Department of Physics and Astronomy
University of Heidelberg

Bachelor Thesis in Physics
submitted by

Nils Fischer

born in Hannover, Germany

handed in on

July 22, 2015

Gravitational Waves in Modified Gravity

This Bachelor Thesis has been carried out by Nils Fischer at the
Institute for Theoretical Physics in **Heidelberg**
under the supervision of
Dr. Valeria Pettorino & **Prof. Dr. Luca Amendola**

Gravitational Waves in Modified Gravity

Nils Fischer

Abstract   Modified gravity theories generally aim to solve part of the *cosmological constant problem* by providing self-accelerating cosmological solutions without a cosmological constant. Such modifications of general relativity also affect the evolution of gravitational waves in the proposed theory. Instead of focussing on an explicit model, I introduce parametric modifications to the evolution equation of gravitational waves in both unimetric and bimetric settings and investigate their effect on the evolution of tensor perturbation modes. In particular, I argue that any modified gravity theory that exhibits *growing tensor modes* in cosmological evolution can be in tension with experiments. Therefore, parametric constraints for the physical viability of a general modified gravity theory can be found such that tensor modes remain within limits set by observations.

Zusammenfassung   Viele Gravitationstheorien, die über die Allgemeine Relativitätstheorie hinausgehen, werden entwickelt um die beschleunigte Expansion unseres Universums ohne eine kosmologische Konstante zu erklären. Solche Veränderungen betreffen im Allgemeinen auch das Verhalten von Gravitationswellen in der vorgeschlagenen Theorie. Anstatt ein explizites Modell anzunehmen führe ich parametrische Anpassungen in der Gravitationswellengleichung ein und untersuche ihre Auswirkungen auf Tensorstörungen sowohl in unimetrischen als auch bimetrischen Gravitationstheorien. Insbesondere argumentiere ich, dass Modelle mit im Zeitverlauf des Universums anwachsenden Tensormoden in Konflikt mit experimentellen Beobachtungen stehen können. Somit können parametrische Einschränkungen für die physikalische Viabilität von Gravitationstheorien gefunden werden, sodass Tensormoden innerhalb der experimentellen Grenzen verbleiben.

Contents

List of Figures

1. Introduction

For a long time, physicists believed space and time were both flat and fundamentally different concepts. In classical *Newtonian spacetime*, it is therefore straight forward to define notions such as the *length of a curve* and *simultaneity* [53].

With Maxwell's theory of electromagnetism and experimental observations regarding the speed of light, however, many of these concepts had to be abandoned. Physicists realized the necessity to reconsider fundamental assumptions about space and time. Albert Einstein addressed many of these issues with his theory of special relativity in 1905 [21]. But particularly one striking observation remained unexplained:

Newtonian mechanics postulates that particles accelerate under the influence of any force $\boldsymbol{F}$ proportional to their *inertial mass m_i* with [36]

$$m_i \cdot \boldsymbol{a} = \boldsymbol{F} \quad \text{where } \boldsymbol{a} \equiv \frac{\mathrm{d}^2 \boldsymbol{x}}{\mathrm{d}t^2} \text{ denotes acceleration.} \tag{1.1}$$

Simultaneously, the gravitational force

$$|\boldsymbol{F}_{\mathrm{G}}| = \mathrm{G_N} \frac{m_{g,1} m_{g,2}}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|^2} \quad \text{with the } \textit{gravitational constant } \mathrm{G_N} \tag{1.2}$$

two particles $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ induce on each other is proportional to their *gravitational mass m_g* that is completely unrelated to the inertial mass in this theory. Strikingly, however, experiments find inertial and gravitational mass indistinguishable from another such that physicists identify both as *mass $m = m_i = m_g$* instead. This *weak equivalence principle* has the well-known implication that any two objects will fall with the same velocity in vacuum, regardless of their weight, and has been precisely tested to orders of 10^{-13} [2, 55].

The weak equivalence principle constitutes the basis of Einstein's 1915 theory of *general relativity* where gravity is not regarded as an external force but rather a phenomenon of the geometry of spacetime itself, that is curved by its matter content [19].

Therefore, the trajectories of freely moving particles appear affected by a gravitational force although they actually follow straight lines in curved spacetime, known as *geodesics*. In other words, gravity is only an inertial force observed in a cartesian reference frame that is induced by the curvature of spacetime in the presence of energy.

General relativity not only succeeded in explaining astronomical phenomena that stood in conflict with Newtonian gravity, such as the perihelion shift of Mercury and gravitational lensing [18], but also enabled the development of many high-precision technologies. Among them, navigational systems such as GPS are arguably the most prominent. Most remarkably, however, it allowed physicists to scientifically approach questions that concern the evolution of the entire universe for the first time in the field of *cosmology*. Assuming only that, at sufficiently large scales, our universe looks the same everywhere and in all directions, notions such as the *beginning* and *age of the universe* became computable in cosmological models that could be compared to experimental data. For instance, distance measurements of galaxies and stars provided evidence of a recent accelerated expansion of our universe [39]. One of the most important cosmological observations remains the high precision measurements of the *cosmic microwave background* [38, 44], that is thermal radiation originating in the early universe. These discoveries played a crucial role in the development of the ΛCDM (Lambda Cold Dark Matter) standard model of cosmology. In this model, our universe went through a phase of exponential expansion, known as *cosmic inflation*, at very early times, followed by several cosmological eras where each is dominated by one of the different matter species in the universe. These include radiation, baryonic matter and the elusive *dark matter* and *dark energy*.

Gravitational waves are an intrinsic part of general relativity and the standard model of cosmology. They occur when the geometry of spacetime is perturbed in a tensorial manner. For instance, binary systems of stellar objects such as stars or black holes of different mass orbiting each other generate such perturbations that propagate through spacetime. Gravitational waves that originated in the early universe and could be measured in the cosmic microwave background are known as *primordial gravitational waves*. This thesis begins with a detailed discussion of gravitational waves in general relativity in **chapter 2**. Since attempts to measure gravitational waves, either directly by dedicated ground-based experiments or indirectly by precise measurements of the cosmic microwave background, have not been successful yet, their amplitude is constrained to be extremely small by the data at hand [30, 44].

Although the ΛCDM standard model of cosmology agrees with observational data remarkably well, it does not offer an entirely conclusive model of our universe. One of the shortcomings this model exhibits is related to its dark energy component that provides the accelerated expansion of our universe at recent times. It is given by a free parameter Λ, known as the *cosmological constant* in this theory. In fact, the *cosmological constant problem* connects unsolved phenomena of both quantum field theory and general relativity and remains one of the greatest mysteries in theoretical physics of our time. It is formulated in **chapter 3**, which also introduces the concept of modified theories of gravity as an approach for solving part of it. Such theories generally propose alterations to general relativity that have self-accelerating late-time cosmological solutions without relying on a cosmological constant.

Any theory of gravity that goes beyond general relativity must also predict gravitational waves that agree with the experimental constraints mentioned above. In particular, modified gravity theories where gravitational waves grow in amplitude in cosmological evolution can easily be in tension with experiments. The main idea of this thesis is to find properties of modified gravity theories such that growing gravitational waves, that could render the theory physically unviable, are avoided. To achieve this, I introduce appropriate parameters in the evolution equation of gravitational waves that can result from general modifications of gravity. I will explore the effect these parametric modifications have on the evolution of gravitational waves in modified gravity theories in **chapter 4**. In addition to numerical solutions, I will solve the parametrized evolution equation analytically for appropriate assumptions to find constraints for the parameters such that the theory does not exhibit gravitational waves that grow in amplitude. Subsequently in **chapter 5**, I will focus this parametric approach on the extended class of *bimetric* models where a second space-time metric in our universe interacts with the gravity-inducing physical metric and both metrics can exhibit gravitational waves. Finally, **chapter 6** summarizes the results of this thesis.

Throughout the thesis, I choose natural units with the speed of light $c \equiv 1$ and a mostly-positive metric signature $(-1, 1, 1, 1)$. Unless otherwise noted, repeated indices in a product are automatically summed on, where greek letters run from zero to three and latin letters run from one to three.

2. Gravitational Waves in General Relativity

This chapter first gives a brief introduction to general relativity and standard FLRW cosmology that focusses on the results required for a discussion of gravitational waves. Then, the evolution equation of gravitational waves, that will be the main object of consideration for the remainder of this thesis, is derived from metric tensor perturbations in an FLRW universe. Since the main idea of the thesis is to find constraints for modified gravity theories such that tensor perturbations remain within experimental bounds, an overview on the detection of gravitational waves follows. In this context, also the sensitivity of current detectors and measurements of the cosmic microwave background are discussed that give observational limits on the amplitude of gravitational waves.

2.1. The Spacetime Metric in General Relativity

In general relativity, the *Einstein equations*

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi \mathrm{G_N} T_{\mu\nu} \tag{2.1}$$

$$\text{with the } \textit{Einstein Tensor} \quad G_{\mu\nu} = R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \mathcal{R}, \tag{2.2}$$

$$\text{the } \textit{Ricci Scalar} \quad \mathcal{R} = g^{\mu\nu} R_{\mu\nu}, \tag{2.3}$$

$$\text{the } \textit{Ricci Tensor} \quad R_{\mu\nu} = R^{\alpha}_{\alpha\mu\nu}, \tag{2.4}$$

$$\text{the } \textit{Riemann Tensor} \quad R^{\alpha}_{\beta\mu\nu} = \frac{\partial \Gamma^{\alpha}_{\beta\nu}}{\partial x^{\mu}} - \frac{\partial \Gamma^{\alpha}_{\beta\mu}}{\partial x^{\nu}} + \Gamma^{\alpha}_{\gamma\mu} \Gamma^{\gamma}_{\beta\nu} + \Gamma^{\alpha}_{\gamma\nu} \Gamma^{\gamma}_{\beta\mu} \tag{2.5}$$

$$\text{and the } \textit{Christoffel Symbols} \quad \Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(\frac{\partial g_{\mu\beta}}{\partial x^{\gamma}} + \frac{\partial g_{\mu\gamma}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\mu}} \right) \tag{2.6}$$

relate the geometry of spacetime, encoded in the metric tensor $g_{\mu\nu}$, to the energy content of the universe $T_{\mu\nu}$. The *cosmological constant* Λ is a free parameter in this theory and is discussed in detail in section 3.1.

We can obtain the Einstein equations through variation of the *Einstein-Hilbert action*

$$S[g] = \int \mathrm{d}^4 x \sqrt{-|g|} \left(\mathcal{R} - 2\Lambda \right) + \int \mathrm{d}^4 x \sqrt{-|g|} \mathcal{L}_{\mathrm{matter}}(g, \Phi) \quad \text{with} \quad |g| = \det g \quad (2.7)$$

with respect to $g_{\mu\nu}$. The matter Lagrangian denoted by $\mathcal{L}_{\mathrm{matter}}(g, \Phi)$, where Φ represents all matter fields, is only *minimally coupled* to gravity through the measure $\sqrt{-|g|}\, \mathrm{d}^4 x$ that arises from the spacetime metric.

Given a stress-energy tensor $T_{\mu\nu}$, the Einstein equations constitute ten highly non-linear partial differential equations for the spacetime metric g, since it is a symmetric, four-dimensional tensor. The metric, in turn, restores the notion of the length of a curve in spacetime and thus allows us to formulate postulates for the dynamics of matter in the universe. In particular, as a generalization of Newton's law, particles without any external forces acted upon will move on *geodesics* in spacetime where their curve γ is stationary with respect to the length functional $L[\gamma]$ such that

$$\delta L[\gamma] = 0. \tag{2.8}$$

Many solutions to the Einstein equations have been found for particular choices of matter distributions given by $T_{\mu\nu}$. To make an exact solution possible, they often assume strong symmetries. The metric tensor is usually given in some choice of coordinates x^μ in its matrix notation or as line element

$$\mathrm{d}s^2 = g_{\mu\nu}\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu. \tag{2.9}$$

For instance, a trivial vacuum solution to the Einstein equations is the flat *Minkowskian* spacetime metric of special relativity

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{bmatrix} -1 & 0 \\ 0 & \mathbb{1}_3 \end{bmatrix}_{\mu\nu} \quad \text{or} \quad \mathrm{d}s^2 = -\,\mathrm{d}t^2 + \delta_{ij}\, \mathrm{d}x^i\, \mathrm{d}x^j \tag{2.10}$$

in cartesian coordinates.

2.2. Standard FLRW Cosmology

When we strive to understand how the entire universe evolves, we can impose a number of symmetry conditions that are based on the *cosmological principle*. The pos-

tulate states that the universe is *spatially homogeneous and isotropic* at the largest scales since there is no apparent reason to believe otherwise [1]. This gives rise to six spatial symmetries and leads to the *Friedmann-Lemaître-Robertson-Walker (FLRW)* metric

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2 \gamma_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j \tag{2.11a}$$

$$\text{where} \quad \gamma_{ij}(r,\theta,\phi) = \begin{bmatrix} \frac{1}{1-\kappa r^2} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2(\theta) \end{bmatrix}_{ij} \tag{2.11b}$$

that is a particular solution to the Einstein equations describing a smooth, expanding universe [17, 49]. The *scale factor* $a(t)$ is the only freedom left after considering the symmetries and scales the time-independent metric γ_{ij} of a spatial, three-dimensional subspace with constant curvature κ.

