NE 155/255, Fall 2019

Source Iteration in 1D

October 25, 2019

Let's revisit "source iteration" and how that works to get a better handle on within-group iteration. Assume

- steady state
- one spatial dimension
- azimuthal symmetry
- S_N angular discretization ($a = 1, \dots, N(N+2)$; $N(N+2) = M$)
- P_N with $N = 1$: linearly anisotropic scattering ($\ell = 0, 1$ so $\Sigma_s = \Sigma_{s0} + 3\mu_a\mu_{a'}\Sigma_{s1}$)
- multigroup in energy ($g = 0, \dots, G-1$)

With that, we have

$$\mu_a \frac{d}{dx}\psi_a^g(x) + \Sigma_t(x)\psi_a^g(x) = 2\pi \sum_{g'=0}^{G-1}\sum_{\ell=0}^{N} \Sigma_{s\ell}^{g'\to g}(x)\tilde{\psi}_\ell^g(x) + q_a^g(x) \tag{1}$$

$$\tilde{\psi}_0^g(x) = \sum_{a=1}^{M} w_a \psi_a^g(x)\,; \quad P_0 = 1 \tag{2}$$

$$\tilde{\psi}_1^g(x) = \sum_{a=1}^{M} w_a \mu_a \psi_a^g(x)\,; \quad P_1 = \mu \tag{3}$$

We've

- converted the angular integration in the source term to a quadrature integration ($\sum_{a=1}^{M} w_a f_a$)
- expanded the double differential scattering cross section in Legendre polynomials ($\sum_{\ell=0}^{N} P_\ell f_\ell$)
- applied groups (y^g)

Next, let's add the iteration index for source iteration.

$$\mu_a \frac{d}{dx}\psi_a^{g,(k+1)}(x) + \Sigma_t^g(x)\psi_a^{g,(k+1)}(x) = 2\pi \sum_{g'=0}^{G-1}\sum_{\ell=0}^{N} \Sigma_{s\ell}^{g'\to g}(x)\tilde{\psi}_\ell^{g',(k)}(x) + q_a^g(x) \tag{4}$$

$$\tilde{\psi}_0^{g,(k)}(x) = \sum_{a=1}^{M} w_a \psi_a^{g,(k)}(x)\,; \quad P_0 = 1 \tag{5}$$

$$\tilde{\psi}_1^{g,(k)}(x) = \sum_{a=1}^{M} w_a \mu_a \psi_a^{g,(k)}(x)\,; \quad P_1 = \mu \tag{6}$$

You can see that because the *source* depends on the flux from angles and energies other than the one we are in, we need to *iterate*. Hence, **source iteration**. We still need to iterate with only one group because of the angle component. If we don't have scattering, we don't need to iterate.

To add spatial discretization, define the right hand side as s_i:

$$\psi_{i,a}^{g,(k+1)} = \frac{s_{i,a}^{g,(k)} + \frac{2}{(1\pm\alpha_i)}\frac{|\mu_a|}{\Delta_i}\bar{\psi}_{a,i\mp1/2}^{g,(k+1)}}{\Sigma_i^g + \frac{2}{(1\pm\alpha_i)}\frac{|\mu_a|}{\Delta_i}} \tag{7}$$

$$\psi_{a,i\pm1/2}^{g,(k+1)} = \frac{2}{(1\pm\alpha_i)}\psi_{i,a}^{g,(k+1)} - \frac{(1\mp\alpha_i)}{(1\pm\alpha_i)}\bar{\psi}_{a,i\mp1/2}^{g,(k+1)} \tag{8}$$

where the overbar indicates incoming flux (based on direction). With all of this, we can see the iteration levels. We have an initial guess $\psi^{(0)}$ for each group, angle, and cell. For a given group, do the following <u>until convergence of the flux in that group</u>:

1. For each angle
   (a) Start at the appropriate cell where we have known incoming flux moments, $\bar{\psi}_{i\mp1/2}^g$
   (b) Calculate the source guess in that cell using the past flux value (because it is known at all angles and energies):

   $$s_{a,i}^{g,(k)} = 2\pi \sum_{g'=0}^{G-1}\sum_{\ell=0}^{N} \Sigma_{s\ell}^{g'\to g}(x)\tilde{\psi}_\ell^g(x) + q_a^g(x)$$

   (c) Calculate and store the updated flux in that cell, $\psi_{i,a}^{g,(k+1)}$, using Equation 7
   (d) Calculate the updated outgoing flux in that cell, $\psi_{i\pm1/2}^{g,(k+1)}$, using Equation 8
   (e) Set the incoming flux in the next cell to be the outgoing flux in the previous cell
   (f) move to the next cell
2. move to the next angle

Algorithm 1 1-D WDD (within group)

initialize values and vectors (flux, mesh spacing, etc.)
while not converged **do**
  # sweep from left to right, assuming flux incident on left boundary
  for a in angles where $a > 0$: **do**
    $Q_{scatter} = \sum_{\ell=0}^{N} \Sigma_{s,\ell} \tilde{\psi}_\ell^{old}$
    $Q_{total} = Q_{ext} + Q_{scatter}$
    for x in cells (starting at left) **do**
      get incoming flux (set by BC or upwind cell)
      # calculate cell-center flux and outgoing flux

$$\psi_{a,i}^{new} = \frac{Q_{total}^{old} + \frac{2}{(1+\alpha)} \frac{|\mu_a|}{\Delta_i} \bar{\psi}_{a,i-1/2}^{new}}{\Sigma_t + \frac{2}{(1+\alpha_i)} \frac{|\mu_a|}{\Delta_i}}$$

$$\psi_{a,i+1/2}^{new} = \frac{2}{(1+\alpha_i)} \psi_{a,i}^{new} - \frac{(1-\alpha_i)}{(1+\alpha_i)} \bar{\psi}_{a,i-1/2}^{new}$$

    end for
  end for
  # sweep from right to left, no flux incident on right boundary
  for a in angles where $a < 0$: **do**
    $Q_{scatter} = \sum_{\ell=0}^{N} \Sigma_{s,\ell} \tilde{\psi}_\ell^{old}$
    $Q_{total} = Q_{ext} + Q_{scatter}$
    for x in cells (starting at right) **do**
      get incoming flux (set by BC or upwind cell)
      # calculate cell-center flux and outgoing flux

$$\psi_{a,i}^{new} = \frac{Q_{total}^{old} + \frac{2}{(1-\alpha)} \frac{|\mu_a|}{\Delta_i} \bar{\psi}_{a,i+1/2}^{new}}{\Sigma_t + \frac{2}{(1-\alpha_i)} \frac{|\mu_a|}{\Delta_i}}$$

$$\psi_{a,i-1/2}^{new} = \frac{2}{(1-\alpha_i)} \psi_{i,a}^{new} - \frac{(1+\alpha_i)}{(1-\alpha_i)} \bar{\psi}_{a,i+1/2}^{new}$$

    end for
  end for
  $\Phi(x) = \sum_{a=1}^{M} w_a \psi_a(x)$
  check for scalar flux convergence
end while