Calculate Slope

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = 0/0 \text{ or undefined}$$

~~Introduction to Limits.~~

Limits: A limit describes the ~~future~~ value ~~of a~~ a function ~~at it~~ approaches when the ~~in~~ input variable to a function approaches a specific value. In our case, the input variable is x_2 and our function is (see above).

Calc for machine learning / Understandin Limits

→ Mission 07 Undefind Limit to defined limit

$$f(x) = -x^2 + 3x - 1$$

$$\lim_{x_2 \to 3} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$f(x_1) = -1$

$x_1 = +3$

~~expanch:~~
expand:

$$\lim_{x_2 \to 3} \frac{-(x_2)^2 + 3x_2 - 1 + 1}{x_2 - 3} \Rightarrow \frac{-x_2^2 + 3x_2}{x_2 - 3} \Rightarrow \frac{x_2(-x_2 + 3)}{x_2 - 3}$$

$$\Rightarrow \frac{-x_2 \cancel{(x_2 - 3)}}{\cancel{x_2 - 3}} \Rightarrow -x_2$$

$$\lim_{x_2 \to 3} -x_2 = -3$$

Sub chapter: Finding Extreme Points (maxima)

~~03 Differentiation~~

02 Introduction to Derivatives

Confusion: Derivative is defined as a function that can determine the slope of a tangent line for any x value along the function.

The widget is confusing as the green line is only tangent when $h = 0$. I guess my confusion is because the axes are not labeled.

03 Differentiation

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \quad f(x) = -(x)^2 + 3x - 1$$

$$\lim_{h \to 0} \frac{\left(-(x+h)^2 + 3(x+h) - 1\right) - \left(-(x)^2 + 3x - 1\right)}{h}$$

$$-(x^2 + 2xh + h^2) = -x^2 + -2xh - h^2$$

$$\lim_{h \to 0} \frac{\cancel{-x^2} - 2xh - h^2 + \cancel{3x} + 3h \cancel{-1} + \cancel{x^2} - \cancel{3x} \cancel{+1}}{h}$$

$$\lim_{h \to 0} \frac{-2xh - h^2 + 3h}{h} = -2x - h + 3$$

to simpilify this using direct subsitution do the following see green line above

$$f(x) = -2x - 0 + 3 = -2x + 3$$

or

$y = -2x + 3$ this is the derivative which is expressed as y ~~prim~~ prime or...

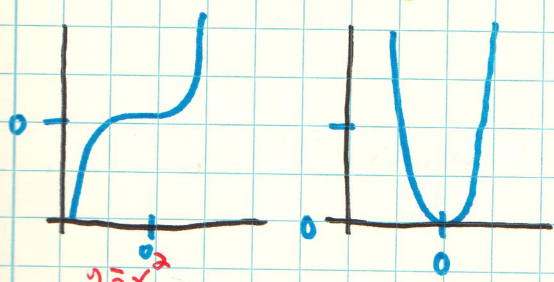
$y' = -2x + 3$ or $f'(x) = -2x + 3$

The last common notation is the following which can be read as "the derivative w/ respect to x is"

$$\frac{d}{dx} = \left[-x^2 + 3x - 1\right] = -2x + 3$$

04 Critical Points: a point on a curve where the gradient = 0

function: ~~y=x~~ $y = x^2$



slope of the tangent @ different x values

- Critical points are interesting in data science when they represent extreme values, which can be split into two categories: maximum & minimum.
    1) Maximums → when slope transitions from positive to negative.
    2) Minimum → When slope transitions from negative to positive
    3) When slope doesn't ~~ta~~ transition ~~...~~ signs ~~the~~ than it is not an extreme. See the graphs above.

06 Power Rule

$f'(x) = r x^{r-1}$

so...

