Telescopic method with Chebyshev interpolation

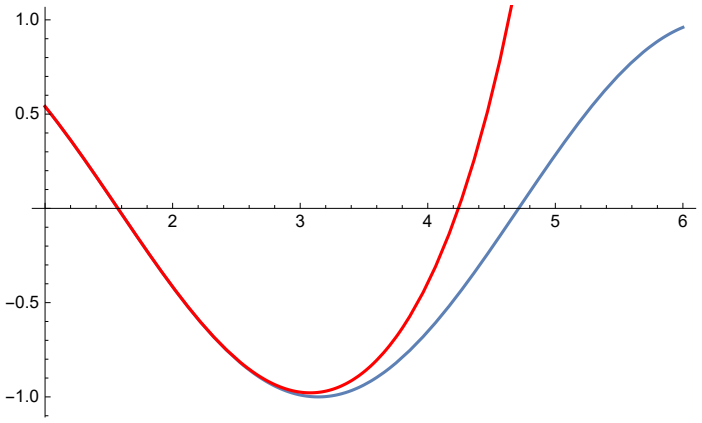
```
chebyshev1[0, x_] = 1;
chebyshev1[1, x_] = x;
chebyshev1[n_, x_] := (2 x chebyshev1[n - 1] - chebyshev1[n - 2])

interpolation[pol_, n_, a_, b_] := Module[{pol1 = pol},
  pol1 = pol - Coefficient[pol, x^n] * (b - a)^n / 2^(2 n-1) * chebyshev1[n, (2 x + a + b) / (b - a)];
  pol1]

telescopM[fun_, a_, b_, n_] :=
 Module[{pol = Normal@Series[fun, {x, 0, n}]}, Do[pol = interpolation[pol, n, a, b], n];
  pol]
```

Test1

```
pol1 = telescopM[Cos[x], 1, 6, 9] // N // Expand
```

$$1. - 0.5\, x^2 + 0.0416667\, x^4 - 0.00138889\, x^6 + 0.0000248016\, x^8$$

```
Show[{Plot[Cos[k], {k, 1, 6}], Plot[pol1 /. x → k, {k, 1, 6}, PlotStyle → Red]}]
```



Test2

```
pol2 = telescopM[Cos[x], 1, 6, 15] // N // Expand
```

$$1. - 0.5\, x^2 + 0.0416667\, x^4 - 0.00138889\, x^6 + 0.0000248016\, x^8 - 2.75573 \times 10^{-7}\, x^{10} + 2.08768 \times 10^{-9}\, x^{12} - 1.14707 \times 10^{-11}\, x^{14}$$

```
Show[{Plot[Cos[k], {k, 1, 6}], Plot[pol2 /. x → k, {k, 1, 6}, PlotStyle → Red]}]
```