# Contents

# 1. Introduction

write intro

## 1.1. Gravitational Waves

write intro

### 1.1.1. Gravity and Spacetime in General Relativity

For a long time, physicists believed space and time were both flat and fundamentally different concepts. In *Newtonian spacetime*, it is therefore straight forward to define notions such as the *length of a curve* and *simultaneity*.

cite [Tolish]

With Maxwell's theory of electromagnetism and experimental observations regarding the speed of light, however, many of these concepts had to be abandoned. Physicists realized the necessity to reconsider fundamental assumptions about space and time. Albert Einstein addressed many of these issues with his theory of special relativity in 1905, but, in particular, one striking observation remained unexplained:

summarize SR briefly, mention GR->SR for minkowskian metric

Newtonian mechanics postulates that particles accelerate under the influence of **any** force proportional to their *inertial mass* $m_i$. At the same time, the gravitational force a particle induces on another is proportional to its *gravitational mass* $m_g$ that is completely unrelated to the former in this theory. Strikingly, however, experiments find inertial and gravitational mass indistinguishable from another — to this end, any two objects will fall with the same velocity, no matter their weight — and physicists identify both as *mass* $m = m_i = m_g$ instead. This *equivalence principle* constitutes the basis of Einstein's 1915 theory of *general relativity* where gravity is **not regarded as an external force** acting on particles in spacetime but rather a phenomenon of the geometry of spacetime itself, with **particles moving freely in curved spacetime**.

order of magnitude?

In general relativity, the *Einstein equations*

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi \mathrm{G_N} T_{\mu\nu} \tag{1.1}$$

with the *Einstein Tensor* $$G_{\mu\nu} = R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \mathcal{R}, \tag{1.2}$$

the *Ricci Scalar* $$\mathcal{R} = g^{\mu\nu} R_{\mu\nu}, \tag{1.3}$$

the *Ricci Tensor* $$R_{\mu\nu} = R^{\alpha}_{\alpha\mu\nu}, \tag{1.4}$$

the *Riemann Tensor* $$R^{\alpha}_{\beta\mu\nu} = \Gamma^{\alpha}_{\beta\nu,\mu} - \Gamma^{\alpha}_{\beta\mu,\nu} + \Gamma^{\alpha}_{\gamma\mu}\Gamma^{\gamma}_{\beta\nu} + \Gamma^{\alpha}_{\gamma\nu}\Gamma^{\gamma}_{\beta\mu} \tag{1.5}$$

and the *Christoffel Symbols* $$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left( g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu} \right) \tag{1.6}$$

relate the geometry of spacetime, encoded in the metric tensor $g_{\mu\nu}$, to the energy content of the universe $T_{\mu\nu}$. The *cosmological constant* $\Lambda$ is a free parameter in this theory and is discussed in detail in subsection 1.2.1.

We can obtain the Einstein equations through variation of the *Einstein-Hilbert action*

$$S[g] = \int \mathrm{d}^4 x \sqrt{-|g|} \, (\mathcal{R} - 2\Lambda) \quad \text{with} \quad |g| = \det g \tag{1.7}$$

with respect to $g_{\mu\nu}$. A detailed derivation is done in section A.1.

Given a stress-energy tensor $T_{\mu\nu}$, the Einstein equations constitute ten highly non-linear partial differential equations for the spacetime metric $g$. The metric, in turn, restores the notion of the length of a curve in spacetime and thus allows us to formulate postulates for the dynamics of matter in the universe.

why ten? > symmetric

In particular, as a generalization of Newton's law, particles without any external forces acted upon will move on *geodesics* in spacetime, i.e. their curve is stationary with respect to the length functional.

explain better

A simple solution to the Einstein equations is the flat *Minkowskian* spacetime metric

in vacuum?

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{bmatrix} -1 & 0 \\ 0 & \mathbb{1}_3 \end{bmatrix}_{\mu\nu} \tag{1.8}$$

of special relativity.

include this here, or elaborate?