It is important to note that distances described by the FLRW coordinates are merely *coordinate distances* or *comoving distances* that remain constant even in an expanding universe. Comoving distances are scaled by the time-dependent scale factor $a(t)$ to obtain the physical distances. Similarly, one can define the *comoving* or *conformal time*

$$\eta = \int_0^t \frac{\mathrm{d}t'}{a(t')} \tag{2.12}$$

as the comoving distance light (with $\mathrm{d}s^2 = 0$) could have traveled since $t = 0$. The conformal time thus defines the causal structure in comoving coordinates and is also called the *comoving horizon*. It is often convenient to parametrize the evolution of the universe in conformal time η instead of cosmic time t. Another parametrization we will use are *e-foldings* $\ln a$. For the remainder of this thesis, derivatives by conformal time and e-foldings will be denoted by dots and primes, respectively, as in

$$\frac{\mathrm{d}}{\mathrm{d}\eta} \equiv \dot{} \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}\ln a} \equiv {}'. \tag{2.13}$$

Given the particular form of the spacetime metric, the Einstein tensor (2.2) can be explicitly computed. When we also assume the universe is filled with matter that, at large scales, resembles a perfect fluid with stress-energy tensor

$$T^{\mu\nu} = (\rho + p)\,u^\mu u^\nu + pg^{\mu\nu} \quad \text{in time direction} \quad u^\mu = (1, 0, 0, 0)^T \tag{2.14}$$

---

[1]Of course, the cosmological principle can be questioned, and has been. See for example Schwarz et al. [51] and references therein.

with *density ρ, pressure p* and linear *equation of state*

$$p = w\rho \quad \text{with } \textit{equation of state parameter } w, \tag{2.15}$$

the Einstein equations (2.1) reduce to two ordinary, coupled differential equations [17, 49]

$$\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} = -\frac{4\pi}{3} G_N \left(\rho + 3p\right) a + \frac{\Lambda}{3} \tag{2.16a}$$
acceleration equation

$$H^2 = \frac{8\pi}{3} G_N \rho - \frac{\kappa}{a^2} + \frac{\Lambda}{3} \tag{2.16b}$$
Friedmann equation

with the *Hubble function*

$$H \equiv \frac{\mathrm{d}a}{\mathrm{d}t} \frac{1}{a} \quad \text{or} \quad \mathcal{H} \equiv \frac{\dot{a}}{a} = aH \quad \text{in conformal time } \eta. \tag{2.17}$$

The equation of state parameter w for various fluid species is summarized in Table 2.1. For a universe filled with any such fluid, we can solve the Friedmann equations (2.16) to find both the time evolution of the density

$$\rho \propto a^{-n(w)} \tag{2.18}$$

and of the scale factor

$$a(t) \propto \begin{cases} t^{\frac{2}{n(w)}} & \text{for } w \neq -1 \\ e^{Ht} & \text{for } w = -1 \end{cases} \quad \text{with} \quad n(w) := 3\left(1 + w\right). \tag{2.19}$$

This result is derived in Appendix A.1 and already exhibits a wealth of cosmological implications. In a universe filled with fluid of $w \neq -1$ we find $H \propto \frac{1}{t}$, for example, and can therefore identify it with a notion of the *age of the universe*. Furthermore, the relation $\rho \propto t^{-2}$ that follows from (2.18) in such a universe implies a singularity at $t = 0$ where the density becomes infinite. This is called the *big bang*.

Multiple non-interacting fluid species in the universe combine to the total density

$$\rho(t) = \sum_i \rho_i(t) \quad \text{with equation of state} \quad p_i = w_i \rho_i \quad \text{each,} \tag{2.20}$$

where we can include the effect of a spatial curvature κ and a cosmological constant

	i	w	$n(w)$	$\tilde{n}(w)$
radiation	γ	$1/3$	4	1
dust	m	0	3	2
cosmological constant	Λ	-1	0	-1
spatial curvature	κ	$-1/3$	-2	∞

Table 2.1.: **Overview of cosmological properties for an FLRW universe filled with various fluid species**   Radiation (relativistic particles) and dust (non-interacting matter) are also denoted as *ordinary matter*.

Λ through the definition of additional effective densities

$$\rho_\kappa(t) = -\frac{3}{8\pi G_N}\frac{\kappa}{a^2} \quad \text{and} \quad \rho_\Lambda(t) = \frac{\Lambda}{8\pi G_N}. \tag{2.21}$$

When we then define the dimensionless *density parameters*

$$\Omega_i(t) := \frac{\rho_i(t)}{\rho_{\text{crit}}} \quad \text{with} \quad \rho_{\text{crit}} := \frac{3H_0^2}{8\pi G_N} \tag{2.22}$$

where H_0 denotes the value of the Hubble function today at cosmic time $t = t_0$, the Friedmann equation (2.16b) becomes

$$\frac{H^2}{H_0^2} = \sum_i \Omega_i(t) \tag{2.23}$$

and we find from (2.18) the relation

$$\Omega_\Lambda \propto a^2\Omega_\kappa \propto a^3\Omega_m \propto a^4\Omega_\gamma. \tag{2.24}$$

Because the universe expands monotonically with time for any of these fluid species, as given by (2.19), this result allows us to consider successive *cosmological regimes* or *eras* in an FLRW universe with a dominant species each. Radiation dominates in the early universe and is followed by a regime of dust domination, also known as *matter domination*. The spatial curvature κ is measured to be zero very precisely today and is therefore assumed to vanish in the following. At recent times, the universe is in a regime dominated by a cosmological constant known as *de Sitter space* that is discussed in more detail in section 3.1.

2.3. Tensor Perturbations in an FLRW Universe

At smaller scales, the universe is not homogeneous and isotropic at all, of course. Galaxies, stars and planets, as well as radiation or, in fact, any energy content of the universe perturb the spacetime metric locally. It is therefore insightful to consider perturbations δg around the smooth FLRW metric and their evolution.

When we assume an exact solution g to the unperturbed Einstein equations and consider a sufficiently small perturbation to the stress-energy tensor $\delta T_{\mu\nu}$, then the metric perturbation δg that solves

$$G_{\mu\nu}[g + \delta g] = T_{\mu\nu}[g] + \delta T_{\mu\nu}[g] \quad \text{for} \quad 8\pi G_N \equiv 1 \tag{2.25}$$

will also be small and we obtain

$$\delta G_{\mu\nu}[g, \delta g] = \delta T_{\mu\nu}[g] \tag{2.26}$$

with $\delta G_{\mu\nu}[g, \delta g]$ linear in δg in linear perturbation theory [49].

Because of its symmetry condition, δg has 10 degrees of freedom that we can parametrize as

$$\delta g = -2A \, \mathrm{d}x^0 \otimes \mathrm{d}x^0 + B_i \left(\mathrm{d}x^0 \otimes \mathrm{d}x^i + \mathrm{d}x^i \otimes \mathrm{d}x^0 \right) + \left(2C\gamma_{ij} + 2E_{ij} \right) \mathrm{d}x^i \otimes \mathrm{d}x^j \tag{2.27}$$

for small spatial *scalar fields* $A(x^0)$ and $C(x^0)$, a *vector field* $B_i(x^0)$ and a symmetric, traceless *tensor field* $E_{ij}(x^0)$ [49].

As of the *Helmholtz theorem*, the parameters uniquely decompose further into scalar, vector and tensor components as [17, 57]

$$\delta g = \delta g^{\text{scalar}} + \delta g^{\text{vector}} + \delta g^{\text{tensor}}. \tag{2.28}$$

with their explicit form given in Appendix A.3. This allows us to study scalar, vector and tensor perturbations separately in linear perturbation theory.

With the tensor field E_{ij} decomposed, the remaining tensor perturbation component

$$\delta g_{ij}^{\text{tensor}} = h_{ij} \tag{2.29}$$

is a *symmetric, traceless and divergence-free tensor field* that satisfies

$$h_{ij} = h_{ji}, \quad \gamma^{ij} h_{ij} = 0 \quad \text{and} \quad k^i h_{ij} = 0 \tag{2.30}$$

where $\boldsymbol{k}$ denotes the *wave vector* in Fourier space. It can therefore be expressed in terms of two functions $h_\times$ and h_+ as

$$h_{ij} = \begin{bmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{bmatrix}_{ij} \tag{2.31}$$

with an implicit choice of the z-axis in direction of the wave vector [17]. This choice of coordinates is also known as *TT-gauge* or *de Donder gauge*.

With this form of the metric we can compute the Einstein tensor perturbation in (2.26) and obtain [17]

$$\delta G_{ij} = \frac{3a^2}{2} H \frac{\mathrm{d} h_{ij}}{\mathrm{d} t} + \frac{a^2}{2} \frac{\mathrm{d}^2 h_{ij}}{\mathrm{d} t^2} + \frac{k^2}{2} h_{ij}. \tag{2.32}$$

The perturbed Einstein equations (2.26) that govern the evolution of the metric tensor perturbations thus reduce to a wave equation

$$\frac{3a^2}{2} H \frac{\mathrm{d} h_{ij}}{\mathrm{d} t} + \frac{a^2}{2} \frac{\mathrm{d}^2 h_{ij}}{\mathrm{d} t^2} + \frac{k^2}{2} h_{ij} = 8\pi G_N \Sigma_{ij} \tag{2.33}$$

that is sourced by the tensor component of the stress-energy perturbation δT, known as *anisotropic stress* Σ.

Its solutions are called *gravitational waves* and occur in two independent *polarizations* $h_\times$ and h_+ that each satisfy the free *evolution equation of gravitational waves*

$$\frac{\mathrm{d}^2 h}{\mathrm{d} t^2} + 3H \frac{\mathrm{d} h}{\mathrm{d} t} + \frac{k^2}{a^2} h = 0 \quad \text{in cosmic time } t \tag{2.34a}$$

$$\text{or} \quad \ddot{h} + 2\mathcal{H} \dot{h} + k^2 h = 0 \quad \text{in conformal time } \eta \tag{2.34b}$$

for $h \in \{h_\times, h_+\}$. Because of their tensorial nature, gravitational waves deform spacetime in a quadrupolar manner [50] that is visualized in Figure 2.1 and also as an animation at the bottom of each page in this thesis.

Neglecting the friction term, harmonic oscillations

$$h \propto \mathrm{e}^{\pm k\eta} \quad \text{with the *wavelength mode* } \quad k \equiv |\boldsymbol{k}| \tag{2.35}$$
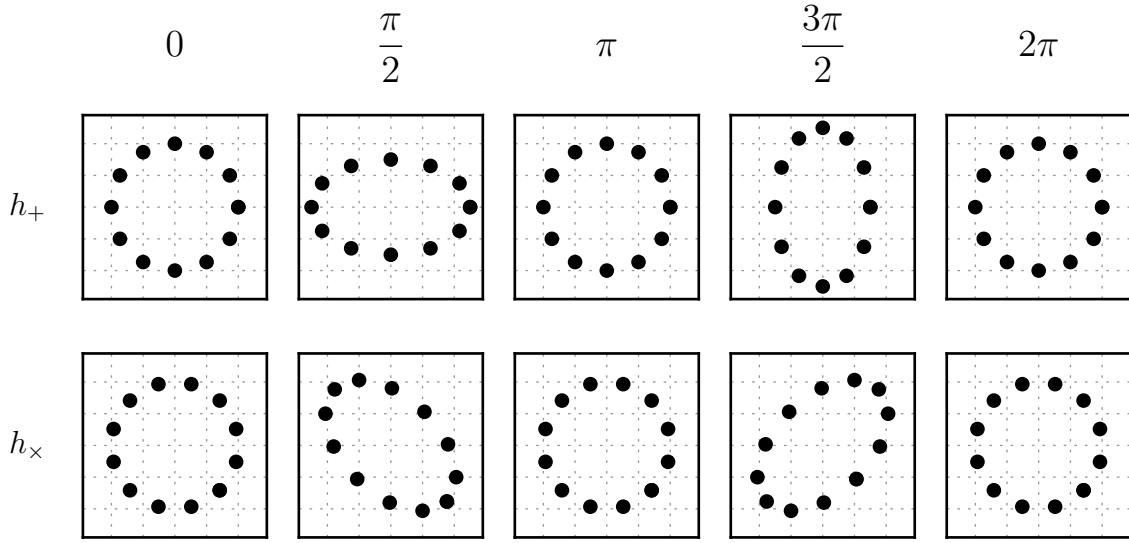
Figure 2.1.: **Effect of gravitational waves on a ring of particles**   A passing gravitational wave alternatingly stretches and compresses the proper distance between two masses (visualized as points) in the plane transverse to its propagation direction [50]. The two polarizations $h_\times$ and h_+ are rotated by 45° relative to one another.

solve (2.34b). However, gravitational waves are damped by the expansion of the universe as exhibited by (2.34), where the friction term is proportional to the Hubble function. In an expanding universe, the Hubble function (2.17) is positive, thus damping the amplitude of gravitational waves. However, the tensor perturbations will remain constant at early times where the conformal time is still smaller than the wavelength scale $\frac{1}{k}$ of the gravitational wave. It begins to oscillate at its *horizon entry* $\eta \simeq \frac{1}{k}$, where cosmological scales on the order of the gravitational wave's wavelength move into causal contact. The horizon entry for larger-scale modes with smaller k thus occurs later. This behaviour is presented in Figure 2.2 where the absolute value $|h(a)|$ of several tensor modes in a universe dominated by the cosmological regimes discussed in section 2.2 is plotted in logarithmic scale. Detailed information about the assumptions, initial conditions and parameters that were chosen to obtain the plot in Figure 2.2 are given in the context of a parametric modification to the evolution equation of gravitational waves (2.34) in section 4.1.
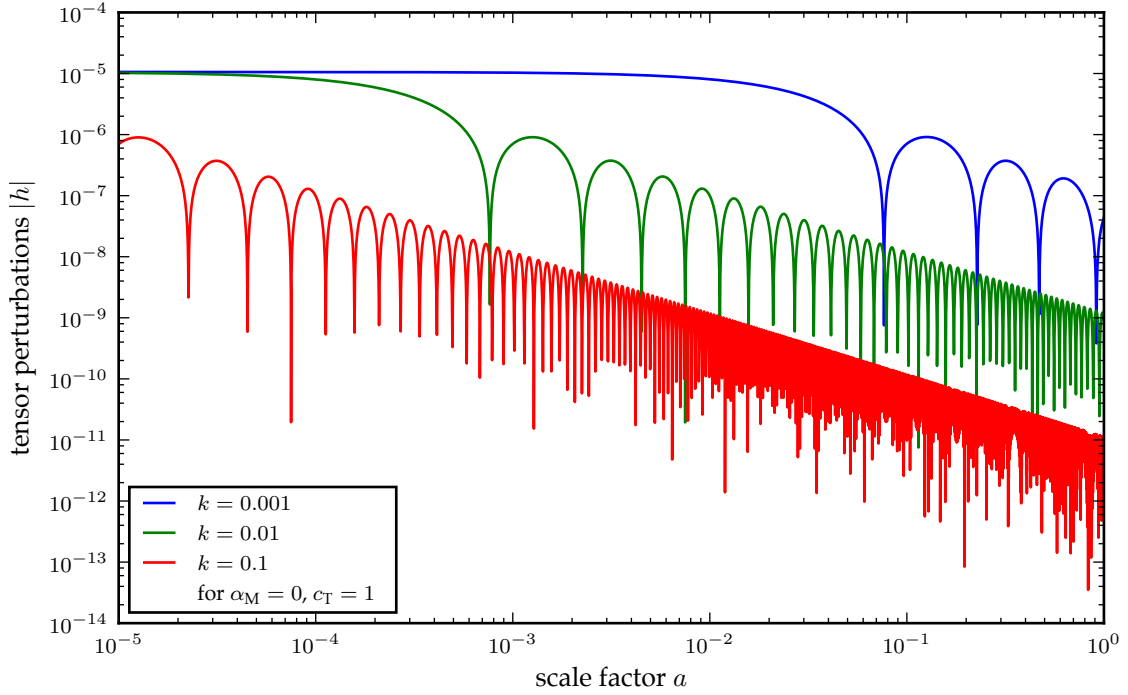
Figure 2.2.: **Tensor perturbations $\left|h(a)\right|$ for different wavelength modes k**   In an expanding universe, tensor perturbations remain constant at early times until cosmological scales on the order of the gravitational wave's wavelength scale $\frac{1}{k}$ move into causal contact. This horizon entry occurs later for large-scale gravitational waves with smaller modes k.

