Algorithms: COMP3121/3821/9101/9801

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TOPIC 4: THE GREEDY METHOD

Activity selection problem

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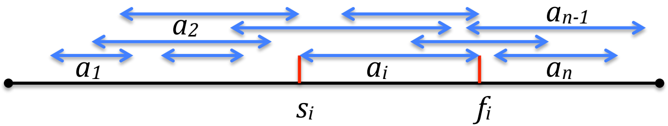

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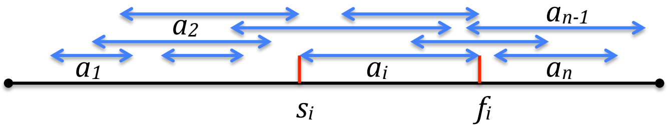

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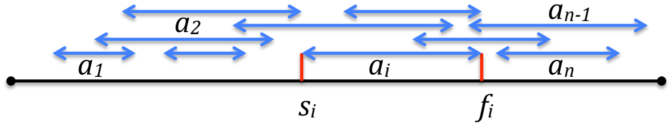

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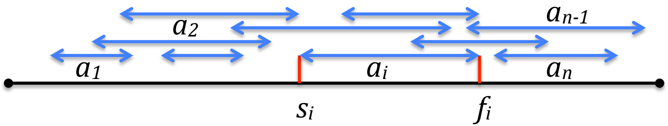

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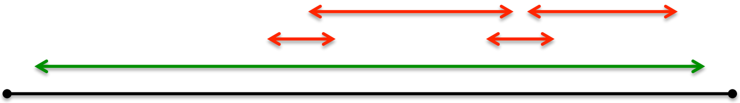

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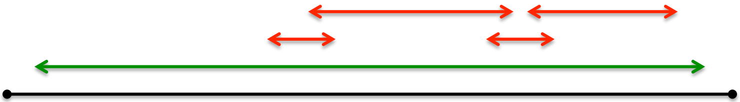

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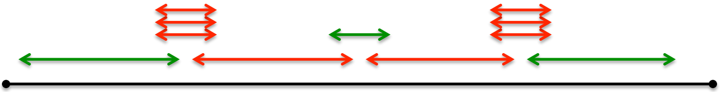

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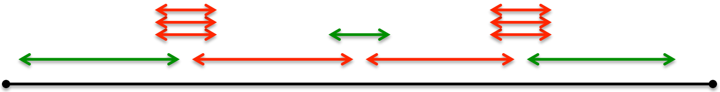

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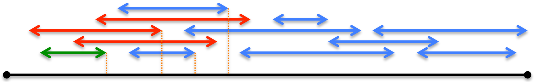

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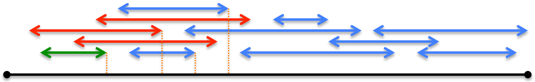

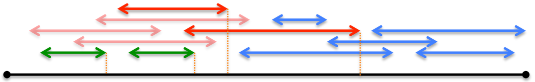

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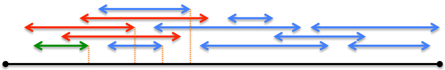

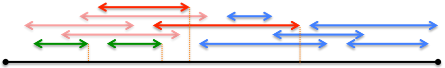

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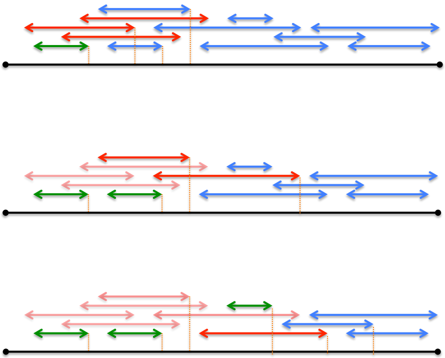

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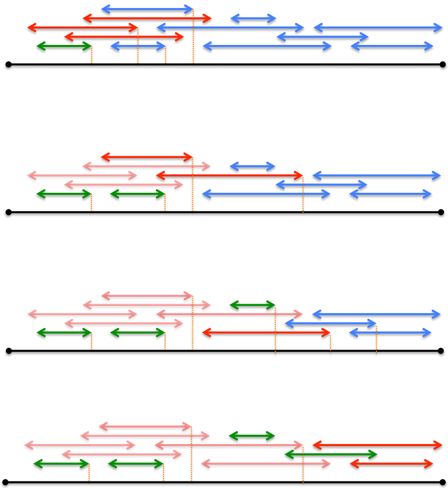
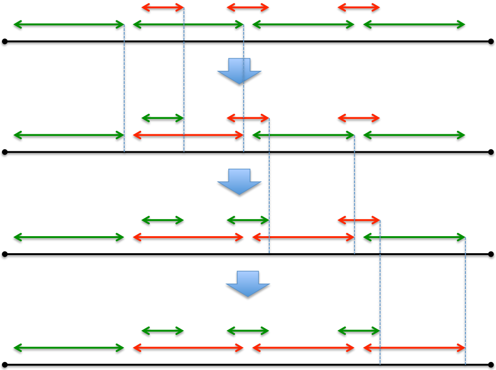


The Greedy Method

- Transforming any optimal solution to the greedy solution with equal number of activities: find the first place where the chosen activity violates the greedy choice and show that replacing that activity with the greedy choice produces a non conflicting selection with the same number of activities. Continue in this manner till you "morph" your optimal solution into the greedy solution, thus proving the greedy solution is also optimal.



