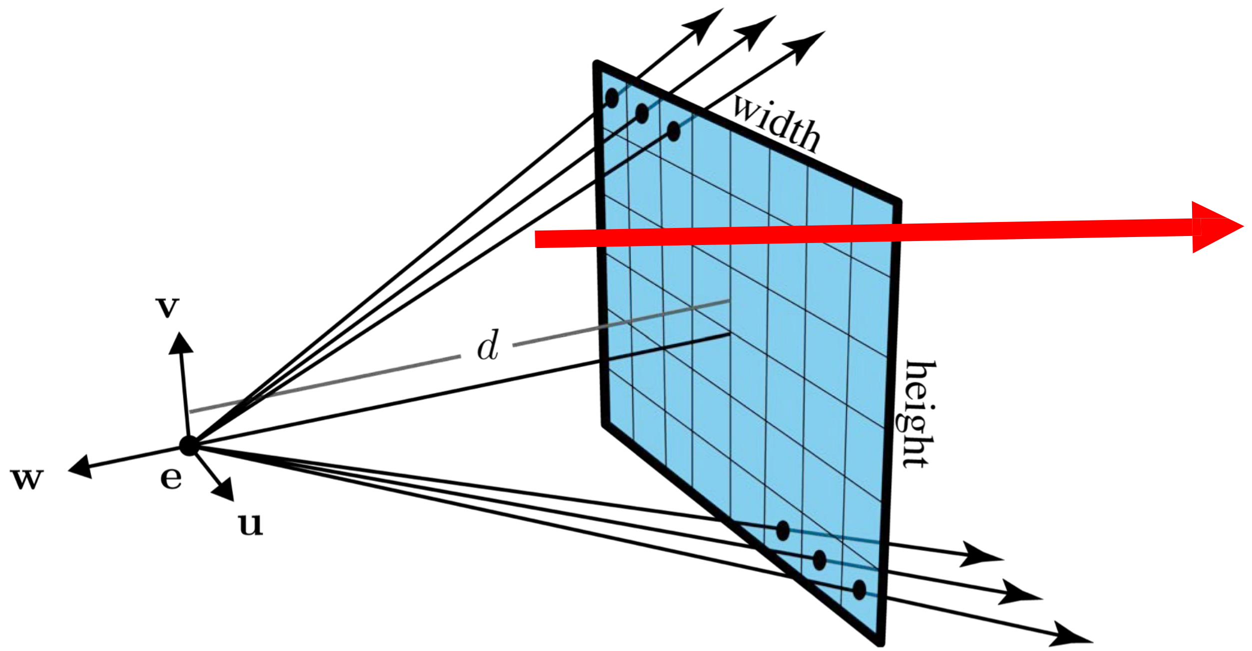
Num Pixels X

Physical Height of Image in 3D Space

$$v = \frac{\text{height}}{n_y} \cdot \left(j + \frac{1}{2}\right) - \frac{\text{height}}{2}$$

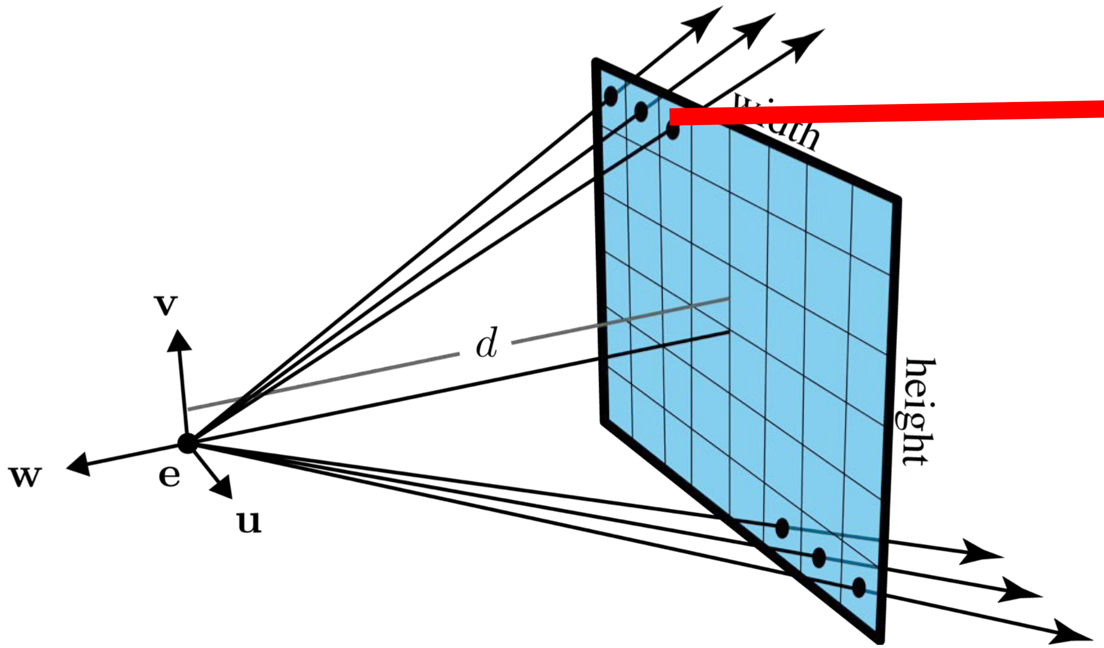
Num Pixels Y

Ray Equation in Camera Space



$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

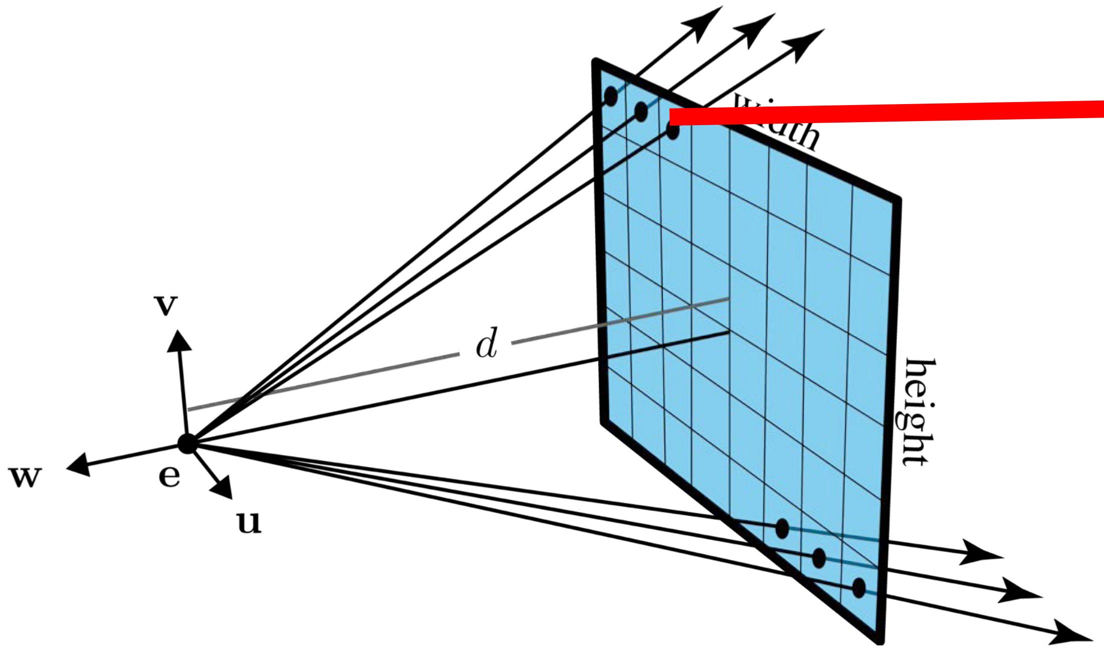
Ray Equation in Camera Space



$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

$$\mathbf{p}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t\left(\begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right)$$

Ray Equation in Camera Space



$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

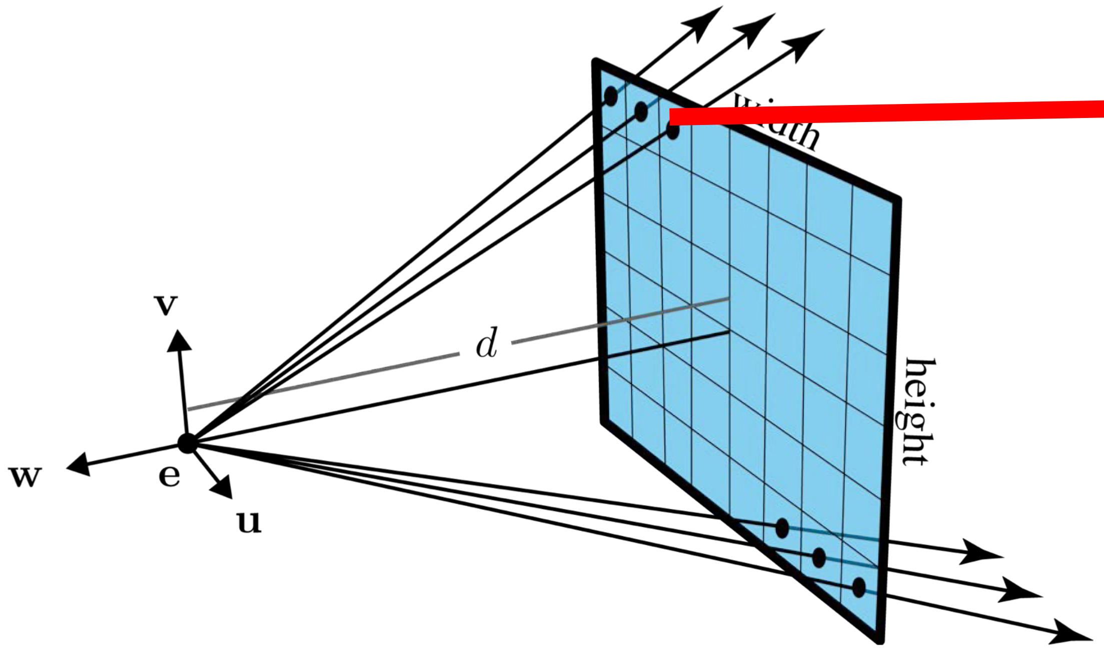
$$\mathbf{p}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t\left(\begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$\mathbf{p}(t) = t\begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix}$$

$$u = \frac{\text{width}}{n_x} \cdot \left(i + \frac{1}{2}\right) - \frac{\text{width}}{2}$$

$$v = \frac{\text{height}}{n_y} \cdot \left(j + \frac{1}{2}\right) - \frac{\text{height}}{2}$$

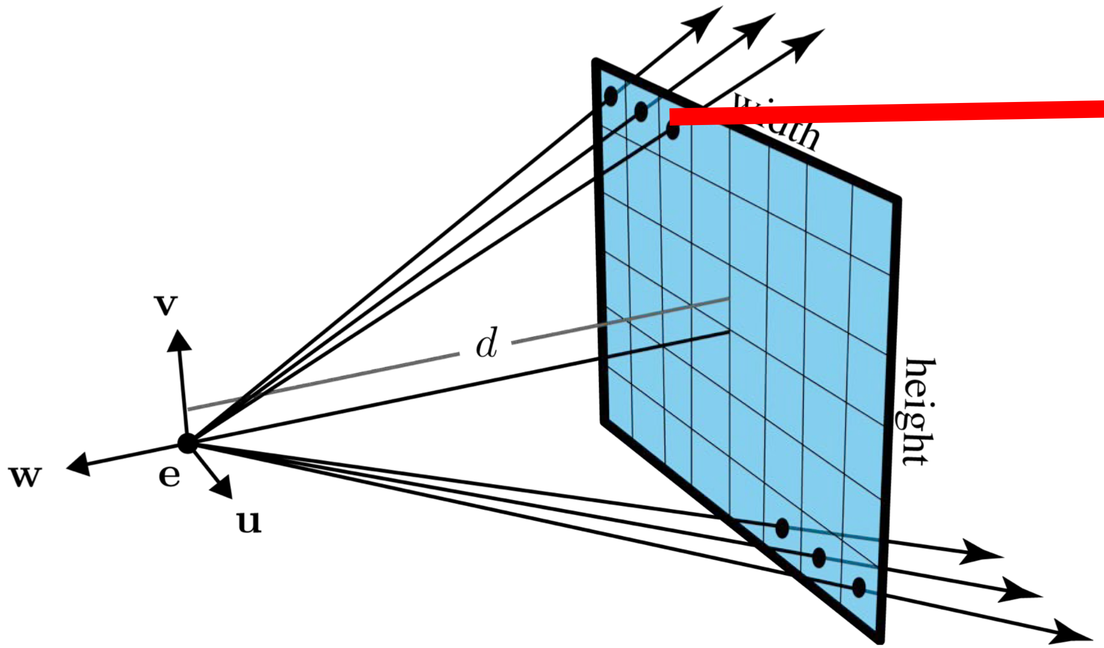
Ray Equation in World Space



$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

$$\mathbf{p}(t) = \mathbf{e} + t\left(u(i)\mathbf{u} + v(j)\mathbf{v} + -d\mathbf{w}\right)$$

Ray Equation in World Space



$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

$$\mathbf{p}(t) = \mathbf{e} + t\begin{bmatrix}\mathbf{u} & \mathbf{v} & \mathbf{w}\end{bmatrix}\begin{bmatrix}u(i) \\ v(j) \\ -d\end{bmatrix}$$

Camera Transformation Matrix

Ray Casting

```
for each pixel in the image {
    Generate a ray
    for each object in the scene {
        if (Intersect ray with object){
            Set pixel colour
        }
    }
}
```