2.4. Detection of Gravitational Waves

Since quadrupole motions, as depicted in Figure 2.1, generate gravitational waves, they also distort spacetime in the same tensorial manner when passing through. The change in proper distance ΔL that two masses with an equilibrium proper distance L experience when oscillating under the influence of a gravitational wave can be measured to obtain its amplitude

$$h \propto \frac{\Delta L}{L}. \tag{2.36}$$

It is a common misconception that the effect of gravitational waves is unobservable since any ruler to measure changes in distance would also be distorted. But in a local reference frame that initially coincides with one particle, the passing gravitational wave merely induces an inertial force on another particle, known as *tidal force* [50]. Other forces acting on the particles must also be considered, however. Since the electromagnetic interaction that is the cause for a conventional ruler to remain at its

length is several orders of magnitude stronger than gravitation, its deformation by a gravitational wave is negligible in comparison to the change in distance between free masses. Furthermore, changes in proper distance affect the travel time of light, so that interferometers constitute ideal gravitational wave detectors [50].

Since the expected amplitude of gravitational waves for typical sources in astronomy is at most on the order of $h \sim 10^{-16}$ [27, 50], sophisticated technology is necessary to measure these tiny changes in distance. For this reason, the detection of gravitational waves was far out of reach when Einstein predicted their existence in 1918 [20]. The experimental search for gravitational waves did not begin until several decades later with the pioneering work of Joseph Weber on *resonant bar detectors*[2] [56]. In 1984, the expected loss of energy through gravitational radiation was measured[3] for the first time and provided a precision test of general relativity and the existence of gravitational waves [58].

More recent resonant-bar detectors such as *ALLEGRO* and *NAUTILUS* [7, 41] reached sensitivities on the order of 10^{-18} in their respective bandwidth [27]. Many modern gravitational wave detectors are based on laser interferometry, where the change in length of the interferometer's arms is detected as a change in interference intensity [27]. Among detectors of this type such as *VIRGO, GEO 600* and *TAMA 300*, the interferometer LIGO has reported a sensitivity on the order of 10^{-22} at a 100 Hz band without detecting any gravitational waves and is currently in the process of being upgraded to increase its sensitivity by a factor of order 10^3 [29, 30].

Through increasingly sophisticated technology, detectors are just about to reach the sensitivity necessary for a direct detection of gravitational waves at a number of experiments around the world and in space. Since both the sensitivity and maximum wavelength scale of interferometers increase with the length of their arms, that in turn is limited by the curvature of earth, several space-based gravitational wave detectors have been proposed. The *LISA* project is planned to feature an interferometer that consists of a triangular constellation of satellites on a solar orbit trailing the earth and is scheduled to launch its proof-of-concept *Pathfinder* later this year [54]. Since massive astronomical sources in our galaxy and beyond are expected

---

[2]Any two masses connected by a spring can form a *resonant* gravitational wave detector that operates at the resonance frequency of the spring, since a passing gravitational waves that oscillates at the same frequency will induce vibrations between the masses. The original *Weber bar* detectors [56] consisted of large, cylindrical bars of aluminum with piezoelectric sensors to detect changes in length.

[3]The binary pulsar PSR 1913+16 that was observed to obtain these measurements had been discovered nearly two decades earlier by Hulse and Taylor [25], who received the 2003 Nobel Price in Physics for this discovery.

to also emit gravitational radiation of wavelengths larger than any experimentally accessible length scale in our solar system, the *pulsar timing* method [23, and references therein] promises to provide sensitivity for such low-frequency gravitational waves.

In addition to astronomical sources of gravitational waves and the quest to detect them, our understanding of the early universe predicts the existence of *primordial gravitational waves* imprinted in the polarization of the cosmic microwave background (CMB). First discovered accidentally in 1964 [38] and measured to extreme precision ever since [44], the CMB is a thermal radiation of temperature $(2.7260 \pm 0.0013)\,\mathrm{K}$ [22] originating at the cosmological epoch of *recombination*. At this point in cosmological evolution, the expanding universe became cold enough for neutral atoms to form from the hot hydrogen plasma the universe was filled with. This event is also known as *decoupling* or *last scattering*, since photons that previously continuously scattered through interaction with the free protons and electrons began to propagate freely through the now transparent universe.

Although the CMB radiation is extremely homogeneous and isotropic, it exhibits both temperature fluctuations on the order of 10^{-5} as well as polarization anisotropies that are of particular interest when gravitational waves are concerned [15, 24]. Thomson scattering processes in regions with quadrupolar temperature anisotropies in the hot plasma are the primary source for the polarization of the CMB [24]. Since this polarization component originates from density perturbations, it can be described by a scalar field with vanishing curl, known as *E-mode*, that complements another *B-mode* scalar field with vanishing divergence. B-mode polarizations are not generated by scattering processes, but both E- and B-modes are produced by gravitational waves that pass through the plasma at decoupling. Measurements of the CMB polarization can therefore provide evidence of primordial gravitational waves that are predicted by models of *inflation*[4] [1, 52]. However, secondary gravitational lensing effects can generate B-modes from primary E-mode polarizations, posing an additional challenge in the search for gravitational waves in the CMB B-mode polarization data [15]. The abundance of tensor perturbations in the CMB is conventionally quantified by its *tensor-to-scalar ratio*

$$r_{0.05} \equiv \frac{A_\mathrm{T}}{A_\mathrm{S}} < 0.12 \quad \text{at } 95\,\% \text{ confidence,} \tag{2.37}$$

---

[4]A detailed discussion of cosmological inflation is out of scope for this thesis but can be found in Baumann [8], for instance.

where A_T and A_s denote the primordial tensor and scalar perturbation amplitude, respectively, the subscript in $r_{0.05}$ corresponds to the *pivot scale* $k_0 = 0.05\,\mathrm{Mpc}^{-1}$ where the values are taken and the experimental bounds are obtained from BICEP2/Keck and Planck Collaborations [10]. With this choice of parameters, the physical amplitude of primordial tensor perturbations becomes

$$h_\mathrm{phys} \propto \sqrt{A_\mathrm{T}\left(\frac{k}{k_0}\right)^{n_\mathrm{T}}}, \tag{2.38}$$

where the amplitude is assumed to be weakly sensitive to the *tensor spectral index* n_T that obeys the theoretically motivated single-field inflation consistency relation [42]

$$n_\mathrm{T} = -\frac{r}{8}. \tag{2.39}$$

We will further discuss gravitational waves in the context of *modified gravity* in chapter 4 and the remainder of the thesis. However, to understand the reason why a modification of general relativity may be necessary, chapter 3 first formulates the *cosmological constant problem* and an approach for solving it.

3. The Cosmological Constant Problem

Both quantum field theory and general relativity are extremely well-tested theories and constitute the basis of modern physics in their respective fields. Whereas quantum field theory succeeds remarkably well in predicting particle physics phenomena, general relativity celebrates an equal success in large-scale astronomical and cosmological observations. Both theories and particularly their interplay are not without mysteries, however.

This section strives to formulate the *cosmological constant problem* and particularly emphasizes its problem of *radiative instability*. The quest for solutions will lead us to consider modified theories of gravity that will be the focus for the remainder of this thesis.

3.1. The Cosmological Constant

Lovelock's theorem [16, 32, 35] states that general relativity as it emerges from the Einstein-Hilbert action 2.7 is, in fact, the unique metric theory of gravity in four spacetime dimensions

- involving only one metric tensor field g

- that gives rise to second-order field equations $G_{\mu\nu}(g) + \Lambda g_{\mu\nu} = 0$

- where $G_{\mu\nu}$ is of rank $(2, 0)$, symmetric and divergence-free.

This includes a free parameter Λ of the theory that is called the *cosmological constant*. Its value does not follow from the theory and thus it can only be constrained experimentally.

It appears in the Einstein equations (2.1) as a contribution

$$T_{\Lambda,\mu\nu} = -\Lambda g_{\mu\nu} \tag{3.1}$$

to the stress-energy tensor that corresponds to a homogeneous energy density penetrating the entire spacetime. In standard FLRW cosmology, such a contribution results in an accelerating expansion of our universe as derived in detail in Appendix A.2.

In fact, cosmological observations clearly suggest such an accelerated expansion taking place at recent cosmic times [39]. The ΛCDM (Λ *cold dark matter*) standard model of cosmology therefore includes a cosmological constant determined by experiments and agrees with all cosmological observations extremely well [44]. The value of the cosmological constant is conventionally given in terms of its corresponding density parameter that recent measurements found to be $\Omega_\Lambda = 0.686 \pm 0.020$, suggesting that it makes up almost 70 % of the density in our universe today [43]. However, the physical origin of the homogeneous energy contribution that the cosmological constant represents remains a mystery in this theory and is given the elusive name *dark energy*. Furthermore, it is already remarkable at this point that the dark energy contribution is measured to be on the same order of magnitude as the contribution of matter to the total density in our universe, since their corresponding density parameters change rapidly with time as given by (2.18).

The fact that the cosmological constant must be fixed by observations is not remarkable on its own, of course, since also the gravitational constant G_N must be determined experimentally. Theories such as the standard model of particle physics include a number of such free parameters. This poses an entirely different problem that is often answered using anthropic arguments. This is discussed in detail in Appendix B.

3.2. Contributions to the Cosmological Constant

The cosmological constant problem arises when we consider both classical and quantum phenomena that, to our knowledge, should contribute to the cosmological constant.

Even in basic quantum mechanics, the uncertainty principle requires every physical system to have a zero-point energy. This immediately carries over to quantum field theory, where a (free) field is an infinite collection of coupled quantum mechanical harmonic oscillators. With a zero-point energy each, they combine to an infinite *vacuum energy* ρ_{vac} [33].

In quantum field theory, the vacuum energy is largely ignored as only differences in energy determine the dynamics of the system. In general relativity, however, all energy content gravitates, including the vacuum energy [13, 33, 37].

Remarkably, assigning an importance not only to energy differences, but also to absolute energy values gives rise to another, entirely classical contribution to the stress-energy tensor, namely the *zero-point potential* V_0. This is the energy

$$T_{\mu\nu} = -V_0 g_{\mu\nu} \tag{3.2}$$

where the kinetic energy vanishes and the potential assumes its minimum value V_0. This zero-point potential is usually chosen arbitrarily when only energy differences are considered, but must be taken into account in general relativity. Particularly in presence of phase transitions, one can generally not choose the zero-point potential such that it always vanishes [33].

The free parameter Λ in the Einstein equations therefore combines with both the quantum vacuum energy ρ_{vac} and the classical zero-point potential V_0 of every quantum field in the universe to an *effective* cosmological constant

$$\Lambda_{\text{eff}} = \Lambda + \rho_{\text{vac}} + V_0 \tag{3.3}$$

that we measure as dark energy. In comparison to the small value for Λ_{eff} we observe today, however, both contributions are extremely large [33]. This already suggests a severe *fine-tuning* problem, where the value of the original cosmological constant Λ must precisely cancel the other contributions up to the small value we measure today. Fine-tuning is also discussed in more detail in Appendix B.

3.3. Radiative Instability

The full scope of the problem arises when we consider in more detail the vacuum energy that we found to be infinite before. The mechanism to make sense of divergences like this in quantum field theory is the framework of *renormalization*. In the process to find a finite, *renormalized* value for ρ_{vac} one generally adds counterterms for every order in perturbation theory that each depend on an *arbitrary subtraction scale*.

Generally, successive orders are not significantly suppressed by a sufficiently small perturbation parameter λ, however. In fact, for the standard model Higgs field the self-coupling parameter of perturbation λ is of the order 10^{-1}, for instance, and therefore every order in perturbation theory must be renormalized independently [37]. This *radiative instability* requires us to fine-tune the cosmological constant repeatedly for every order in perturbation theory and thus makes it sensitive even to small-scale physics, where we assume our theory to break down [37].

This also prohibits us from finding an *effective theory* where the full structure of perturbation theory is encoded in one finite, renormalized value by means of a *Wilson effective action*. Again, we find the renormalized vacuum energy is unstable against changes in the unknown UV-regime of the theory [37].

