NE 155/255
Numerical Simulations in Radiation Transport
Introduction to Monte Carlo

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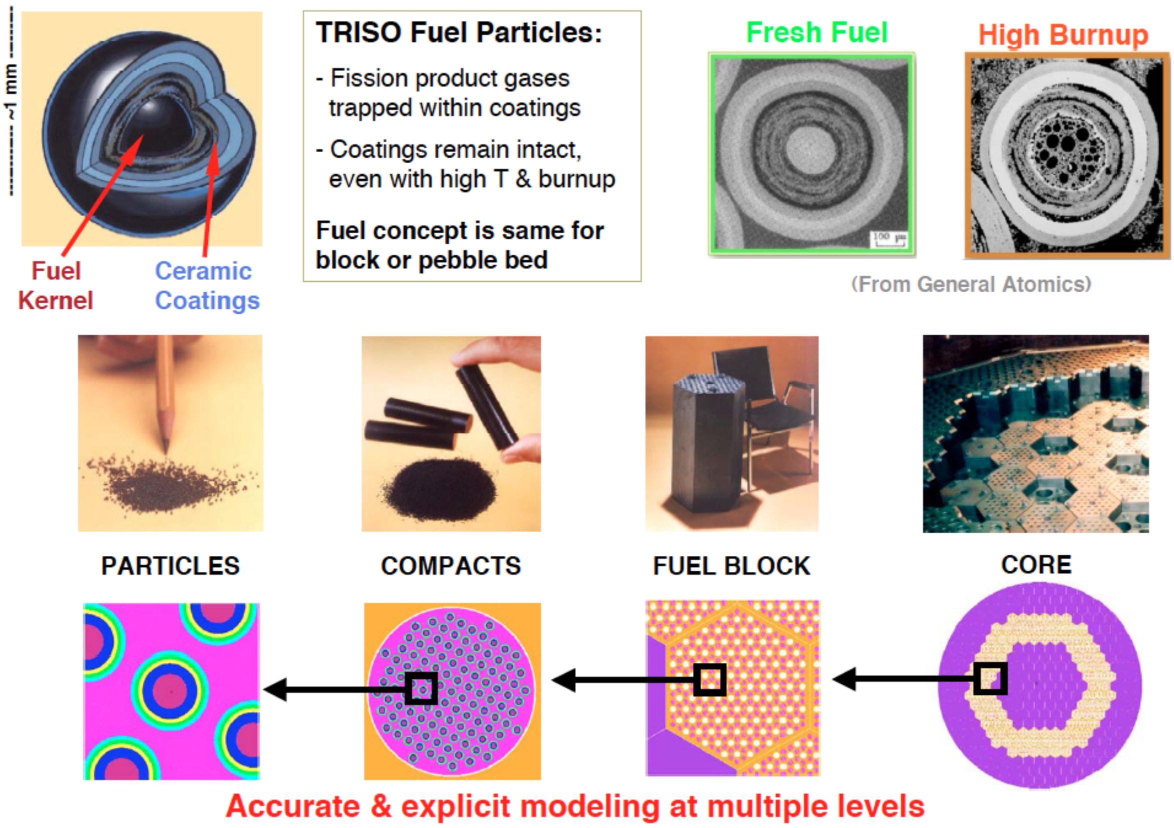
November 6, 2019

Mathematical Validity

- Consider particles with a phase space describing position, $\vec{r}$, and velocity, $\vec{v}$
- A neutral particle can be transmitted from one position to another at a constant velocity
$$T(\vec{r}\,' \to \vec{r}, \vec{v})$$
- A particle can undergo a collision at a single position that changes its velocity
$$C(\vec{r}, \vec{v}\,' \to \vec{v})$$
- Integral TE:

$$\psi(\vec{r}, \vec{v}) = \int_{\vec{r}\,'} \left[\int_{\vec{v}\,'} \psi(\vec{r}\,', \vec{v}\,') C(\vec{r}\,', \vec{v}\,' \to \vec{v}) d\vec{v}\,' \right] T(\vec{r}\,' \to \vec{r}, \vec{v}) d\vec{r}\,'$$

