Machine Learning CS 726, Fall 2011 *Christoph Schulze, Michael Whidby, and Kotaro Hara*

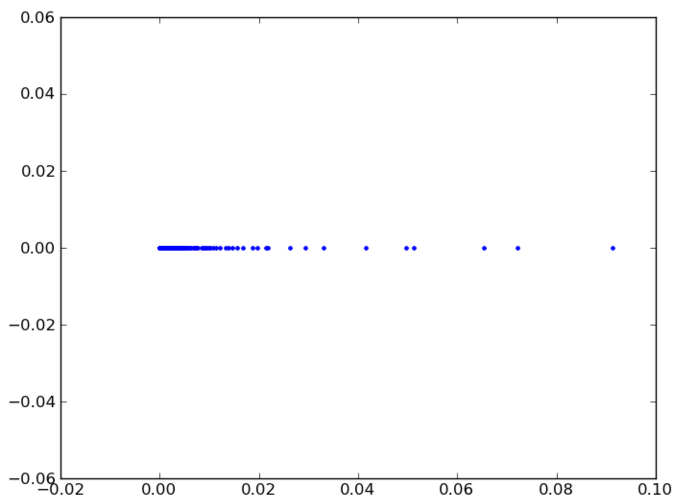
Project 3: Basic classification

WU1

Two eigen vectors both pass through the origin, perpendicular to each other. However, depending on your random data, slopes will vary.

WU2

Figure 1: wu2



To account for 90% variance, we have to include 82 eigenvectors. To account for 95% variance, we have to include 136 eigenvectors.

WU3

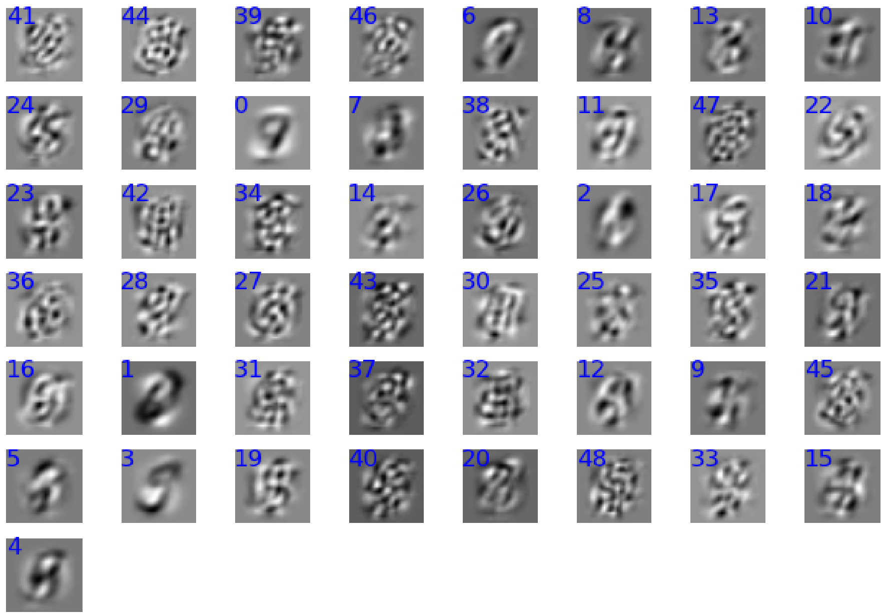
No, they do not look like digits.

WU4

Vanilla PCA will find this data difficult because variance of all directions are similar, which means that eigenvectors can point in any direction.

Large eigenvalues means there is no correlation between any of the axis.

Figure 2: wu3



WU5

PCA did not do what we want it to do :'(In addition to the reasons listed in WU4, vanilla PCA did not do what we wanted to do because the data is not linearly separable.