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- Thus, the algorithm runs in time $O(n \log n)$.
- Think about the details how you would implement this algorithm efficiently.

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- **Instance:** A list of activities a_i, $(1 \leq i \leq n)$ with starting times s_i and finishing times $f_i = s_i + d$; thus, all activities are of the same duration. No two activities can take place simultaneously.

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- Greedy strategy no longer works - we will need a more sophisticated technique.

Minimising job lateness

- **Instance:** A start time T_0 and a list of jobs a_i, $(1 \leq i \leq n)$, with duration times t_i and deadlines d_i. Only one job can be performed at any time; all jobs have to be completed. If a job a_i is completed at a finishing time $f_i > d_i$ then we say that it has incurred lateness $l_i = f_i - d_i$.

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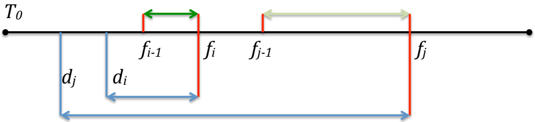
- **Optimality:** Consider any optimal solution. We say that jobs a_i and jobs a_j form an inversion if job a_i is scheduled before job a_j but $d_j < d_i$.



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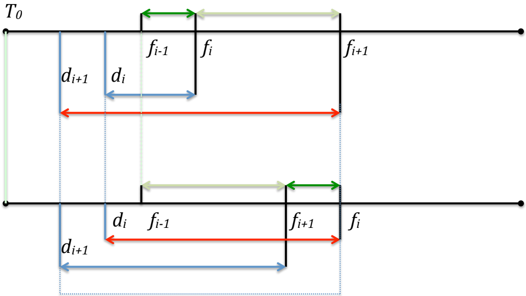

- Note that swapping adjacent inverted jobs reduces the larger lateness!

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- **Proof of optimality:** Let d be the distance associated with the first edge of the minimal spanning tree which was not added to our k trees; it is clearly the minimal distance between two vertices belonging to two of our k trees. Clearly, all the edges included in k many trees produced by our algorithm are of length smaller or equal to d.

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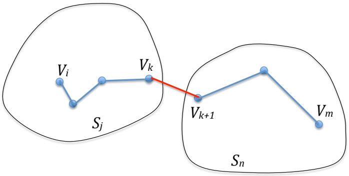
- Consider any partition $\mathcal{S}$ into k subsets different from the one produced by our algorithm. This means that there is a tree produced by our algorithm which contains vertices V_i and V_m from two different subsets from $\mathcal{S}$, say S_j and S_n.



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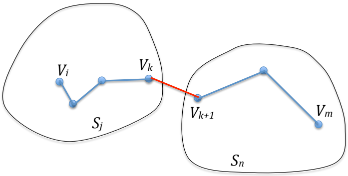

- Since V_i and V_m belong to the same tree, there is a path along the edges of that tree connecting V_i and V_m. Let V_k and V_{k+1} be two consecutive vertices on that path such that V_k belongs to S_j and V_{k+1} belongs to S_n. Note that $d(V_k, V_{k+1}) \leq d$ which implies that the minimal distance between clusters of $\mathcal{S}$ is smaller or equal to the minimal distance d between the k trees produced by our algorithm. Thus, such a partition cannot be a more optimal clustering than the one produced by our algorithm.

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- Thus, in total running the (partial) Kruskal algorithm also takes $O(n^2 \log n^2) = O(n^2 \log n)$.
- So the grand total for the whole algorithm is $O(n^2 \log n)$ many steps.

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- There is a line of 111 stalls, some of which need to be covered with boards. You can use up to 11 boards, each of which may cover any number of consecutive stalls. Cover all the necessary stalls, while covering as few total stalls as possible.

Tape storrage

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- This is minimised if $l_1 \leq l_2 \leq l_3 \leq \ldots \leq l_n$.

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Tape storrage II

- **Instance:** A list of n files f_i of lengths l_i and probabilities to be needed p_i, $\sum_{i=1}^{n} p_i = 1$, which have to be stored on a tape. To retrieve a file, one must start from the beginning of the tape and scan it until the tape is fund and read.
- **Task:** Order the files on the tape so that the expected retrieval time is minimised.
- **Solution:** If the files are stored in order $l_1, l_2, \ldots l_n$, then the expected time is proportional to

$$p_1 l_1 + (l_1 + l_2)p_2 + (l_1 + l_2 + l_3)p_3 + \ldots + (l_1 + l_2 + l_3 + \ldots + l_n)p_n$$

- We now show that this is minimised if the files are ordered in a decreasing order of values of the ratio p_i/l_i.