Next class

Ray Intersections

Ray Casting

Some Slides/Images adapted from Marschner and Shirley and David Levin

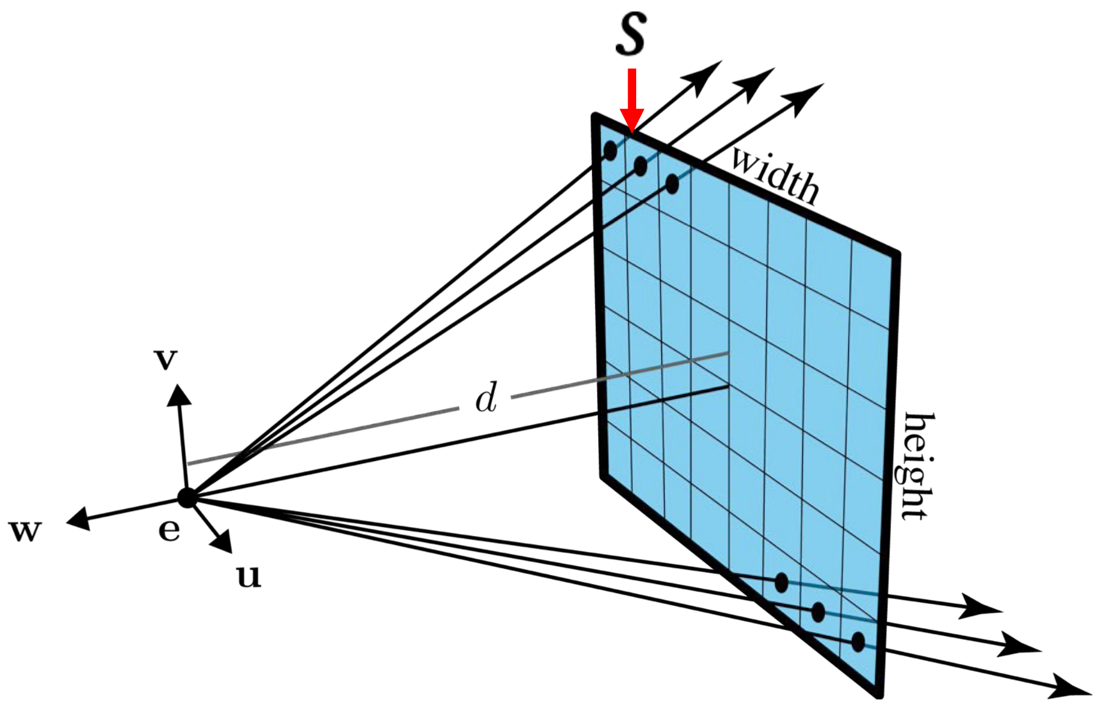
Ray Casting

- Ray Casting and Change of Coordinates Review
- Ray-Object Intersection
    - Ray-Plane Intersection
    - Ray-Sphere Intersection
    - Ray-Triangle Intersection

Ray Casting

```
for each pixel in the image {
    Generate a ray
    for each object in the scene {
        if (Intersect ray with object){
            Set pixel colour
        }
    }
}
```

Ray Equation



$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

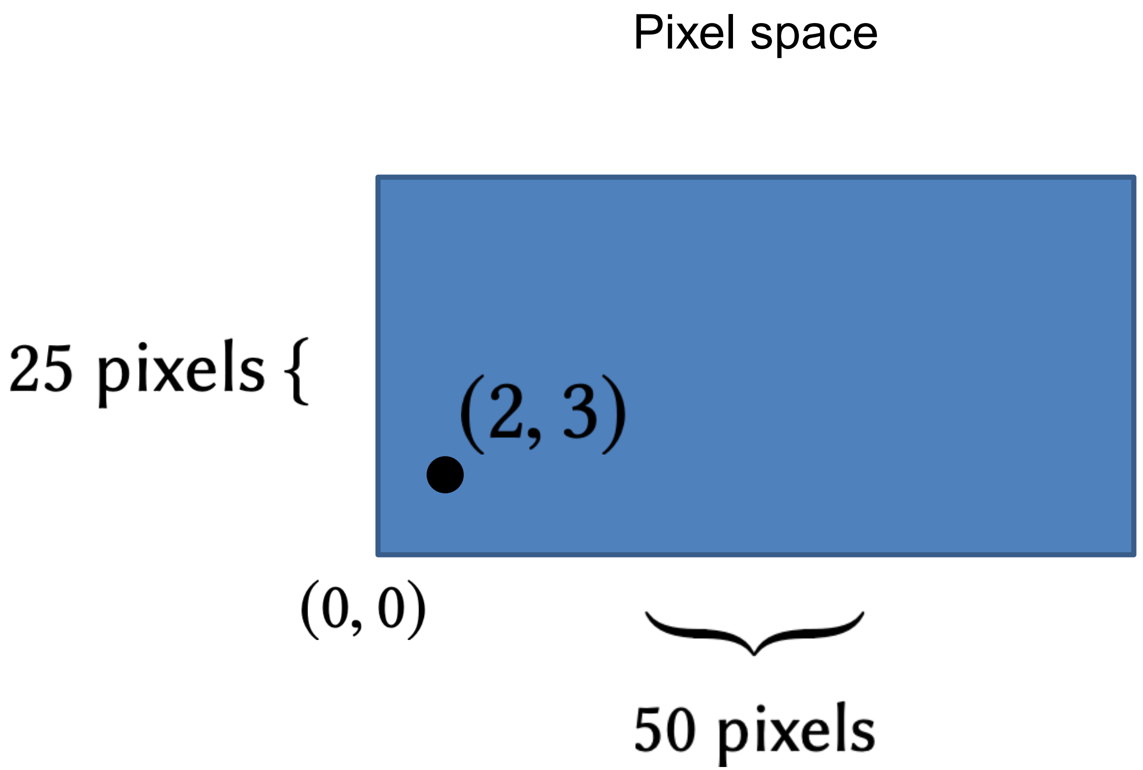
Example – Pixel Space

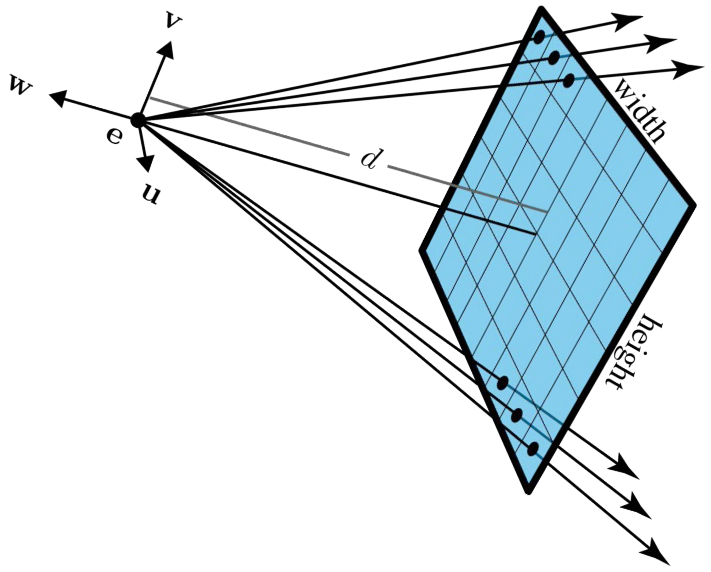
$$\text{width} = 2$$

$$\text{height} = 1$$

$$n_x = 50$$

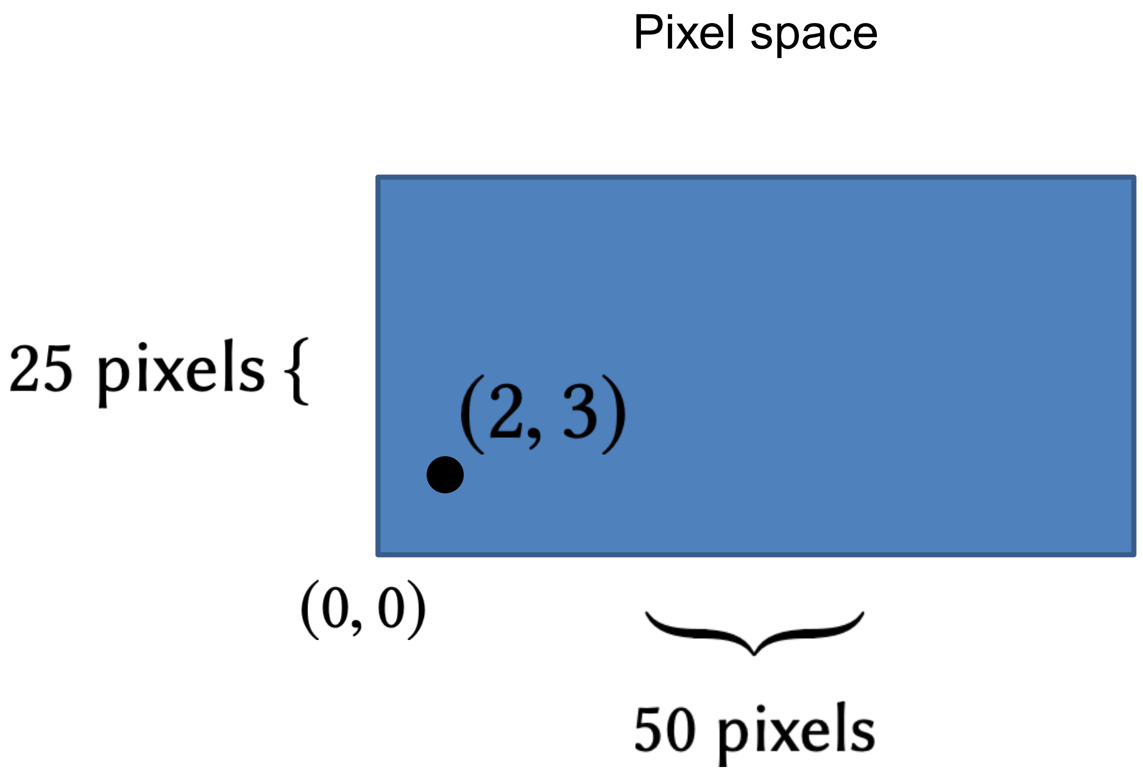
$$n_y = 25$$





Example – Pixel Space

$$\text{width} = 2$$
$$\text{height} = 1$$
$$n_x = 50$$
$$n_y = 25$$

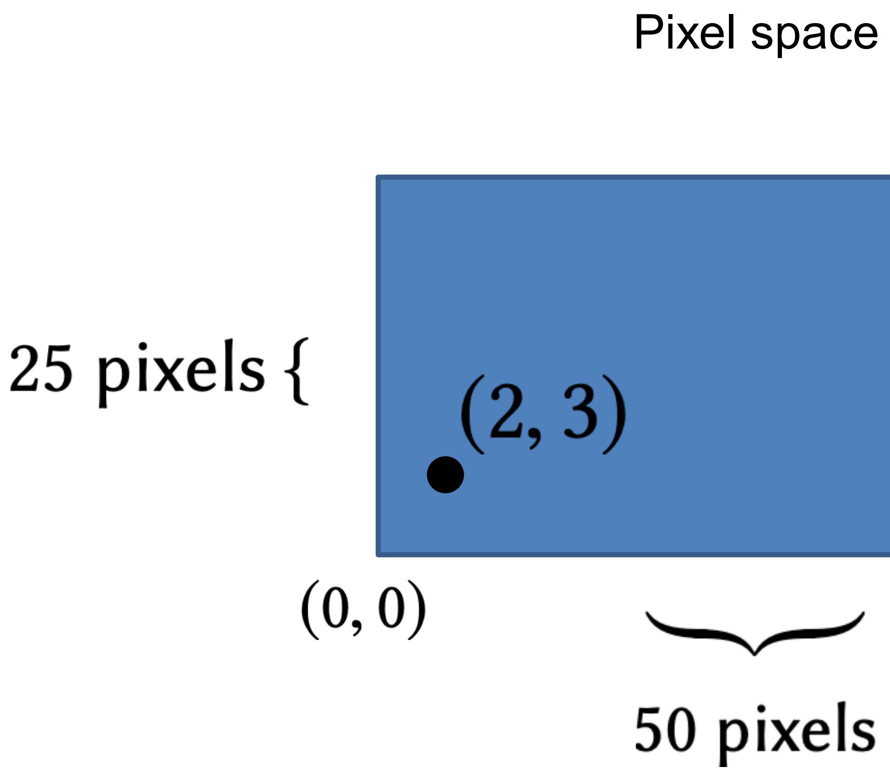


$$u = \frac{\text{width}}{n_x} \cdot \left(i + \frac{1}{2}\right) - \frac{\text{width}}{2}$$

$$v = \frac{\text{height}}{n_y} \cdot \left(j + \frac{1}{2}\right) - \frac{\text{height}}{2}$$

