

Shift-share Designs

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Setting

- Observations $i = 1, \dots, N$ corresponding to regions (commuting zones (CZs), counties etc). Interested in effect of some treatment D_i on outcome Y_i .
- Construct regional instruments X_i by combining national-level **shifters** (or shocks) \mathcal{X}_s to different sectors s (e.g. industries) with regional **shares** (or weights) w_{is} ,

$$X_i = \sum_{s=1}^S w_{is} \mathcal{X}_s, \quad w_{is} \geq 0.$$

So X_i is a **shift-share instrument**.

Key issues

How do we think about: (i) identification and (ii) inference?

Examples

- Bartik (1991) or Blanchard and Katz (1992), want to estimate inverse labor supply elasticity
 - Y_i : log change in wages; D_i : log change in employment. Need instrument for labor demand. X_s : employment growth of industry s ; w_{is} : lagged employment shares.
- Autor, Dorn, and Hanson (2013, ADH) want effect of Chinese imports on US labor markets.
 - Y_i : local labor market outcome (employment, wages, ...); D_i measures rise of Chinese imports to location i . Weak labor markets may be more likely to import. X_s : growth of Chinese exports to non-US countries in industry s ; w_{is} : lagged industry shares.
- Effects of immigration (e.g. Card 2001) on natives, or other outcomes
 - D_i : local immigration. i can be region-skill group cell, national-level education-experience cell, or simply region. w_{is} : share of immigrants from country s ; X_s : (normalized) change or growth in # of immigrants from country s . Very similar to the logic of Bartik (1991).
- Also: effects on local labor markets of trade competition, technological change, credit supply; impact of sectoral shocks on marriage patterns, crime levels, innovation... See, e.g., Adão, Kolesár, and Morales (2019) for partial list.

Identification

Estimation and Inference

Empirical Applications

Need a sense in which X_i is as good as randomly assigned. Two approaches:

1. View w_{is} as randomly assigned. Explored in Goldsmith-Pinkham, Sorkin, and Swift (2020). Can then think of setup as overidentified instrumental variables (IV) model, with X_i as one of many possible ways of combining “instruments” w_{is} .
 - If we believe exogeneity of w_{is} , no issues with estimation of inference, so long as (i) $n \rightarrow \infty$, and (ii) we can form groups of regions across which w_{is} are independent.
 - Can try to “open the black box” of the constructed instrument by looking at the implicit weights X_i puts on each share, called “Rotemberg weights” in this context; IV analog to leverage analysis in ordinary least squares (OLS).
 - But shares w_{is} not plausibly exogenous in many applications: worry there are shift-share terms with structure similar to X_i in the structural residual (will explore below).
2. View X_s as randomly assigned. First proposed by a working-paper version of Borusyak, Hull, and Jaravel (2022); advocated for by Adão, Kolesár, and Morales (2019). Focus on here.

Stylized economic model: Setup i

To serve as background to help think through econometric issues, useful to consider stylized model (taken from Adão, Kolesár, and Morales 2019).

- Economy with multiple sectors $s = 1, \dots, S$ and regions $i = 1, \dots, J$
- Labor demand and supply depend on wage ω_i :

$$\text{Demand of } s \text{ in } i: \quad \log L_{is} = -\sigma_s \log \omega_i + \log D_{is}, \quad \sigma_s > 0,$$

$$\text{Supply in } i: \quad \log L_i = \phi \log \omega_i + \log S_i, \quad \phi > 0,$$

σ_s and ϕ : labor demand and supply elasticities; ω_i : wage; S_i : supply shifter;

- Decompose labor demand shifter D_{is} into observed shifter of interest χ_s and potentially unobserved components: $\log D_{is} = \rho_s \log \chi_s + \log u_s^D + \log u_{is}^D$.

