

# Mat 354

## Homework 3

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1. A box of hard candy contains 19 white candies, 8 red, and 15 blue. If you select 6 pieces of candy randomly from the box, without replacement, give the probability that...

a) Exactly 3 of the candies are white:

$$P = \frac{\binom{19}{3} \binom{15+8}{3}}{\binom{42}{6}} = 0.3271386$$

b) There are two candies of each color:

$$P = \frac{\binom{19}{2} \binom{8}{2} \binom{15}{2}}{\binom{42}{6}} = 0.09583693$$

c) Every one of the sampled candies are either red or white:

$$P = \frac{\binom{19+8}{6}}{\binom{42}{6}} = 0.05642815$$

d) Each color is represented:

$$P = 1 - \left( \frac{\binom{19+8}{6}}{\binom{42}{6}} + \frac{\binom{19+15}{6}}{\binom{42}{6}} + \frac{\binom{15+8}{6}}{\binom{42}{6}} \right) = 0.6679504$$

2. You are meeting two people at the *Gare du Nord* in Paris, one arriving from Amsterdam, the other arriving from Brussels. The trains are scheduled to arrive at the same time. Let  $A$  and  $B$  denote the events that the trains you are meeting are on time. The probability the Amsterdam train is on time is  $P(A) = 0.93$ . The Brussels train is on time with probability 0.89; the probability both trains are late is 0.05. Find the probability that...

a) At least one train is on time:

$$\begin{aligned} P(A \cup B) &= 1 - P(\bar{A} \cap \bar{B}) \\ &= 1 - 0.05 \\ &= 0.9500 \end{aligned}$$

b) Both trains are on time:

$$\begin{aligned} P(A \cap B) &= 1 - P(\bar{A} \cup \bar{B}) \\ &= 1 - (P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B})) \\ &= 1 - (0.07 + 0.11 - 0.05) \\ &= 0.8700 \end{aligned}$$

c) Exactly one train is on time:

$$\begin{aligned} P(A \cup B - A \cap B) &= P(A \cup B) - P(A \cap B) \\ &= 0.95 - 0.87 \\ &= 0.0800 \end{aligned}$$

d) At least one train is late:

$$\begin{aligned} P(A \bar{\cap} B) &= 1 - P(A \cap B) \\ &= 1 - 0.87 \\ &= 0.1300 \end{aligned}$$

e) The Brussels train is on time given that the Amsterdam train is on time:

$$\begin{aligned} P(B|A) &= \frac{P(A \cap B)}{P(A)} \\ &= \frac{0.87}{0.93} \\ &= 0.9355 \end{aligned}$$

f) Given the Brussels train is late, the Amsterdam train is on time:

$$\begin{aligned}
 P(A|\bar{B}) &= \frac{P(A \cap \bar{B})}{P(\bar{B})} \\
 &= \frac{P(A - A \cap B)}{P(\bar{B})} \\
 &= \frac{P(A) - P(A \cap B)}{P(\bar{B})} \\
 &= \frac{0.93 - 0.87}{1 - 0.89} \\
 &= \frac{0.06}{0.11} \\
 &= 0.5455
 \end{aligned}$$

g) Given the Brussels train is late, the Amsterdam train is late:

$$\begin{aligned}
 P(\bar{A}|\bar{B}) &= \frac{\bar{A} \cap \bar{B}}{P(\bar{B})} \\
 &= \frac{P(A \cup B)}{P(\bar{B})} \\
 &= \frac{1 - P(A \cap B)}{P(\bar{B})} \\
 &= \frac{1 - 0.95}{1 - 0.89} \\
 &= \frac{0.05}{0.11} \\
 &= 0.4545
 \end{aligned}$$

h) The Brussels train is on time given that at least one of the trains is on time.

$$\begin{aligned}
 P(B|A \cup B) &= \frac{P(B \cap (A \cup B))}{P(A \cup B)} \\
 &= \frac{P(B)}{P(A \cup B)} \\
 &= \frac{0.89}{0.95} \\
 &= 0.9368
 \end{aligned}$$

- i) The Brussels train is on time given that exactly one of the trains is on time.

$$\begin{aligned}
 P(B|(A \cup B - A \cap B)) &= \frac{P(B \cap (A \cup B - A \cap B))}{P(A \cup B - A \cap B)} \\
 &= \frac{P(B - A \cap B)}{P(A \cup B - A \cap B)} \\
 &= \frac{P(B) - P(A \cap B)}{P(A \cup B - A \cap B)} \\
 &= \frac{0.89 - 0.87}{0.08} \\
 &= 0.2500
 \end{aligned}$$

- j) The Brussels train is late given that at least one of the trains is late.

$$\begin{aligned}
 P(\bar{B}|\bar{A} \cup \bar{B}) &= \frac{P(\bar{B} \cap (\bar{A} \cup \bar{B}))}{P(\bar{A} \cup \bar{B})} \\
 &= \frac{P(\bar{B})}{P(\bar{A} \cup \bar{B})} \\
 &= \frac{0.11}{0.13} \\
 &= 0.8462
 \end{aligned}$$

- k)  $A$  and  $B$  are independent events. Explain.

$$P(A \text{ and } B \text{ are independent events}) = 0$$

Since  $P(A \cap B) = 0.87 \neq 0.8277 = P(A)P(B)$  we know that  $A$  and  $B$  are not independent.

### 3. Some combinations:

- a) Compute the following pairs of values:

- i)  $\binom{7}{2}$  and  $\binom{7}{5}$

$$\binom{7}{2} = \frac{7!}{(7-2)!2!} = 21 = \frac{7!}{(7-5)!5!} = \binom{7}{5}$$

- ii)  $\binom{13}{5}$  and  $\binom{13}{8}$

$$\binom{13}{5} = \frac{13!}{(13-5)!5!} = 1287 = \frac{13!}{(13-8)!8!} = \binom{13}{8}$$

b) What value  $j$  not equal to 123456789 makes this true?

$$\binom{(9876543210)}{(123456789)} = \binom{(9876543210)}{j}$$

The value  $j = 9876543210 - 123456789 = 9753086421$

c)  $k$  is a positive integer. What integer other than  $k - 1$  correctly fills in the first blank? Simplify the expression to correctly fill in the second blank.

$$\binom{k}{k-1} = \binom{k}{\quad} = \underline{\hspace{1cm}}$$

The integer 1 will correctly fill in the first blank. The second blank is filled in by  $k$ , that is

$$\binom{k}{k-1} = \binom{k}{1} = k$$

d) Determine exact values for the following:

i)  $\sum_{i=0}^3 \binom{3}{i}$

ii)  $\sum_{i=0}^4 \binom{4}{i}$

iii)  $\sum_{i=0}^8 \binom{8}{i}$

$$\text{iv)} \quad \sum_{i=0}^{1000} \binom{1000}{i}$$

$$\text{v)} \quad \sum_{i=0}^8 \binom{i+1}{i}$$