Example – Pixel Space

$$\text{width} = 2$$
$$\text{height} = 1$$
$$n_x = 50$$
$$n_y = 25$$



$$u = \frac{2}{50} \cdot \left(2 + \frac{1}{2}\right) - \frac{2}{2}$$

$$v = \frac{1}{25} \cdot \left(3 + \frac{1}{2}\right) - \frac{1}{2}$$

Example – Pixel Space

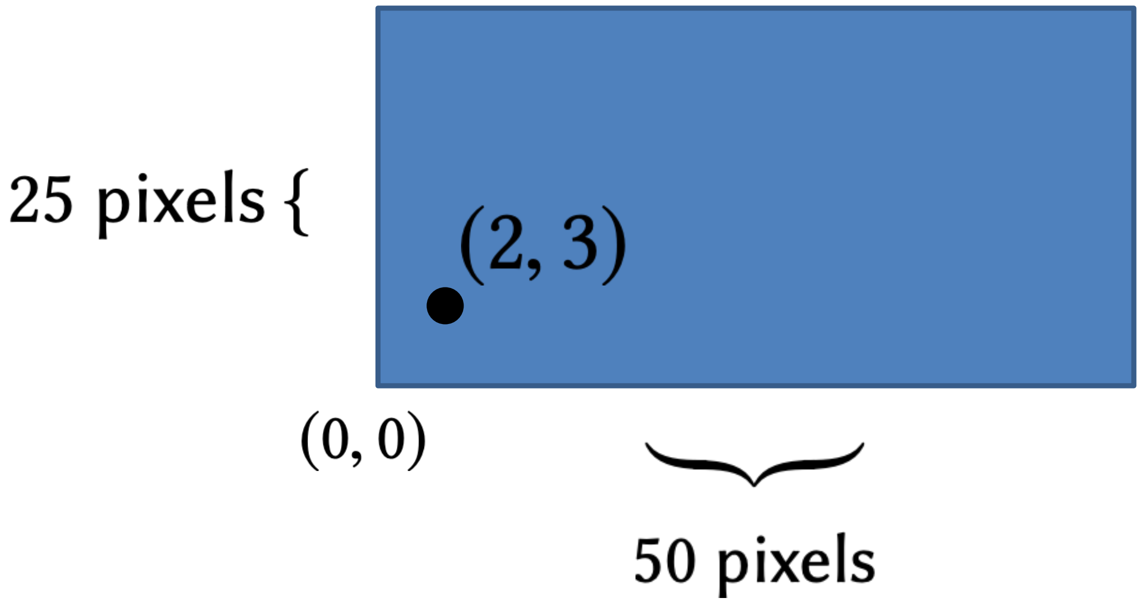
$$\text{width} = 2$$

$$\text{height} = 1$$

$$n_x = 50$$

$$n_y = 25$$

Pixel space

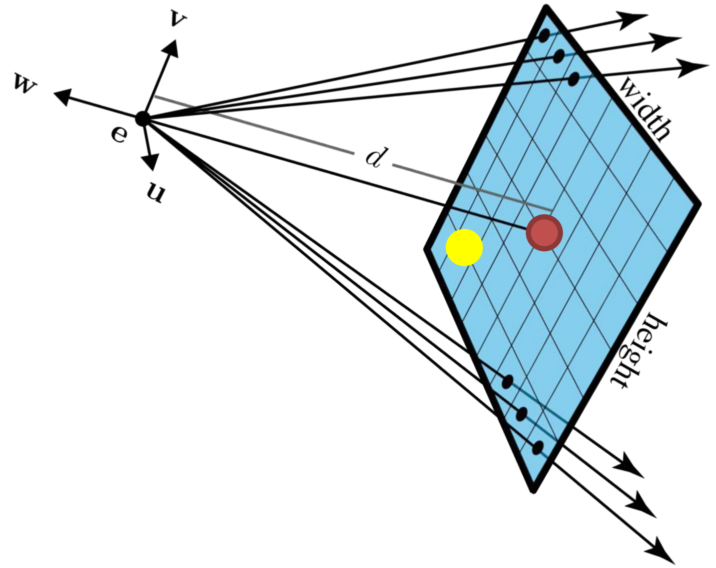


$$u = -0.9$$

$$v = -0.36$$

Example – Pixel Space

Camera space



$$\text{width} = 2$$

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$$n_x = 50$$

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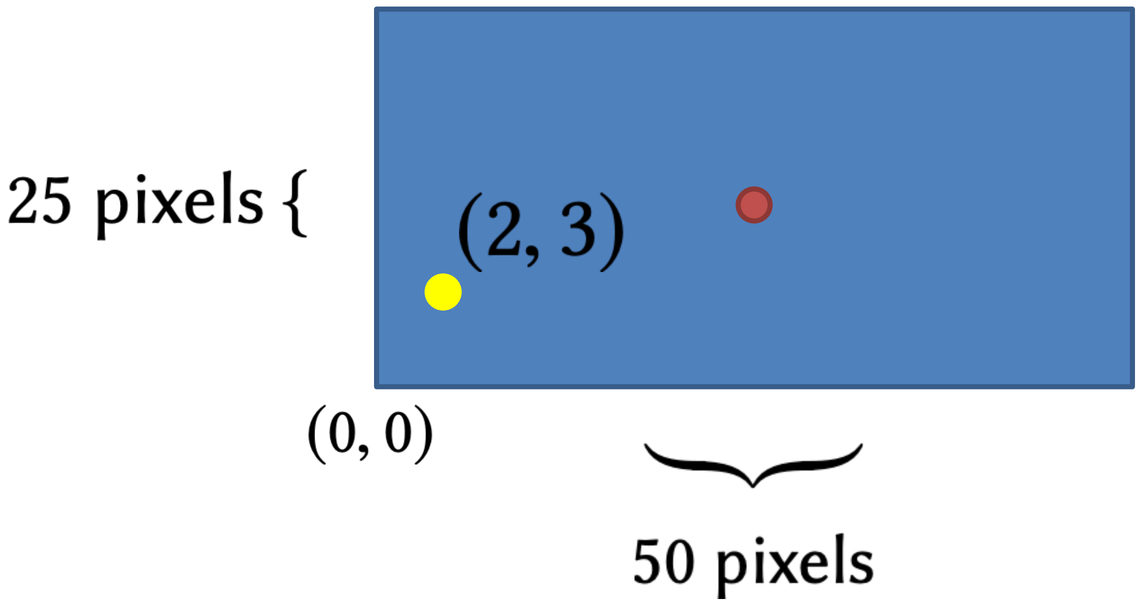
Pixel space

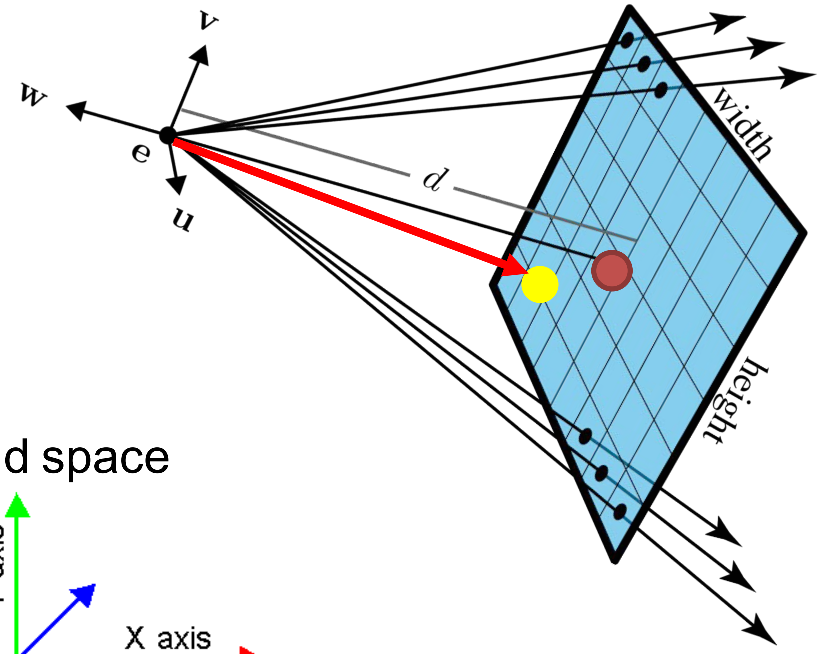


$$u = -0.9$$

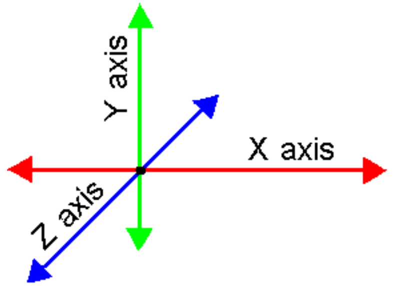
$$v = -0.36$$

Example – Camera Space

Camera space



World space



$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

are the basis vectors for camera space

$$d = 10$$

d is the distance from the viewpoint to the image plane (focal length)

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation

Example – Camera Space

$d = 10$

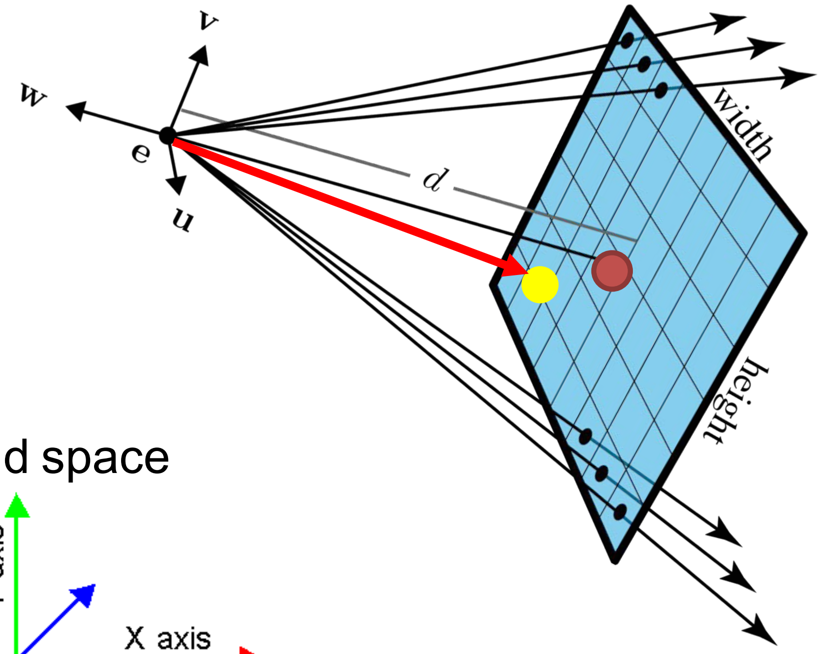
$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

are the basis vectors for camera space

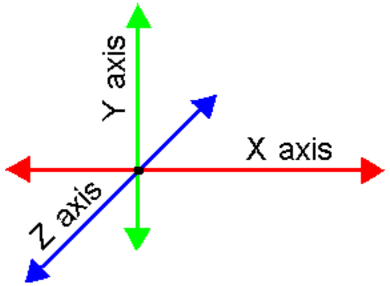
$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation

Camera space



World space



$$\mathbf{p}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t\left(\begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right)$$

Example – Camera Space

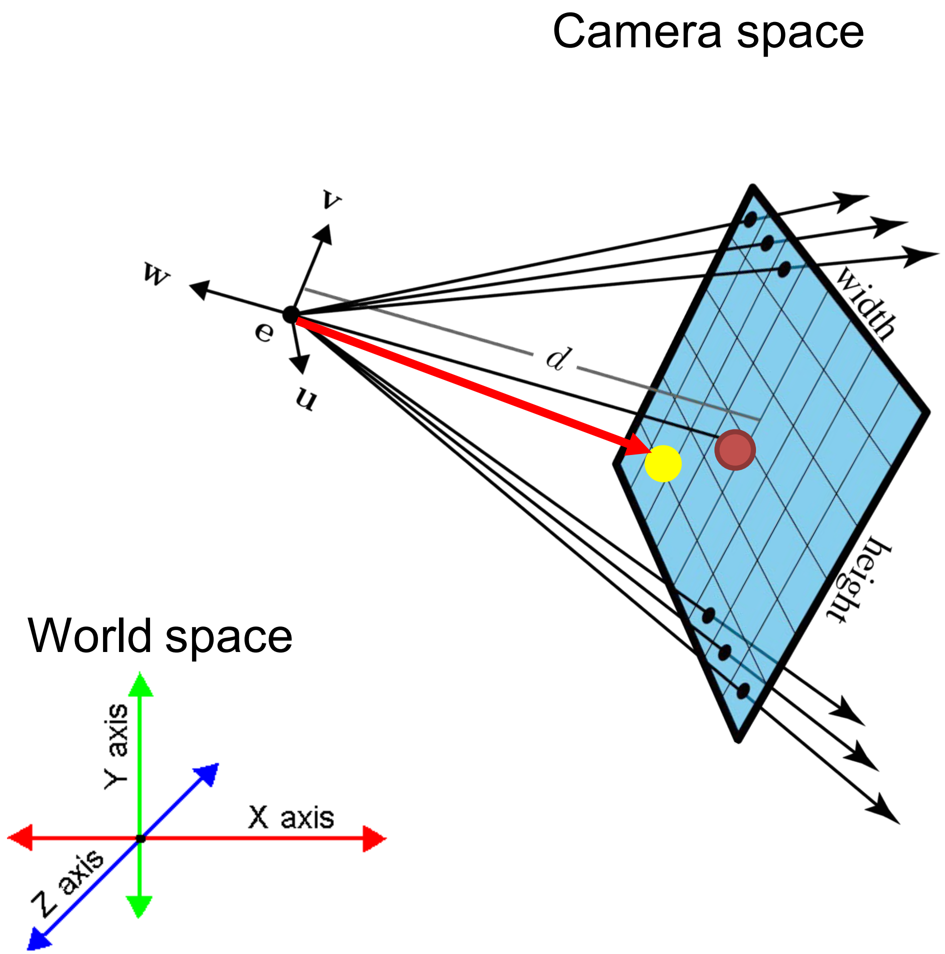
$d = 10$

$\mathbf{u}, \mathbf{v}, \mathbf{w}$

are the basis vectors for camera space

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation



for (i,j) = (2,3)

$$u(2) = -0.9, v(3) = -0.36$$

$$\mathbf{p}(t) = t \begin{bmatrix} -0.9 \\ -0.36 \\ -10 \end{bmatrix}$$

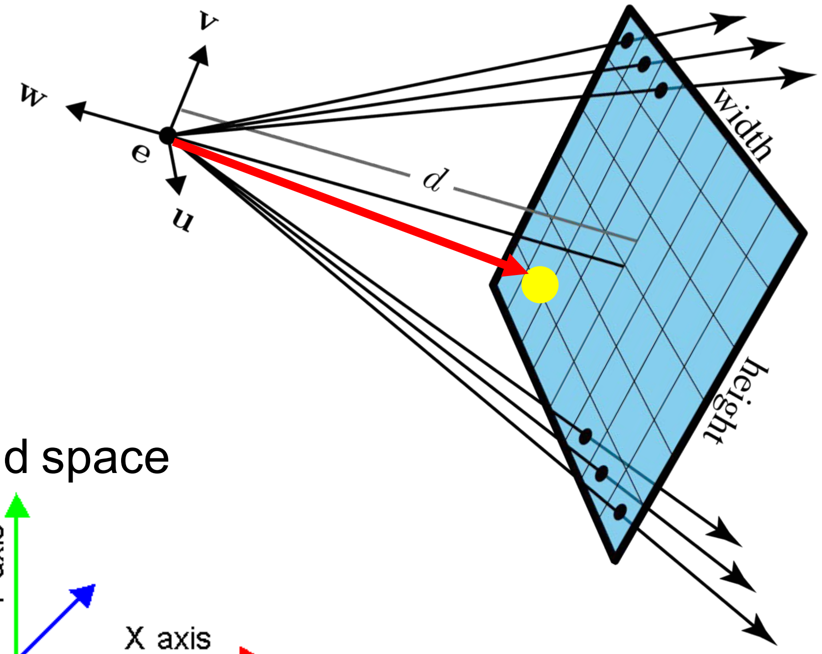
Example – World Space

$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

are the basis vectors for camera space

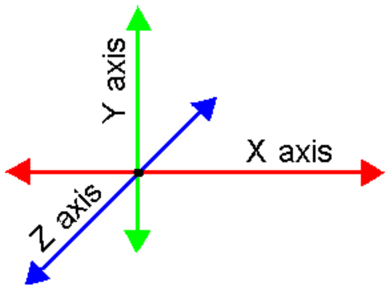
$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation



$$\mathbf{e} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

World space



$$\mathbf{p}(t) = \mathbf{e} + t\left(u(i)\mathbf{u} + v(j)\mathbf{v} + -d\mathbf{w}\right)$$