- Also decompose labor supply shocks into group-specific shifters:

$$\log S_i = \sum_{g=1}^G \tilde{w}_{ig} \log u_g^S + \log u_i^S,$$

- Workers assumed immobile across regions, freely mobile across sectors \implies market clearing:
 $L_i = \sum_{s=1}^S L_{is}$ for $i = 1, \dots, J$.
- Assume $\{\hat{\chi}_s, \hat{u}_s^D, \hat{u}_{is}^D, \hat{u}_g^S, \hat{u}_i^S\}_{i,s,g} \sim F$, where $\hat{z} = \log(z^t/z^0)$.

Equilibrium

- Up to first-order approximation around initial equilibrium

$$\hat{L}_i = \sum_{s=1}^S l_{is}^0 (\gamma_{is} \hat{\chi}_s + \lambda_i \hat{u}_s^D + \lambda_i \hat{u}_{is}^D) + (1 - \lambda_i) \left(\sum_{g=1}^G \tilde{w}_{ig} \hat{u}_g^S + \hat{u}_i^S \right),$$

l_{is}^0 : initial employment share, $\lambda_i \equiv \phi [\phi + \sum_s l_{is}^0 \sigma_s]^{-1}$, and $\gamma_{is} \equiv \rho_s \lambda_i$ (recall ρ_s is elasticity of demand wrt χ_s). Similar to a reduced-form regression considered in Autor, Dorn, and Hanson (2013).

- Solving for wages yields regression like that in Bartik (1991) or Blanchard and Katz (1992)

$$\hat{\omega}_i = \frac{1}{\phi} \hat{L}_i - \frac{1}{\phi} \left(\sum_{g=1}^G \tilde{w}_{ig} \log u_g^S + \log u_i^S \right)$$

Can use demand shifters $\hat{\chi}_s$ to estimate $1/\phi$ with IV, since $\phi^{-1} = (\partial \hat{\omega}_i / \partial \hat{\chi}_s) / (\partial \hat{L}_i / \partial \hat{\chi}_s)$

Key takeaways

1. Change in regional employment and wage changes both combine multiple shift-share terms; complicated correlation structure in residuals
2. Shifter effects depend on parameters that are heterogeneous across i and s .

Focus on reduced-form effect of X_i onto Y_i for clarity of argument.

- As always, first ask: what do we mean by “effect”? What’s the ideal experiment here?
- Use potential outcomes, $Y_i(\chi_1, \dots, \chi_s) = Y_i(0) + \sum_s w_{is} \chi_s \beta_{is}$. Observed outcome is $Y_i(\mathcal{X}_1, \dots, \mathcal{X}_s)$.
 - Exercise: how would one define potential outcomes in Goldsmith-Pinkham, Sorkin, and Swift (2020)?
What’s the idealized experiment? Can we think of an economic model with such a structure?
 - ADH example: What would regional outcomes be if we assign different shocks \mathcal{X}_s to Chinese export growth? Posit effect proportional to regional employment exposure to industry s , w_{is} .
 - For studying effect of demand shocks $\hat{\chi}_s$ in stylized model,

$$Y_i = \hat{L}_i, \quad w_{is} = l_{is}^0, \quad \chi_s = \hat{\chi}_s, \quad \beta_{is} = \gamma_{is},$$

and $Y_i(0) = \lambda_i \sum_{s=1}^S w_{is} (\hat{u}_s^D + \hat{u}_{is}^D) + (1 - \lambda_i) (\sum_{g=1}^G \tilde{w}_{ig} \hat{u}_g^S + \hat{v}_i)$ aggregates all shifters other than $\hat{\chi}_s$.

- w_{is} are equilibrium objects, condition on them throughout.

- With no controls, we need \mathcal{X} to be as good as randomly assigned, in line with discussion in OLS section of the course: $E[\mathcal{X}_s \mid \mathcal{F}_0] = 0$, $\mathcal{F}_0 = (Y(0), B, W)$. We use matrix notation: A has N rows, \mathcal{A} has S rows, B has elements β_{is} , W has elements w_{is} . OLS estimand:

$$\beta = \frac{\sum_i E[X_i Y_i \mid \mathcal{F}_0]}{\sum_i E[X_i^2 \mid \mathcal{F}_0]} = \frac{\sum_{i=1}^N \sum_{s=1}^S \pi_{is} \beta_{is}}{\sum_{i=1}^N \sum_{s=1}^S \pi_{is}}, \quad (1)$$

where $\pi_{is} = w_{is}^2 \text{var}(\mathcal{X}_s \mid W)$.

