Discrete Random Variables

Binomial - Fixed number of trials, success or failure. Y is number of trials until first success.

$$P(y) = \binom{n}{y} p^y (1-p)^{n-y} \qquad F(y) = \text{pbinom}(y\text{,size,prob}) \qquad E(Y) = np \qquad V(Y) = np(1-p)$$

Geometric - Do trials until success. Y is the number of of the first success trial.

$$P(y) = (1-p)^{y-1} p \qquad F(y) = 1 - (1-p)^y \qquad E(Y) = \frac{1}{p} \qquad V(Y) = \frac{1-p}{p^2}$$

Hypergeometric - N objects. Simple Random Sample of n objects. r are type A, $N-r$ are type B. Y is the number of type A in the n.

$$P(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}} \qquad F(y) = \text{phyper}(y, r, N-r, n) \qquad E(Y) = \frac{1}{p} \qquad V(Y) = \frac{1-p}{p^2}$$

Poisson - Defined for $\lambda > 0$. The limit of the binomial distribution as $n \to \infty$ and $p \to 0$. Think of a box of pollutant particles, slicing it into small slices to check if a particle is in each slice.

$$P(y) = \frac{e^{-\lambda}\lambda^y}{y!} \qquad F(y) = \text{ppois}(y, r, N-r, n) \qquad E(Y) = \lambda \qquad V(Y) = \lambda$$

Chebychev's Inequality: $\quad 1 - \frac{1}{k^2} < P(|Y-\mu| < \sigma k) \qquad \frac{1}{k^2} \geq P(|Y-\mu| \geq \sigma k)$

Continuous Random Variables

Quantiles - The p^{th} quantile ($100p^{th}$ percentile) of $F(Y)$ is ϕ_p such that $F(\phi_p) = p$.

Density Function - $f(y) = \frac{\mathrm{d}}{\mathrm{d}y}F(y) \qquad E(Y) = \int_{-\infty}^{\infty} yf(y)dy \qquad V(Y) = E(Y^2) - E(Y)^2$

Exponential - $F(y) = 1 - e^{-\frac{y}{\beta}}$, $\beta = \frac{1}{\lambda}$

Normal - Density function: $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ is an even function $\big(\phi(-a) = \phi(a)\big)$ (This is the standard normal)
Use qnorm to find quantiles. pnorm integrates the density function to give $F(y)$ Define for constants a and b

$$f(y) = \frac{1}{\sqrt{2\pi}b} e^{-\frac{(y-a)^2}{2b^2}}$$

If a random variable Y has this density, then the random variable $Z = \frac{Y-a}{b}$ is standard normal. Y has normal distribution with mean μ and variance $\sigma^2 > 0$ if

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

Probabilities are found by rescaling.

Gamma Function - $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx \qquad$ For an integer k, $\Gamma(k) = (k-1)!$

$$\text{For } \alpha > 0, \beta > 0 \qquad f(y) = \begin{cases} \frac{y^{\alpha-1}e^{-y/\beta}}{\Gamma(\alpha)\beta^\alpha} & y > 0 \\ 0 & y \leq 0 \end{cases} \qquad E(Y) = \alpha\beta \qquad V(Y) = \alpha\beta^2$$