### 1.1.2. FRW Cosmology

In cosmology, we strive to understand **how the entire universe evolves.**

elaborate

As of the *cosmological principle*, it is reasonable to assume the universe is *spatially homogeneous and isotropic* at large scales. This gives rise to six spatial symmetries and leads

include derivation with Killing vector fields?

to the *Friedmann-Robertson-Walker (FRW)* metric

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2\gamma_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j \quad \text{where} \quad \mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu \tag{1.9}$$

$$\text{and} \quad \gamma_{ij}(r,\theta,\phi) = \begin{bmatrix} \frac{1}{1-\kappa r^2} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 sin^2(\theta) \end{bmatrix}_{ij} \tag{1.10}$$

that is a particular solution to the Einstein equations describing a smooth, expanding universe. The *scale factor* $a(t)$ is the only freedom left after considering the symmetries and scales the time-independent metric $\gamma_{ij}$ of a spatial subspace with constant curvature $\kappa$. See section A.2 for a detailed derivation.

can the sub-space(?) be called spatial, or rather three-dim.?

It is important to note that distances described by the FRW coordinates are merely *coordinate distances* or *comoving distances* that remain constant even in an expanding universe. Comoving distances are scaled by the time-dependent scale factor $a(t)$ to obtain the physical distances. Similarly, one can define the *comoving* or *conformal time*

$$\eta = \int_0^t \frac{\mathrm{d}t'}{a(t')} \tag{1.11}$$

as the comoving distance light (with $\mathrm{d}s^2 = 0$) could have traveled since $t = 0$. The conformal time thus defines the causal structure in comoving coordinates and is also called the *comoving horizon*. It is often convenient to parametrize the evolution of the universe in conformal time $\eta$ instead of cosmic time $t$. Another parametrization we will use are *e-foldings* $\log a$. For the remainder of this thesis, derivatives by conformal time and e-foldings will be denoted by dots and primes, respectively, as in

$$\frac{\mathrm{d}}{\mathrm{d}\eta} \equiv ^{\cdot} \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}\log a} \equiv '. \tag{1.12}$$

Given the particular form of the spacetime metric, the Einstein tensor (1.2) can be explicitly computed. This is done in **??**. When we also assume the universe is filled with matter that, at large scales, resembles a perfect fluid with stress-energy tensor

add reference

$$T^{\mu\nu} = (\rho + p)\,u^\mu u^\nu + p g^{\mu\nu} \quad \text{in time direction} \quad u^\mu = (1,0,0,0)^T \tag{1.13}$$

and linear *equation of state*

this should be w equiv p/rho and need
what p and rho are

$$p = \omega\rho,$$

the Einstein equations (1.1) reduce to two ordinary, coupled differential equations

add reference

check lambda dependence

$$\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} = -\frac{4\pi}{3}\mathrm{G_N}\,(\rho + 3p)\,a + \frac{\Lambda}{3} \tag{1.15a}$$

*acceleration equation*

$$\mathrm{H}^2 = \frac{8\pi}{3}\mathrm{G_N}\rho - \frac{\kappa}{a^2} + \frac{\Lambda}{3} \tag{1.15b}$$

*Friedmann equation*

with the *Hubble function*

$$\mathrm{H} \equiv \frac{\mathrm{d}a}{\mathrm{d}t}\frac{1}{a} \quad \text{or} \quad \mathcal{H} \equiv \frac{\dot{a}}{a} = a\mathrm{H} \quad \text{in conformal time } \eta. \tag{1.16}$$

The *equation of state parameter* $\omega$ for various matter types is summarized in Table 1.1. For a universe filled with any such matter, we can solve the Friedmann equations (1.15) to find both the time evolution of the matter density

$$\rho \propto a^{-n(\omega)} \tag{1.17}$$

and of the scale factor

$$a(t) \propto \begin{cases} t^{\frac{2}{n(\omega)}} & \text{for } \omega \neq -1 \\ \mathrm{e}^{\mathrm{H}t} & \text{for } \omega = -1 \end{cases} \quad \text{with} \quad n(\omega) := 3\,(1+\omega)\,. \tag{1.18}$$

This result is derived in section A.3 and already exhibits a wealth of cosmological implications. In a universe filled with matter of $\omega \neq -1$ we find $\mathrm{H} \propto \frac{1}{t}$, for example, and can therefore identify it with a notion of the *age of the universe*. Furthermore, the relation $\rho \propto t^{-2}$ that follows from (1.17) in such a universe implies a singularity at $t = 0$ where the density becomes infinite. This is called the *big bang*.