1. $f(x) = x^2$, r would be 2 : & it derivative would be: $f'(x) = 2x$

2. $f(x) = -(x)^2 + 3x - 1$

   $f'(x) = -2x + 3$ wow so much easier

- - - - - - - -

The actual exercise, calculate the derivative

1. $f(x) = x^5$ $\quad f'(x) = 5x^4$

2. $f(x) = x^9$ $\quad f'(x) = 9x^8$

Next, look up the slope at $x = 2$ for (1) & $x = 0$ for (2)

1. $f'(2) = 5(2)^4 = 80$

2. $f'(0) = 9(0)^8 = 0$

07 Linearity of Differentiation

Consists of 2 rules:

1) The Sum Rule

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

2) The constant factor rule

$$\frac{d}{dx}[c\,f(x)] = c\frac{d}{dx}[f(x)]$$

example:

$$\frac{d}{dx}[-x^3+x^2] = \frac{d}{dx}[-x^3] + \frac{d}{dx}[x^2] = -3x^2+2x$$

The actual exercise...

Calculate the derivative of

$f(x) = x^5 - x$, the determine the slope when $x=1$

$\frac{d}{dx} = 5x^4 - 1$ $\quad f'(1) = 5(1)^4 - 1 = 4$

$f(x) = x^3 - x^2$, slope when $x=2$

$\frac{d}{dx} = 3x^2 - 2x$ $\quad f'(2) = 3(2)^2 - 2(2) = 8$

oops, I used the power rule & not the linearity of differentiation rule.

08 Practicing Finding Extreme Values

1. Find the critical points for $x^3 - x^2$
   To do this I'll need to determine the derivative, set it equal to 0 & solve for x.

$f(x) = x^3 - x^2 \quad \frac{d}{dx} = 3x^2 - 2x$

this is the easy part
I'm not sure how to solve for x, when there are more than 1 solution

$0 = 3x^2 - 2x$

ok, I'm just going to try some stuff here.

$3x^2 = 2x \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$

wholy shit, thats one solution...

divide both sides by x

the other solution is probably the easy one, 0

critical points = $\frac{2}{3}$ & 0

2. Determine if a critical point is a relative maximum or minimum
   Recall if slope goes from (+) to (−) its a max & vice versa if its a min

01 Overview of Linear Algebra

- Optimal Salary Problem

1. $y = 1000 + 30x$

2. $y = 100 + 50x$

base per/week	\$/hour
\$1000/wk	\$30/hr
\$100/wk	\$50/hr

y = the total earned each week

If we know how much we'd like to ~~earn~~ earn each week.

02 Solving Linear Systems By Elimination

The point where both functions intersect is called the solution to the system.

Using equations 1 & 2 above we can solve for x.

$$1000 + 30x = 100 + 50x \Rightarrow 1000 - 100 = 50x - 30x$$

$$900 = 20x$$

$$x = 45$$

The same can be done to solve for y.

$$y = 30(45) + 1000 = 2{,}350$$

$$y = 50(45) + 100 = 2{,}350$$

This illustrates that after 45 hours we will have earned the same amount of \$2,350.00

03 Representing Functions In General Form &
04 Representing An Augmented Array Matrix In NumPy

In linear algebra we usually represent linear functions in the general form.

$$Ax + By = c$$

In the general form, variables & their coefficients are on the left side & the constant term is on the ~~to~~ right. We can switch from point-slope form to the general form by rearranging the terms:

$$y = mx + b \Rightarrow mx - y = -b$$

function 1 in general form

$$30x - y = -1000$$

function 2 in general form

$$50x - y = -100$$

To represent both linear functions in a system we use an augmented matrix:

$$\left[\begin{array}{cc|c} 30 & -1 & -1000 \\ 50 & -1 & -100 \end{array}\right]$$

06 (Matrix) Row Operations

1) Any two rows can be swapped
(continue from previous matrix)

$$\xrightarrow{\text{Swap R1 \& R2}} \cancel{\begin{matrix} 30 & -1 & -1000 \\ 50 & -1 & -100 \end{matrix}} \left[\begin{array}{cc|c} 50 & -1 & -100 \\ 30 & -1 & -1000 \end{array}\right]$$

2) Any row can be multiplied
by a nonzero constant.