With these large and radiatively unstable contributions to the effective cosmological constant and its measured value that is fine-tuned to many orders of magnitude below [33], we must admit a severe flaw in our most successful descriptions of nature. Since the only observable effect the cosmological constant has on our universe is through gravity, we can suspect that it may be necessary to reconsider or modify general relativity's underlying assumptions in order to solve the cosmological constant problem.

3.4. The *New* Cosmological Constant Problem and Modified Gravity

The cosmological constant problem is deeply rooted in our inability to find a renormalized vacuum energy that is stable against changes in its effective description. Therefore, one approach to address the problem is to assume that some mechanism makes the vacuum energy vanish altogether instead and then find another theory that explains the non-zero cosmological constant we observe today.

In fact, unbroken *supersymmetry* would accomplish just that. In supersymmetry, bosons and fermions are related by a symmetry and share the same mass. Supersymmetric partners contribute to the vacuum energy with opposite signs, thus precisely canceling each other [34].

With the cosmological constant set to zero, the cosmological observation of accelerated expansion remains to be explained by a different mechanism. Dark energy models such as the *quintessence* theory [45, 59] postulate further contributions to the

stress-energy tensor that have the same accelerating effect as a cosmological constant. A different approach is to modify the theory of gravity instead.

Every attempt towards a modified theory of gravity has to take Lovelock's theorem into consideration and break at least one of its assumptions that make general relativity unique. The concept of the entire class of $f(\mathcal{R})$ *theories*, for example, is to replace the Ricci scalar $\mathcal{R}$ in the Einstein-Hilbert action (2.7) by a function $f(\mathcal{R})$. $f(\mathcal{R})$ theories can break Lovelock's assumption of only second-order equations of motion or are in many cases equivalent to an additional scalar field contribution to the matter Lagrangian that is non-minimally coupled to gravity [6, 16].

Alternatively, the hypothesis of a *massive graviton* requires a second, arbitrary *reference metric f* in addition to the physical metric g [47]. The additional metric is necessary to construct a mass term because a single metric allows only trivial self-interaction terms $g^{\mu\sigma}g_{\sigma\nu} = \delta^{\mu}_{\nu}$ and $g^{\mu\nu}g_{\mu\nu} = 4$. By postulating a symmetric action where the reference metric f behaves dynamically just like g, we arrive at the theory of *massive bigravity*. The cosmology in this theory allows solutions with late-time acceleration without a cosmological constant [3, 28].

4. Parametrized Evolution of Gravitational Waves in Modified Gravity

Many theories of modified gravity are discussed in the literature [6, 16, 47] and this thesis will not focus on one specific model. Instead, the main objective is to derive parametric constraints for general modified gravity theories from our observational knowledge of tensor perturbation modes.

Since even the most precise detectors discussed in section 2.4 have not found evidence for gravitational waves so far, their sensitivity constrains observable tensor perturbations to extremely small amplitudes. Modifications to general relativity that affect the evolution of gravitational waves can exceed these experimental bounds, thus rendering the theory physically unviable. In particular, any modified gravity theory that exhibits growing tensor modes in cosmological evolution can easily be in tension with experiments, since their amplitude would likely be large enough today to be within the sensitivity of existing gravitational wave detectors.

To explore the evolution of gravitational waves in modified gravity theories in general, and to identify constraints such that growing tensor modes are avoided in particular, the remainder of the thesis will investigate various parametric modifications to the evolution equation of gravitational waves (2.34) that can arise from modified gravity theories. Suitable parametrizations have recently been discussed in Amendola et al. [4, 5], Bellini and Sawicki [9], Linder [31], Pettorino and Amendola [40], Raveri et al. [46], Saltas et al. [48], and Xu [60] and references therein, where common modifications include a modified friction contribution α_{M} and a deviation c_{T} of the tensor propagation speed from the speed of light. In this parametrization, the

evolution equation of gravitational waves (2.34) becomes [4, 40, 46]

$$\frac{\mathrm{d}^2 h}{\mathrm{d}t^2} + (3 + \alpha_{\mathrm{M}})\,\mathrm{H}\frac{\mathrm{d}h}{\mathrm{d}t} + \frac{c_{\mathrm{T}}^2 k^2}{a^2}h = 0 \quad \text{in cosmic time } t, \tag{4.1a}$$

$$\ddot{h} + (2 + \alpha_{\mathrm{M}})\,\mathcal{H}\dot{h} + c_{\mathrm{T}}^2 k^2 h = 0 \quad \text{in conformal time } \eta \tag{4.1b}$$

$$\text{or} \quad h'' + \left(\frac{\mathcal{H}'}{\mathcal{H}} + 2 + \alpha_{\mathrm{M}}\right)h' + \frac{c_{\mathrm{T}}^2 k^2}{\mathcal{H}^2}h = 0 \quad \text{in e-foldings } \ln a \tag{4.1c}$$

where both α_{M} and c_{T} may in general be functions of time and depend on the specific model. For instance, Hwang and Noh [26] show that the modified friction contribution in an $f(\mathcal{R})$ theory is given by

$$\alpha_{\mathrm{M}} = \frac{\mathrm{d}\ln F}{\mathrm{d}\ln a} \quad \text{with} \quad F := \frac{\mathrm{d}f(\mathcal{R})}{\mathrm{d}\mathcal{R}}. \tag{4.2}$$

Many examples for modified gravity theories where the propagation speed of tensor perturbations differs from the speed of light are found in Amendola et al. [4], Linder [31], and Raveri et al. [46]. The standard behaviour of general relativity (2.34) is recovered for $\alpha_{\mathrm{M}} = 0$ and $c_{\mathrm{T}} = 1$. This chapter will explore the evolution of gravitational waves in a modified gravity theory that reduces to a modification of one or both of these parameters and identifies constraints such that growing tensor modes are avoided.

4.1. Constant modified friction

To analyze the effect of modified friction on gravitational waves, I will first consider the case $\alpha_{\mathrm{M}} = const$ in the parametrized evolution equation (4.1). Since the parameter α_{M} appears as an additive contribution to the damping term in the wave equation, it is expected to adjust the slope of the tensor perturbation amplitude. Therefore it is of crucial importance when constraints for growing tensor modes are of consideration.

4.1.1. Numerical solution

A solution of the parametrized evolution equation (4.1) for various values of $\alpha_{\mathrm{M}} = const$ was obtained numerically with the `ParametricNDSolve` function in Mathematica under the following assumptions:

- First, initial conditions

$$h(a \to 0) = 1 \quad \text{and} \quad h'(a \to 0) = 0 \tag{4.3}$$

  were chosen that coincide with the standard super-horizon evolution at early times discussed in section 2.3.

- Furthermore, a standard FLRW background was assumed where the Hubble function is given by the Friedmann equation (2.23)

$$\mathrm{H}(t) = \mathrm{H}_0 \sqrt{\sum_i \Omega_i(t)}. \tag{4.4}$$

- Only matter and dark energy density contributions

$$\Omega_{\mathrm{m}}(t) = \Omega_{\mathrm{m}}(t_0)a^{-3} \quad \text{and} \quad \Omega_\Lambda(t) = 1 - \Omega_{\mathrm{m}}(t_0) \tag{4.5}$$

  were considered here, because they dominate in late-time cosmological eras as discussed in section 2.2.

- The physical amplitude (2.38)

$$h_{\mathrm{phys}}(a) = \sqrt{A_{\mathrm{T}} \left(\frac{k}{k_0} \right)^{n_{\mathrm{T}}}} \cdot h(a) \tag{4.6}$$

  of primordial gravitational waves was also computed according to section 2.4.

- A tensor-to-scalar ratio of $r = 0.05$ was assumed that agrees with the observational constraints given in (2.37).

- Experimental values for H_0, $\Omega_{\mathrm{m}}(t_0)$ and A_{s} were obtained from the Planck Collaboration [42] results.

In Figure 4.1, the absolute value $\left| h(a) \right|$ is plotted in logarithmic scale for solutions with both positive and negative values of α_{M}. The amplitude of the gravitational wave clearly decreases more rapidly for larger α_{M} since additional friction is introduced. However, also the horizon entry is delayed for larger α_{M}, thus introducing a competing effect. The same behaviour was found by Pettorino and Amendola [40] and could be related to an observable effect on the B-mode spectrum of the cosmic
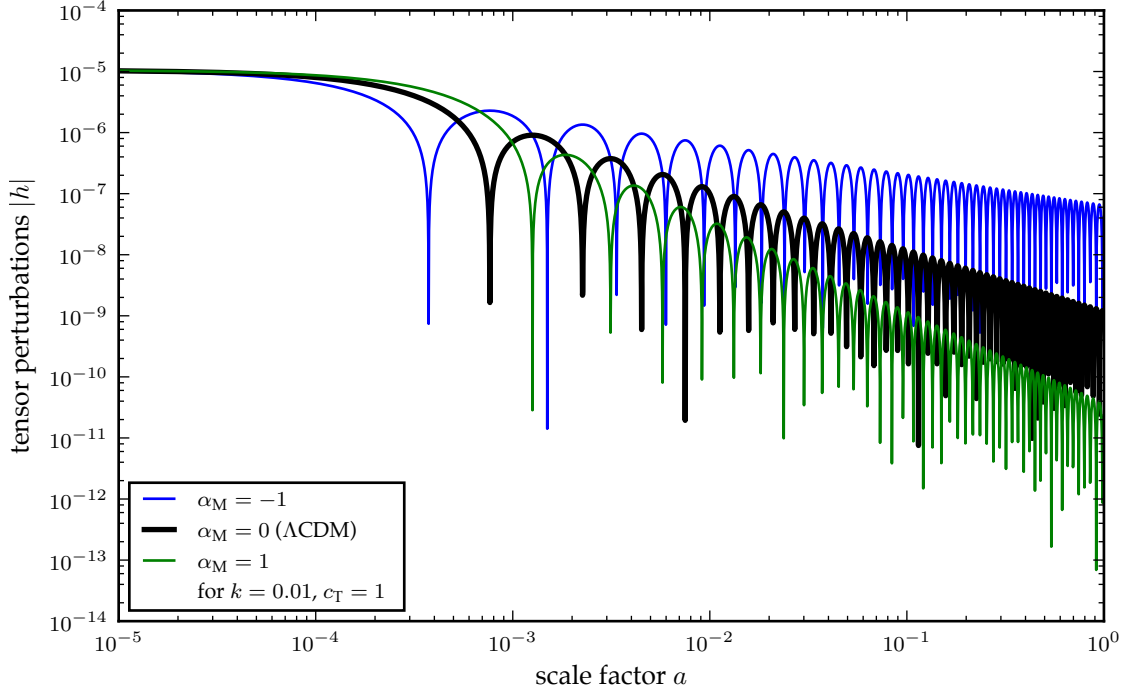
Figure 4.1.: **Tensor perturbations $|h(a)|$ with modified friction $\alpha_{\text{M}} = const$** Increasing α_{M} introduces more friction, so that the amplitude of the gravitational wave decreases more rapidly, but also delays the horizon entry.

microwave background. In particular, since the horizon entry is delayed for additional friction, smaller-scale tensor perturbations would be observed in the CMB polarization [40].

4.1.2. Analytic solution in cosmological regimes

The parametrized evolution equation (4.1) can be solved analytically in cosmological regimes dominated by one of the fluid species discussed in section 2.2. The expansion of an FLRW universe dominated by matter or radiation is given by (2.19) or

$$a(\eta) \propto \eta^{\tilde{n}(w)} \quad \text{with} \quad \tilde{n}(w) \coloneqq \frac{2}{1 + 3w} \quad \text{in conformal time.} \tag{4.7}$$

This allows us to find an expression $\mathcal{H} = \frac{\dot{a}}{a} = \frac{\tilde{n}(w)}{\eta}$ for the Hubble function so that (4.1) becomes

$$\ddot{h} + (2 + \alpha_{\text{M}}) \frac{\tilde{n}(w)}{\eta} \dot{h} + c_{\text{T}}^2 k^2 h = 0. \tag{4.8}$$

This is a Bessel differential equation that has solutions in terms of Bessel functions as discussed in Appendix A.4. Thus, (4.8) is solved by

$$h(\eta) = \eta^{-p} \left[C_1 J_p(c_{\text{T}} k\eta) + C_2 Y_p(c_{\text{T}} k\eta) \right] \quad \text{with} \quad p = \tilde{n}(w)\left(1 + \frac{\alpha_{\text{M}}}{2}\right) - \frac{1}{2}, \tag{4.9}$$

where $J_p(x)$ and $Y_p(x)$ denote the Bessel functions of first and second kind, respectively. Their asymptotic behaviour for large modes k or late times η is given in Appendix A.4 and both correspond to oscillations with an amplitude decreasing as $\eta^{-\frac{1}{2}}$. Therefore, the amplitude of gravitational waves with modified friction α_{M} behaves as

$$h(\eta) \propto \eta^{-p-\frac{1}{2}} = \eta^{-\tilde{n}(w)\left(1+\frac{\alpha_{\text{M}}}{2}\right)} \tag{4.10a}$$

$$\text{or} \quad h(a) \propto a^{-\frac{1}{\tilde{n}(w)}\left(p-\frac{1}{2}\right)} = a^{-\left(1+\frac{\alpha_{\text{M}}}{2}\right)} \tag{4.10b}$$

times fast oscillation in this regime.

This result immediately gives a constraint for the α_{M} parameter such that the model does not exhibit growing tensor modes. Since (4.10b) exhibits a growing amplitude of $h(a)$ for a positive exponent $-\left(1 + \frac{\alpha_{\text{M}}}{2}\right) > 0$ in cosmological expansion, any theory of gravity that modifies the evolution equation of gravitational waves (4.1) by an additional friction term $\alpha_{\text{M}} = const$ should fulfill

$$\alpha_{\text{M}} \geq -2 \tag{4.11}$$

in order to avoid growing tensor modes.