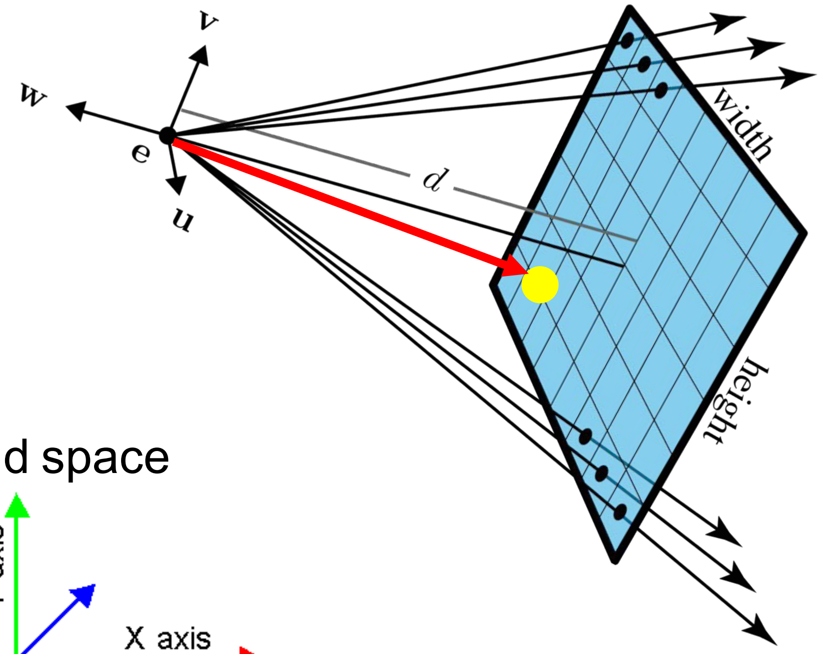
Example – World Space

$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

are the basis vectors for camera space

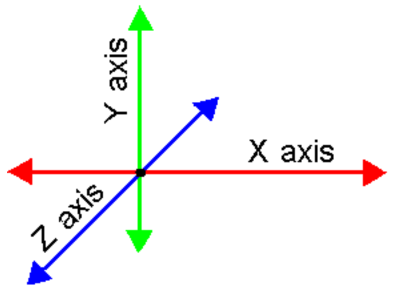
$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation



$$\mathbf{e} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

World space



$$\mathbf{p}(t) = \mathbf{e} + t \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} \end{bmatrix} \begin{bmatrix} u(i) \\ v(j) \\ -d \end{bmatrix}$$

Example – World Space

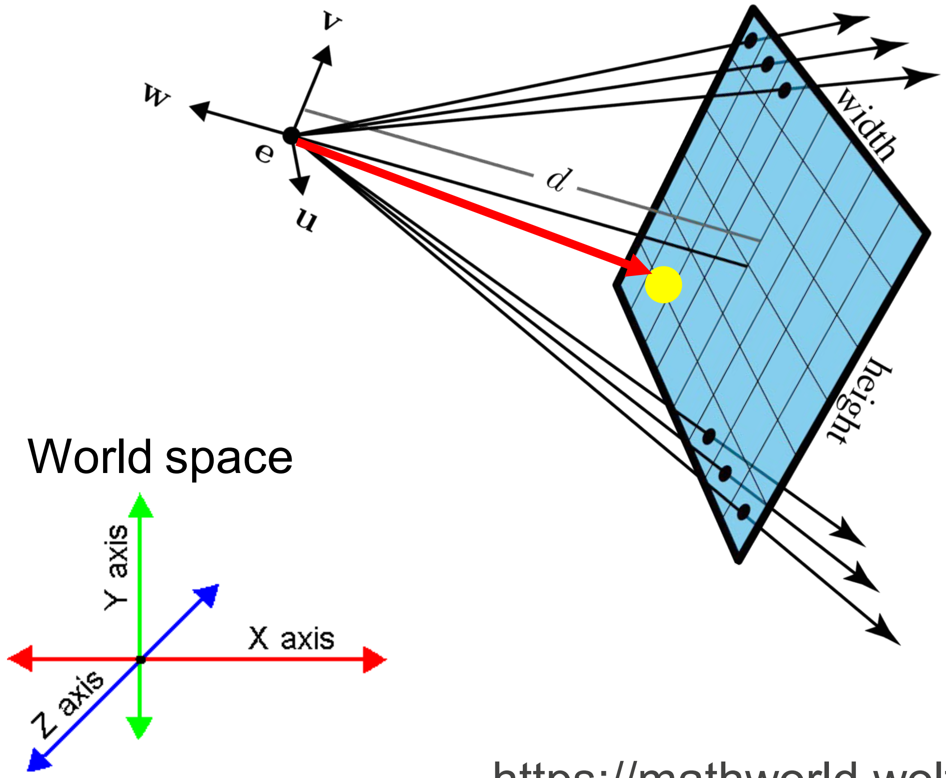
$$\mathbf{e} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

are the basis vectors for camera space

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

our ray equation



$$\mathbf{p}(t) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -0.9 \\ -0.36 \\ -10 \end{bmatrix}$$

