Modified gravity theories generally aim to solve part of the *cosmological constant problem* by providing self-accelerating cosmological solutions without a cosmological constant. Such modifications of general relativity also affect the evolution of gravitational waves in the proposed theory. Instead of focussing on a specific model, I introduce parametric modifications to the evolution equation of gravitational waves in both unimetric and bimetric settings and investigate their effect on the evolution of tensor perturbation modes. In particular, I argue that any modified gravity theory that exhibits *growing tensor modes* in cosmological evolution can be in tension with experiments. Therefore, parametric constraints for the physical viability of a general modified gravity theory can be found such that tensor modes remain within limits set by observations.

Contents

1. Introduction

For a long time, physicists believed space and time were both flat and fundamentally different concepts. In *Newtonian spacetime*, it is therefore straight forward to define notions such as the *length of a curve* and *simultaneity* [Tolish].

With Maxwell's theory of electromagnetism and experimental observations regarding the speed of light, however, many of these concepts had to be abandoned. Physicists realized the necessity to reconsider fundamental assumptions about space and time. Albert Einstein addressed many of these issues with his theory of special relativity in 1905, but, particularly, one striking observation remained unexplained:

summarize SR briefly, mention GR->SR for minkowskian metric

Newtonian mechanics postulates that particles accelerate under the influence of **any** force proportional to their *inertial mass* m_i. At the same time, the gravitational force a particle induces on another is proportional to its *gravitational mass* m_g that is completely unrelated to the former in this theory. Strikingly, however, experiments find inertial and gravitational mass indistinguishable from another — to this end, any two objects will fall with the same velocity in vacuum, no matter their weight — and physicists identify both as *mass* $m = m_i = m_g$ instead. This *equivalence principle* constitutes the basis of Einstein's 1915 theory of *general relativity* where gravity is not regarded as an external force but rather a phenomenon of the geometry of spacetime itself that is curved by its matter content. Therefore, the trajectories of freely moving particles appear affected by a gravitational force although they actually follow straight lines, or *geodesics*, in curved spacetime. In other words, gravity is only an inertial force observed in a cartesian reference frame that is induced by the curvature of spacetime in presence of mass.

order of magnitude?

General relativity not only succeeded in explaining phenomena at astronomical scales that stood in conflict with Newtonian gravity, such as the perihelion shift of Mercury and gravitational lensing, but also enabled the development of many high-precision technologies. Among them, navigational systems such as GPS are arguably the most prominent. Most remarkably, however, it allowed physicists to scientifically approach questions that concern the evolution of the entire universe

ref.

ref.

itself for the first time in the field of *cosmology*. Assuming only that, at sufficiently large scales, our universe looks the same everywhere and in all directions, notions such as the *beginning* and *age of the universe* became computable in cosmological models that could be compared to experimental data. For instance, distance measurements of galaxies and stars provided evidence of a recent accelerated expansion of our universe. One of the most important cosmological observations remains the high precision measurements of the *cosmic microwave background* that is thermal radiation originating in the early universe. These discoveries, together ref. with measurements of the matter composition in out universe and many more, lead to the ΛCDM (Lambda Cold Dark Matter) standard model of cosmology. In ref. this model, our universe went through a phase of exponential expansion, known as *cosmic inflation*, at very early times, followed by several cosmological eras where each is dominated by one of the different matter species in the universe. These include radiation, ordinary matter and the elusive *dark matter* and *dark energy*.

Gravitational waves are an intrinsic part of general relativity and the standard model of cosmology that occur when the geometry of spacetime is disturbed in a tensorial manner. For instance, binary systems of stellar objects such as stars or black holes of different mass orbiting each other generate such perturbations that will propagate through spacetime. Gravitational waves that originate in the early universe and could be measured in the cosmic microwave background are known as *primordial gravitational waves*. This thesis begins with a detailed discussion of gravitational waves in general relativity in section 1.1. Since attempts to measure gravitational waves, either directly by dedicated ground-based experiments or indirectly by precise measurements of the cosmic microwave background, have not been successful yet, their amplitude is constrained to be extremely small by the data at hand. ref.

Although the ΛCDM standard model of cosmology agrees with observational data remarkably well, it does not offer an entirely conclusive model of our universe. One of the shortcoming this model exhibits is related to its dark energy component that provides the accelerated expansion of our universe at recent times. It is given by a free parameter Λ known as the *cosmological constant* in this theory. In fact, the *cosmological constant problem* connects unsolved phenomena of both quantum field theory and general relativity and remains one of the greatest mysteries in theoretical physics of our time. It is formulated in section 1.2 and also introduces the concept of modified theories of gravity as an approach for solving part of it. Such theories generally propose alterations to general relativity that have self-accelerating late-time cosmological solutions without relying on a cosmological constant.