The numerical result obtained before reflects this behaviour as presented in Figure 4.2. The plot shows the evolution of tensor modes for both $\alpha_{\text{M}} = -2$ and $\alpha_{\text{M}} < -2$ using the same numerical solution as in Figure 4.1 as well as the slope of the amplitude obtained analytically in (4.10). As expected from the analytic considerations, the gravitational wave with $\alpha_{\text{M}} = -2$ remains stable whereas the gravitational wave with $\alpha_{\text{M}} < -2$ grows in amplitude.

4.2. Deviating propagation speed

To explore the effect a modified propagating speed has on the evolution of gravitational waves in a modified gravity theory, I consider a parametric deviation c_{T} from
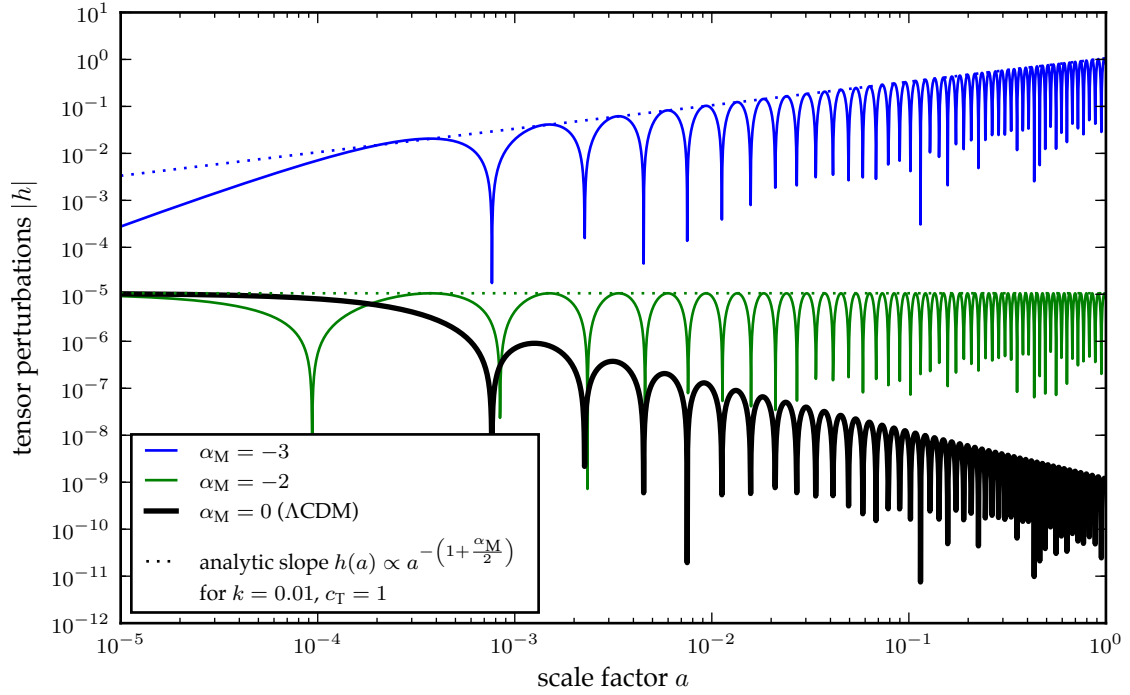
Figure 4.2.: **Tensor perturbations $\left|h(a)\right|$ with modified friction $\alpha_{\mathrm{M}} \leq -2$**   The numerical solution (solid lines) agrees with the slope of the amplitude obtained analytically (dotted lines). Gravitational waves with $\alpha_{\mathrm{M}} = -2$ remain stable but grow in amplitude for $\alpha_{\mathrm{M}} < -2$.

the speed of light in the parametrized evolution equation (4.1) that was proposed by Amendola et al. [4] and Raveri et al. [46]. The deviation of the propagation speed as it appears in (4.1) changes the effective wavelength scale $\frac{1}{c_{\mathrm{T}}k}$ such that the horizon entry of the gravitational wave occurs later for smaller values of c_{T}. The horizon entry is discussed in detail in the context of general relativity in section 2.2.

Figure 4.3 shows the evolution of gravitational waves for several values of $c_{\mathrm{T}} = const$. The plots were obtained numerically with the `ParametricNDSolve` function in Mathematica, where the same assumptions and initial conditions as in section 4.1 were chosen.

Because gravitational waves with lower propagation speed c_{T} exhibit a delayed horizon entry, their amplitude today is closer to their initial value than for gravitational waves that propagate faster. A modified friction contribution α_{M}, as discussed in section 4.1, therefore is less effective for gravitational waves with lower c_{T} that enter the horizon later and thus have less time until today to decrease or increase in amplitude. This suggests a degeneracy between a modified propagation speed and a modified friction contribution, as both affect the amplitude of tensor perturbations
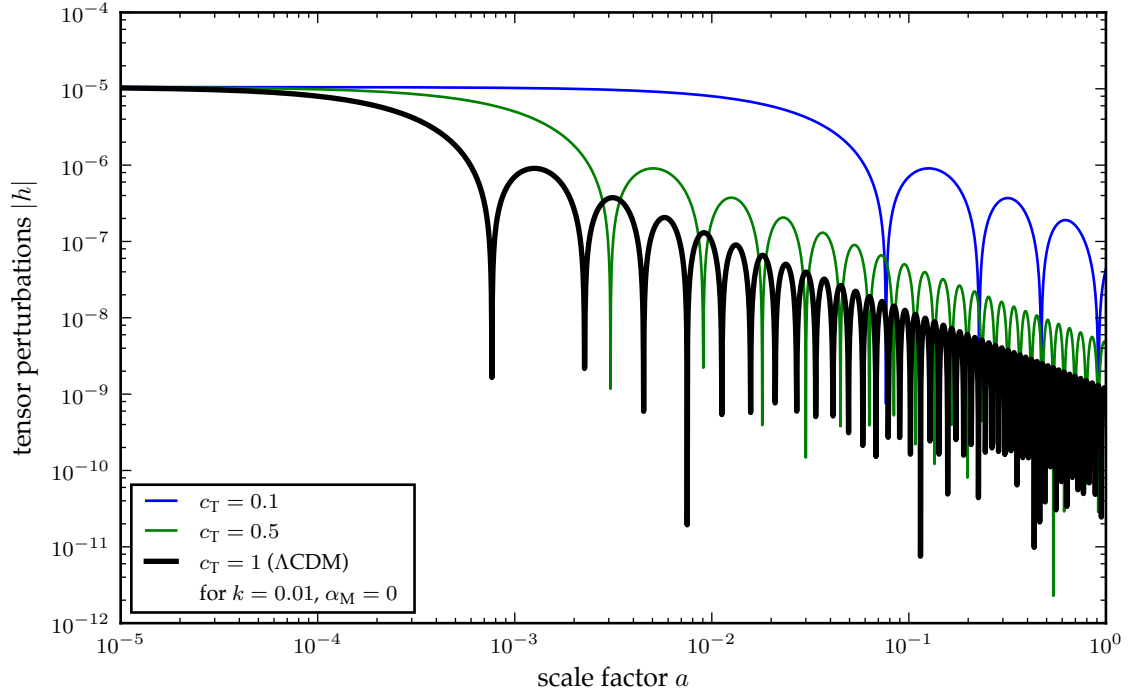
Figure 4.3.: **Tensor perturbations $\left|h(a)\right|$ with a modified propagation speed** $c_\mathrm{T} = const$ — Gravitational waves with lower propagation speed c_T exhibit a delayed horizon entry.

measured today. A positive α_M that results in a faster decrease in amplitude can be compensated by a lower propagation speed c_T that in turn delays the horizon entry and thus gives the tensor mode less time to decrease. The amount by which growing tensor modes increase in amplitude is reduced by a lower propagation speed for the same reasons.

An increase in propagation speed of gravitational waves relative to the speed of light is also in agreement with observational data [40, 46]. In such a theory, an observer would measure the gravitational wave before light emitted by the same source had time to reach it. Since there is no apparent reason prohibiting gravitational waves from carrying information, the causal structure of this theory must be reconsidered.

4.3. Late-time modified friction

Many theories of modified gravity aim to solve the *new cosmological constant problem* presented in section 3.4. This requires such theories to model the accelerated

expansion of our universe today as described in section 3.1. Proposed modifications to general relativity thus generally only affect late-time cosmological regimes where the ΛCDM standard model of cosmology relies on dark energy to accelerate the universe. Therefore, suitable parametrized modifications to the evolution equation of gravitational waves (4.1) are time-dependent and vanish in early-time cosmological regimes to reflect this behaviour of modified gravity theories.

In particular, I consider for the modified friction α_{M} the time-dependent parametrization

$$\alpha_{\mathrm{M}}(t) = \alpha_{\mathrm{M0}} \cdot a(t)^{\beta} \quad \text{with} \quad \alpha_{\mathrm{M0}} = const \quad \text{and} \quad \beta > 0. \tag{4.12}$$

In an expanding universe where $a(t) \in (0,1]$ monotonically increases with time, the modified friction contribution α_{M} vanishes at early times to recover general relativity, but deviates from ΛCDM today.

Figure 4.4 shows numerical solutions to the evolution equation (4.1) with this particular parametrization for α_{M} for various values of both α_{M0} and β. It was obtained with the `ParametricNDSolve` function in Mathematica, where the same initial conditions and assumption as in section 4.1 were chosen. The special case $\beta = 0$ corresponds to constant friction $\alpha_{\mathrm{M}}(t) = \alpha_{\mathrm{M0}}$ as investigated in section 4.1. Increasing β to positive values reduces the modified friction at early times where $a(t) < 1$. Larger values for β amplify this effect.

At late times, tensor modes will increase or decrease in amplitude with a slope approximating the behaviour of constant modified friction α_{M0} discussed in section 4.1. However, tensor modes that grow in this regime according to (4.11) can be suppressed by a sufficiently large value of β to remain physically viable.
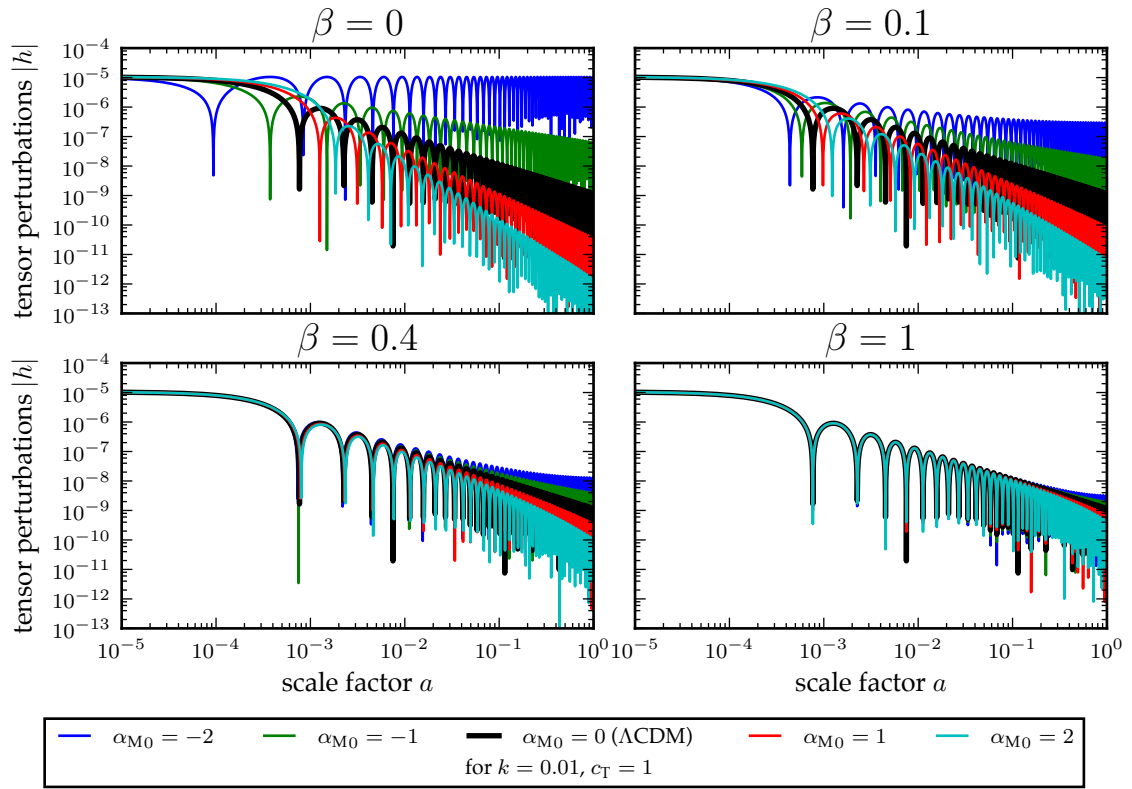
Figure 4.4.: **Tensor perturbations** $\left|h(a)\right|$ **with parametrized modified friction** $\alpha_{\mathrm{M}}(t) = \alpha_{\mathrm{M0}} \cdot a(t)^{\beta}$ For positive β, the evolution matches ΛCDM at early times and deviates at late times. Increasing $|\alpha_{\mathrm{M0}}|$ yields larger deviations from ΛCDM at late times.

