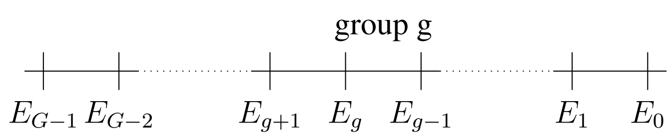
NE 155/255, Fall 2019
Equation Discretization
September 2019

We'll start from the general time-independent NTE without delayed neutrons, with 6 independent variables. We need to discretize each variable.

$$\begin{aligned}
&\hat{\Omega} \cdot \nabla \psi(\vec{r}, E, \hat{\Omega}) + \Sigma_t(\vec{r}, E)\psi(\vec{r}, E, \hat{\Omega}) \\
&\quad = \int_0^\infty \int_{4\pi} \Sigma_s(\vec{r}, E' \to E, \hat{\Omega}' \to \hat{\Omega})\psi(\vec{r}, E', \hat{\Omega}')d\hat{\Omega}'dE' \\
&\qquad + \frac{\chi_p(E)}{4\pi} \int_0^\infty \int_{4\pi} \nu(E')\Sigma_f(\vec{r}, E')\psi(\vec{r}, E', \hat{\Omega}')d\hat{\Omega}'dE' \\
&\qquad + S(\vec{r}, E, \hat{\Omega})
\end{aligned} \tag{1}$$

Energy Discretization

We'll handle the energy dimension by breaking continuous energy into G groups, where group g is $[E_g, E_{g-1}]$:



We will solve for group-integrated values in each energy bin using the following definitions:

$$\psi_g(\vec{r}, \hat{\Omega}) \equiv \int_{E_g}^{E_{g-1}} dE\, \psi(\vec{r}, \hat{\Omega}, E) \qquad \phi_g(\vec{r}) \equiv \int_{E_g}^{E_{g-1}} dE\, \phi(\vec{r}, E)$$

$$S_g(\vec{r}, \hat{\Omega}) \equiv \int_{E_g}^{E_{g-1}} dE\, S(\vec{r}, \hat{\Omega}, E) \qquad \chi_g \equiv \int_{E_g}^{E_{g-1}} dE\, \chi(E)$$

To perform these integrals, we need to introduce approximations. We *assume that each item is separable in energy*.

For example:

$$\psi(\vec{r},\hat{\Omega},E) \approx f(E)\psi_g(\vec{r},\hat{\Omega})\,, \quad E_g < E \le E_{g-1}\,,$$

where $f(E)$ is normalized such that $\int_g dE\, f(E) = 1$.

Next, we need a way to create multigroup cross sections. Options for how to do that in more detail are covered in NE 250. Here we will do the most common/generic approach: weight with the angular flux:

$$\Sigma_{t,g}(\vec{r}) \equiv \frac{\int_{E_g}^{E_{g-1}} dE\, \Sigma_t(\vec{r},E)f(E)}{\int_{E_g}^{E_{g-1}} dE\, f(E)} = \frac{\int_{E_g}^{E_{g-1}} dE\, \Sigma_t(\vec{r},E)\psi(\vec{r},\hat{\Omega},E)}{\int_{E_g}^{E_{g-1}} dE\, \psi(\vec{r},\hat{\Omega},E)}$$

We do the same thing for fission (not shown here). Scattering requires an extra integral:

$$\begin{aligned}\Sigma_s^{g'\to g}(\vec{r},\hat{\Omega}'\cdot\hat{\Omega}) &\equiv \frac{\int_{E_g}^{E_{g-1}} dE \int_{E_{g'}}^{E_{g'-1}} dE'\, \Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\cdot\hat{\Omega})f(E')}{\int_{E_{g'}}^{E_{g'-1}} dE'\, f(E')} \\ &\equiv \frac{\int_{E_g}^{E_{g-1}} dE \int_{E_{g'}}^{E_{g'-1}} dE'\, \Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\cdot\hat{\Omega})\psi(\vec{r},\hat{\Omega}',E')}{\int_{E_{g'}}^{E_{g'-1}} dE'\, \psi(\vec{r},\hat{\Omega}',E')}\end{aligned}$$

Using these definitions, we can write a transport equation for each group, $g = 0,\dots,G-1$:

$$\begin{aligned}[\hat{\Omega}\cdot\nabla + \Sigma_{t,g}(\vec{r})]\psi_g(\vec{r},\hat{\Omega}) = \sum_{g'=0}^{G-1}\int_{4\pi} d\hat{\Omega}'\, &\Sigma_s^{g'\to g}(\vec{r},\hat{\Omega}'\cdot\hat{\Omega})\psi_{g'}(\vec{r},\hat{\Omega}') \\ &+ \frac{\chi_g}{4\pi}\sum_{g'=0}^{G-1}\nu_{g'}\Sigma_{f,g'}(\vec{r})\phi_{g'}(\vec{r}) + S_g(\vec{r},\hat{\Omega}),\end{aligned}$$

giving G coupled equations. Note that the coupling may cause iteration because of the exchange of particles between energy groups (upscattering!).

These equations are **exact** in the unique case where (1) the separability in energy holds and (2) the cross sections are constant within each energy group.