Discrete Random Variables

Binomial - Fixed number of trials, success or failure. Y is number of trials until first success.

$$P(y) = \binom{n}{y} p^y (1-p)^{n-y} \qquad F(y) = \text{pbinom}(y\text{,size,prob}) \qquad E(Y) = np \qquad V(Y) = np(1-p)$$

Geometric - Do trials until success. Y is the number of of the first success trial.

$$P(y) = (1-p)^{y-1}p \qquad F(y) = 1-(1-p)^y \qquad E(Y) = \frac{1}{p} \qquad V(Y) = \frac{1-p}{p^2}$$

Hypergeometric - N objects. Simple Random Sample of n objects. r are type A, $N-r$ are type B. Y is the number of type A in the n.

$$P(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}} \qquad F(y) = \text{phyper}(y, r, N-r, n) \qquad E(Y) = \frac{1}{p} \qquad V(Y) = \frac{1-p}{p^2}$$

Poisson - Defined for $\lambda > 0$. The limit of the binomial distribution as $n \to \infty$ and $p \to 0$. Think of a box of pollutant particles, slicing it into small slices to check if a particle is in each slice.

$$P(y) = \frac{e^{-\lambda}\lambda^y}{y!} \qquad F(y) = \text{ppois}(y, r, N-r, n) \qquad E(Y) = \lambda \qquad V(Y) = \lambda$$

Chebychev's Inequality: $\quad 1 - \frac{1}{k^2} < P(|Y-\mu| < \sigma k) \qquad \frac{1}{k^2} \geq P(|Y-\mu| \geq \sigma k)$

Continuous Random Variables

Quantiles - The p^{th} quantile ($100p^{th}$ percentile) of $F(Y)$ is ϕ_p such that $F(\phi_p) = p$.

Density Function - $f(y) = \frac{\mathrm{d}}{\mathrm{d}y}F(y) \qquad E(Y) = \int_{-\infty}^{\infty} yf(y)dy \qquad V(Y) = E(Y^2) - E(Y)^2$

Exponential - $F(y) = 1 - e^{-\frac{y}{\beta}}$, $\beta = \frac{1}{\lambda}$

Normal - Density function: $\phi(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$ is an even function $\big(\phi(-a) = \phi(a)\big)$ (This is the standard normal)
Use qnorm to find quantiles. pnorm integrates the density function to give $F(y)$ Define for constants a and b

$$f(y) = \frac{1}{\sqrt{2\pi}b}e^{-\frac{(y-a)^2}{2b^2}}$$

If a random variable Y has this density, then the random variable $Z = \frac{Y-a}{b}$ is standard normal. Y has normal distribution with mean μ and variance $\sigma^2 > 0$ if

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

Probabilities are found by rescaling.

Gamma Function - $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx \qquad$ For an integer k, $\Gamma(k) = (k-1)!$

$$\text{For } \alpha > 0, \beta > 0 \qquad f(y) = \begin{cases} \frac{y^{\alpha-1}e^{-y/\beta}}{\Gamma(\alpha)\beta^\alpha} & y > 0 \\ 0 & y \leq 0 \end{cases} \qquad E(Y) = \alpha\beta \qquad V(Y) = \alpha\beta^2$$

Lack of Memory Property - The following holds for continous exponential and discrete geometric distributions. Let $a > 0, b > 0$. Then

$$P(Y > a + b | Y > a) = P(Y > a)$$

Join Probability Functions

Expectation of a function -

$$E\left(g\left(y_1, y_2\right)\right) = \int\int_{\mathbb{R}^2} g\left(y_1, y_2\right) f\left(y_1, y_2\right) dy_1 dy_2$$

Covariance - The covariance of two random variables Y_1, Y_2 is

$$\text{cov}\left(Y_1, Y_2\right) = E\left(Y_1 Y_2\right) - E(Y_1)E(Y_2)$$

Correlation - The Correlation between two random variables Y_1, Y_2 ranges from -1 to 1, and is given by

$$\rho = \frac{\text{cov}\left(Y_1, Y_2\right)}{\sqrt{V(Y_1)V(Y_2)}}$$

If two random variables Y_1 and Y_2 are independent, then their covariance is 0, and for functions g and h it is true that

$$E\left[g(Y_1)h(Y_2)\right] = E\left[g(Y_1)\right] E\left[h(Y_2)\right]$$

Other facts

$\text{cov}\left(a_1 Y_1 + b_1, a_2 Y_2 + b_2\right) = a_1 a_2 \text{cov}\left(Y_1, Y_2\right)$

$\text{cov}\left(Y_1 + Y_2, Y_3 + Y_4\right) = \text{cov}\left(Y_1, Y_3\right) + \text{cov}\left(Y_1, Y_4\right) + \text{cov}\left(Y_2, Y_3\right) + \text{cov}\left(Y_2, Y_4\right)$

$V\left(Y_1 + Y_2\right) = V(Y_1) + V(Y_2) + \text{cov}\left(Y_1, Y_2\right)$

Five Number Summary: $\{\min, Q_1, Q_2, Q_3, \max\}$

Outliers are identified as values more than 1.5 IQR (Inter-Quartile Range) outside Q_1 and Q_3.

Sample Variance: $\delta^2 = \frac{\sum_{i=1}^{n}(9y_i - \overline{y})^2}{n-1}$

Law of Total Probability (Stratified Sampling Theorem): Let $B_1, B_2, ...B_k$ partition S, and let A be some event in S. Then

$$P(A) = \sum_{i=1}^{k} P(B_i)P(A|B_i)$$

Bayes' Theorem: For a partition $B_1, B_2, ...B_k$ of S, an event A and the particular event B_j out of the partition,

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$$

$E(g_1(Y) + g_2(Y)) = E(g_1(Y)) + E(g_2(Y))$

$E(aY + b) = aE(Y) + b$

$V(aY + b) = a^2 V(Y)$

$SD(aY + b) = |a| SD(Y)$

If a random variable has a moment generating function , $m(t)$, in a neighborhood of $t = 0$ with $m(t) < \infty$ for all $|t| < b$ for some $b > 0$, then

$$E(Y^k) = m^{(k)}(0) \qquad\qquad k^{th} \text{ derivative}$$