5. Gravitational Waves in Parametrized Bigravity

Bigravity is a class of modified gravity theories introduced in section 3.4 where a second *reference metric* f in addition to the physical metric g is considered with its own dynamics to obtain a theory of *massive gravity*. Only the physical g-metric induces physical gravity, but both metrics are coupled such that the evolution equation of gravitational waves in this bimetric setting becomes [5]

$$\ddot{h}_n + (2 + \alpha_{\mathrm{M}n})\, \mathcal{H}\dot{h}_n + \left(\mathcal{H}^2 m_n^2 + c_{\mathrm{T}n}^2 k^2\right) h_n = \mathcal{H}^2 q_n h_m \quad \text{in conformal time } \eta$$

(5.1a)

$$\text{or} \quad h_n'' + \left(\frac{\mathcal{H}'}{\mathcal{H}} + 2 + \alpha_{\mathrm{M}n}\right) h_n' + \left(m_n^2 + \frac{c_{\mathrm{T}n}^2 k^2}{\mathcal{H}^2}\right) h_n = q_n h_m \quad \text{in e-foldings } \ln a$$

(5.1b)

where the indices $n, m \in \{g, f\}$ with $n \neq m$ refer to the perturbations and parameters associated with the physical metric g and the reference metric f, respectively. The modified friction terms $\alpha_{\mathrm{M}n}$ and the deviations from the speed of light $c_{\mathrm{T}n}$ for each of the two metrics were discussed in chapter 4 in the context of unimetric modified gravity. Each metric also has an associated *mass parameter* m_n and a *coupling* q_n to the other metric. In the theory of bigravity, these parameters have specific, time-dependent forms that are given in [5]. The effect the additional metric and mass parameters have on the evolution of gravitational waves will be discussed in this chapter.

5.1. Coupling to the reference metric

Since strict experimental bounds apply to the amplitude of physical tensor perturbations h_g, we can expect the source term $q_f h_g$ of the reference metric f to be negligible

with respect to the other terms [5]. Thus, I explore solutions of the parametrized bimetric evolution equation (5.1) with $q_f = 0$, but with a non-zero coupling parameter $q_g = const.$ The non-standard behaviour of the reference metric that is governed by a choice of its associated parameters will therefore modify the evolution of the physical tensor perturbations h_g.

Figure 5.1 depicts several solutions of the parametrized bimetric evolution equation (5.1) that were obtained numerically with the `ParametricNDSolve` function in Mathematica. For both g and f, the same assumptions and initial conditions as in section 4.1 were chosen. In addition to the coupling $q_g = 1$, only the modified friction parameter of the f-metric was given a non-standard value such that its tensor perturbations grow in amplitude for $\alpha_{\mathrm{M}f} < -2$ as discussed in section 4.1.
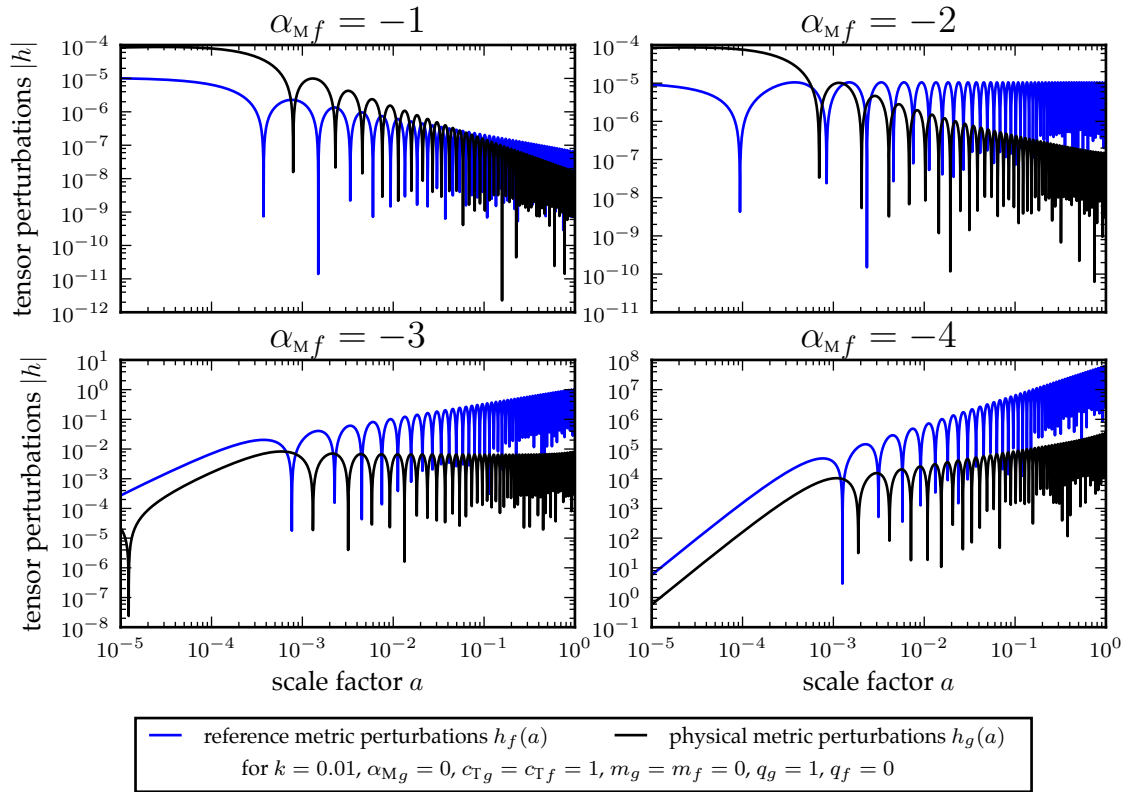
Figure 5.1.: **Physical and reference metric perturbations with parametrized coupling** $q_g = 1$ The amplitude of the physical metric perturbations follows the slope of the reference metric perturbations at late times. Note that the coupling q_g and the modified friction $\alpha_{\mathrm{M}f}$ of the f-metric are the only non-standard parameters. The coupling is also the cause of the initial offset that appears in the plots in the depicted regime despite the same choice of initial conditions for both metrics.

Because matter only couples to the physical metric g whereas the reference metric f is not directly observable, tensor modes for the reference metric are not constrained by experiments. However, the coupling q_g also leads to non-standard behaviour of the physical metric, although its other associated parameters remain unmodified. Figure 5.1 shows that the physical metric perturbations follow the slope of the reference metric perturbations at late times and also exhibit a growing amplitude for sufficiently negative $\alpha_{\mathrm{M}f}$ when coupled to the reference metric. In the specific bigravity model investigated in [5], a similar behaviour is found for explicit, time-dependent expressions of the parameters that were considered constant here.

5.2. Non-zero mass parameter

The numerical solutions of the parametrized bimetric evolution equation (5.1) obtained in section 5.1 also allow for a brief discussion of the mass parameter m_n for both metrics $n \in \{g, f\}$. Figure 5.2 shows the evolution of the physical metric perturbations $\left|h_g(a)\right|$ for various values of its mass parameter m_g and all other parameters set to their standard value. Because the mass parameter appears as an additive contribution to the effective wavelength mode $c_{\mathrm{T}}k$ in (5.1), larger values of m_n will advance the horizon entry of the tensor mode. A degeneracy between the mass parameter and a deviating propagation speed c_{T} is to be expected.

5.3. Decoupling at early times

Bimetric modified gravity theories, like unimetric theories, usually deviate from ΛCDM only at late times to provide an accelerated expansion of the universe without a cosmological constant. This is discussed in more detail in section 4.3. For physically viable bimetric theories it is therefore reasonable to assume that the metrics decouple in early-time cosmological regimes. This allows the physical metric g to behave unmodified whereas the reference metric can exhibit arbitrary dynamics.

A specific bimetric model is investigated in [5], for example, where the physical and reference metric decouple at early times. Amendola et al. [5] found that for subhorizon modes in radiation and matter dominated eras also the mass parameters become negligible in this model. Therefore, the evolution of tensor perturbations
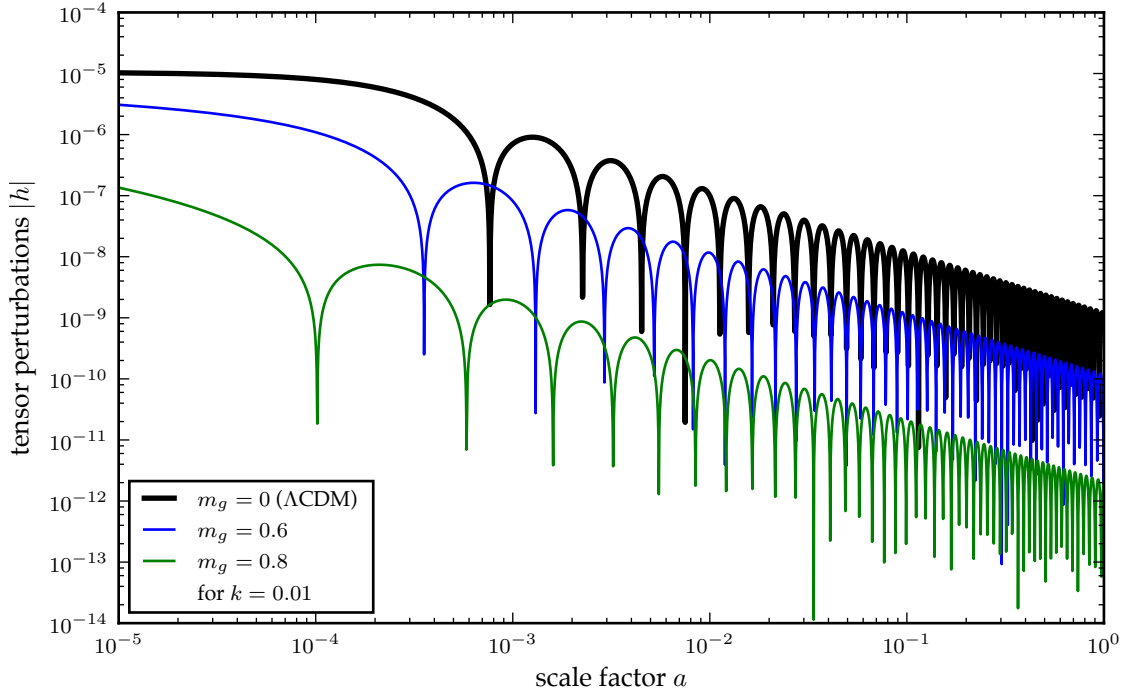
Figure 5.2.: **Tensor perturbations $\left| h(a) \right|$ with non-zero mass parameter** m_g Due to the additive contribution to the effective wavelength mode $c_\mathrm{T} k$, increasing the mass parameter m_g will advance the horizon crossing of the gravitational wave.

for both metrics reduce to the unimetric parametrized evolution equation (4.1) with parameters [5]

$$\alpha_{\mathrm{M}g} = 0 \qquad\qquad c_{\mathrm{T}g} = 1 \qquad\qquad (5.2a)$$

$$\alpha_{\mathrm{M}f} = -3(1+w) \qquad\qquad c_{\mathrm{T}f} = \frac{(3w+1)^2}{4} \qquad\qquad (5.2b)$$

where the physical metric remains entirely standard such that the amplitude of tensor perturbations h_g decreases like $\frac{1}{a}$ according to (4.10). However, the reference metric tensor perturbations h_f with $\alpha_{\mathrm{M}f} < -2$ in (5.2b) grow in amplitude like a^1 in radiation domination and $a^{\frac{1}{2}}$ in matter domination.

With a non-zero coupling q_g of the g- to the growing f-metric also at early times, as discussed in section 5.1 and depicted in Figure 5.1, the observable g-metric acquires a similar trend and tensions with experimental constraints are foreseeable. A decoupling of the metrics at early times can therefore delay growing physical tensor modes in a bimetric theory where the reference metric perturbations grow in amplitude.

6. Summary

Many theories of modified gravity were proposed to explain the late-time accelerated expansion of our universe without a cosmological constant. The cosmological constant is a free parameter in general relativity that resembles a homogeneous energy density penetrating the entire spacetime and is known as *dark energy* in the ΛCDM standard model of cosmology. To our knowledge, other energy components such as the classical zero-point potential and the vacuum energy of all quantum fields should also contribute to the effective cosmological constant. Since a radiatively stable vacuum energy can not be found, a severe fine-tuning problem exists that is known as the *cosmological constant problem*. An approach for solving this problem is to assume a mechanism that sets the cosmological constant to zero and to find a modified theory of gravity that allows self-accelerating cosmological solutions.

Any modified gravity theory must respect the experimental bounds for gravitational waves in order to be considered physically viable. Theories that predict tensor perturbations with growing amplitudes in cosmological expansion can therefore easily be in tension with observations. Common modifications to the evolution of gravitational waves include modified friction contributions and a deviating propagation speed that I investigated in chapter 4 by introducing appropriate parameters α_{M} and c_{T} [4, 40]. Numerical solutions showed a more rapid decay of tensor perturbation amplitudes and the competing effect of a delayed horizon entry for larger additional friction contributions $\alpha_{\mathrm{M}} = const$. An analytic solution could also be found in a universe dominated by matter or radiation. The slope of the tensor perturbation amplitude in this regime is given by (4.10)

$$h(a) \propto a^{-\left(1+\frac{\alpha_{\mathrm{M}}}{2}\right)} \tag{6.1}$$

such that gravitational waves remain stable for (4.11)

$$\alpha_{\mathrm{M}} \geq -2. \tag{6.2}$$

However, the effect a modified friction contribution has on the amplitude of gravitational waves today is reduced by a decrease in propagation speed c_T as shown in section 4.2. Since the horizon entry of tensor modes is delayed for smaller c_T, they have less time until today to decrease or increase in amplitude.

Generally, time-dependent modified friction and propagation speed parameters, such as the parametrization for α_M explored in section 4.3, can suppress the modifications at early times so that the model only recently deviates from ΛCDM. A thorough analysis of the evolution of tensor modes in a specific modified gravity theory is necessary to determine whether their amplitudes remain within experimental bounds. Particularly theories that predict late-time modified friction in conflict with the stability condition (4.11) can easily be in tension with observational data.

To formulate theories of massive gravity, a second, arbitrary *reference metric f* must be coupled to the physical metric g. This additional metric is given dynamics similar to g in the theory of *bigravity*, so that both can exhibit gravitational waves. Since the metrics are coupled, the parametrized evolution equation of tensor perturbations is extended by general mass and coupling parameters m_n and q_n for both metrics $n \in \{g, f\}$ in chapter 5. A non-zero mass parameter will advance the horizon entry of the tensor mode and therefore lead to an expected degeneracy with a deviating propagation speed c_T. The parameter q_n denotes the strength of the coupling to the other metric's tensor perturbations h_m.