https://mathworld.wolfram.com/ChangeofCoordinatesMatrix.html

Basic Components of Ray Casting

Ray

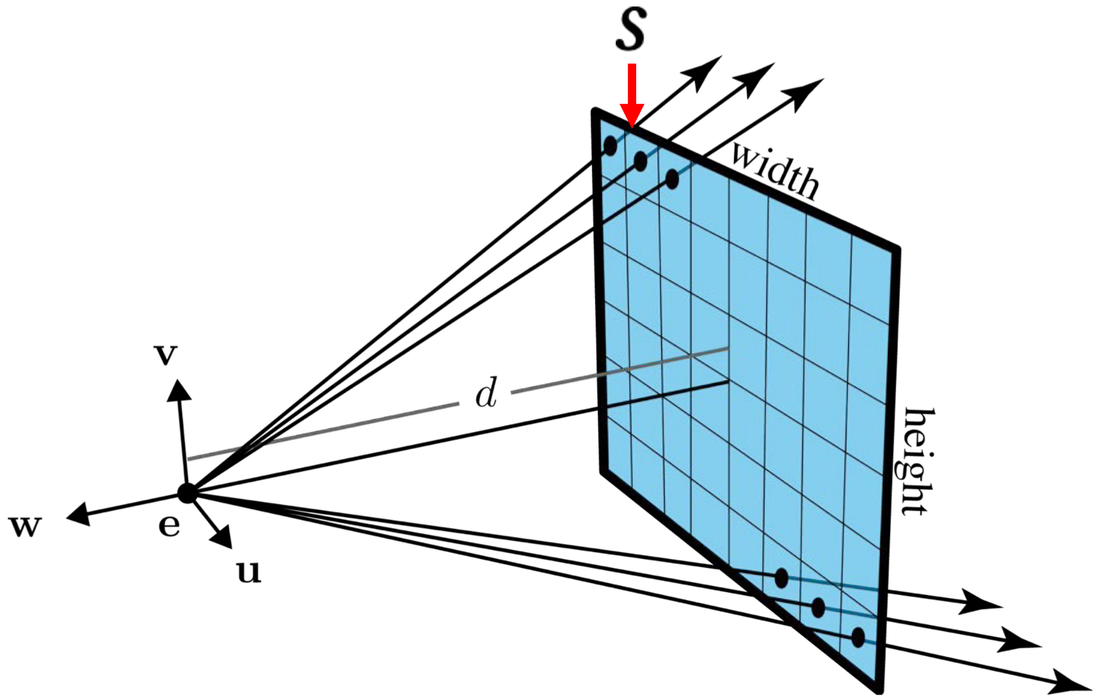
Camera

Intersection Tests

Intersection Tests

- Plane
- Sphere
- Triangle

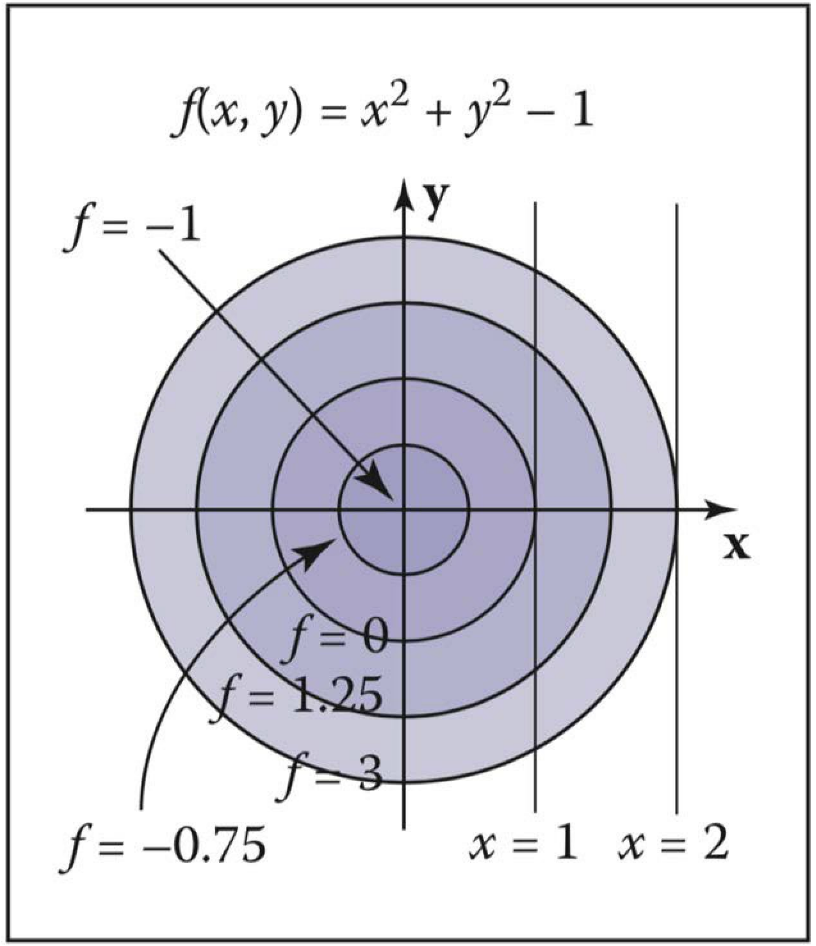
Ray Equation



$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

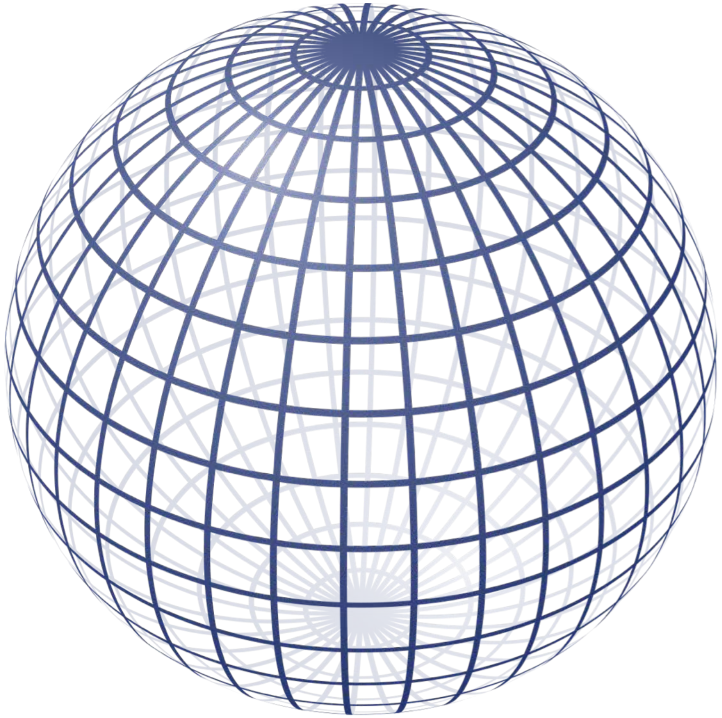
Aside: Types of Surface

Implicit Surface



Parametric Surface

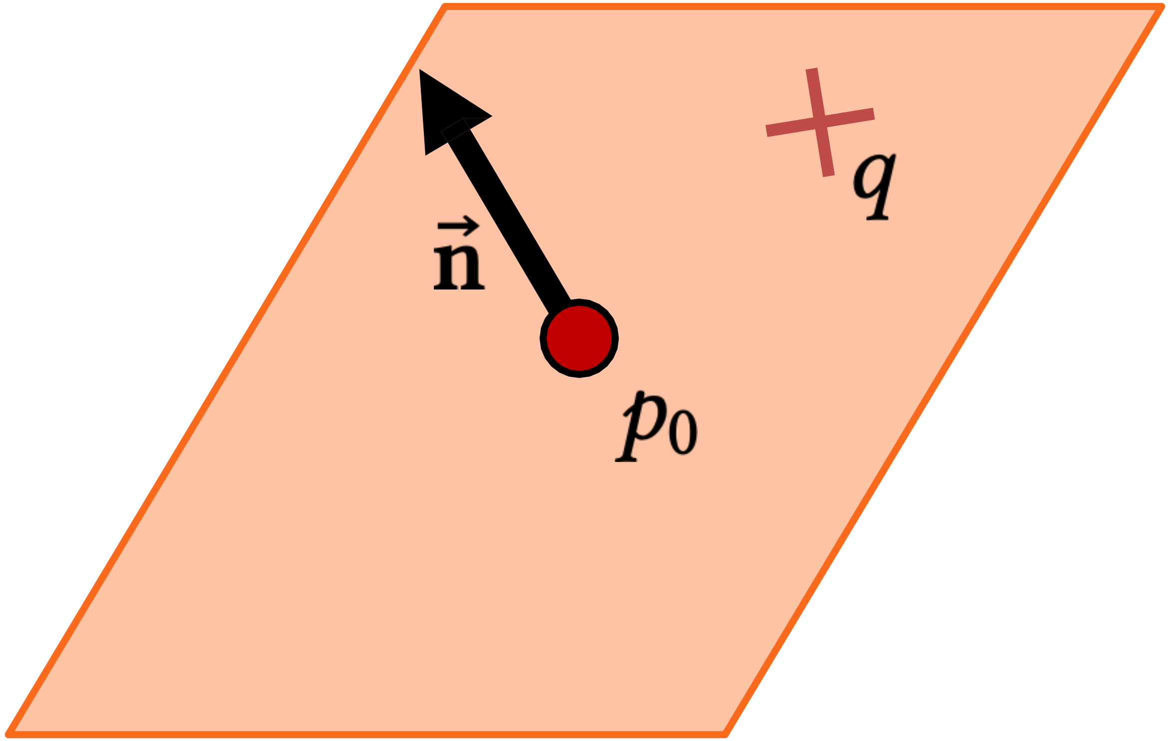
$$x = r\cos\phi\sin\theta,$$
$$y = r\sin\phi\sin\theta,$$
$$z = r\cos\theta.$$