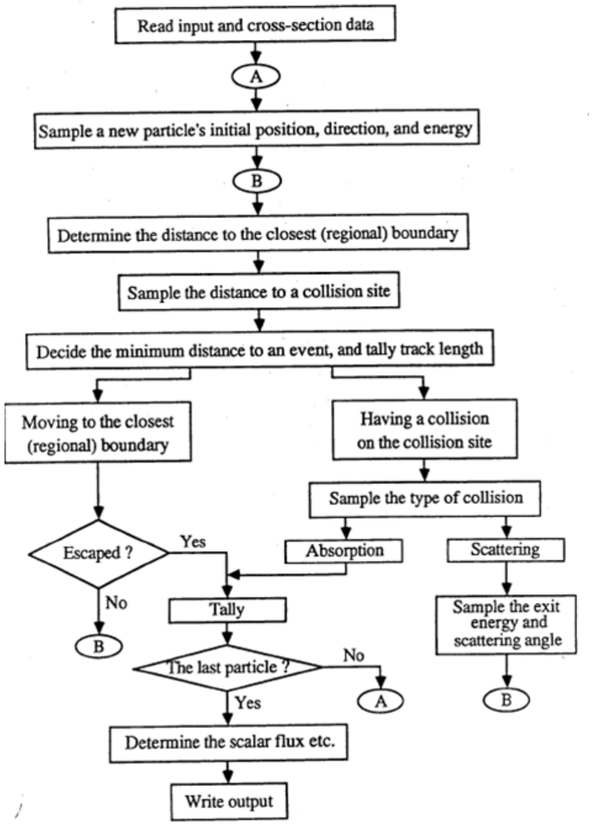
Can Model Very Complex Things



Major Components of MC Algorithm

- **PDFs**: the physical/mathematical system must be described by a set of pdfs.
- **Random number generator**: a source of random #s uniformly distributed on the unit interval.
- **Sampling rule**: prescription for sampling the pdf (given having random #s)
- **Scoring**: the outcomes must be accumulated/tallied for quantities of interest
- **Error estimation**: an estimate of the statistical error (variance) of the solution
- **Variance Reduction**: methods for reducing the variance and computation time simultaneously
- **Parallelization**: efficient use of computers

Basic Event-Based Algorithm



Let's Get Started With...

1. Physics as Probability
2. Definitions: PDF & CDF
3. Motivation & Goal of Random Sampling
4. Basic Random Sampling Techniques
   - Direct Discrete Sampling
   - Direct Continuous Sampling
   - Rejection Sampling

Notes derived from Rachel Slaybaugh, Jasmina Vujic, and Paul Wilson.

Learning Objectives

1. Provide examples of probabilistic representations of physics
2. Distinguish between a PDF and CDF
3. Distinguish between a *discrete* PDF (CDF) and a *continuous* PDF (CDF)
4. Describe the goal of random sampling
5. Identify and implement the best random sampling technique for a given distribution

Physics as Probability

Various physical phenomena can be represented by probability distributions

- Photon emission energy
  - Each possible energy has a different probability (intensity)
- Scattering cross-sections
  - Each possible scattering angle has a different probability as a function of the energy
- Transmission through a medium
  - Probability of reaching a particular position depends on the cross-section

Probability Density Functions

All variables, x, have a Probability Density Function (PDF), $p(x)$, with the following characteristics:

Continuous

$$p\{a \le x \le b\} = \int_a^b p(x)dx$$

$$p(x) \ge 0$$

$$\int_{-\infty}^{\infty} p(x)dx = 1$$

Discrete

$$p(x = x_k) = p_k \equiv p(x_k)$$
$$k = 1, \ldots, N$$

$$p_k \ge 0$$

$$\sum_{k=1}^{N} p_k = 1$$

Cumulative Distribution Functions

All PDFs, $p(x)$, have an associated Cumulative Distribution Function (CDF), $P(x)$, with the following properties:

Continuous

$$P\{x' \le x\} = P(x) = \int_{-\infty}^{x} p(x')dx'$$

$$P(-\infty) = 0\,, \quad P(\infty) = 1$$
$$0 \le P(x) \le 1$$
$$\frac{dP(x)}{dx} \ge 0$$

Discrete

$$P\{x' \le x\} = P_k \equiv P(x_k) = \sum_{j=1}^{k} p_j$$

$$k = 1, \ldots, N$$
$$P_0 = 0\,, \quad P_N = 1$$
$$0 \le P_k \le 1$$
$$P_k \ge P_{k-1}$$