Without any coupling, both metrics evolve independently. Since only the g-metric induces physical gravity, the f-metric perturbations are unobservable. Therefore they are not subject to experimental constraints and can exhibit arbitrary behaviour including growing tensor modes. The unobservable reference metric can, however, interact with the physical metric through the coupling parameters q_n to modify its evolution. In particular, the amplitude of the physical metric perturbations will follow the slope of the reference metric perturbations as shown for $q_g = const$ in section 5.1.

A coupling between the metrics must exist for the theory to provide any modifications to observable gravity, but it is generally time-dependent such that the g-metric deviates from ΛCDM only recently and the theory has self-accelerating late-time cosmological solutions. When the coupling q_g becomes relevant at late times, the behaviour explored in section 5.1 must be taken into consideration such that the physical metric perturbations do not exceed observational constraints by coupling to growing reference metric tensor modes.

In conclusion, numerous parametric modifications to the evolution of gravitational waves in modified gravity theories were explored and constraints could be found in (4.11) such that tensor modes remain stable for constant modified friction. The interplay between the parameters for both unimetric as well as bimetric theories leads to degeneracies as soon as multiple contributions are taken into consideration. Also time-dependent parametrizations were investigated that can generally suppress the modifications. A systematic comparison to the experimental data is therefore necessary to reveal definite constraints for the individual parameters such that both the required self-acceleration is obtained and the observational bounds on tensor modes are respected. In the quest to formulate a theory of gravity without a cosmological constant, the effects explored in this thesis can guide further analysis to gain crucial insight about the physical viability of modified gravity theories.

7. Acknowledgements

First and foremost, I would like to thank Dr. Valeria Pettorino and Prof. Dr. Luca Amendola for the opportunity to work in their research group and for the supervision of my bachelor project, as well as Dr. Matteo Martinelli for his invaluable support. My thanks also goes to the rest of the *cosmici* group for the numerous and fruitful discussions about our universe and beyond. I must, however, apologize to anyone in the group who feels their personal opinion on the cosmological constant problem is not reflected explicitly enough in this thesis and suggest to reach a concensus over another barbecue, be it with lobsters or without.

I am also most grateful for the lecturers in theoretical physics that sparked my fascination in the fundamental theories of nature, both at the University of Heidelberg and at the *Winter School on Gravity and Light 2015* in Linz, particularly Prof. Dr. Carlo Ewerz and Dr. Frederic Schuller for their excellent lectures on classical electrodynamics and general relativity, respectively. It was also in great parts Prof. Dr. Bernhard Schutz who awakened my interest in gravitational waves with his insightful lecture in Linz.

Furthermore, I wish to express my appreciation for the work done by the plethora of contributors to scientific toolkits such as the Python programming language, Mathematica and LaTeX and their associated packages, since their importance in modern science is not acknowledged nearly enough as they deserve. Taken for granted in their daily use by many, these computational tools have had an unmeasurable impact on the advancement of science.

Finally, I want to thank everybody who spent their time in discussion with me, be it about physics or philosophy, or the fine line in between. I believe discussions have the unique nature that they provide insights one can not reach on one's own, and that any question that is voiced out of curiosity is worthy of being asked. Thus, I am grateful for the quality many physicists posses to reconsider even the most fundamental assumptions in our quest to understand the universe we live in.

Appendices

A. Mathematical Appendix

A.1. Time evolution of the Scale Factor in an FLRW Universe

A time derivate of the Friedmann equation (2.16b) leads to

$$0 = \frac{d\rho}{dt} + 3H\left(\rho + p\right) = \frac{d\rho}{dt} + H\rho \cdot n(w). \tag{A.1}$$

Substituting e-folds for cosmic time with

$$\frac{d\rho}{dt} = \frac{d\rho}{d\ln a}\frac{d\ln a}{dt} = H\frac{d\rho}{d\ln a} \tag{A.2}$$

gives then a differential equation

$$\frac{d\rho}{d\ln a} = -\rho \cdot n(w) \tag{A.3}$$

for $\rho(a)$ that is solved by

$$\rho(a) \propto a^{-n(w)}. \tag{A.4}$$

Inserting (2.18) into the Friedmann equation (2.16b) gives another differential equation for the scale factor $a(t)$

$$\frac{da}{dt} \propto a^{-\frac{n(w)}{2}+1} \tag{A.5}$$

assuming the case $\kappa = 0$. It has the solutions

$$a(t) \propto \begin{cases} t^{\frac{2}{n(w)}} & \text{for } w \neq -1 \\ e^{Ht} & \text{for } w = -1 \end{cases} \tag{A.6}$$

that are discussed in section 2.2.

A.2. Accelerating effect of a Cosmological Constant

Considering the acceleration equation (2.16a), any fluid species with

$$\rho + 3p < 0 \iff w < -\frac{1}{3} \tag{A.7}$$

will accelerate the expansion of the universe.

The cosmological constant in the Einstein equations (2.1) resembles a homogeneous energy density with stress-energy tensor

$$T_{\Lambda,\mu\nu} = -\Lambda g_{\mu\nu} \tag{A.8}$$

that, compared to the stress-energy tensor of a perfect fluid (2.14), corresponds to

$$p_\Lambda = -\Lambda \quad \text{and} \quad p_\Lambda = -\rho_\Lambda. \tag{A.9}$$

Thus, it has the phenomenology of a negative pressure or equation of state parameter

$$w_\Lambda = -1 \tag{A.10}$$

with the accelerating effect found in (A.7).

A.3. Decomposition of Metric Perturbations

The parametrization for the metric tensor perturbation

$$\delta g = -2A \, \mathrm{d}x^0 \otimes \mathrm{d}x^0 + B_i \left(\mathrm{d}x^0 \otimes \mathrm{d}x^i + \mathrm{d}x^i \otimes \mathrm{d}x^0 \right) + \left(2C\gamma_{ij} + 2E_{ij} \right) \mathrm{d}x^i \otimes \mathrm{d}x^j \tag{A.11}$$

for small spatial scalar fields $A(x^0)$ and $C(x^0)$, a vector field $B_i(x^0)$ and a symmetric, traceless tensor field $E_{ij}(x^0)$ is a choice to describe its 10 degrees of freedom. γ_{ij} denotes a Riemannian metric for the tree-dimensional spatial subspace.

The Helmholtz theorem allows us to decompose the parameters as [49, 57]

$$A = \tilde{A} \tag{A.12}$$

$$B_i = \nabla_i \tilde{B} + \tilde{B}_i \tag{A.13}$$

$$C = \tilde{C} \tag{A.14}$$

$$E_{ij} = \left(\nabla_i \nabla_j - \frac{1}{3} \gamma_{ij} \gamma^{\mu\nu} \nabla_\mu \nabla_\nu \right) \tilde{E} + 2\nabla_{(i} \tilde{E}_{j)} + \tilde{E}_{ij} \tag{A.15}$$

where ∇ denotes the Levi-Civita covariant derivative with respect to γ. $\tilde{B}$, $\tilde{C}$ and $\tilde{E}$ are scalar fields, $\tilde{B}_i$ and $\tilde{E}_i$ are divergence-free vector fields with

$$\nabla_i \tilde{B}^i = 0 \quad \text{and} \quad \nabla_i \tilde{E}^i = 0 \tag{A.16}$$

and $\tilde{E}_{ij}$ is a symmetric, traceless and divergence-free tensor field with

$$\tilde{E}_{ij} = \tilde{E}_{ji}, \quad \gamma^{ij} \tilde{E}_{ij} = 0 \quad \text{and} \quad \nabla^i \tilde{E}_{ij} = 0. \tag{A.17}$$

Reordering the metric tensor perturbation in this decomposition then gives

$$\delta g = \delta g^{\text{scalar}} + \delta g^{\text{vector}} + \delta g^{\text{tensor}} \tag{A.18}$$

$$\text{with} \quad \delta g^{\text{scalar}} = -2\tilde{A}\, \mathrm{d}x^0 \otimes \mathrm{d}x^0 + \nabla_i \tilde{B} \left(\mathrm{d}x^0 \otimes \mathrm{d}x^i + \mathrm{d}x^i \otimes \mathrm{d}x^0 \right)$$
$$+ 2 \left[\tilde{C} \gamma_{ij} + \left(\nabla_i \nabla_j - \frac{1}{3} \gamma_{ij} \gamma^{\mu\nu} \nabla_\mu \nabla_\nu \right) \tilde{E} \right] \mathrm{d}x^i \otimes \mathrm{d}x^j \tag{A.19}$$

$$\delta g^{\text{vector}} = \tilde{B}_i \left(\mathrm{d}x^i \otimes \mathrm{d}x^0 + \mathrm{d}x^0 \otimes \mathrm{d}x^i \right) + 4\nabla_{(i} \tilde{E}_{j)}\, \mathrm{d}x^i \otimes \mathrm{d}x^j \tag{A.20}$$

$$\text{and} \quad \delta g^{\text{tensor}} = 2\tilde{E}_{ij}\, \mathrm{d}x^i \otimes \mathrm{d}x^j. \tag{A.21}$$

When considering general coordinate transformation, only two scalar, one vector and the tensor contribution remain unaffected and constitute the *gauge invariant* quantities [49, 57].

Throughout the thesis, the tensor perturbations denoted by h_{ij} are related to the parametrization above by

$$h_{ij} = 2\tilde{E}_{ij} \tag{A.22}$$

and therefore fulfill the same conditions (A.17).

A.4. Bessel functions

The *Bessel differential equation*

$$f''(x) + (2p + 1)\frac{1}{x}f'(x) + \left(\alpha^2 + \frac{\beta^2}{x^2}\right)f(x) = 0 \tag{A.23}$$

has solutions [11, 12]

$$f(x) = x^{-p}\left[C_1 J_q(\alpha x) + C_2 Y_q(\alpha x)\right] \quad \text{with} \quad q = \sqrt{p^2 - \beta^2} \tag{A.24}$$

where $J_q(x)$ and $Y_q(x)$ denote the *Bessel functions of first and second kind*, respectively, that are given by [11, 12]

$$J_q(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)\Gamma(n+1+q)}\left(\frac{x}{2}\right)^{2n+q} \tag{A.25}$$

$$\text{and} \quad Y_q(x) = \frac{\cos(\pi q)\, J_q(x) - J_{-q}(x)}{\sin(\pi q)}. \tag{A.26}$$

Their asymptotic behaviour for large x is [11]

$$J_q(x) \to \sqrt{\frac{2}{\pi x}}\cos\left(x - \frac{2q+1}{4}\pi\right) + \mathcal{O}(x^{-\frac{3}{2}}) \tag{A.27}$$

$$Y_q(x) \to \sqrt{\frac{2}{\pi x}}\sin\left(x - \frac{2q+1}{4}\pi\right) + \mathcal{O}(x^{-\frac{3}{2}}). \tag{A.28}$$

B. On the Anthropic Principle

The anthropic principle states that if we wish to explain why our universe exists the way it does, the answer is that it must have qualities that allow intelligent creatures to arise who are capable of asking the question. As I am doing so eloquently right now.

—Sheldon Cooper, *The Big Bang Theory*[1]

In our quest to understand nature, we postulate theories. A good theory, from a physicist's point of view, is based on a number of assumptions that give rise to many observable predictions. The more opportunities we have to falsify a theory, the more robust and trustworthy it is considered, given none of the tests require us to discard it. The most accepted theories of nature share *first postulates*, such as the *principle of least action*, that have withstood falsification so consistently, we believe they form fundamental principles of nature. Whether such principles can *always* be traced back to an even more fundamental understanding of nature is subject of many discussions and a separate issue.

However, even our most successful physical theories, general relativity among them, include degrees of freedom that do not arise from first principles but must be determined experimentally. Particularly *fundamental physical constants*, such as the fine-structure constant[2] and the cosmological constant, are considered free parameters that would drastically change the appearance of the universe to the point where observers like us were unable to exist, if they were not *fine-tuned* to the particular value we observe. This problem is answered by employing a manifestation of the *anthropic principle* that requires our universe to assume these values based on the concept that *only a universe can be observed that allows observers to arise*.

---

[1]Quote from season 6, episode 1 of the CBS television series *The Big Bang Theory*

[2]The fine-structure constant, or Sommerfeld constant, determines the coupling strength of charged particles to the electromagnetic field in the standard model of particle physics.

Although the tautology in this statement is clear, it raises crucial questions about our ability to find fundamental laws of nature from our viewpoint within nature itself. However, the anthropic principle is easily abused and must be applied with care, since it inhibits any further scientific inquiries and provides no means of falsification for the theory when stated in the above sense. Historically, for instance, the *flatness problem* of non-inflationary cosmology prompted many anthropic arguments until the theory of cosmological inflation revealed opportunities for a natural solution. An anthropic approach is still a viable theory, although difficult to test, granted that nature allows for a sufficient number of domains with varying conditions such that the required environmental selection can occur [14].

Instead of resorting to the anthropic principle in search for answers, fine-tuning problems present the opportunity to reconsider the most fundamental premises of our theory. Particularly those assumed implicitly, such as the classical notions of space and time, are challenging to identify but crucial to reach a more profound understanding of nature.

Bibliography

[1] L.F. Abbott and Mark B. Wise. "Constraints on generalized inflationary cosmologies". In: *Nuclear Physics B* 244.2 (1984), pp. 541–548. ISSN: 05503213. DOI: `10.1016/0550-3213(84)90329-8`.

[2] E G Adelberger. "New tests of Einstein's equivalence principle and Newton's inverse-square law". In: *Classical and Quantum Gravity* 18.13 (July 2001), pp. 2397–2405. ISSN: 0264-9381. DOI: `10.1088/0264-9381/18/13/302`.