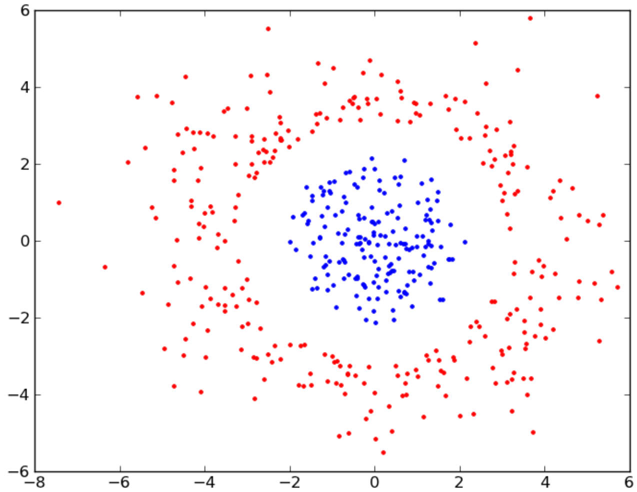
WU6

Eignevalues here is [0.08228617 0.05882765], which is significantly smaller than the eigenvalues for linear: [6.00010675 5.57361851]

WU7

```
linear: evals [ 6.00010675  5.57361851]
poly2: [ 60.36960773  57.61047579]
poly3: [ 2316.48011949  2139.88680252]
rbf0_2: evals [ 0.15469386  0.10178237]
rbf0_5: evals [ 0.1194342   0.06754455]
rbf2: evals [ 0.05293176  0.04460633]
rbf5: evals [ 0.02906865  0.02521906]
```

Figure 3: wu4



WU8

The Observation vectors [1,1,1,2] and [2,1,1,2] are different in only one place and lead to the outputs [0,0,0,1] and [1,1,1,1] that differ in more than one place.

```
(a,b,pi) = datasets.getHMMData()
print hmm.viterbi(array([1,1,1,2]), a, b, pi)    -> [0 0 0 1]
print hmm.viterbi(array([2,1,1,2]), a, b, pi)    -> [1 1 1 1]
```

WU9

It seems to have hit a local maximum at the log probability -2.77259 since there was no increase in the log probabilities over 20 iterations)

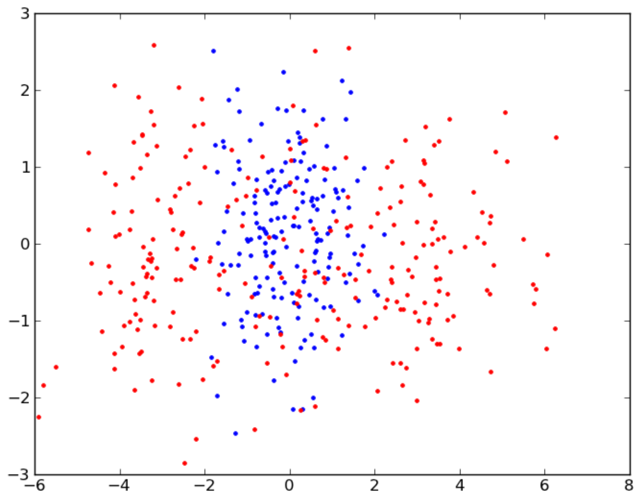
With three states the log error decreases and the model is much more likely to produce the observed emissions.

With four states it does something akin to overfitting where it creates a transition matrix that perfectly fits the observed emissions. The log probability that the observed sequence is generated by this model is 4.16334e-17 which basically means that it is impossible to create any other sequence. It now perfectly emits the observation sequence but gets stuck in state 4 after the third transition and will continue to emit Wet with a very high probability. (95%) So it perfectly fits the observation but looses the ability to generalize.

Results for 2 States:

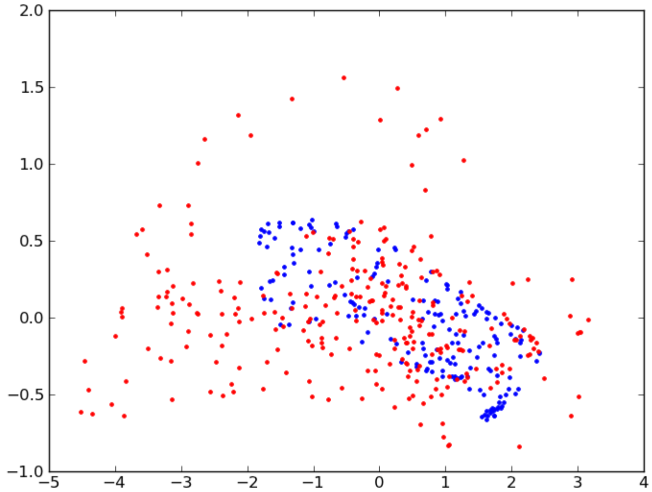
```
iteration 50    ... log probability -2.77259
a_em [[  1.20100122e-15   1.00000000e+00]
 [  1.00000000e+00   0.00000000e+00]]

b_em [[ 0.   0.5  0.5]
 [ 0.5  0.5  0. ]]
```