- Treats population of interest to be sample at hand. Answers what we'd be estimating in idealized experiment where \mathcal{X}_s randomly assigned.
- Although shares endogenous, we exploit that variation in shares w_{is} generates variation in exposure to exogenous shocks \mathcal{X}_s : this is basis for identification.

- If have controls with exact shift-share structure, $Z_i = \sum_s w_{is} \mathcal{Z}_s$, then need to assume (why?)

$$E[\mathcal{X}_s \mid \mathcal{Z}, Y(0), B, W] = \mathcal{Z}'_s \gamma.$$

Important: implies that if $\sum_s w_{is} \neq 1$, need to include $\sum_s w_{is}$ as control (why?)

With controls, weights in (1) become $\pi_{is} = w_{is}^2 \text{var}(\mathcal{X}_s \mid W, \mathcal{Z})$, and X_i is replaced by residual after projecting it off Z_i .

- What if controls don't have exact shift-share structure? One option is to assume that such controls “proxy” for unobserved sectoral shocks, see Adão, Kolesár, and Morales (2019) for formalization of this idea.
- **Research question:** is this the best way of thinking about the issue? Can we give some other explanation for why not run shift-share regressions at sectoral level?

1. Is β an interesting object? Is it policy relevant?
 - Analogous to policy relevance of OLS estimand in OLS lecture.
2. What are we estimating in the presence of cross-regional spillovers?
 - Analogous to relevance of comparing treated and control outcomes in an experiment where we worry about peer effects or other types of spillovers. See discussion of Stable unit treatment value assumption (SUTVA) in lecture on OLS.

We will not address these concerns here

Identification

Estimation and Inference

Empirical Applications

- In stylized economic model, $Y_i(0)$ (and hence regression residual) incorporates terms that have shift-share structure, with shares that could be identical to (other demand shocks u_s^D) or different from (supply shocks to occupational groups, u_g^S) w_{is} . Correct inference requires taking into account potential cross-regional correlation in residuals across observations with similar values of w_{is} .
- Regions with similar sectoral employment shares $\{w_{is}\}_{s=1}^S$ also tend to have similar regression residuals and hence similar $X_i \epsilon_i$. Correlation independent of regions' geographic location \implies not captured by clustering on geographic units.

Placebo evidence

- Use 2000–2007 observed changes in labor market outcomes $\{Y_i\}_{i=1}^n$ and 1990 employment shares $\{w_{is}\}_{i=1,s=1}^{N,S}$ for US commuting zones.
- $N = 722$ regions; $S = 397$ sectors corresponding to 4-digit Standard Industrial Classification (SIC) codes
- For each simulation draw m , generate $\mathcal{X}_s^m \sim \mathcal{N}(0, 5)$, and estimate

$$Y_i = \delta + \beta \sum_{s=1}^S w_{is} \mathcal{X}_s^m + \epsilon_i.$$

Computer-generated shocks cannot have impacted US labor mkt outcomes $\implies \beta = 0$.

- Consider Eicker-Huber-White (EHW) (*Robust*) and state-clustered standard errors (*Cluster*), commonly used in practice.

Standard errors and rejection rate of $H_0: \beta = 0$ at 5% level, from Adão, Kolesár, and Morales (2019).