define all species (d, lambda, ecc) and symbol

| | $i$ | $\omega$ | $n(\omega)$ |
|---|---|---|---|
| radiation | $\gamma$ | $\frac{1}{3}$ | 4 |
| dust | d | 0 | 3 |
| cosmological constant | $\Lambda$ | $-1$ | 0 |
| spatial curvature | $\kappa$ | $-\frac{1}{3}$ | $-2$ |

Table 1.1.: Overview of cosmological properties for a universe filled with various matter types

Written like this, this is pressure, not the equation of state

Multiple matter types in the universe combine to the total matter density

$$\rho(t) = \sum_i \rho_i(t) \quad \text{with equation of state} \quad p_i = \omega_i \rho_i \quad \text{each,} \tag{1.19}$$

where we can include the effect of a spatial curvature $\kappa$ and a cosmological constant $\Lambda$ through the definition of additional pseudo-densities

$$\rho_\kappa(t) = -\frac{3}{8\pi\mathrm{G_N}}\frac{\kappa}{a^2} \quad \text{and} \quad \rho_\Lambda(t) = \frac{\Lambda}{8\pi\mathrm{G_N}}. \tag{1.20}$$

When we then define the relative matter densities

explain

$$\Omega_i(t) := \frac{\rho_i(t)}{\rho_\mathrm{crit}(t)} \quad \text{with} \quad \rho_\mathrm{crit}(t) := \frac{3\mathrm{H}^2}{8\pi\mathrm{G_N}}, \tag{1.21}$$

the Friedmann equation (1.15b) becomes

$$\sum_i \Omega_i(t) = 1 \tag{1.22}$$

and we find from (1.17) the relation

and thus you assume it to be zero in the following?

$$\Omega_\Lambda \propto a^2 \Omega_\kappa \propto a^3 \Omega_\mathrm{d} \propto a^4 \Omega_\gamma . \tag{1.23}$$

Because the universe expands monotonically with time for any of these matter types, as given by (1.18), this result allows us to consider successive *cosmological regimes* in an FRW universe with a dominant matter type each. Radiation dominates in the early universe and is followed by a regime of dust domination (also called *matter domination*). The spatial curvature $\kappa$ is measured to be zero very precisely today, ~~thus eliminating the corresponding regime~~. Today, the universe is in a regime dominated by a cosmological constant (also called *de Sitter space*) that is discussed in more detail in subsection 1.2.1.

### 1.1.3. Tensor Perturbations in an FRW Universe

At smaller scales, the universe is not homogeneous and isotropic at all, of course. Galaxies, stars and planets, as well as radiation or, in fact, any energy content of the universe disturb the spacetime metric locally. It is therefore reasonable to consider perturbations $\delta g$ around the smooth FRW metric and their evolution.

also primordial / early universe / inflation and such

When we assume an exact solution $g$ to the unperturbed Einstein equations and consider a sufficiently small perturbation to the stress-energy tensor $\delta T_{\mu\nu}$, then the metric perturbation $\delta g$ that solves

$$G_{\mu\nu}[g + \delta g] = T_{\mu\nu}[g] + \delta T_{\mu\nu}[g] \quad \text{for} \quad 8\pi \mathrm{G_N} = 1 \tag{1.24}$$

..and lead to gravitational interaction

will also be small and we obtain

motivate better > baccigalupi p. 22

$$\delta G_{\mu\nu}[g, \delta g] = \delta T_{\mu\nu}[g] \tag{1.25}$$

with $\delta G_{\mu\nu}[g, \delta g]$ linear in $\delta g$ in linear perturbation theory.