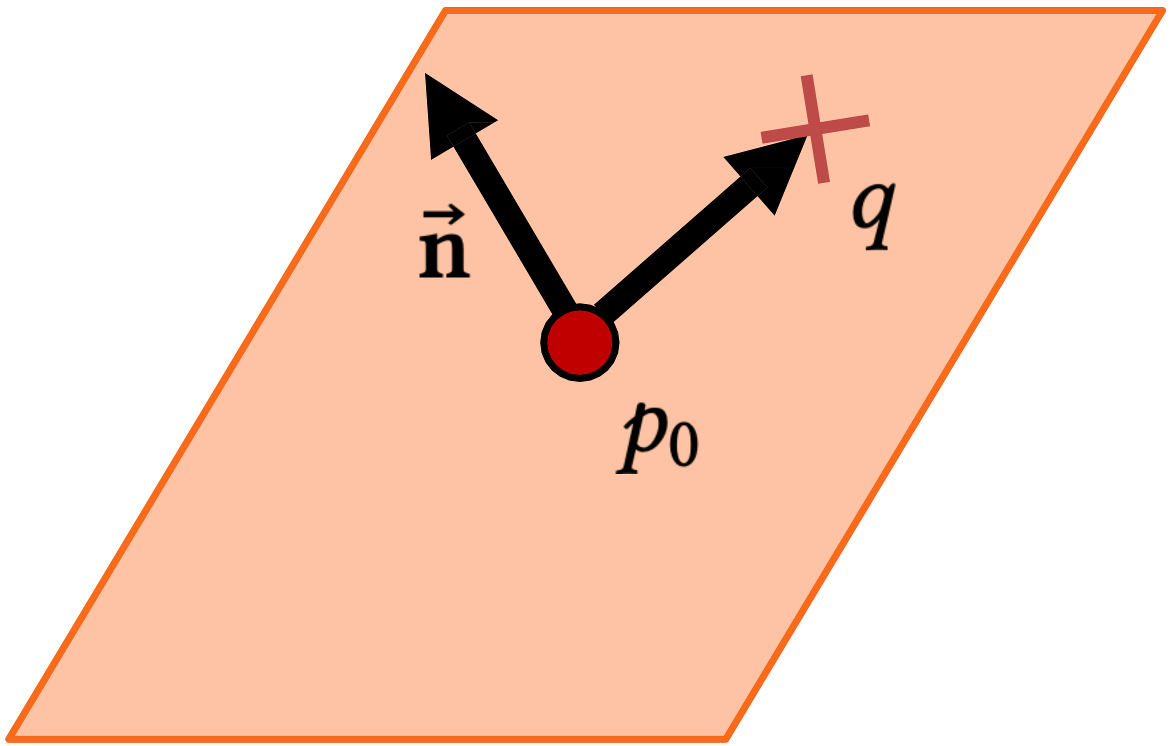
Intersection Tests

- Plane
- Sphere
- Triangle

Plane Equation

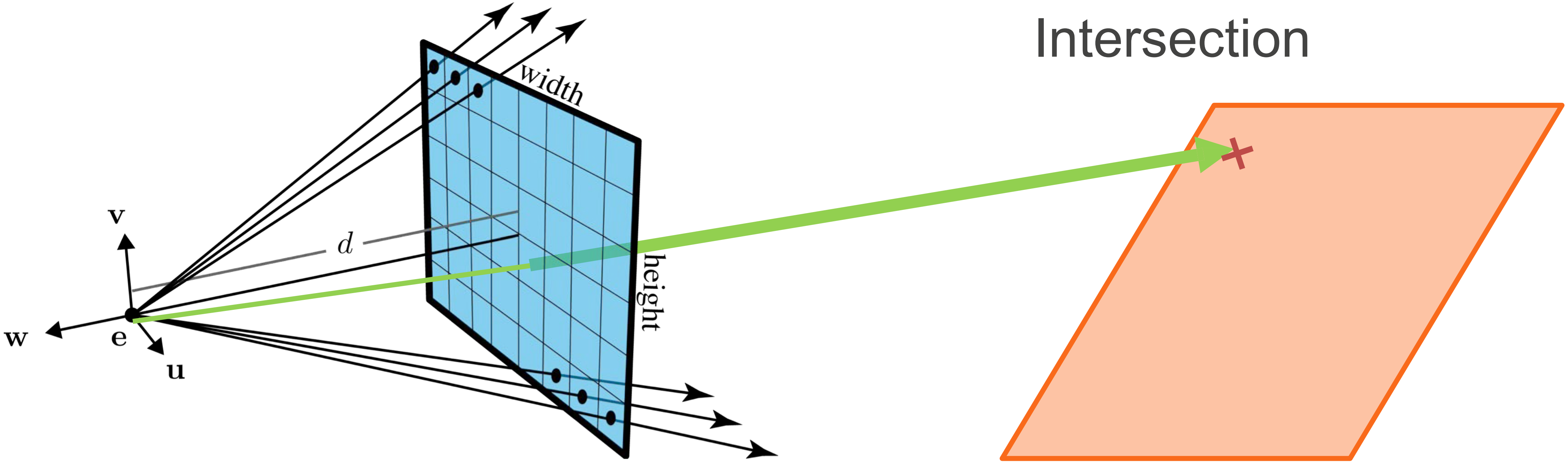


Plane Equation



$$\vec{\mathbf{n}} \cdot (q - p_0) = 0$$

Ray-Plane Intersection

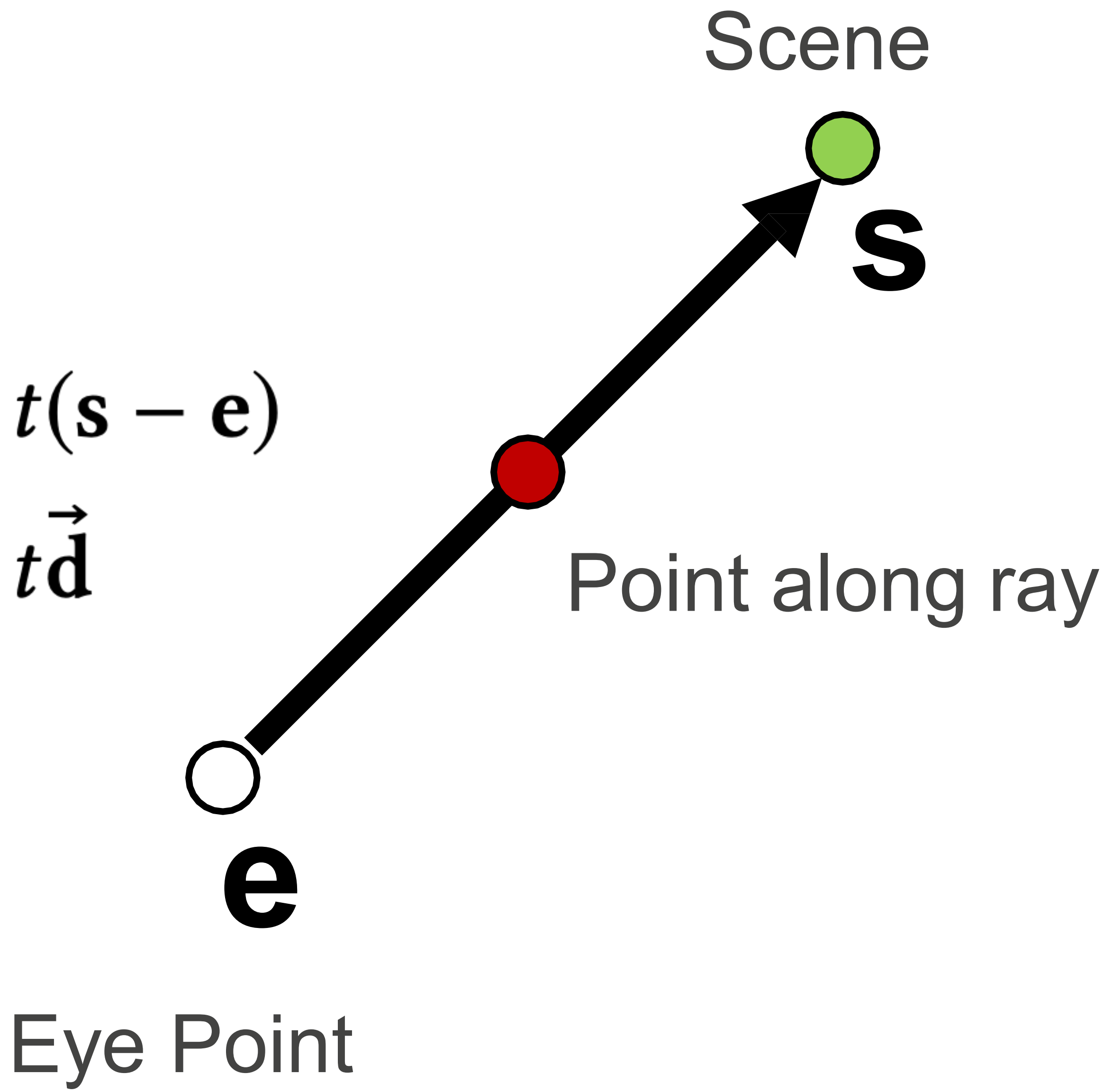


Ray-Plane Intersection

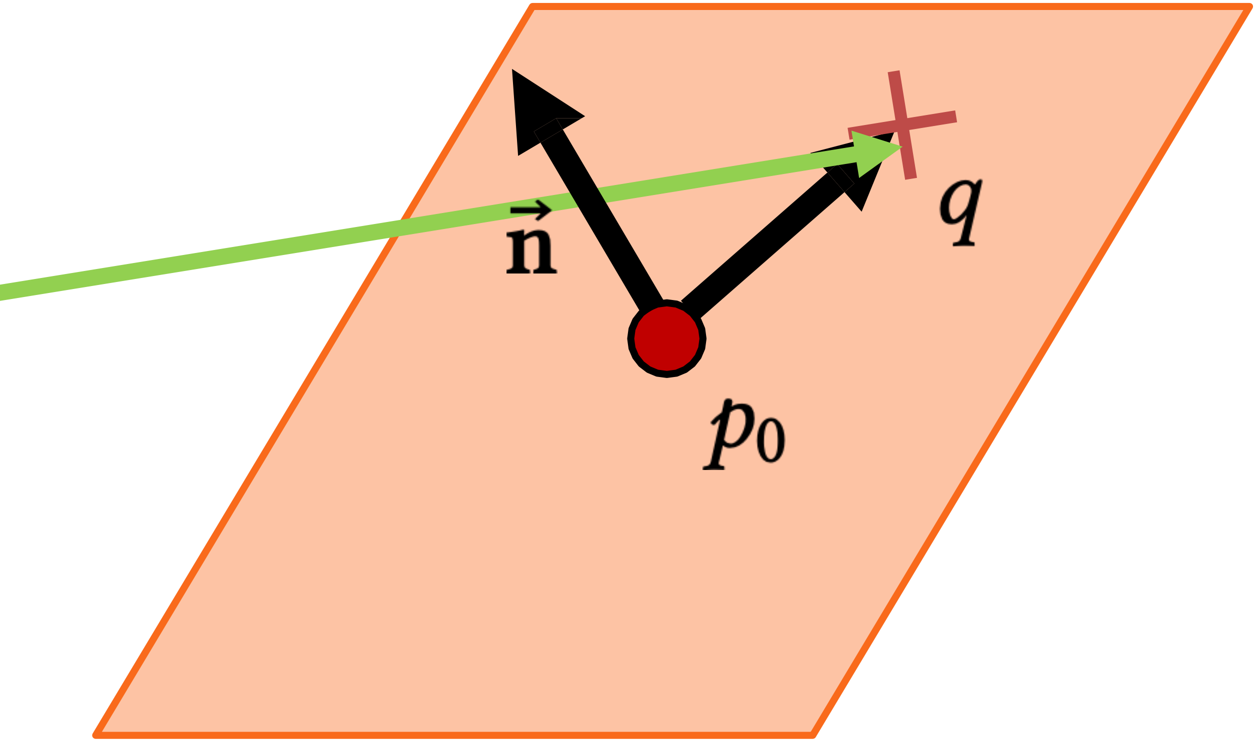
Ray equation

$$\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$

$$\mathbf{p}(t) = \mathbf{e} + t\vec{\mathbf{d}}$$



Plane Equation

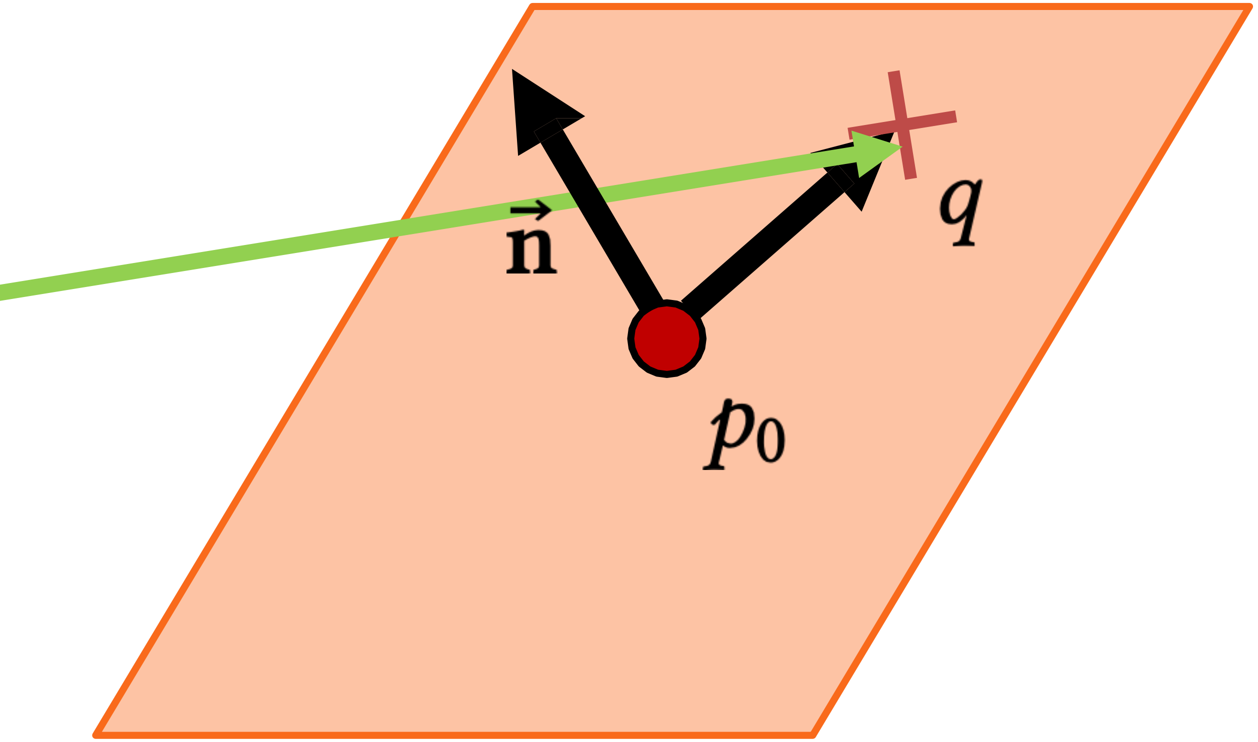


Plane equation
Substitute ray equation into it

$$\vec{\mathbf{n}} \cdot (\mathbf{p}(t) - p_0) = 0$$

$$\vec{\mathbf{n}} \cdot ((\mathbf{e} + t\vec{\mathbf{d}}) - p_0) = 0$$

Plane Equation



Plane equation
Substitute ray equation into it

$$\vec{\mathbf{n}} \cdot (\mathbf{p}(t) - p_0) = 0$$

$$\vec{\mathbf{n}} \cdot ((\mathbf{e} + t\vec{\mathbf{d}}) - p_0) = 0$$

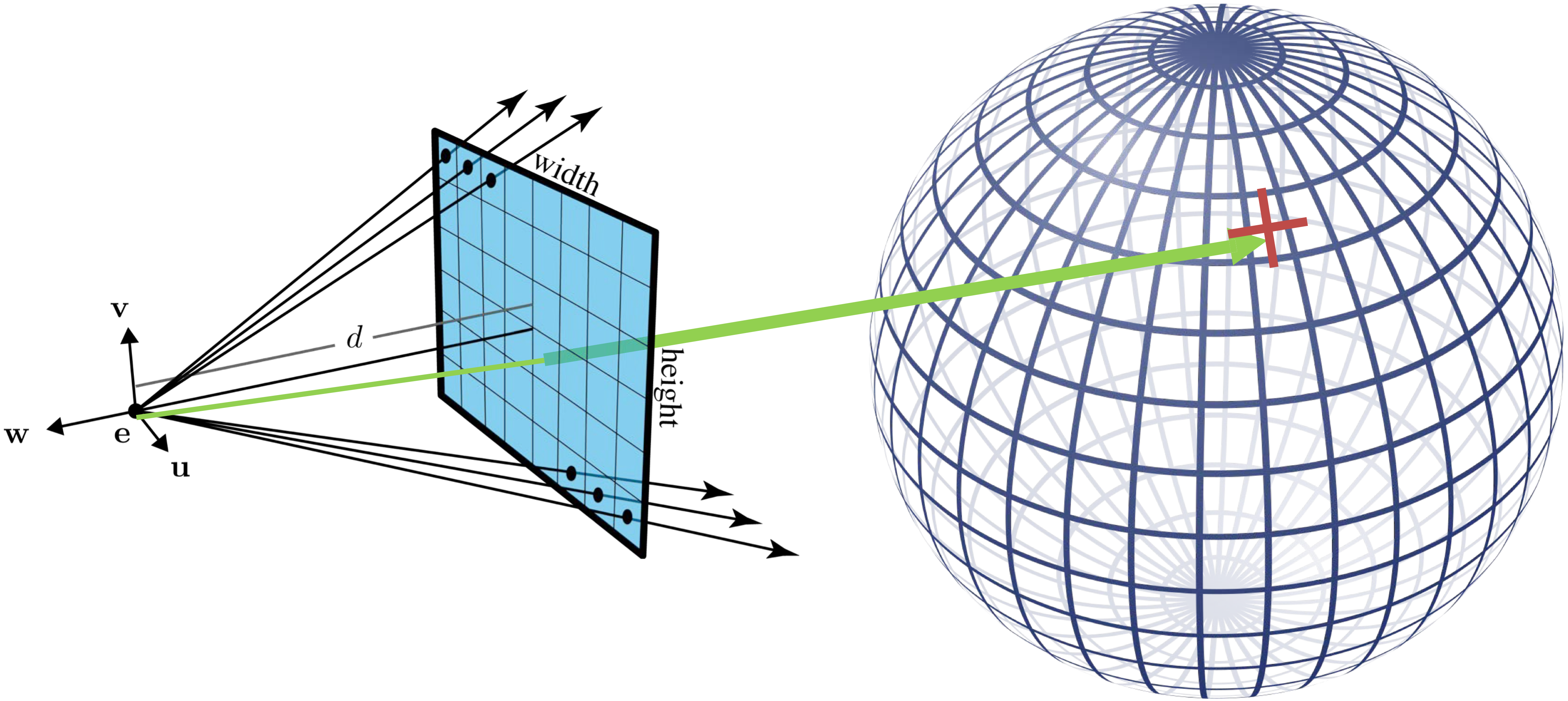
Solve for t

$$\boxed{t = \frac{-\vec{\mathbf{n}} \cdot (\mathbf{e} - p_0)}{\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}}}$$

Intersection Tests

- Plane
- Sphere
- Triangle

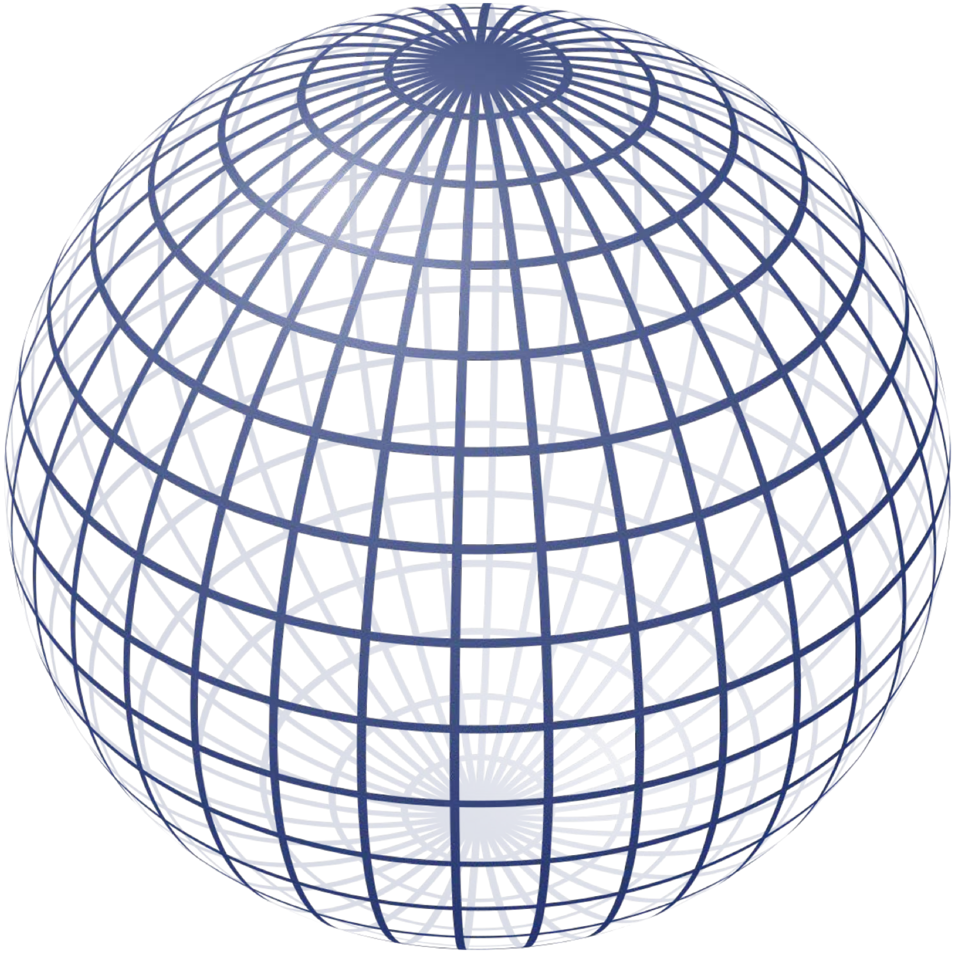
Ray-Sphere Intersection



Implicit Equation of a Sphere

$$(q - \mathbf{c}) \cdot (q - \mathbf{c}) - r^2 = 0$$

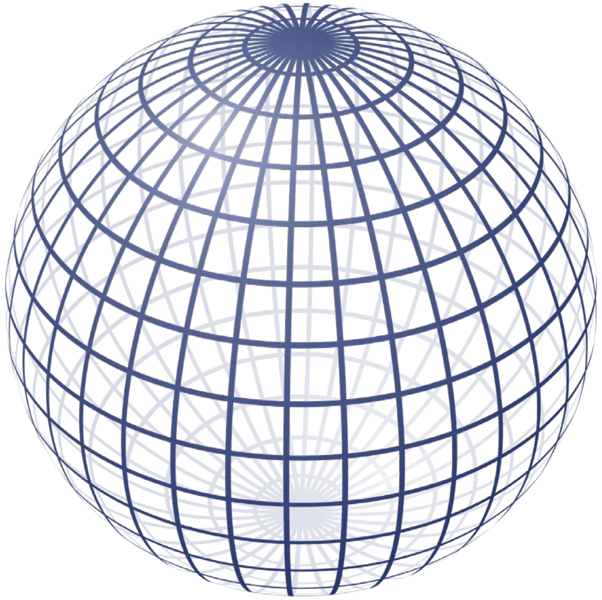
*Sphere centered at **c** with radius r*



Ray-Sphere Intersection

Substitute ray equation into implicit equation for sphere

$$(\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) \cdot (\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) - r^2 = 0$$