	Estimate		Median std. error		Rejection rate	
	Mean	SD	Robust	Cluster	Robust	Cluster
	(1)	(2)	(3)	(4)	(5)	(6)

Panel A: Change in the share of working-age population

Employed	-0.01	2.00	0.73	0.92	48.5%	38.1%
Employed in mfg	-0.01	1.88	0.60	0.76	55.7%	44.8%
Employed in non-mfg	0.00	0.94	0.58	0.67	23.2%	17.6%

Panel B: Change in average log weekly wage

Employed	-0.03	2.66	1.01	1.33	47.3%	34.2%
Employed in mfg	-0.03	2.92	1.68	2.11	26.7%	16.8%
Employed in non-mfg	-0.02	2.64	1.05	1.33	45.4%	33.7%

Correct standard errors: setup

- To help focus on key issues, assume $\beta_{is} = \beta$, and abstract from controls. See Adão, Kolesár, and Morales (2019) for results with controls. Then

$$\hat{\beta} - \beta = \frac{\sum_i X_i Y_i(0)}{\sum_i X_i^2} = \frac{\sum_s \mathcal{X}_s \sum_i w_{is} Y_i(0)}{\sum_{s,t} \mathcal{X}_s \mathcal{X}_t \sum_i w_{is} w_{it}}.$$

- Use design-based framework where we condition on $\mathcal{F}_0 = (Y(0), W)$, and consider repeated sampling of \mathcal{X}_s . This is implicit in placebo. Would want only 5 of 100 researchers find a non-zero treatment effect if each of them comes up with a different set of irrelevant shifters.
- Key substantive assumption: \mathcal{X}_s independent across s (but perhaps not identically distributed). No restriction on correlation structure of objects in \mathcal{F}_0 .

Regularity conditions

- Need $S \rightarrow \infty$ for consistency.
- To ensure low “leverage”, also need sectors to be small in the sense that $\max_s n_s/N \rightarrow 0$, where $n_s = \sum_i w_{is}$ is total share of sector s (and also regularity conditions on $Y_i(0)$)
Intuition: consider “concentrated sectors”, $w_{is} = \mathbb{1}\{s = s(i)\}$, where $s(i)$ is concentration of region i .
Then setup equivalent to OLS in randomized controlled trials with cluster-level treatment, assumption equivalent to the largest cluster asymptotically negligible.
- In contrast, Goldsmith-Pinkham, Sorkin, and Swift (2020) can live with small, finite S .

AKM Standard error formula

- Sandwich form with bread as usual, but meat is different today:

$$\widehat{se}(\hat{\beta}) = \frac{\sqrt{\sum_{s=1}^S X_s^2 \hat{R}_s^2}}{\sum_{i=1}^N X_i^2}, \quad \hat{R}_s = \sum_{i=1}^N w_{is} \hat{\epsilon}_i.$$

With “concentrated sectors”, $w_{is} = \mathbb{1}\{s = s(i)\}$, $\widehat{se}(\hat{\beta})$ equivalent to clustering regions that specialize in same sector—very different from clustering on state!

- In line with rule of thumb that one should “cluster” at level of variation of regressor of interest.
- Formula essentially forms sectoral clusters, using w_{is} to weight residuals.
- When effective # of clusters/sectors small, can impose H_0 when estimating the residual (call it AKMo). AKM vs AKMo analogous to Wald vs LM tests in likelihood models. In IV context, AKMo generalizes Anderson and Rubin (1949) CI to allow structural errors to be correlated. Robust to weak instruments.

Identification

Estimation and Inference

Empirical Applications

- Interested in effect of Chinese exports on US labor mkt outcomes. i : CZ; s : 4-digit SIC code, as in placebo. Use 1990–2007 data (2-period panel). $N = 1,444$ ($722 \text{ CZs} \times 2 \text{ time periods}$).
- Estimate shift-share IV regression with:
 - Outcome: 10-year equivalent change in employment share
 - Endogenous variable: shift-share regressor, Chinese exports to US aggregated using beginning-of-period employment shares
 - Shift-share instrument: Chinese exports to rest of world, aggregated using beginning-of-period employment shares
 - Largest set of controls used in ADH; AKM and AKMo methods cluster at 3-digit SIC level.
- CIs on average 23% (AKM) or 66% (AKMo) wider, significance not affected

	All	Manuf.	Non-Manuf.
	(1)	(2)	(3)