[3] Yashar Akrami, Tomi Sebastian Koivisto, and Marit Sandstad. "Accelerated expansion from ghost-free bigravity: A statistical analysis with improved generality". In: *Journal of High Energy Physics* 2013.3 (2013). ISSN: 11266708. DOI: `10.1007/JHEP03(2013)099`. arXiv: `1209.0457`.

[4] Luca Amendola, Guillermo Ballesteros, and Valeria Pettorino. "Effects of modified gravity on B-mode polarization". In: *arXiv preprint arXiv:1405.7004* 043009 (2014), pp. 1–14. ISSN: 1550-7998. DOI: `10.1103/PhysRevD.90.043009`. arXiv: `arXiv:1405.7004v1`.

[5] Luca Amendola, Frank Koennig, Matteo Martinelli, Valeria Pettorino, and Miguel Zumalacarregui. "Surfing gravitational waves: can bigravity survive growing tensor modes?" In: *astro-ph.CO* (2015), pp. 1–18. arXiv: `1503.02490`.

[6] Luca Amendola et al. "Cosmology and fundamental physics with the Euclid satellite". In: *Living Reviews in Relativity* 16.1 (2013). ISSN: 14338351. DOI: `10.12942/lrr-2013-6`. arXiv: `1206.1225`.

[7] P. Astone et al. "The gravitational wave detector NAUTILUS operating at T = 0.1 K". In: *Astroparticle Physics* 7.3 (Aug. 1997), pp. 231–243. ISSN: 09276505. DOI: `10.1016/S0927-6505(97)00023-6`.

[8] Daniel Baumann. "TASI Lectures on Inflation". In: (2009), p. 160. ISSN: <null>. DOI: `10.1142/9789814327183\_0010`. arXiv: `0907.5424`.

[9]   Emilio Bellini and Ignacy Sawicki. "Maximal freedom at minimum cost: linear large-scale structure in general modifications of gravity". In: *Journal of Cosmology and Astroparticle Physics* 2014.07 (2014), pp. 050–050. ISSN: 1475-7516. DOI: `10.1088/1475-7516/2014/07/050`. arXiv: `1404.3713`.

[10]   BICEP2/Keck and Planck Collaborations. "Joint Analysis of BICEP2/Keck Array and Planck Data". In: *Physical Review Letters* 114.10 (2015), pp. 1–17. ISSN: 0031-9007. DOI: `10.1103/PhysRevLett.114.101301`. arXiv: `arXiv:1502.00612v2`.

[11]   Mary L Boas. *Mathematical Methods in the Physical Sciences*. 3rd ed. John Wiley & Sons, 2005. ISBN: 0471198269.

[12]   Frank Bowman. *Introduction to Bessel Functions*. Dover Publications, 2010. ISBN: 0486604624.

[13]   C P Burgess. "The Cosmological Constant Problem: Why it's hard to get Dark Energy from Micro-physics". In: August 2013 (2013), p. 52. arXiv: `1309.4133`.

[14]   Sean M. Carroll. "Why is the Universe Accelerating?" In: 2 (2003), p. 22. ISSN: 0094243X. DOI: `10.1063/1.1848314`. arXiv: `0310342 [astro-ph]`.

[15]   Anthony Challinor. "CMB anisotropy science: a review". In: *Proceedings of the International Astronomical Union* 8.S288 (Aug. 2012), pp. 42–52. ISSN: 1743-9213. DOI: `10.1017/S1743921312016663`.

[16]   Timothy Clifton, Pedro G. Ferreira, Antonio Padilla, and Constantinos Skordis. *Modified gravity and cosmology*. 2012. DOI: `10.1016/j.physrep.2012.01.001`. arXiv: `1106.2476`.

[17]   Scott Dodelson. *Modern Cosmology*. 1st ed. Academic Press, 2003. ISBN: 0122191412.

[18]   Albert Einstein. "Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie". In: *Preussische Akademie der Wissenschaften, Sitzungsberichte* 2 (1915), pp. 831–839.

[19]   Albert Einstein. "Feldgleichungen der Gravitation". In: *Preussische Akademie der Wissenschaften, Sitzungsberichte* 2 (1915), pp. 844–847.

[20]   Albert Einstein. "Gravitationswellen". In: *Preussische Akademie der Wissenschaften, Sitzungsberichte* 1 (1918), pp. 154–167.

[21]   Albert Einstein. "Zur Elektrodynamik bewegter Körper". In: *Annalen der Physik* 17 (1905), pp. 891–921. URL: `http://onlinelibrary.wiley.com/store/10.1002/andp.19053221004/asset/19053221004%5C_ftp.pdf`.

[22] D. J. Fixsen. "The Temperature of the Cosmic Microwave Background". In: (2009), p. 6. DOI: `10.1088/0004-637X/707/2/916`. arXiv: `0911.1955`.

[23] G. Hobbs et al. "The international pulsar timing array project: using pulsars as a gravitational wave detector". In: (2009), pp. 1–11. ISSN: 0264-9381. DOI: `10.1088/0264-9381/27/8/084013`. arXiv: `0911.5206`.

[24] Wayne Hu. "Lecture Notes on CMB Theory: From Nucleosynthesis to Recombination". In: (2008), p. 39. arXiv: `0802.3688`.

[25] R. A. Hulse and J. H. Taylor. "Discovery of a pulsar in a binary system". In: *The Astrophysical Journal* 195 (Jan. 1975), p. L51. ISSN: 0004-637X. DOI: `10.1086/181708`.

[26] Jai-chan Hwang and Hyerim Noh. "Cosmological perturbations in generalized gravity theories". In: *Physical Review D* 54.2 (1996). ISSN: 05562821.

[27] L Ju, D G Blair, and C Zhao. "Detection of gravitational waves". In: *Reports on Progress in Physics* 63.9 (Sept. 2000), pp. 1317–1427. ISSN: 0034-4885. DOI: `10.1088/0034-4885/63/9/201`. arXiv: `0005058` [`arXiv:gr-qc`].

[28] F Könnig, Yashar Akrami, and Luca Amendola. "Stable and unstable cosmological models in bimetric massive gravity". In: *astro-ph.CO* l (2014), pp. 1–17. ISSN: 15502368. DOI: `10.1103/PhysRevD.90.124014`. arXiv: `arXiv:1407.4331v2`.

[29] LIGO Scientific Collaboration. "Advanced LIGO". In: *Classical and Quantum Gravity* 32.7 (2015), p. 074001. ISSN: 0264-9381. DOI: `10.1088/0264-9381/32/7/074001`.

[30] LIGO Scientific Collaboration and VIRGO Collaboration. "An upper limit on the stochastic gravitational-wave background of cosmological origin." In: *Nature* 460.7258 (2009), pp. 990–994. ISSN: 0028-0836. DOI: `10.1038/nature08278`. arXiv: `0910.5772`.

[31] Eric V. Linder. "Are Scalar and Tensor Deviations Related in Modified Gravity?" In: 2 (2014), p. 6. ISSN: 15502368. DOI: `10.1103/PhysRevD.90.083536`. arXiv: `1407.8184`.

[32] David Lovelock. "The Einstein Tensor and Its Generalizations". In: *Journal of Mathematical Physics* 12.3 (1971), p. 498. ISSN: 00222488. DOI: `10.1063/1.1665613`.

[33] JérÔme Martin. "Everything you always wanted to know about the cosmolog-
      ical constant problem (but were afraid to ask)". In: *Comptes Rendus Physique*
      13.6-7 (2012), pp. 566–665. ISSN: 16310705. DOI: `10.1016/j.crhy.2012.04.`
      `008`. arXiv: `1205.3365`.

[34] Stephen P. Martin. "A Supersymmetry Primer". In: September (1997), p. 152.
      DOI: `10.1142/9789814307505\_0001`. arXiv: `9709356 [hep-ph]`.

[35] Alberto Navarro and José Navarro. "Lovelock's theorem revisited". In: *Journal
      of Geometry and Physics* 61.10 (2011), pp. 1950–1956. ISSN: 03930440. DOI: `10.`
      `1016/j.geomphys.2011.05.004`. arXiv: `1005.2386`.

[36] Isaac Newton. "Philosophiae naturalis principia mathematica". In: (1686).

[37] Antonio Padilla. "Lectures on the Cosmological Constant Problem". In: *Inter-
      national Journal of Modern Physics D* 05.04 (2015), p. 31. ISSN: 0218-2718. DOI:
      `10.1142/S0218271896000278`. arXiv: `1502.05296`.

[38] A. A. Penzias and R. W. Wilson. "A Measurement of Excess Antenna Temper-
      ature at 4080 Mc/s." In: *The Astrophysical Journal* 142 (July 1965), p. 419. ISSN:
      0004-637X. DOI: `10.1086/148307`.

[39] Saul Perlmutter. "Supernovae, dark energy, and the accelerating universe". In:
      Physics Today 56.4 (2003), pp. 53–60. ISSN: 0031-9228. DOI: `10.1063/1.1580050`.

[40] Valeria Pettorino and Luca Amendola. "Friction in Gravitational Waves: a test
      for early-time modified gravity". In: *Physics Letters B* 742 (2014), p. 4. ISSN:
      03702693. DOI: `10.1016/j.physletb.2015.02.007`. arXiv: `1408.2224`.

[41] G Pizzella. "Resonant detectors for the search for gravitational waves". In:
      Classical and Quantum Gravity 14.6 (1999), pp. 1481–1485. ISSN: 0264-9381. DOI:
      `10.1088/0264-9381/14/6/014`.

[42] Planck Collaboration. "13 Planck 2015 results. XIII. Cosmological parameters".
      In: *arXiv* 1502.01589 (2015). arXiv: `1502.01589v2`.

[43] Planck Collaboration. "Planck 2013 results. XVI. Cosmological parameters".
      In: (2013), pp. 1–69. DOI: `10.1051/0004-6361/201321591`. arXiv: `1303.5076`.

[44] Planck Collaboration. "Planck 2015 results. I. Overview of products and scien-
      tific results". In: (Feb. 2015), p. 36. arXiv: `1502.01582`.

[45] Bharat Ratra and P. J. E. Peebles. "Cosmological consequences of a rolling ho-
      mogeneous scalar field". In: *Physical Review D* 37.12 (June 1988), pp. 3406–3427.
      ISSN: 0556-2821. DOI: `10.1103/PhysRevD.37.3406`.

[46] Marco Raveri, Alessandra Silvestri, and Shuang-Yong Zhou. "Measuring the speed of cosmological gravitational waves". In: 061501 (2014), p. 5. ISSN: 15502368. DOI: `10.1103/PhysRevD.91.061501`. arXiv: `1405.7974`.

[47] Claudia de Rham. "Massive Gravity". In: (2014), pp. 1–178. ISSN: 1433-8351. DOI: `10.12942/lrr-2014-7`. arXiv: `1401.4173`.

[48] Ippocratis D. Saltas, Ignacy Sawicki, Luca Amendola, and Martin Kunz. "Scalar anisotropic stress as signature of correction to gravitational waves". In: 1 (2014), p. 4. ISSN: 10797114. DOI: `10.1103/PhysRevLett.113.191101`. arXiv: `1406.7139`.

[49] Frederic Schulz. *Lecture Notes on General Relativity*. Winter School on Gravity and Light 2015, Linz, 2015.

[50] Bernard Schutz. *A First Course in General Relativity*. 2nd ed. Cambridge University Press, 2009. ISBN: 0521887054.

[51] Dominik J. Schwarz, David Bacon, Song Chen, Chris Clarkson, Dragan Huterer, Martin Kunz, Roy Maartens, Alvise Raccanelli, Matthias Rubart, and Jean-Luc Starck. "Testing foundations of modern cosmology with SKA all-sky surveys". In: (2015), pp. 1–11. arXiv: `1501.03820`.

[52] Alexei a. Starobinsky. *Spectrum of relict gravitational radiation and the early state of the universe*. 1979.

[53] Alexander Tolish. "General Relativity and the Newtonian Limit". In: (2010). URL: `http://www.math.uchicago.edu/~may/VIGRE/VIGRE2010/REUPapers/Tolish.pdf`.

[54] Stefano Vitale. "Space-borne gravitational wave observatories". In: *General Relativity and Gravitation* 46.5 (2014), p. 1730. ISSN: 0001-7701. DOI: `10.1007/s10714-014-1730-2`. arXiv: `1404.3136`.

[55] T a Wagner, S Schlamminger, J H Gundlach, and E G Adelberger. "Torsion-balance tests of the weak equivalence principle". In: *Classical and Quantum Gravity* 29.18 (2012), p. 184002. ISSN: 0264-9381. DOI: `10.1088/0264-9381/29/18/184002`. arXiv: `1207.2442`.

[56] J. Weber. "Detection and generation of gravitational waves". In: *Physical Review* 117.1 (1960), pp. 306–313. ISSN: 0031899X. DOI: `10.1103/PhysRev.117.306`.

[57] Steven Weinberg. *Cosmology*. Oxford University Press, 2008. ISBN: 0198526822.

[58] J. M. Weisberg and J. H. Taylor. "Observations of post-newtonian timing effects in the binary pulsar PSR 1913+16". In: *Physical Review Letters* 52.15 (1984), pp. 1348–1350. ISSN: 00319007. DOI: `10.1103/PhysRevLett.52.1348`.

[59] C. Wetterich. "Cosmology and the fate of dilatation symmetry". In: *Nuclear Physics B* 302.4 (June 1988), pp. 668–696. ISSN: 05503213. DOI: `10.1016/0550-3213(88)90193-9`.

[60] Lixin Xu. "Gravitational Waves: A Test for Modified Gravity". In: (2014), pp. 1–9. DOI: `10.1103/PhysRevD.91.103520`. arXiv: `1410.6977`.

Statement of Authorship

I herewith declare that this thesis was solely composed by myself and that it consti-
tutes my own work unless otherwise acknowledged in the text. I confirm that any
quotes, arguments or concepts developed by another author and all sources of in-
formation are referenced throughout the thesis. This work has not been accepted in
any previous application for a degree.

Heidelberg, July 22, 2015

_____

Signature