Any theory of gravity that goes beyond general relativity must also predict gravitational waves that agree with the experimental constraints mentioned above, however. In particular, modified gravity theories where gravitational waves grow in amplitude in cosmological evolution can easily be in tension with experiments. The main idea of this thesis is to find properties of modified gravity theories such that growing gravitational waves are avoided that could render the theory physically unviable. To achieve this, I introduce appropriate parameters in the evolution equation of gravitational waves that can result from general modifications of gravity. I will explore the effect these parametric modifications have on the evolution of gravitational waves in modified gravity theories in chapter 2. In addition to numerical solutions, I will solve the parametrized evolution equation analytically for appropriate assumptions to find constraints for the parameters such that the theory does not exhibit gravitational waves that grow in amplitude. Subsequently, I will focus this parametric approach on the extended class of *bimetric* models where a second spacetime metric in our universe interacts with the gravity-inducing physical metric and both metrics can exhibit gravitational waves. Finally, chapter 4 summarizes the results of this thesis.

Throughout the thesis, I choose natural units with the speed of light

$$c \equiv 1 \tag{1.1}$$

and a mostly-positive metric signature $(-1, 1, 1, 1)$.

1.1. Gravitational Waves

In this section, I will first give a brief introduction to general relativity and standard FLRW cosmology. The evolution equation for gravitational waves that is the main object of consideration in the remainder of this thesis then follows from metric tensor perturbations in such a universe.

2. Parametrization of Modified Gravitational Wave Evolution

Many theories of modified gravity are discussed in the literature and this thesis will not focus on one specific model. Instead, the remainder of the thesis will explore various parametric modifications to the evolution equation of gravitational waves (1.34) that can result from a modified gravity theory. For such a modified gravity theory to be physically viable, the metric tensor perturbations must remain within constraints set by observations. In particular, any theory that exhibits growing tensor modes in cosmological evolution should be regarded with serious doubt, as their amplitude would likely be large enough today to be excluded by experiments.

ref.

mention observations

In an $f(\mathcal{R})$ theory without anisotropic stress, for example, the evolution equation becomes [Xu, Hwang and Noh, 1996]

ref.

introduce better

check Xu ref.

$$\ddot{h} + \left(2 + \frac{\mathrm{d}\log F}{\mathrm{d}\log a}\right)\mathcal{H}\dot{h} + k^2 h = 0 \quad \text{with} \quad F := \frac{\mathrm{d}f(\mathcal{R})}{\mathrm{d}\mathcal{R}}, \tag{2.1}$$

thus adding an additional friction term. Furthermore, the propagation speed of gravitational waves in a modified gravity theory can deviate from the speed of light c [Amendola et al., 2014, Raveri et al., 2014]. I investigate the effects such modifications have on the evolution of gravitational waves by introducing appropriate parameters in (1.34). A reasonable parametric modification motivated by the above considerations is

$$\frac{\mathrm{d}^2 h}{\mathrm{d}t^2} + (3 + \alpha_{\mathrm{M}})\,\mathrm{H}\frac{\mathrm{d}h}{\mathrm{d}t} + \frac{{c_{\mathrm{T}}}^2 k^2}{a^2} h = 0 \quad \text{in cosmic time } t, \tag{2.2a}$$

$$\ddot{h} + (2 + \alpha_{\mathrm{M}})\,\mathcal{H}\dot{h} + {c_{\mathrm{T}}}^2 k^2 h = 0 \quad \text{in conformal time } \eta \tag{2.2b}$$

$$\text{or} \quad h'' + \left(\frac{\mathcal{H}'}{\mathcal{H}} + 2 + \alpha_{\mathrm{M}}\right) h' + \frac{{c_{\mathrm{T}}}^2 k^2}{\mathcal{H}^2} h = 0 \quad \text{in e-foldings } \log a \tag{2.2c}$$

where α_{M} denotes an additional friction term and c_{T} a deviation of the propagation speed from the speed of light. Both parameters may in general be time-dependent.

The standard behaviour of general relativity discussed in subsection 1.1.3 is recovered for $\alpha_{\rm M} = 0$ and $c_{\rm T} = 1$. This chapter will explore the evolution of gravitational waves in a modified gravity theory that reduces to a modification of one or both of these parameters to find constraints such that growing tensor modes are avoided.

2.1. Constant additional friction

I will first consider a constant friction term $\alpha_{\rm M} = const.$ in the parametrized evolution equation (2.2).

2.1.1. Numerical solution

Figure 2.1 shows the numerical solution of the parametrized evolution equation (2.2) for various values of $\alpha_{\rm M}$. The result was obtained with the `ParametricNDSolve` function in Mathematica where several assumptions were made:

First, initial conditions were chosen such that

$$h(a \to 0) = 1 \quad \text{and} \quad h'(a \to 0) = 0. \tag{2.3}$$

Furthermore, a standard FLRW background was assumed where the Hubble function is given by the Friedmann equation (1.24)

write the limit like this?

$$\mathrm{H}(t) = \mathrm{H}_0 \sqrt{\sum_i \Omega_i(t)}. \tag{2.4}$$

Only matter and dark energy contributions $\Omega_{\rm d}$ and Ω_Λ were considered here because they dominate in late-time cosmological regimes as discussed in subsection 1.1.2. Values for H_0 and $\Omega_{\rm d}(t_0)$ in

$$\Omega_{\rm d}(t) = \Omega_{\rm d}(t_0) a^{-3} \quad \text{and} \quad \Omega_\Lambda(t) = 1 - \Omega_{\rm d}(t_0) \tag{2.5}$$

were obtained from the **?** results given in **??**. Lastly, the physical amplitude

add reference to planck

$$h_{\rm phys}(a) = \sqrt{A_{\rm T} \left(\frac{k}{k_0}\right)^{n_{\rm T}}} \cdot h(a) \tag{2.6}$$

for gravitational waves was also computed from the **?** results given in **??**.

ref.

explain phys. ampl.!