Because of its symmetry condition, $\delta g$ has 10 degrees of freedom that we can parametrize as [Schulz]

really?

$$\delta g = -2A\, \mathrm{d}x^0 \otimes \mathrm{d}x^0 + B_i \left( \mathrm{d}x^0 \otimes \mathrm{d}x^i + \mathrm{d}x^i \otimes \mathrm{d}x^0 \right) + \left( 2C\gamma_{ij} + 2E_{ij} \right) \mathrm{d}x^i \otimes \mathrm{d}x^j \tag{1.26}$$

for small spatial *scalar fields* $A = A(x^0)$ and $C = C(x^0)$, a *vector field* $B_i = B_i(x^0)$ and a symmetric, trace-free *tensor field* $E_{ij} = E_{ij}(x^0)$.

add reference or derive

notation

As of the *Helmholtz theorem*, the parameters uniquely decompose further into scalar, vector and tensor components as

$$\delta g = \delta g^\text{scalar} + \delta g^\text{vector} + \delta g^\text{tensor} . \tag{1.27}$$

This is only true for linear perturbations

What do you mean?

This is shown in detail in section A.4 and allows us to study scalar, vector and tensor perturbations separately.

add info about scalar,

This paragraph is a bit confused

*Gravitational waves* now arise when we only consider **unsourced tensor perturbations** of the metric. This is analogous to the propagation of electromagnetic waves in vacuum, for example. In fact, scalar and vector perturbations can only arise from stress-energy perturbation. Therefore, only tensor perturbations remain for $\delta T_{\mu\nu} = 0$.

elaborate

add reference

It is shown in section A.4 that

$$\delta g_{ij}^{\text{tensor}} = h_{ij} \tag{1.28}$$

is a *symmetric, traceless, divergence-free tensor field* and can therefore be expressed in terms of two functions $h_\times$ and $h_+$ as

$$h_{ij} = \begin{bmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{bmatrix}_{ij} \tag{1.29}$$

with an implicit choice of axis.

elaborate choice of axis, show div/trace-free with wave-vector k

The perturbed line element becomes

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2 \left(\gamma_{ij} + h_{ij}\right) \mathrm{d}x^i\, \mathrm{d}x^j \tag{1.30}$$

and with this explicit form of the metric we can compute the Einstein tensor perturbation in (1.25). This is done in **??** and we obtain

$$\delta G_{ij} = \delta R_{ij} = \frac{3a^2}{2} \mathrm{H} \frac{\mathrm{d}h_{ij}}{\mathrm{d}t} + \frac{a^2}{2} \frac{\mathrm{d}^2 h_{ij}}{\mathrm{d}t^2} + \frac{k^2}{2} h_{ij}. \tag{1.31}$$

The perturbed Einstein equations (1.25) that govern the evolution of the metric perturbations then become a wave equation

damped oscillator, not wave!?

$$\frac{\mathrm{d}^2 h}{\mathrm{d}t^2} + 3\mathrm{H}\frac{\mathrm{d}h}{\mathrm{d}t} + \frac{k^2}{a^2} h = 0 \quad \text{in cosmic time } t \tag{1.32a}$$

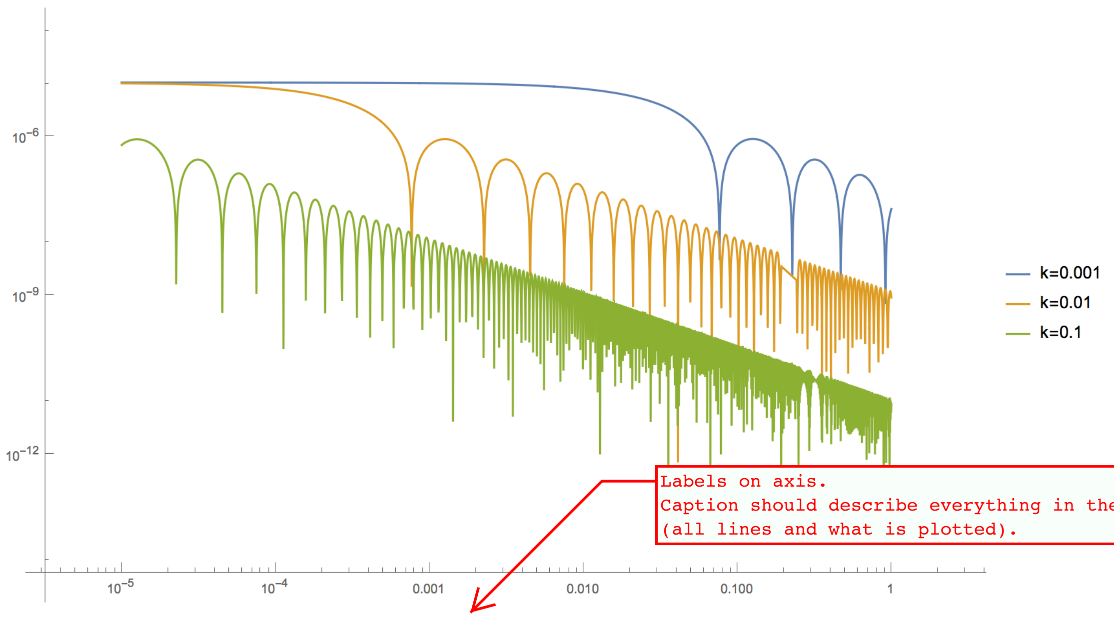
$$\text{or} \quad \ddot{h} + 2\mathcal{H}\dot{h} + k^2 h = 0 \quad \text{in conformal time } \eta \tag{1.32b}$$

