Gradient methods

KZ

Minimal residuals

```
solve[x0_, A_, f_, k_] := Module[{r, x = x0, α}, r = f - A.x;
  α = (A.r.r)/(A.r.A.r);
  Do[x = x + α r;
   r = f - A.x;
   α = (A.r.r)/(A.r.A.r), {i, 1, k}];
  x]
```

Test

```
A = {{1, .42, .54, .66}, {.42, 1, .32, .44}, {.54, .32, 1, .22}, {.66, .44, .22, 1}};
f = {.3, .5, .7, .9};
x = {0, 0, 0, 0};
```

```
solve[x, A, f, 15]
```

```
{-1.25422, 0.0440003, 1.03732, 1.47966}
```

```
Inverse[A].f
```

```
{-1.25779, 0.0434873, 1.03917, 1.48239}
```

Gradient descent

```
solve2[x0_, A_, f_, k_] := Module[{r, x = x0, α}, r = f - A.x;
  α = (r.r)/(A.r.r);
  Do[x = x + α r;
   r = f - A.x;
   α = (r.r)/(A.r.r), {i, 1, k}];
  x]
```

Test

```
solve2[x, A, f, 15]
```

```
{-1.24436, 0.0455961, 1.03255, 1.47257}
```

```
Inverse[A].f
```

```
{-1.25779, 0.0434873, 1.03917, 1.48239}
```

Conjugate gradients

```
solve3[x0_, A_, f_, k_] := Module[{r, r1, s, x = x0, α, β}, r = f - A.x;
  s = r;
  Do[r1 = r;
   α = r1.r1 / A.s.s;
   x = x + α s;
   r = r - α A.s;
   β = r.r / r1.r1;
   s = r + β s, {i, 1, k}];
  x]
```

Test

```
solve3[x, A, f, 5]
```

```
{-1.25779, 0.0434873, 1.03917, 1.48239}
```

```
Inverse[A].f
```

```
{-1.25779, 0.0434873, 1.03917, 1.48239}
```