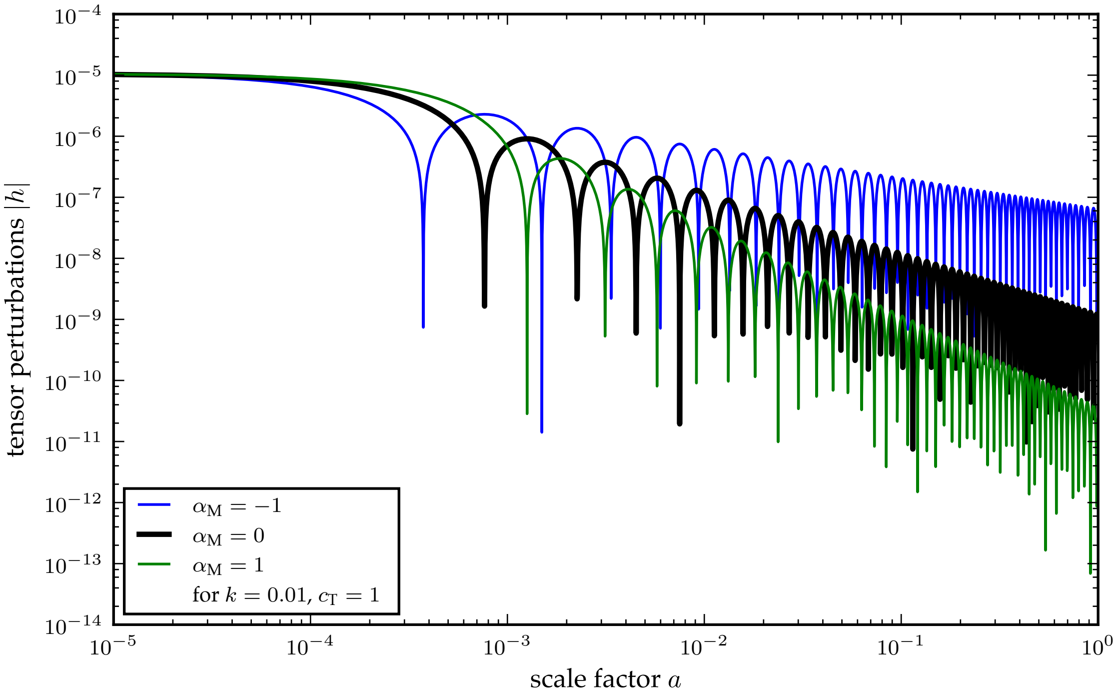

Figure 2.1.: **Tensor perturbations** $|h(a)|$ **with additional friction** $\alpha_M = const.$ Increasing α_M introduces more friction, so that the amplitude of the gravitational wave decreases more rapidly, but also delays the horizon entry.

In Figure 2.1, the absolute value $|h(a)|$ is plotted in double-logarithmic scale for both positive and negative values of α_M. The amplitude of the gravitational wave clearly decreases more rapidly for larger α_M since additional friction is introduced. However, also the horizon entry is delayed for larger α_M thus introducing a competing effect. This reproduces the behaviour found in Pettorino and Amendola [2014].

better word?

elaborate

write this here?

2.1.2. Analytic solution in cosmological regimes

The parametrized evolution equation (2.2) can even be solved analytically in cosmological regimes dominated by one of the matter types discussed in subsection 1.1.2. The expansion of an FLRW universe dominated by matter or radiation is given by (1.20) or

$$a(\eta) \propto \eta^{\tilde{n}(w)} \quad \text{with} \quad \tilde{n}(w) := \frac{2}{1+3w} \quad \text{in conformal time.} \tag{2.7}$$

This allows us to find an expression $\mathcal{H} = \frac{\dot{a}}{a} = \frac{\tilde{n}(w)}{\eta}$ for the Hubble function so that (2.2) becomes

$$\ddot{h} + (2 + \alpha_{\mathrm{M}}) \frac{\tilde{n}(w)}{\eta} \dot{h} + {c_{\mathrm{T}}}^2 k^2 h = 0. \tag{2.8}$$

This is a Bessel differential equation that has solutions in terms of Bessel functions as discussed in Appendix A.6. Thus, (2.8) is solved by

$$h(\eta) = \eta^{-p} \left[C_1 J_p(c_{\mathrm{T}} k\eta) + C_2 Y_p(c_{\mathrm{T}} k\eta) \right] \quad \text{with} \quad p = \tilde{n}(w) \left(1 + \frac{\alpha_{\mathrm{M}}}{2} \right) - \frac{1}{2} \tag{2.9}$$

where $J_p(x)$ and $Y_p(x)$ denote the Bessel functions of first and second kind, respectively. Their asymptotic behaviour for large modes k or late times η is given in Appendix A.6 and both correspond to oscillations with an amplitude decreasing as $\eta^{-\frac{1}{2}}$. Therefore, the amplitude of gravitational waves with additional friction α_{M} behaves as

$$h(\eta) \propto \eta^{-p-\frac{1}{2}} = \eta^{-\tilde{n}(w)\left(1+\frac{\alpha_{\mathrm{M}}}{2}\right)} \tag{2.10a}$$

$$\text{or} \quad h(a) \propto a^{-\frac{1}{\tilde{n}(w)}\left(p-\frac{1}{2}\right)} = a^{-\left(1+\frac{\alpha_{\mathrm{M}}}{2}\right)} \tag{2.10b}$$

times fast oscillation in this regime.