for $h \in \{h_\times, h_+\}$. Its solutions are called *gravitational waves* and occur in two independent *polarizations* $h_\times$ and $h_+$. Gravitational waves are damped by the expansion of the universe as exhibited by (1.32).

improve plot!

observations, limits

We will further discuss gravitational waves in the context of *modified gravity* in **??** and the remainder of the thesis. However, to understand the reason why a modification of general relativity may be necessary, section 1.2 first formulates the *cosmological constant problem* and an approach for solving it.




Labels on axis.
Caption should describe everything in the plot
(all lines and what is plotted).

Figure 1.1.: The gravitational wave enters the horizon earlier for larger modes k.

## 1.2. The Cosmological Constant Problem

Both quantum field theory and general relativity are extremely well-tested theories and constitute the basis of modern physics, both in their respective fields. Whereas quantum field theory succeeds remarkably well in predicting particle physics phenomena, general relativity celebrates an equal success in large-scale cosmological observations. Both theories and particularly their interplay are not without mysteries, however.

better: not fully understood?

This section strives to formulate the *cosmological constant problem* and particularly emphasizes its problem of *radiative instability*. The quest for solutions will lead us to consider modified theories of gravity that will be the focus for the remainder of this thesis.

### 1.2.1. The Cosmological Constant

*Lovelock's theorem* states that general relativity as it emerges from the Einstein-Hilbert action 1.7 is, in fact, the **unique** metric theory in four spacetime dimensions that

add reference

- gives rise to second-order equations of motion
- for only one symmetric rank-2 tensor
- that is local and lorentz-invariant.

Bianchi-identities?

according to how you wrote, all sentences are
supposed to start with 'that'

would be good to explain what local and lorentz
invariant means

This includes a free parameter $\Lambda$ of the theory that is called the *cosmological constant*. Its value does not follow from the theory and thus it can only be constrained experimentally.

It appears in the Einstein equations 1.1 as a contribution

$$T_{\Lambda,\mu\nu} = -\Lambda g_{\mu\nu} \tag{1.33}$$

to the stress-energy tensor that corresponds to a **homogeneous energy density** penetrating the entire spacetime. In standard FRW cosmolo[...] an accelerating expansion of our universe as derived i[...]

Need to detail more on where this estimate comes from and under which assumption one gets to 120 orders of magnitude

In fact, cosmological observations clearly suggest such an accelerated expansion taking place at recent cosmic times. The $\Lambda$CDM ($\Lambda$ *cold dark matter*) standard model of cosmology therefore includes a cosmological constant on the order of $10^{-122}$ in Planck units and agrees with all cosmological observations remarkably well. In this theory, the physical origin of the homogeneous energy contribution that the cosmological constant represents remains a mystery, however, and is given the elusive name *dark energy*.

add references

really?

good word?

The fact that the cosmological constant must be fixed by observations is not remarkable on its own, of course, since also the gravitational constant $G_N$ must be determined experimentally. Theories such as the standard model of particle physics include a number of such free parameters. This poses an entirely different, ~~somewhat philosophical~~ problem that is often naively answered using anthropic arguments. This is discussed in detail in B.

add reference

### 1.2.2. Contributions to the Cosmological Constant

The cosmological constant problem arises when we consider both classical and quantum phenomena that, to our knowledge, should contribute to the cosmological constant.