Figure 4: wu5



```
pi_em [ 0.  1.]


Results for 3 States:

iteration 50    ... log probability -1.21641
a_em [[  0.00000000e+000   1.00000000e+000   7.50925376e-038]
 [  7.62540047e-298   3.33436869e-001   6.66563131e-001]
 [  2.66946976e-008   2.58112825e-005   9.99974162e-001]]

b_em [[  1.00000000e+000   0.00000000e+000   9.10012371e-227]
 [  0.00000000e+000   1.00000000e+000   5.21870817e-018]
 [  0.00000000e+000   3.33229765e-001   6.66770235e-001]]

pi_em [ 1.  0.  0.]


Results for 4 States:

iteration 50    ... log probability 4.16334e-17
a_em [[ 0.          0.          0.          1.        ]
 [ 1.          0.          0.          0.        ]
 [ 0.          1.          0.          0.        ]
 [ 0.00970598  0.02388482  0.00708391  0.95932529]]

b_em [[ 0.  1.  0.]
 [ 0.  1.  0.]
 [ 1.  0.  0.]
 [ 0.  0.  1.]]

pi_em [ 0.  0.  1.  0.]
```

Figure 5: wu6



WU10

It learned how likely certain letters appear and in what pairs of combinations they are likely to appear.

- It learned that _,e,a,o,i and t,r,n,s are the most frequent letters. (This can be calculated by combining the emission probabilities with the likelyhood of beeing in state 0 andor 1.)

-It learned that it is not likely for two blanks to appear after each other, but single blanks are very common. You can see this by looking at the transition probabilities in combination with the emission probabilities. The model emits a _ with aprobability of 37% if it is in state 1 but only with 0.0014% if it is in state 0. Since it is much more likely to transition from state 1 to 0 than to stay in state 0 the combination of two _ is not very likely. This matches with the observed text where double blanks don't appear at all.

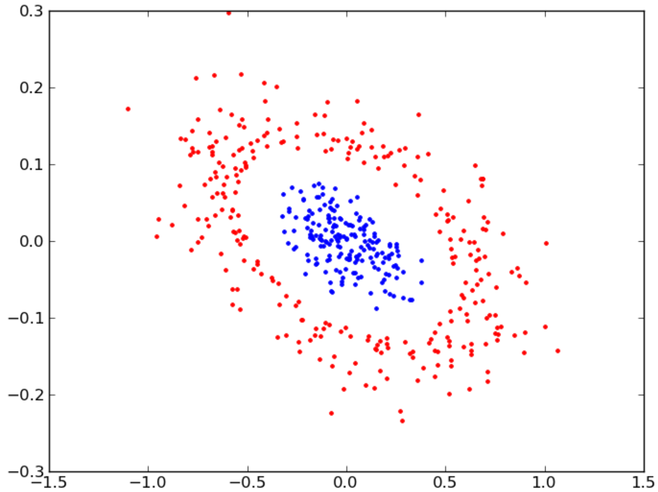
- Similar to the blanks you can see that two vowels in a row are much less likely than other combinations and that words tend to start more often without vowels.

- The start state indicates that it also learned that the text is unlikely to start with a blank.

However with only two states the model cannot really emit the observed sequence anymore since it is much too general for that.