This result immediately gives a constraint for the α_{M} parameter such that growing tensor modes are avoided. Since (2.10b) exhibits a growing amplitude of $h(a)$ for a positive exponent $-\left(1 + \frac{\alpha_{\mathrm{M}}}{2}\right) > 0$ in cosmological expansion, any theory of gravity that modifies the evolution equation of gravitational waves (2.2) by an additional friction term $\alpha_{\mathrm{M}} = \mathit{const.}$ should fulfill

$$\alpha_{\mathrm{M}} \geq -2 \tag{2.11}$$

or else be regarded with serious doubt.

"in tension with exp."?

The numerical result obtained before reflects this behaviour as presented in Figure 2.2. The plot shows the evolution of tensor modes for both $\alpha_{\mathrm{M}} = -2$ and $\alpha_{\mathrm{M}} < -2$ using the same numerical solution as in Figure 2.1 as well as the slope of the amplitude obtained analytically in (2.10). As expected from the analytic considerations, the gravitational wave with $\alpha_{\mathrm{M}} = -2$ remains stable whereas the gravitational wave with $\alpha_{\mathrm{M}} < -2$ grows in amplitude.

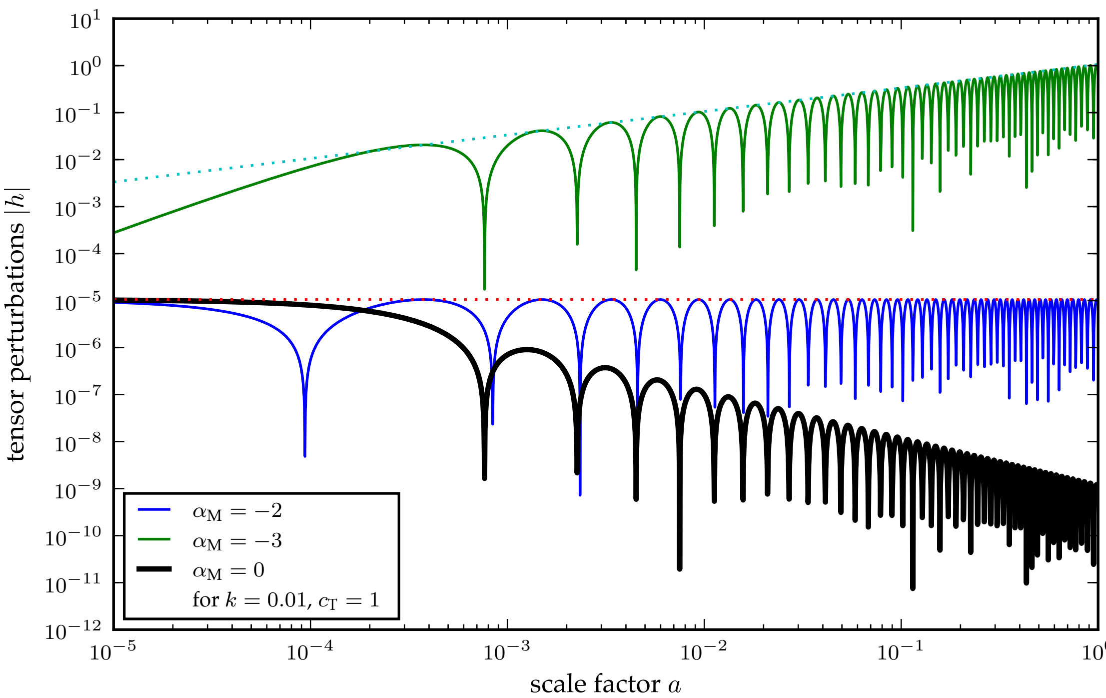

Figure 2.2.: **Tensor perturbations $|h(a)|$ with additional friction $\alpha_M \leq -2$** The numerical solution (solid lines) agrees with the slope of the amplitude obtained analytically (dotted lines). Gravitational waves with $\alpha_M = -2$ remain stable but grow in amplitude for $\alpha_M < -2$.

2.2. Deviating propagation speed

To explore the effect a modified propagating speed has on the evolution of gravitational waves in a modified gravity theory, I consider a parametric deviation c_T from the speed of light in the parametrized evolution equation (2.2). The deviation of the propagation speed as it appears in (2.2) changes the effective wavelength scale $\frac{1}{c_T k}$ such that the horizon entry of the gravitational wave occurs later for smaller values of c_T. The horizon entry is discussed in detail in the context of general relativity in subsection 1.1.2.