$$\left[\begin{array}{cc|c} 30 & -1 & -1000 \\ 50 & -1 & -100 \end{array}\right] \rightarrow \begin{matrix} 2 \cdot R1 \\ 3 \cdot R2 \end{matrix} \rightarrow \left[\begin{array}{cc|c} 60 & -2 & -2000 \\ 150 & -3 & -300 \end{array}\right]$$

3) Any row can be added to another row

$$\left[\begin{array}{cc|c} 30 & -1 & -1000 \\ 50 & -1 & -100 \end{array}\right] \rightarrow R2 = R2 + R1 \rightarrow \left[\begin{array}{cc|c} 30 & -1 & -1000 \\ 80 & -2 & -1100 \end{array}\right]$$

07 Simplifying Matrix To Echelon Form

Two Steps to find the solution of a matrix

1) Rearrange the matrix into echelon form. In this form the values on the diagonal locations are all equal to (1) and the values below the diagonal locations are all equal to (0).

$$\left[\begin{array}{cc|c} 1 & ? & ? \\ 0 & 1 & ? \end{array}\right]$$

First, divide R1 by 30 so that the diagonal value in the 1st row is 1:

$$\left[\begin{array}{cc|c} 30 & -1 & -1000 \\ 50 & -1 & -100 \end{array}\right] \xrightarrow{} R1 = R1/30 \xrightarrow{} \left[\begin{array}{cc|c} 1 & -\frac{1}{30} & -\frac{1000}{30} \\ 50 & -1 & -100 \end{array}\right]$$

Then, subtract 50 times the 1st row from the second:

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{30} & -\frac{1000}{30} \\ 50 & -1 & -100 \end{array}\right] \rightarrow R2 = R2 - (R1 \times 50)$$

$$\left(-50 + \frac{50}{30} \;\middle|\; \frac{50,000}{30}\right)$$

R1 × 50

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{30} & -\frac{1000}{30} \\ 0 & \frac{20}{30} & \frac{47,000}{30} \end{array}\right]$$

Next, multiply R2 by 1.5 to get the diagonal value in the second row to be (1).

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{30} & -\frac{1000}{30} \\ 0 & \frac{20}{30} & \frac{47,000}{30} \end{array}\right] \rightarrow R2 = R2 \cdot \frac{3}{2} \rightarrow \left[\begin{array}{cc|c} 1 & -\frac{1}{30} & -\frac{1000}{30} \\ 0 & 1 & \frac{141,000}{60} \end{array}\right]$$

To complete the transformation we'll have to zero out the second value in the first row. To do this lets ~~subtract R2 x~~ add R2 x $\frac{1}{30}$ to R1.

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{30} & -\frac{1000}{30} \\ 0 & 1 & \frac{141{,}000}{60} \end{array}\right] \rightarrow R1 = R1 + \underbrace{\left(R2 \times \frac{1}{30}\right)}_{0 \; + \; \frac{1}{30} \;|\; \frac{141{,}000}{1800}}$$

$$\left[\begin{array}{cc|c} 1+0 & & -\frac{2250}{30} \\ 0 & 1 & \frac{141{,}000}{60} \end{array}\right]$$

not sure my math is right here, let me run it through python

$$\rightarrow \left[\begin{array}{cc|c} 1 & 0 & -\frac{2250}{30} \\ 0 & 1 & \frac{2350}{30} \end{array}\right]$$

WITNESSED AND UNDERSTOOD

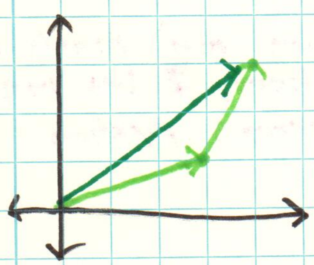
 DATE 03/13/2018 SUBJECT Math Refresher - Vectors PROJECT NO.

03 [redacted] Vector Operations

Vector Summation

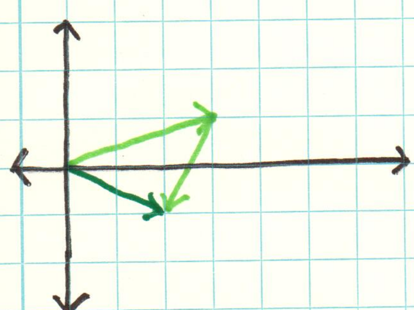
Stop putting the horizontal line in there!