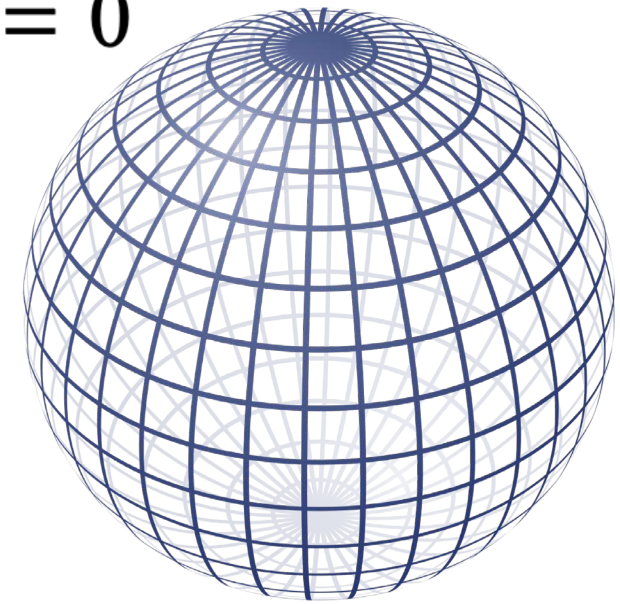
Ray-Sphere Intersection

Substitute ray equation into implicit equation for sphere

$$(\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) \cdot (\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) - r^2 = 0$$

Rearrange

$$(\vec{\mathbf{d}} \cdot \vec{\mathbf{d}})t^2 + 2\vec{\mathbf{d}} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$$



Ray-Sphere Intersection

Substitute ray equation into implicit equation for sphere

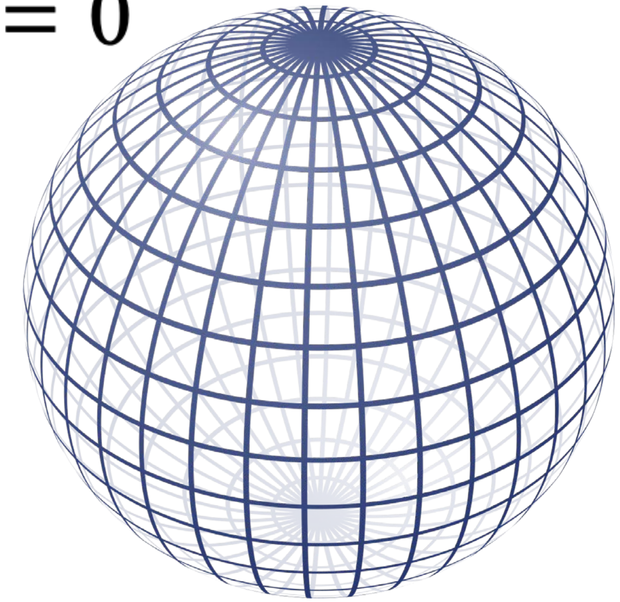
$$(\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) \cdot (\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) - r^2 = 0$$

Rearrange

$$(\vec{\mathbf{d}} \cdot \vec{\mathbf{d}})t^2 + 2\vec{\mathbf{d}} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$$

Looks familiar…

$$At^2 + Bt + C = 0$$



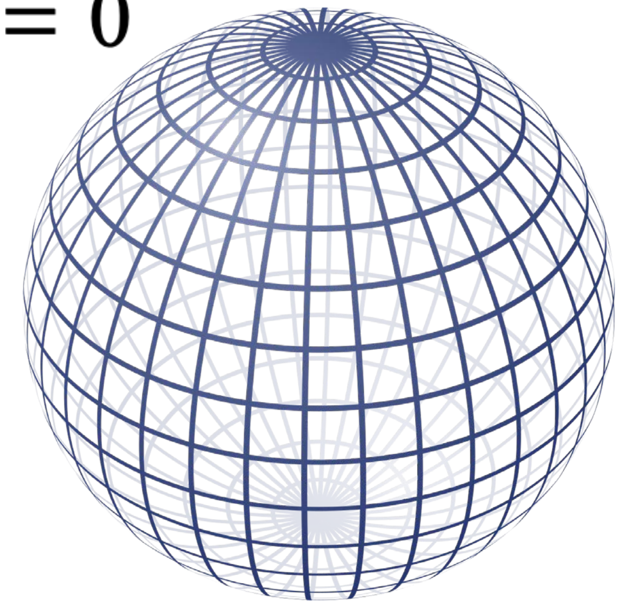
Ray-Sphere Intersection

Substitute ray equation into implicit equation for sphere

$$(\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) \cdot (\mathbf{e} + t\vec{\mathbf{d}} - \mathbf{c}) - r^2 = 0$$

Rearrange

$$(\vec{\mathbf{d}} \cdot \vec{\mathbf{d}})t^2 + 2\vec{\mathbf{d}} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2 = 0$$



Looks familiar…

$$At^2 + Bt + C = 0$$

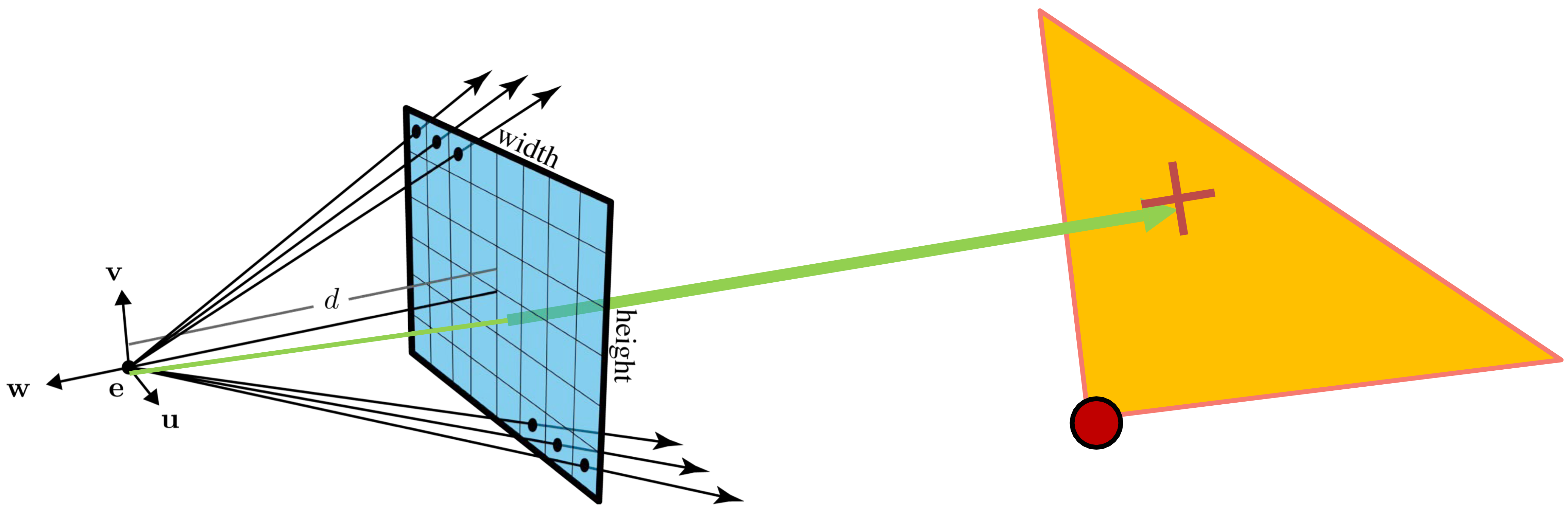
It’s a quadratic! (can use the quadratic equation)

Hint: the discriminant tells us what kinds of roots the equation has.