Figure 2.3 shows the evolution of gravitational waves for several values of $c_T =$ *const.*. The plots were obtained numerically with the `ParametricNDSolve` function in Mathematica where the same assumptions and initial conditions as in section 2.1 were chosen.

Because gravitational waves with lower propagation speed c_T exhibit a delayed horizon entry, their amplitude today is closer to their initial value than for gravitational waves that propagate faster. An additional friction contribution α_M as discussed in section 2.1 therefore is less effective for gravitational waves with

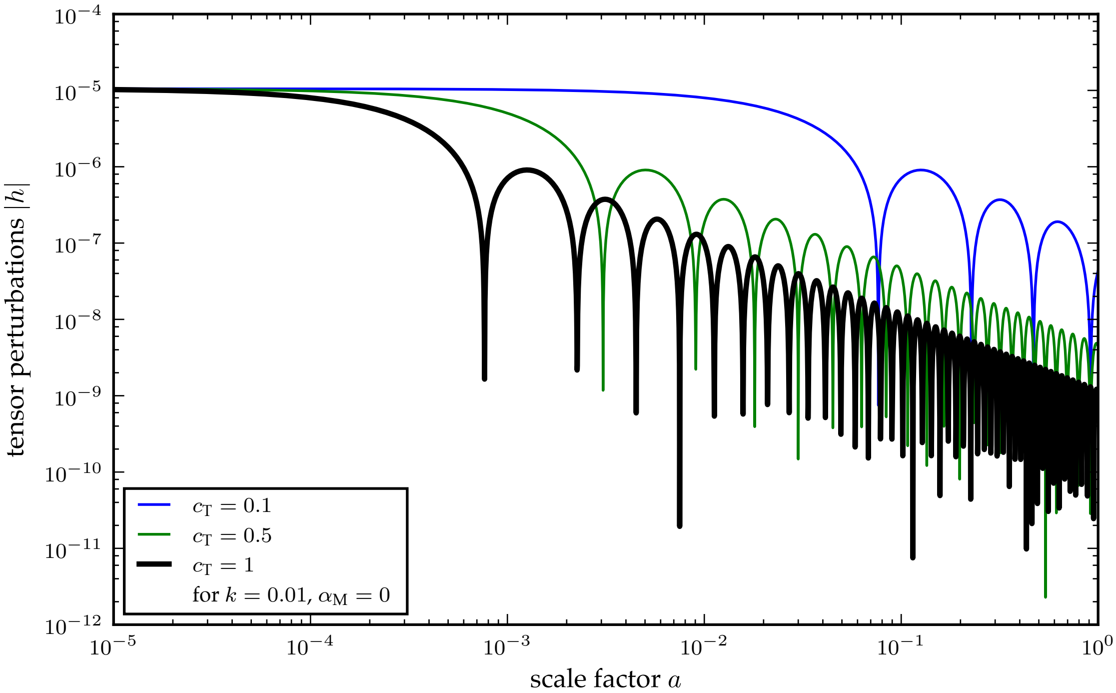

Figure 2.3.: **Tensor perturbations $|h(a)|$ with a modified propagation speed** $c_T =$ *const.* Gravitational waves with lower propagation speed c_T exhibit a delayed horizon entry.

lower c_T that enter the horizon later and thus have less time until today to decrease or increase in amplitude. This suggests a degeneracy between a modified propagation speed and an additional friction contribution, as both affect the amplitude of tensor perturbations measured today. A positive α_M that results in a faster decrease in amplitude can be compensated by a lower propagation speed c_T that in turn delays the horizon entry and thus gives the tensor mode less time to decrease. The amount growing tensor modes increase in amplitude is reduced by a lower propagation speed for the same reasons.

is cT > 1 possible?

2.3. Late-time additional friction

Many theories of modified gravity aim to solve the *new cosmological constant problem* presented in subsection 1.2.4. This requires such theories to model the accelerated expansion of our universe today as described in subsection 1.2.1. Proposed modifications to general relativity thus generally only affect late-time cosmological regimes where the ΛCDM standard model of cosmology relies on dark energy to accelerate the universe. Therefore, suitable parametrized modifications to the

add reference/examp maybe specific f(R)

evolution equation of gravitational waves (2.2) are time-dependent and vanish in early-time cosmological regimes to reflect this behaviour of modified gravity theories.

In particular, I consider for the additional friction α_{M} the time-dependent parametrization

$$\alpha_{\mathrm{M}}(t) = \alpha_{\mathrm{M0}} \cdot a(t)^{\beta} \quad \text{with} \quad \alpha_{\mathrm{M0}} = const. \quad \text{and} \quad \beta > 0. \tag{2.12}$$

In an expanding universe where $a(t) \in (0, 1]$ monotonically increases with time, the additional friction contribution α_{M} vanishes at early times to recover general relativity, but deviates from ΛCDM today.

Figure 2.4 shows numerical solutions to the evolution equation (2.2) with this particular parametrization for α_{M} for various values of both α_{M0} and β. It was obtained with the `ParametricNDSolve` function in Mathematica where the same initial conditions and assumption as in section 2.1 were chosen. The special case $\beta = 0$ corresponds to constant friction $\alpha_{\mathrm{M}}(t) = \alpha_{\mathrm{M0}}$ as investigated in section 2.1. Increasing β to positive values reduces the additional friction at early times where $a(t) < 1$. Larger values for β amplify this effect.

include Figure 4.1 with varying β for const α_{M} at all?