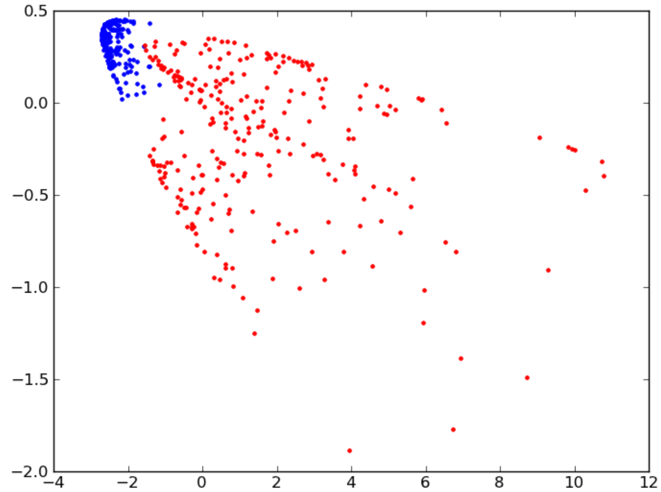
```
iteration 1      ... log probability -11135.1
...
iteration 50     ... log probability -9220.35

Initial state probabilities:
       state(0):       1
       state(1):       3.07405e-18

Transition probabilities:
FROM\TO
       0        1
0       0.270982        0.729018
1       0.73666         0.26334
```

Figure 6: wu7 linear




```
Emission probabilities:
        State 0:
                t   0.146579
                r   0.10483
                n   0.0989395
                s   0.0983503
                h   0.0894233
                d   0.0683162
                l   0.0645881
                c   0.04888
                m   0.0447589
                y   0.0358725
                f   0.035336
                p   0.0335527
                w   0.0312134
                b   0.0265008
                g   0.0253234
                v   0.0206126
                k   0.0172418
                a   0.00509005
                j   0.0017668
                x   0.0017668
                z   0.000588933
                e   0.000413468
                u   4.23781e-05
                _   1.35474e-05
                q   1.17165e-08
                o   1.076e-09
                i   7.3767e-16
        State 1:
                _   0.372007
```

Figure 7: wu7 poly2




```
e  0.228152
a  0.12164
o  0.118451
i  0.102975
u  0.0445997
h  0.00723833
y  0.00183861
k  0.00162122
q  0.00119046
l  0.000196595
t  6.60738e-05
p  1.66102e-05
c  1.47489e-06
s  1.4587e-06
b  1.22527e-06
n  1.16083e-06
g  7.58508e-07
r  3.44083e-07
d  1.86647e-09
x  2.4489e-13
f  8.2358e-17
m  2.43184e-17
w  4.14047e-20
v  2.7307e-25
j  2.9153e-39
z  1.77391e-40
```