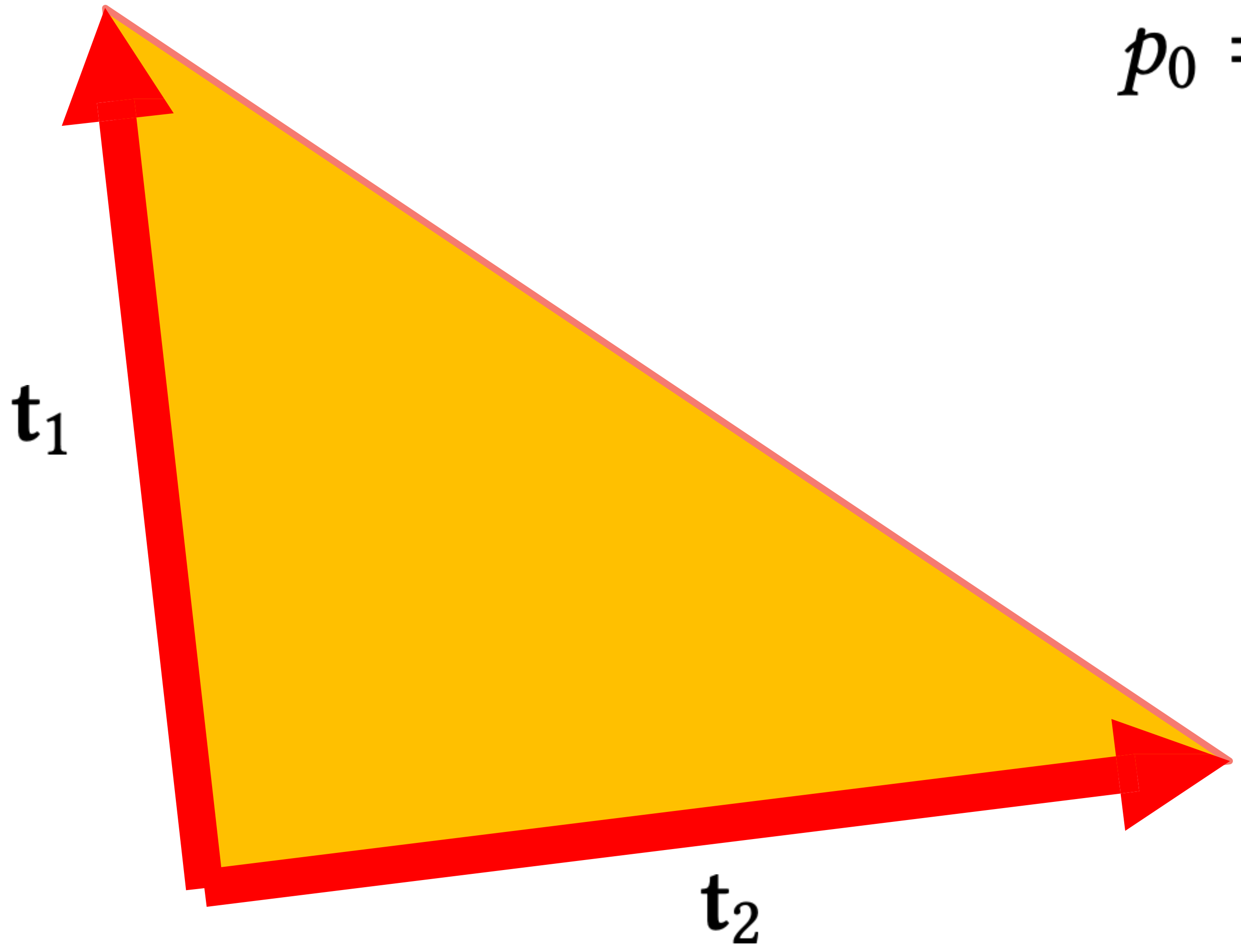
Intersection Tests

- Plane
- Sphere
- Triangle

Ray-Triangle Intersection

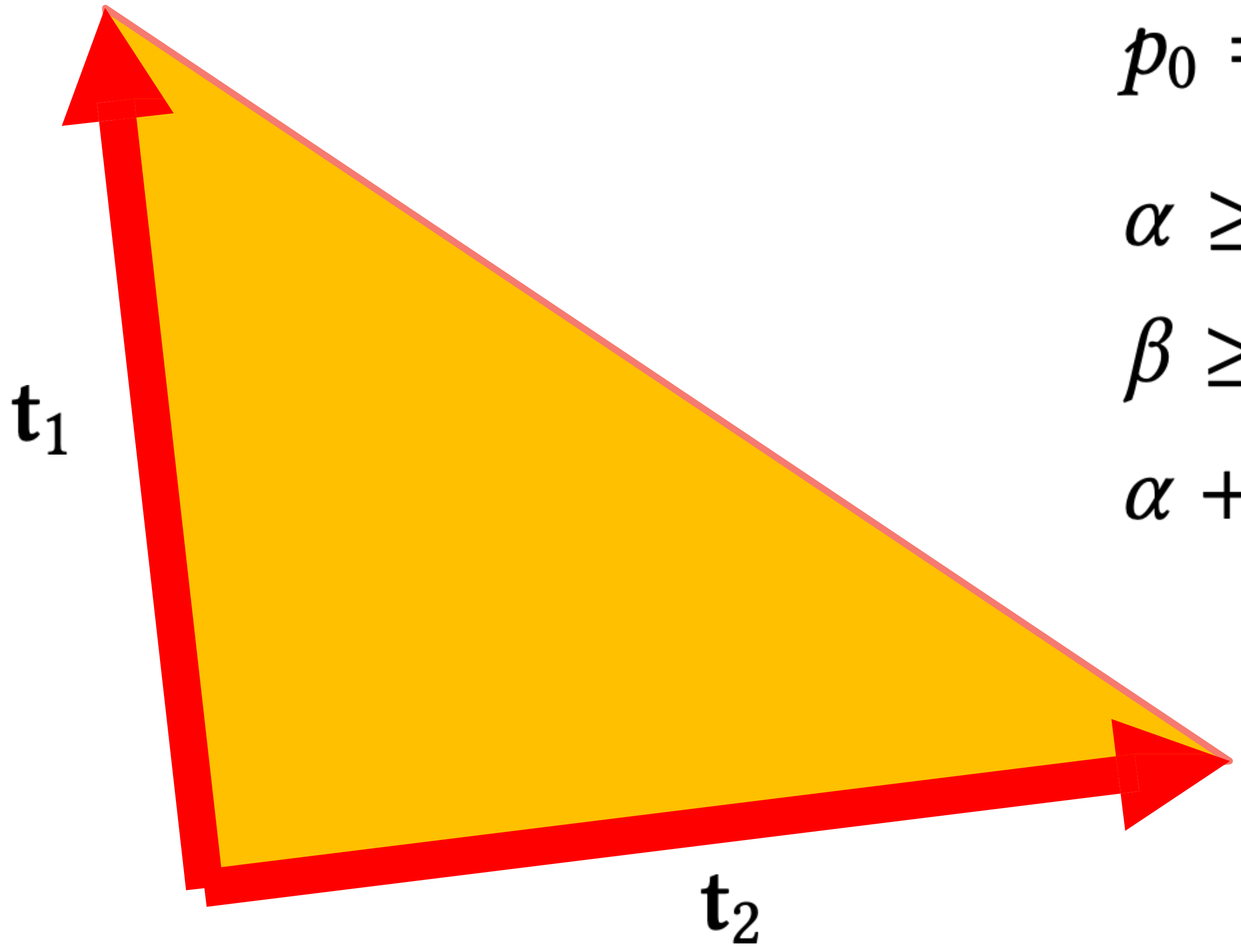


Equations for a Triangle



$$p_0 = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2$$

Equations for a Triangle



$$p_0 = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2$$

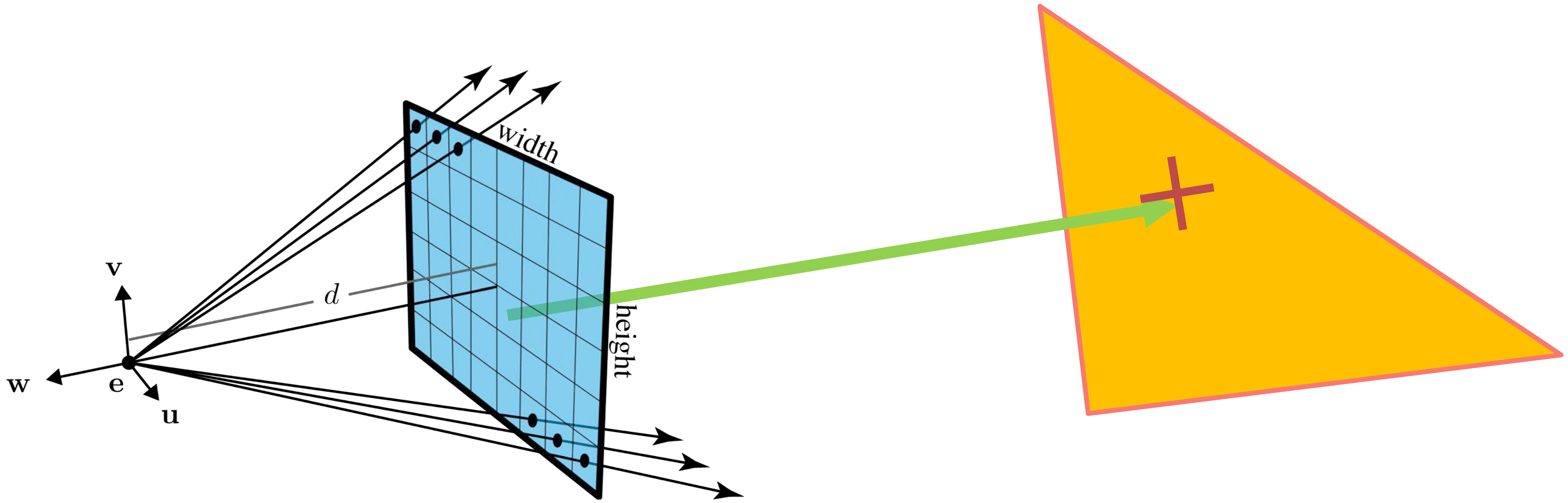
$$\alpha \geq 0$$

$$\beta \geq 0$$

$$\alpha + \beta \leq 1$$

Intersection with a Triangle (Parametric Surface)

Check via equating point on surface with point on ray



Intersection with a Triangle (Parametric Surface)

Check via equating point on surface with point on ray

$$\mathbf{p}(t) = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2$$

$$\mathbf{e} + t\vec{\mathbf{d}} = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2$$

$$\mathbf{e} = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2 - t\vec{\mathbf{d}}$$

Intersection with a Triangle (Parametric Surface)

Check via equating point on surface with point on ray

$$\mathbf{e} = \alpha \mathbf{t}_1 + \beta \mathbf{t}_2 - t\vec{\mathbf{d}}$$

$$\mathbf{e} = \begin{bmatrix} \mathbf{t}_1 & \mathbf{t}_2 & -\mathbf{d} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ t \end{bmatrix}$$

Check values of α, β, t

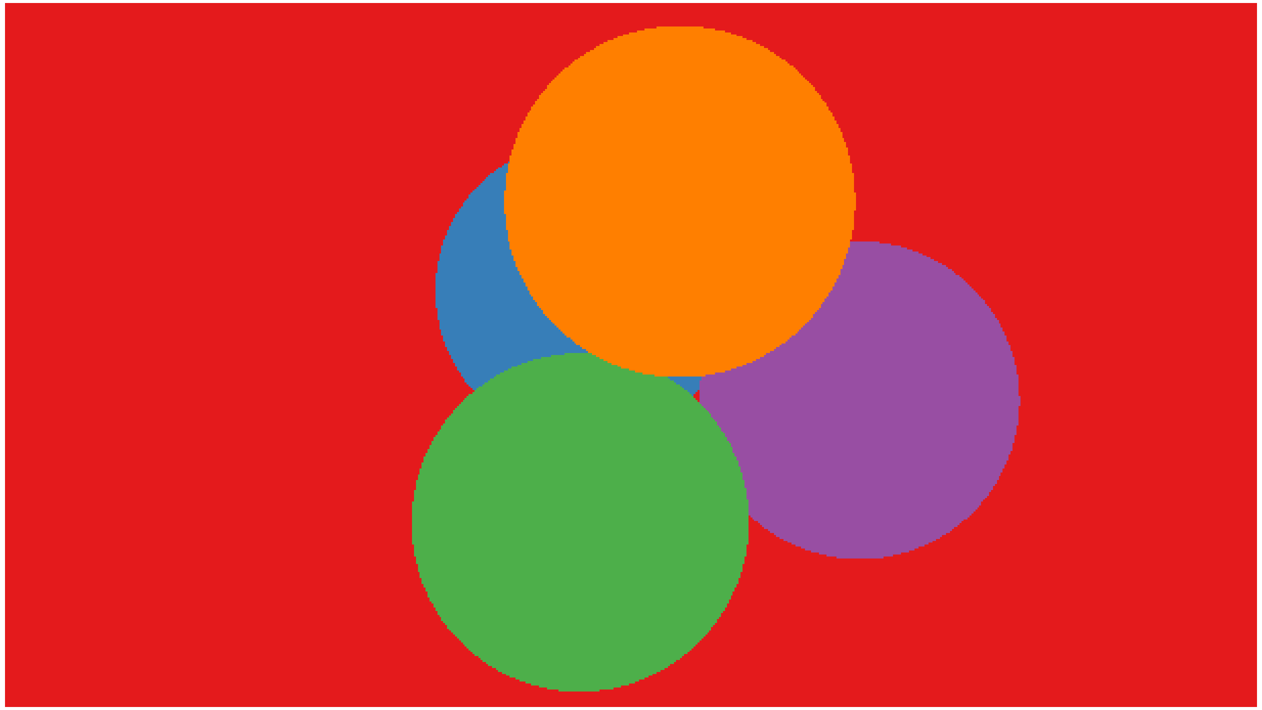
Ray Casting

```
for each pixel in the image {
    Generate a ray

    for each object in the scene {

        if (Intersect ray with object)

            {  Set pixel colour

        }
    }
}
```

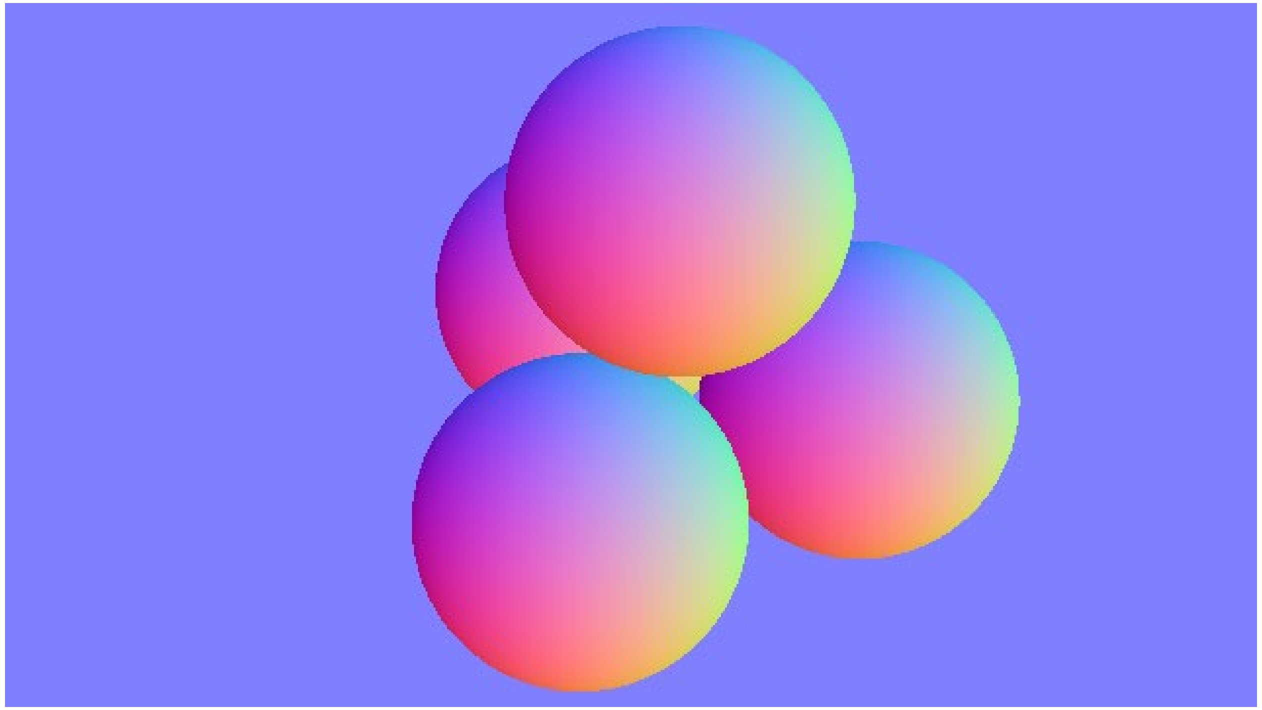
Output Type

- Object ID
- Surface Normal
- Depth

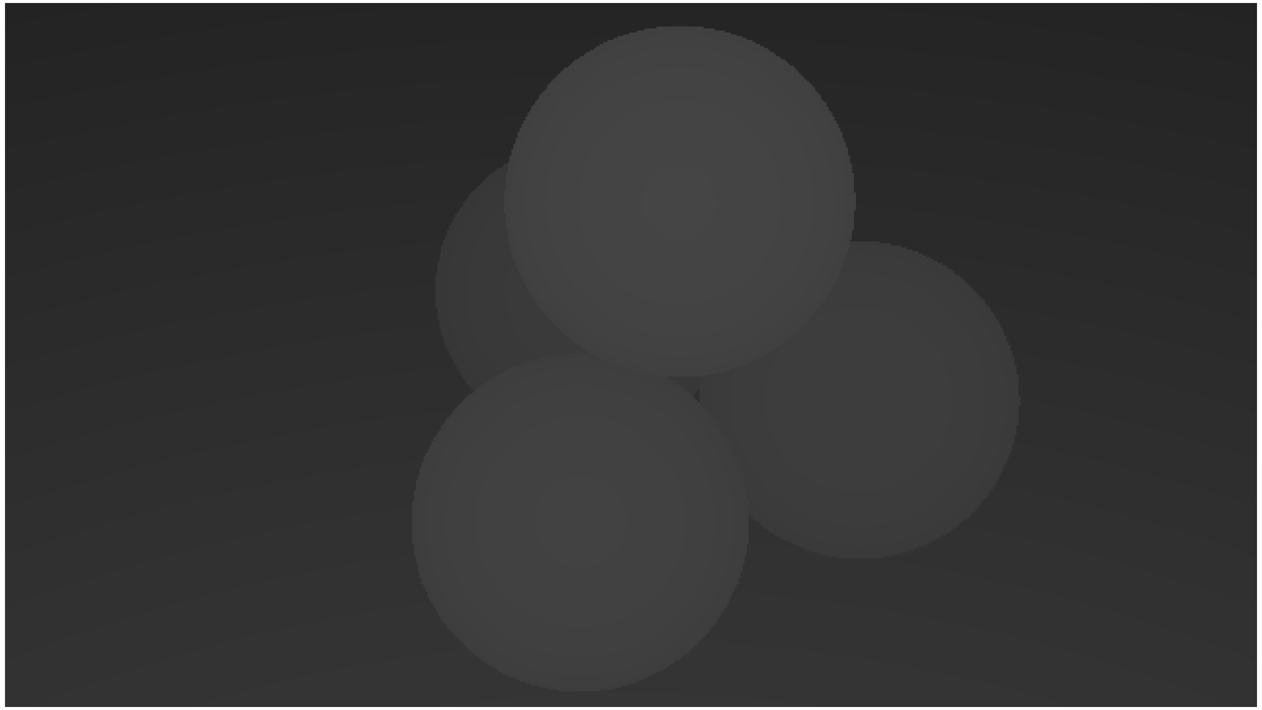
Output Type



Output Type



Output Type



Ray Casting

```
for each pixel in the image {
    Generate a ray

    for each object in the scene {

        if (Intersect ray with object)

            {  Set pixel colour

        }

    }
}
```

Next Week

Ray Tracing