At late times, tensor modes will increase or decrease in amplitude with a slope approximating the behaviour of constant additional friction α_{M0} discussed in section 2.1. However, tensor modes that grow in this regime according to (2.11) can be suppressed by a sufficiently large value of β to remain physically viable.

explain more what this means

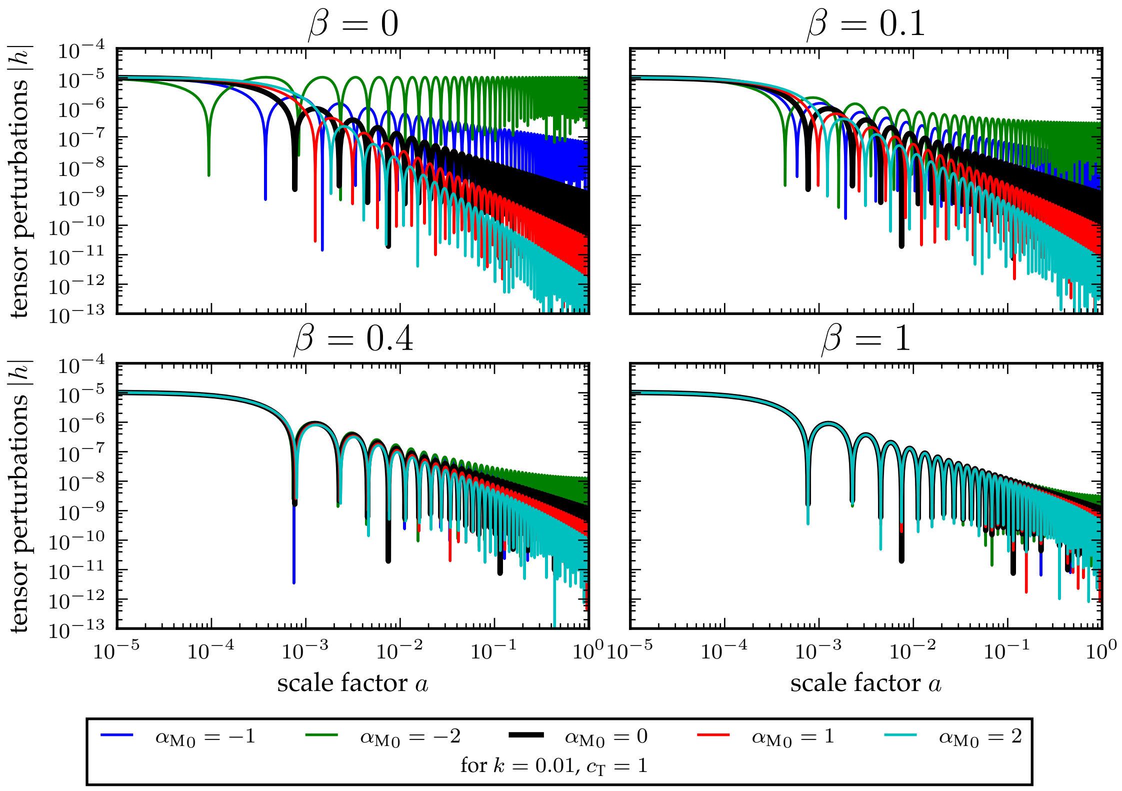

Figure 2.4.: **Tensor perturbations $|h(a)|$ with parametrized additional friction** $\alpha_M(t) = \alpha_{M0} \cdot a(t)^\beta$ For positive β, the evolution matches ΛCDM at early times and deviates at late times. Increasing $|\alpha_{M0}|$ yields larger deviations from ΛCDM at late times.

3. Gravitational Waves in Parametrized Bigravity

Bigravity is a class of modified gravity theories introduced in subsection 1.2.4 where a second *reference metric* f in addition to the physical metric g is considered. Matter only couples to the physical g-metric, but both metrics are coupled such that the evolution equation of gravitational waves in this bimetric setting becomes [Amendola et al., 2015]

$$\ddot{h}_n + (2 + \alpha_{\mathrm{M}n})\,\mathcal{H}\dot{h}_n + \left(\mathcal{H}^2 m_n^2 + c_{\mathrm{T}n}^{\,2} k^2\right) h_n = \mathcal{H}^2 q_n h_m \quad \text{in conformal time } \eta \tag{3.1a}$$

$$\text{or} \quad h_n'' + \left(\frac{\mathcal{H}'}{\mathcal{H}} + 2 + \alpha_{\mathrm{M}n}\right) h_n' + \left(m_n^2 + \frac{c_{\mathrm{T}n}^{\,2} k^2}{\mathcal{H}^2}\right) h_n = q_n h_m \quad \text{in e-foldings } \log a \tag{3.1b}$$

where the indices $n, m \in \{g, f\}$ with $n \neq m$ refer to the perturbations and parameters associated with the physical metric g and the reference metric f, respectively. The additional friction terms $\alpha_{\mathrm{M}n}$ and the deviations from the speed of light $c_{\mathrm{T}n}$ for each of the two metrics were discussed in chapter 2 in the context of unimetric modified gravity. Each metric also has an associated *mass parameter* m_n and a *coupling* q_n to the other metric. In the theory of bigravity, these parameters have specific, time-dependent forms that are given in Amendola et al. [2015]. The effect the additional metric and mass parameters have on the evolution of gravitational waves will be discussed in this chapter.