Already in basic quantum mechanics, the uncertainty principle requires every physical system to have a zero-point energy. This immediately carries over to quantum field theory, where a (free) field is an infinite collection of coupled quantum mechanical harmonic oscillators. With a zero-point energy each, they combine to an infinite *vacuum energy* $\rho_{\text{vac}}$.

reference

add some derivation?

In quantum field theory, the vacuum energy is largely ignored as only differences in energy determine the dynamics of the system. In general relativity, however, all energy content gravitates, including the vacuum energy.

reference

how about Lamb shift, Casimir effect?

Remarkably, assigning an importance to not only energy differences, but also absolute energy values gives rise to another, entirely classical contribution to the stress-energy tensor, namely the *zero-point potential* $V_0$. This is the energy

$$T_{\mu\nu} = -V_0 g_{\mu\nu} \tag{1.34}$$

1.2.2 and 1.2.3 seem a bit less linearly explained then the others and stand a bit separated from the rest of the discussion. Integrate them more within the text and simplify where the kinetic energy vanishes and the potential assumes its minimum value $V_0$. This zero-point potential is usually chosen arbitrarily when only energy differences are considered but must be taken into account in general relativity. Particularly in presence of phase transitions, one can generally not choose the zero-point potential such that it always vanishes.

The free parameter $\Lambda$ in the Einstein equations therefore combines with both the quantum vacuum energy $\rho_{\text{vac}}$ and the classical zero-point potential $V_0$ of every quantum field in the universe to an *effective* cosmological constant

$$\Lambda_{\text{eff}} = \Lambda + \rho_{\text{vac}} + V_0 \tag{1.35}$$

that we measure as dark energy. In comparison to the small value for $\Lambda_{\text{eff}}$ we observe today, however, both contributions are extremely large [Martin, 2012]. This already suggests a severe *fine-tuning* problem where the value of the original cosmological constant $\Lambda$ must precisely cancel the other contributions up to the small value we measure today. This is not the entire cosmological constant problem yet, though.

add reference

elaborate fine-tuning

### 1.2.3. Radiative Instability

The full scope of the problem arises when we consider in more detail the vacuum energy that we found to be infinite before. The mechanism to make sense of divergencies like this in quantum field theory is the framework of *renormalization*. In the process to find a find finite, *renormalized* value for $\rho_{\text{vac}}$ one generally adds counterterms for every order in perturbation theory that each depend on an *arbitrary subtraction scale*.

Generally, successive orders are not significantly suppressed by a sufficiently small perturbation parameter $\Lambda$, however. In fact, for the standard model Higgs field the self-coupling parameter of perturbation $\Lambda$ is of the order $10^{-1}$ and therefore every order in perturbation theory must be renormalized independently. This *radiative instability* requires us to fine-tune the cosmological constant **repeatedly** for every order in perturbation theory and thus makes it sensitive even to small-scale physics where we assume our theory to break down [Datta, 1996].

add reference

This also prohibits us from finding an *effective theory* where the full structure of perturbation theory is encoded in one finite, renormalized value by means of a *Wilson effective action*. Again, we find the renormalized vacuum energy is unstable against changes in the unknown UV-regime of the theory [Datta, 1996].

add reference

how about *why now?*

how about naturalness <-> anthropic Datta [1996]?

ask Wetterich > fixed points

### 1.2.4. The *New* Cosmological Constant Problem and Modified Gravity

The cosmological constant problem is deeply rooted in our inability to find a renormalized vacuum energy that is stable against changes in its effective description. Therefore, an approach to the problem is to assume that some mechanism makes the vacuum energy vanish altogether instead and then find another theory that explains the non-zero cosmological constant we observe today.

In fact, unbroken *supersymmetry* would accomplish just that. In supersymmetry, bosons and fermions are related by a symmetry and share the same mass. Supersymmetric partners contribute to the vacuum energy with opposite signs, however, thus precisely canceling each other.