Panel A: IV Regression

$\hat{\beta}$	-0.77	-0.60	-0.18
Robust	[-1.10, -0.45]	[-0.78, -0.41]	[-0.47, 0.12]
Cluster	[-1.12, -0.42]	[-0.79, -0.40]	[-0.45, 0.10]
AKM	[-1.25, -0.30]	[-0.84, -0.35]	[-0.54, 0.18]
AKM0	[-1.69, -0.39]	[-1.01, -0.36]	[-0.84, 0.14]

Panel B: Reduced-Form Regression

$\hat{\beta}$	-0.49	-0.38	-0.11
Robust	[-0.71, -0.27]	[-0.48, -0.28]	[-0.31, 0.08]
Cluster	[-0.64, -0.34]	[-0.45, -0.30]	[-0.27, 0.05]
AKM	[-0.81, -0.17]	[-0.52, -0.23]	[-0.35, 0.12]
AKM0	[-1.24, -0.24]	[-0.67, -0.25]	[-0.64, 0.08]

- Estimate $1/\phi$ in

$$\hat{\omega}_i = \frac{1}{\phi} \hat{L}_i + Z_i \delta + \epsilon_i$$

Z_i : same vector of controls as in ADH. $N = 1,444$ (722 CZs \times 2 time periods);

- To instrument for \hat{L}_i , use
 1. National employment growth, as in Bartik (1991) (actually, a leave-one-out version of it); or
 2. Increase Chinese imports to high-income countries excl. US, as in ADH
- Two approaches give similar point estimates, but while AKM CIs broadly similar to usual ones with Bartik IV, 20% and 250% wider for AKM and AKM0 with ADH IV (AKM and AKMo methods cluster at 3-digit SIC level).
 - National employment growth absorbs most sector-level shocks, not much shift-share structure left in residual

	First-Stage (1)	Reduced-Form (2)	2SLS (3)
Panel A: Bartik IV—Leave-one-out estimator			
$\hat{\beta}$	0.87	0.71	0.82
Robust	[0.68, 1.06]	[0.53, 0.89]	[0.65, 0.98]
Cluster	[0.62, 1.12]	[0.46, 0.96]	[0.60, 1.03]
AKM (<i>leave-one-out</i>)	[0.59, 1.15]	[0.47, 0.94]	[0.61, 1.02]
AKM0 (<i>leave-one-out</i>)	[0.53, 1.15]	[0.42, 0.94]	[0.59, 1.09]
Panel B: ADH IV			
$\hat{\beta}$	−0.72	−0.48	0.67
Robust	[−1.04, −0.39]	[−0.80, −0.16]	[0.36, 0.98]
Cluster	[−0.93, −0.50]	[−0.78, −0.18]	[0.35, 0.99]
AKM	[−1.19, −0.24]	[−0.88, −0.07]	[0.27, 1.07]
AKM0	[−1.83, −0.35]	[−1.27, −0.10]	[0.18, 1.14]

Open questions

I am not clear on the econometrics in the following settings:

- Some papers use IV regressions where the instrument is an interaction term, $X_{it} = X_t W_i$. See, for instance Nunn and Qian (2014) (interact US wheat production with propensity to receive aid from US as instrument for US food aid), or Kearney and Levine (2015) (instrument to “16 and Pregnant” popularity in the area with predicted popularity: W_i MTV rating; X_t : indicator for show running)
 - Some discussion: Jaeger, Joyce, and Kaestner (2020) with reply, Christian and Barrett (2024), Kahn-Lang and Lang (2020).
- In some settings, the number of sectors is as small as $S = 2$ (Nakamura and Steinsson 2014). How to do inference? Some work-in-progress: Majerovitz and Sastry (2023) and Arkhangelsky and Korovkin (2024)
- Number of structural papers use shift-share instruments (e.g. Diamond 2016), many with non-linear structure. Some progress thinking through these in Borusyak and Hull (2023), but many open questions (inference, identification with controls etc).

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