3.1. Coupling to the reference metric

Since the amplitude of physical tensor perturbations h_g is severely constrained by cosmological observations, I neglect the coupling of the reference metric f to h_g. Instead, I explore solutions of the parametrized bimetric evolution equation (3.1) with a non-zero coupling parameter $q_g = const.$. The non-standard behaviour of the reference metric that is governed by a choice of its associated parameters will therefore modify the evolution of the physical tensor perturbations h_g.

Figure 3.1 depicts several solutions of the parametrized bimetric evolution equation (3.1) that were obtained numerically with the `ParametricNDSolve` function in Mathematica. For both g and f, the same assumptions and initial conditions as in section 2.1 were chosen. In addition to the coupling $q_g = 1$, only the additional friction parameter of the f-metric was given a non-standard value such that its tensor perturbations grow in amplitude for $\alpha_{\mathrm{M}f} < -2$ as discussed in section 2.1.

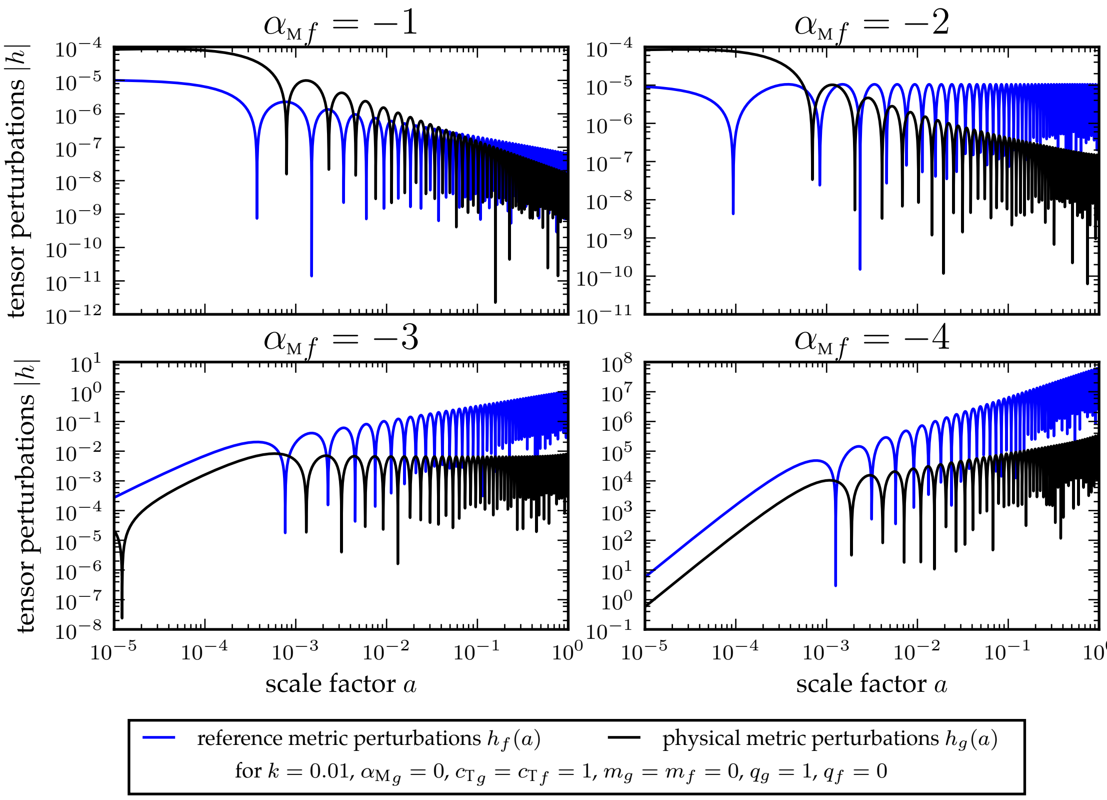

Figure 3.1.: **Physical and reference metric perturbations with parametrized coupling** $q_g = 1$ The amplitude of the physical metric perturbations follows the slope of the reference metric perturbations at late times. Note that the coupling q_g and the additional friction $\alpha_{\mathrm{M}f}$ of the f-metric are the only non-standard parameters.

Because matter only couples to the physical metric g whereas the reference metric f is not directly observable, tensor modes for the reference metric are not constrained by experiments. However, the coupling q_g also leads to non-standard behaviour of the physical metric, although its other associated parameters remain

a coupling between the metrics

unmodified. Figure 3.1 shows that the physical metric perturbations follow the slope of the reference metric perturbations at late times and also exhibit a growing amplitude for sufficiently negative $\alpha_{\mathrm{M}f}$ when coupled to the reference metric. In the specific bigravity model investigated in Amendola et al. [2015], a similar behaviour is found for explicit, time-dependent expressions for the parameters that were considered constant here. elaborate?