With the cosmological constant set to zero, the cosmological observation of accelerated expansion remains to be explained by a different mechanism. Dark energy models such as the *quintessence* theory postulate further contributions to the stress-energy tensor that have the same accelerating effect as a cosmological constant. A different approach is to modify the theory of gravity instead.

other observables?

add reference

Every attempt towards a modified theory of gravity has to take Lovelock's theorem into consideration and break at least one of its assumptions that make general relativity unique. The concept of the entire class of $f(\mathcal{R})$ *theories*, for example, is to replace the Ricci scalar $\mathcal{R}$ in the Einstein-Hilbert action (1.7) by a function $f(\mathcal{R})$ that breaks the assumption of only second-order derivatives.

Alternatively, the ~~very natural~~ hypothesis of a *massive graviton* requires us to introduce a second, arbitrary *reference metric* $f$ in addition to the physical metric $g$ that ~~clearly~~ breaks the second assumption in Lovelock's theorem. By simply assuming the reference metric to behave dynamically just like $g$, we arrive at the theory of *massive bigravity* that allows solutions with late-time acceleration.

add reference, explain: necessary for interaction terms

add reference

connect to next chapter

# 2. Parametrization of Modified Gravitational W

Introduce a bit better this paragraph:
- as discusssed in Amendola etal 2014, the general tensor equatin can be modified in two main ways
- an example is provided by f(R ) theories, where ...
- what we do is to generalize this parametrization and consider ...

Many sed in the literature and this thesis will not focus on one specific model. Instead, the remainder of the thesis will explore various parametric modifications to the evolution equation of gravitational waves (1.32) that can result from a modified gravity theory. For such a modified gravity theory to be physically viable, the metric tensor perturbations must remain within constraints set by observations. In particular, any theory that exhibits **growing** tensor modes in cosmological evolution should be regarded with serious doubt, as their amplitude would likely be large enough today to be ~~detected~~ excluded by experiments.

add reference

Add ref to the bigravity paper we did to test tensor modes, for example.

In an $f(\mathcal{R})$ theory without anisotropic stress, for example, the evolution equation becomes

$$\ddot{h} + \left(2 + \frac{\mathrm{d}\log F}{\mathrm{d}\log a}\right)\mathcal{H}\dot{h} + k^2 h = 0 \quad \text{with} \quad F := \frac{\mathrm{d}f(\mathcal{R})}{\mathrm{d}\mathcal{R}},$$

thus adding an additional friction term [Xu]. Furthermore, the speed of waves in a modified gravity theory can deviate from the speed of light et al., 2014, Raveri et al., 2014]. I investigate the effects such modification evolution of gravitational waves by introducing appropriate parameters in (1.32). A reasonable parametric modification motivated by the above considerations is

Pettorino Amendola 2014

If this Xu is the ref which is most probably wrong that you showed us, then remove it

$$\frac{\mathrm{d}^2 h}{\mathrm{d}t^2} + (3+\alpha_M)\,\mathrm{H}\frac{\mathrm{d}h}{\mathrm{d}t} + \frac{c_\mathrm{T}^2 k^2}{a^2}h = 0 \quad \text{in cosmic time } t, \tag{2.2a}$$

$$\ddot{h} + (2+\alpha_\mathrm{M})\,\mathcal{H}\dot{h} + c_\mathrm{T}^2 k^2 h = 0 \quad \text{in conformal time } \eta \tag{2.2b}$$

$$\text{or} \quad h'' + \left(\frac{\mathcal{H}'}{\mathcal{H}} + 2 + \alpha_\mathrm{M}\right)h' + \frac{c_\mathrm{T}^2 k^2}{\mathcal{H}^2}h = 0 \quad \text{in e-foldings } \log a \tag{2.2c}$$

where $\alpha_\mathrm{M}$ denotes an additional friction term and $c_\mathrm{T}$ a deviation from the speed of light. Both parameters may in general be time-dependent. The standard behaviour of general relativity discussed in subsection 1.1.3 is recovered for $\alpha_\mathrm{M} = 0$ and $c_\mathrm{T} = 1$.

How is this affecting constraints we have on cT? Explain if we are allowed to go to both higher and lower values with respect to cT = 1 Comment more on the effect of this two parameters

## 2.1. Constant Friction

I will first consider a constant friction term $\alpha_\mathrm{M} = \text{const.}$ in (2.2). [Pettorino and Amendola, 2014].