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$



Vector Subtraction

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$



04 Scaling Vectors

$$3 \times \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

05 Vectors in NumPy

see code...

06 Dot Product

$$\vec{a} * \vec{b} = \sum_{i=1}^{n} a_i b_i = a_1 b_1 + a_2 b_2 + \ldots + a_n b_n$$

What this looks like visually

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = (1 \times 3) + (2 \times 0) + (1 \times 1) = 4$$

Unlike the other vector operations, the result of the ~~Scalar value~~ dot product is a scalar value not a vector.

To compute the dot product between 2 vectors, we need to use the numpy.dot() function. This function accepts ~~An~~ NumPy ndarray objects as the required parameters. The main quirk ~~on~~ is that one of the two vectors need to be represented as a row vector while the other a column vector.

~~Vector~~
07 Linear Combination

Using vectors we can determine if a certain vector can be obtained by combining other vectors. For example we may want to know if we can combine the vectors $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ to obtain $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$

Here's what it looks like mathematically:

$$c_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

Being able to scale vectors using scalar multiplication then adding or subtracting these scaled vectors is known as <u>linear combination</u>. This concept is ~~crux~~ crucial to being able to being able to apply linear algebra to solve useful problems.

if

$$w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

08 Linear Combination & Vectors

Circle back to the salary problem represented as a matrix.

$$\left[\begin{array}{cc|c} 30 & -1 & -1000 \\ 50 & -1 & -100 \end{array}\right]$$

We can now ~~tie~~ link this augmented matrix to the linear combination of vectors idea we just discussed.

We want to know if $\begin{bmatrix} -1000 \\ -100 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 30 \\ 50 \end{bmatrix}$ & $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$. To find the solution to this system we need to find the constants x & y where the following equation is true.

$$x \begin{bmatrix} 30 \\ 50 \end{bmatrix} + y \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1000 \\ -100 \end{bmatrix}$$

In the last mission, we solved the augmented ~~matrix~~ matrix by using row operations to obtain the following form.

$$\left[\begin{array}{cc|c} 1 & 0 & x \\ 0 & 1 & y \end{array}\right]$$

09 The Matrix Equation

The matrix equation is the representation of a linear system using only matrices & vectors
Here's the augmented matrix we started out with:

$$\left[\begin{array}{cc|c} 30 & -1 & -1000 \\ 50 & -1 & -100 \end{array}\right]$$

This augmented matrix is the short hand ~~notation~~ representation for the matrix equation:

$$\begin{bmatrix} 30 & -1 \\ 50 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1000 \\ -100 \end{bmatrix}$$

On the left side were multiplying a matrix containing the coefficients w/ the vector containing the variables. The right side ~~were~~ contains the constant values. This separation of coefficients & variables from the constants should be familiar. This is exactly ~~the same~~ what we did in the general form as well.

It's common practice to use $x_1, x_2, \ldots x_n$ instead of x & y to represent the individual values in the solution vector.

$$\begin{bmatrix} 30 & -1 \\ 50 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1000 \\ -100 \end{bmatrix}$$

This allows us to work with vectors w/ any number of elements (instead of just 26 for the number of letters in the english dictionary). We can now introduce the arithmetic representation of the matrix equation:

$$A\vec{x} = \vec{b}$$

Where A represents the coefficient matrix, $\vec{x}$ represents the solution vector, and $\vec{b}$ represents the constants. Note that $\vec{b}$ can't be a vector containing all zero's also known as the zero factor & represented using $\vec{0}$.

DATE SUBJECT PROJECT NO.

Before we can work with this form of the system we need to learn about the following topics in the following mission.

1. The rules that ~~describe~~ describe how matrices can be combined

2. How to multiply a matrix w/ a vector

3. How to calculate the solution vector X w/o using Gaussian elimination.