3.2. Non-zero mass parameter

The numerical solutions of the parametrized bimetric evolution equation (3.1) obtained in section 3.1 also allow for a brief discussion of the mass parameter m_n for both metrics $n \in \{g, f\}$. Figure 3.2 shows the evolution of the physical metric perturbations $|h_g(a)|$ for various values of its mass parameter m_g and all other parameters set to their standard value. Because the mass parameter appears as an additive contribution to the effective wavelength mode $c_{\mathrm{T}}k$ in (3.1), larger values of m_n will advance the horizon entry of the tensor mode. A degeneracy between the mass parameter and a deviating propagation speed c_{T} is to be expected. elaborate?

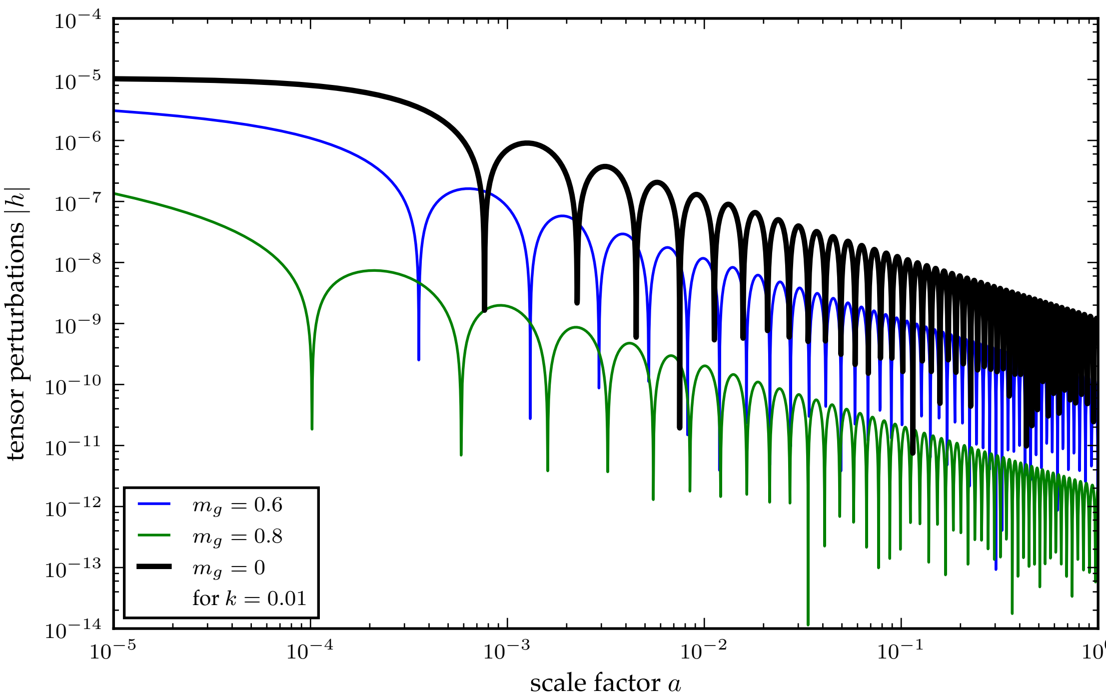

Figure 3.2.: **Tensor perturbations with non-zero mass parameter** m_g Due to the additive contribution to the effective wavelength mode $c_{\mathrm{T}}k$, increasing the mass parameter m_g will advance the horizon crossing of the gravitational wave.

3.3. Decoupling at early times

Bimetric modified gravity theories, like unimetric theories, usually deviate from ΛCDM only at late times to provide an accelerated expansion of the universe without a cosmological constant. This is discussed in more detail in section 2.3. For physically viable bimetric theories it is therefore reasonable to assume that the metrics decouple in early-time cosmological regimes where the physical metric g behaves unmodified but the reference metric can exhibit arbitrary dynamics. ref.

A specific bimetric model is investigated in Amendola et al. [2015], for example, where the physical and reference metric decouple at early times. The authors found that for sub-horizon modes in radiation and matter dominated eras also the mass parameters become negligible in this model. Therefore, the evolution of tensor perturbations for both metrics reduce to the unimetric parametrized evolution equation (2.2) with parameters [Amendola et al., 2015]

$$\alpha_{\mathrm{M}g} = 0 \qquad c_{\mathrm{T}g} = 1 \tag{3.2a}$$

$$\alpha_{\mathrm{M}f} = -3(1+w) \qquad c_{\mathrm{T}f} = \frac{(3w+1)^2}{4} \tag{3.2b}$$

where the physical metric remains entirely standard such that the amplitude of tensor perturbations h_g falls like $\frac{1}{a}$ according to (2.10). However, the reference metric tensor perturbations h_f with $\alpha_{\mathrm{M}f} < -2$ in (3.2b) grow in amplitude like a^1 in radiation domination and $a^{\frac{1}{2}}$ in matter domination.

When the coupling q_g becomes relevant at appropriate late times, the behaviour explored in section 3.1 must be taken into consideration such that the physical metric perturbations do not exceed observational constraints by coupling to growing reference metric tensor modes. A detailed analysis of tensor perturbation can provide crucial insight about the physical viability of bimetric theories. move to summary?