# Mat 354

**Homework 3**

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Due September 16, 2015

1. A box of hard candy contains 19 white candies, 8 red, and 15 blue. If you select 6 pieces of candy randomly from the box, without replacement, give the probability that...

   a) Exactly 3 of the candies are white:

   $$P = \frac{\binom{19}{3}\binom{15+8}{3}}{\binom{42}{6}} = 0.3271386$$

   b) There are two candies of each color:

   $$P = \frac{\binom{19}{2}\binom{8}{2}\binom{15}{2}}{\binom{42}{6}} = 0.09583693$$

   c) Every one of the sampled candies are either red or white:

   $$P = \frac{\binom{19+8}{6}}{\binom{42}{6}} = 0.05642815$$

   d) Each color is represented:

   $$P = 1 - \left(\frac{\binom{19+8}{6}}{\binom{42}{6}} + \frac{\binom{19+15}{6}}{\binom{42}{6}} + \frac{\binom{15+8}{6}}{\binom{42}{6}}\right) = 0.6679504$$

2. You are meeting two people at the *Gare du Nord* in Paris, one arriving from Amsterdam, the other arriving from Brussels. The trains are scheduled to arrive at the same time. Let $A$ and $B$ denote the events that the trains you are meeting are on time. The probability the Amsterdam train is on time is $P(A) = 0.93$. The Brussels train is on time with probability 0.89; the probability both trains are late is 0.05. Find the probability that...

   a) At least one train is on time:

$$\begin{aligned} P(A \cup B) &= 1 - P(\bar{A} \cap \bar{B}) \\ &= 1 - 0.05 \\ &= 0.9500 \end{aligned}$$

   b) Both trains are on time:

$$\begin{aligned} P(A \cap B) &= 1 - P(\bar{A} \cup \bar{B}) \\ &= 1 - \big(P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B})\big) \\ &= 1 - (0.07 + 0.11 - 0.05) \\ &= 0.8700 \end{aligned}$$

   c) Exactly one train is on time:

$$\begin{aligned} P(A \cup B - A \cap B) &= P(A \cup B) - P(A \cap B) \\ &= 0.95 - 0.87 \\ &= 0.0800 \end{aligned}$$

   d) At least one train is late:

$$\begin{aligned} P(A \bar{\cap} B) &= 1 - P(A \cap B) \\ &= 1 - 0.87 \\ &= 0.1300 \end{aligned}$$

   e) The Brussels train is on time given that the Amsterdam train is on time:

$$\begin{aligned} P(B|A) &= \frac{P(A \cap B)}{P(A)} \\ &= \frac{0.87}{0.93} \\ &= 0.9355 \end{aligned}$$

f) Given the Brussels train is late, the Amsterdam train is on time:

$$\begin{aligned}
P(A|\bar{B}) &= \frac{P(A \cap \bar{B})}{P(\bar{B})} \\
&= \frac{P(A - A \cap B)}{P(\bar{B})} \\
&= \frac{P(A) - P(A \cap B)}{P(\bar{B})} \\
&= \frac{0.93 - 0.87}{1 - 0.89} \\
&= \frac{0.06}{0.11} \\
&= 0.5455
\end{aligned}$$

g) Given the Brussels train is late, the Amsterdam train is late:

$$\begin{aligned}
P(\bar{A}|\bar{B}) &= \frac{\bar{A} \cap \bar{B}}{P(\bar{B})} \\
&= \frac{P(A \bar{\cup} B)}{P(\bar{B})} \\
&= \frac{1 - P(A \cup B)}{P(\bar{B})} \\
&= \frac{1 - 0.95}{1 - 0.89} \\
&= \frac{0.05}{0.11} \\
&= 0.4545
\end{aligned}$$

h) The Brussels train is on time given that at least one of the trains is on time.

$$\begin{aligned}
P(B|A \cup B) &= \frac{P(B \cap (A \cup B))}{P(A \cup B)} \\
&= \frac{P(B)}{P(A \cup B)} \\
&= \frac{0.89}{0.95} \\
&= 0.9368
\end{aligned}$$

i) The Brussels train is on time given that exactly one of the trains is on time.

$$\begin{aligned}
P(B|(A \cup B - A \cap B)) &= \frac{P(B \cap (A \cup B - A \cap B)}{P(A \cup B - A \cap B)} \\
&= \frac{P(B - A \cap B)}{P(A \cup B - A \cap B)} \\
&= \frac{P(B) - P(A \cap B)}{P(A \cup B - A \cap B)} \\
&= \frac{0.89 - 0.87}{0.08} \\
&= 0.2500
\end{aligned}$$

j) The Brussels train is late given that at least one of the trains is late.

$$\begin{aligned}
P(\bar{B}|\bar{A} \cup \bar{B}) &= \frac{P(\bar{B} \cap (\bar{A} \cup \bar{B})}{P(\bar{A} \cup \bar{B})} \\
&= \frac{P(\bar{B})}{P(\bar{A} \cup \bar{B})} \\
&= \frac{0.11}{0.13} \\
&= 0.8462
\end{aligned}$$

k) $A$ and $B$ are independent events. Explain.

$$P(\text{A and B are independent events}) = 0$$

Since $P(A \cap B) = 0.87 \neq 0.8277 = P(A)P(B)$ we know that $A$ and $B$ are not independent.

3. Some combinations:

a) Compute the following pairs of values:

i) $\binom{7}{2}$ and $\binom{7}{5}$

$$\binom{7}{2} = \frac{7!}{(7-2)!2!} = 21 = \frac{7!}{(7-5)!5!} = \binom{7}{5}$$

ii) $\binom{13}{5}$ and $\binom{13}{8}$

$$\binom{13}{5} = \frac{13!}{(13-5)!5!} = 1287 = \frac{13!}{(13-8)!8!} = \binom{13}{8}$$

b) What value $j$ not equal to 123456789 makes this true?

$$\binom{9876543210}{123456789} = \binom{9876543210}{j}$$

The value $j = 9876543210 - 123456789 = 9753086421$

c) $k$ is a positive integer. What integer other than $k-1$ correctly fills in the first blank? Simplify the expression to correctly fill in the second blank.

$$\binom{k}{k-1} = \binom{k}{\_\_} = \_\_\_\_\_$$

The integer 1 will correctly fill in the first blank. The second blank is filled in by $k$, that is

$$\binom{k}{k-1} = \binom{k}{1} = k$$

The previous three parts of this problem all fall as a result that the task of choosing $k$ things out of a group of size $n$ is the same as choosing $n-k$ things to leave out, and taking the rest as your choice.

d) Determine exact values for the following:

i) $\sum_{i=0}^{3} \binom{3}{i} = 8$

ii) $\sum_{i=0}^{4} \binom{4}{i} = 16$

iii) $\sum_{i=0}^{8} \binom{8}{i} = 256$

iv) $\sum_{i=0}^{1000} \binom{1000}{i} = 2^{1000}$

v) $\sum_{i=0}^{8} \binom{i+1}{i} = \sum_{i=0}^{8} \binom{i+1}{1} = 9!$