Figure 8: wu7 poly3

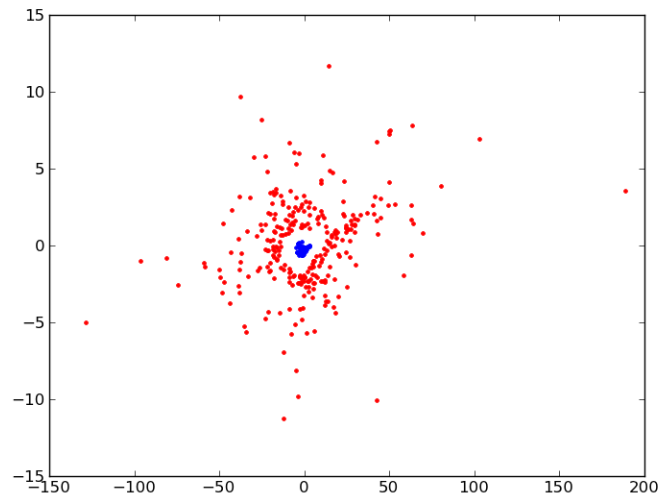


Figure 9: wu7 rbf0.2

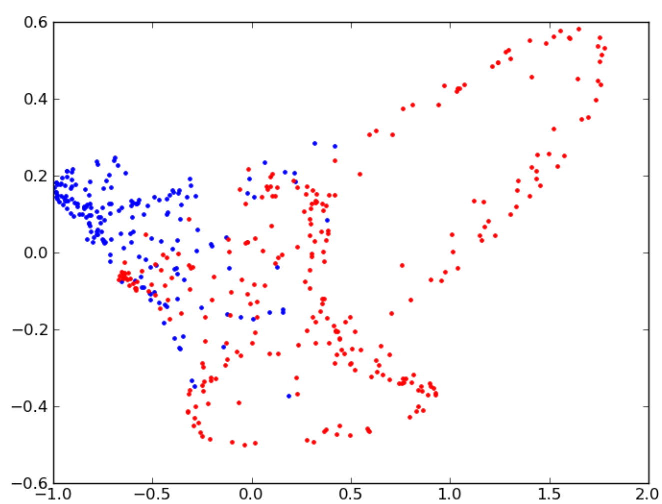


Figure 10: wu7 rbf0.5

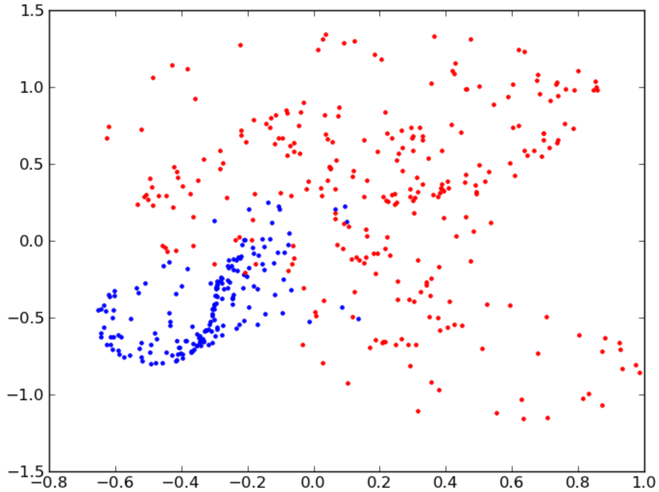


Figure 11: wu7 rbf1

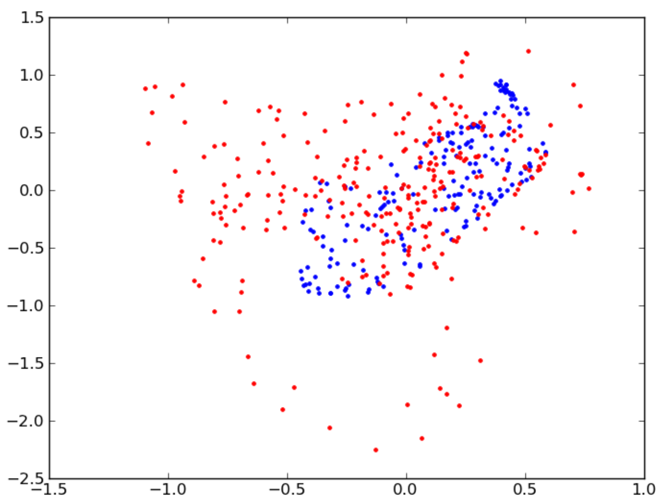


Figure 12: wu7 rbf2

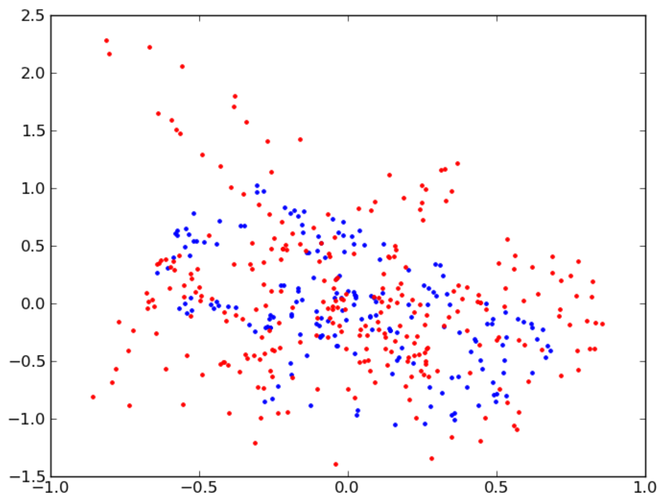


Figure 13: wu7 rbf5