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Best mean-square approximation polynomial

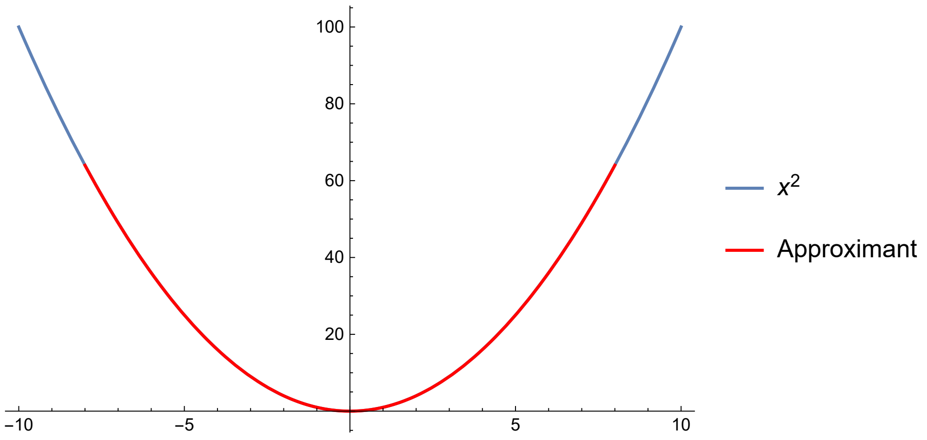
fun - исходная функция

n - степень аппроксиманта

```
hermite[n_] := (-1)^n * e^(x^2) * D[e^(-x^2), {x, n}] / Sqrt[2^n n! Sqrt[Pi]] // FullSimplify

aCoefficient[fun_, n_] := Integrate[fun * e^(-x^2) * hermite[n], {x, -∞, +∞}]

approximant[fun_, n_] :=
 Total[Table[N@hermite[k] * aCoefficient[fun, k] // N, {k, 0, n}]] // FullSimplify
```

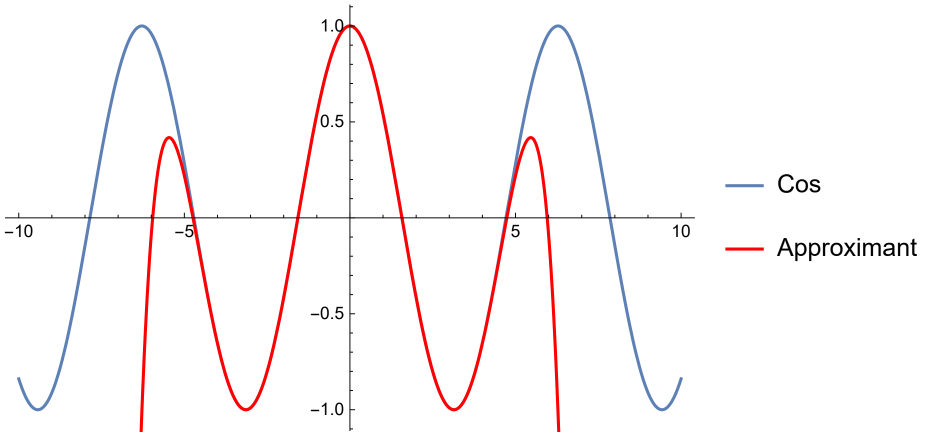
Test 1

```
polynomial = approximant[x^2, 2];

Show[{Plot[x^2, {x, -10, 10}, PlotLegends → {"x^2"}],
  Plot[polynomial /. x → k, {k, -8, 8}, PlotStyle → Red, PlotLegends → {"Approximant"}]}]
```



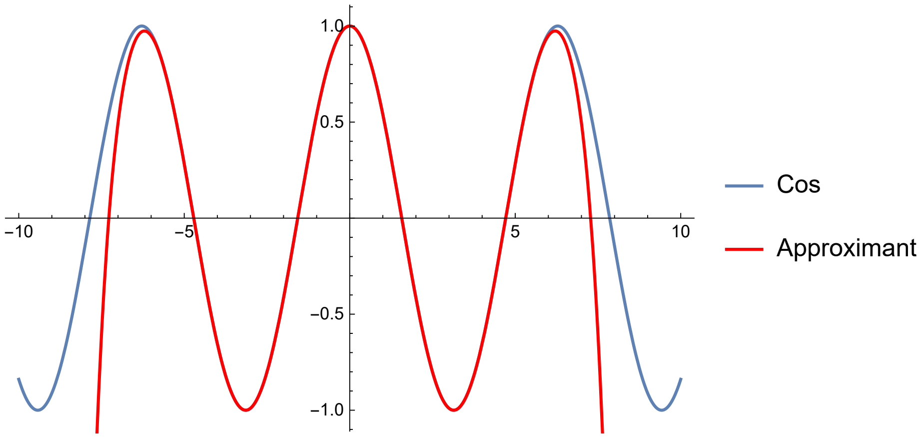
Test 2

```
polynomial1 = approximant[Cos[x], 10];
Show[{Plot[Cos[x], {x, -10, 10}, PlotLegends → {"Cos"}], Plot[
   polynomial1 /. x → k, {k, -8, 8}, PlotStyle → Red, PlotLegends → {"Approximant"}]}]
```



Увеличив степень полинома, получаем большую точность.

```
polynomial12 = approximant[Cos[x], 15];
Show[{Plot[Cos[x], {x, -10, 10}, PlotLegends → {"Cos"}], Plot[
   polynomial12 /. x → k, {k, -8, 8}, PlotStyle → Red, PlotLegends → {"Approximant"}]}]
```



Test 3

```
polynomial2 = approximant[x ⅇ^x, 9];
Show[{Plot[x ⅇ^x, {x, -10, 10}, PlotLegends → {"xⅇ^x"}], Plot[polynomial2 /. x → k,
   {k, -8, 8}, PlotStyle → Red, PlotLegends → {"Approximant"}]}]
```