The Greedy Method

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and

$$\begin{aligned} E' =& p_1 l_1 + (l_1 + l_2)p_2 + (l_1 + l_2 + l_3)p_3 + (l_1 + l_2 + l_3 + \ldots + l_{k-1} + l_{k+1})p_{k+1} + \ldots \\ &+ (l_1 + l_2 + l_3 + \ldots + l_{k-1} + l_{k+1} + l_k)p_k + \ldots + (l_1 + l_2 + l_3 + \ldots + l_n)p_n \end{aligned}$$

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- Thus, $E - E' = l_k p_{k+1} - l_{k+1} p_k$, which is positive just in case $l_k p_{k+1} > l_{k+1} p_k$, i.e., if $p_k / l_k < p_{k+1} / l_{k+1}$.

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- Consequently, $E > E'$ if and only if $p_k / l_k < p_{k+1} / l_{k+1}$, which means that the swap decreases the expected time just in case $p_k / l_k < p_{k+1} / l_{k+1}$, i.e., if there is an inversion: a file f_{i+1} with a larger ratio p_{k+1} / l_{k+1} has been put after a file f_i with a smaller ratio p_k / l_k.

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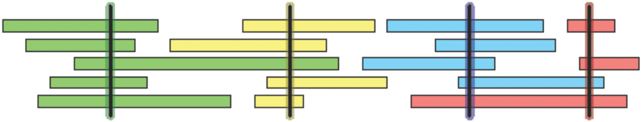
- For as long as there are inversions there will be inversions of consecutive files and swapping will reduce the expected time. Consequently, the optimal solution is the one with no inversions.

The Greedy Method

- Let X be a set of n intervals on the real line. We say that a set P of points stabs X if every interval in X contains at least one point in P; see the figure below. Describe and analyse an efficient algorithm to compute the smallest set of points that stabs X. Assume that your input consists of two arrays $X_L[1..n]$ and $X_R[1..n]$, representing the left and right endpoints of the intervals in X.



A set of intervals stabbed by four points (shown here as vertical segments)

- Assume you are given n sorted arrays of different sizes. You are allowed to merge any two arrays into a single new sorted array and proceed in this manner until only one array is left. Design an algorithm that achieves this task and uses minimal total number of moves of elements of the arrays. Give an informal justification why your algorithm is optimal.

- Along the long, straight road from Loololong to Goolagong houses are scattered quite sparsely, sometimes with long gaps between two consecutive houses. Telstra must provide mobile phone service to people who live alongside the road, and the range of Telstras cell base station is 5km. Design an algorithm for placing the minimal number of base stations alongside the road, that is sufficient to cover all houses.

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- You are given a connected graph with weighted edges. Find a spanning tree such that the largest weight of all of its edges is as small as possible.

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- Design an algorithm which produces a minimum spanning tre T' for the new graph containing the additional vertex v_{n+1} and which runs in time $O(n \log n)$.

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- Once again, along the long, straight road from Loololong (on the West) to Goolagong (on the East) houses are scattered quite sparsely, sometimes with long gaps between two consecutive houses. Telstra must provide mobile phone service to people who live alongside the road, and the range of Telstras cell base station is 5km.

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- Is there a placement of houses for which the associate is right?

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1. Give examples which show that neither Algorithm 1 nor Algorithm 2 solve the problem correctly.
2. Design an algorithm which correctly solves the problem.

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- Assume you have \$2, \$1, 50c, 20c, 10c and 5c coins to pay for your lunch. Design an algorithm that, given the amount that is a multiple of 5c, pays it with a minimal number of coins.

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- Assume denominations of your $n+1$ coins are $1, c, c^2, c^3, \ldots, c^n$ for some integer $c > 1$. Design a greedy algorithm which, given any amount, pays it with a minimal number of coins.

- Give an example of a set of denominations containing the single cent coin for which the greedy algorithm does not always produce an optimal solution.

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- One way of insuring unique readability of codes from a single bitstream is to ensure that no code of a symbol is a prefix of a code for another symbol.
- Codes with such property are called *the prefix codes.*

The Greedy Method - Huffman code

- We can now formulate the problem:

  Given the frequencies (probabilities of occurrences) of each symbol, design an optimal prefix code, i.e., a prefix code such that the expected length of an encoded text is as small as possible.

- Note that this amounts to saying that the *average* number of bits per symbol in an "average" text is as small as possible.

- See the textbook for details; from now on, we will do things on the blackboard which is camera recorded and available on the YouTube.

0-1 knapsack problem

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