Quantum Computing

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1 The Pauli matrices

$$\sigma_0 = I$$

$$\sigma_1 = \sigma_x = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_2 = \sigma_y = Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma_3 = \sigma_z = Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

2 Adjoints and Hermitian operators

Definition 2.1 (Hermitian)**.** An operator A is Hermitian if $A = A^\dagger$.

Theorem 2.2. Two eigenvectors of a Hermitian operator with different eigenvalues are orthogonal.

Definition 2.3. A matrix is normal if $AA^\dagger = A^\dagger A$.

Theorem 2.4. A normal matrix is Hermitian iff it has real eigenvalues.

Definition 2.5 (Unitary)**.** A matrix U is unitary if $U^\dagger U = I$.

Definition 2.6 (Positive and Positive definite)**.** A positive operator A is defined to be an operator such that for any vector $|v\rangle$, $(|v\rangle, A|v\rangle)$ is a real, non-negative number. If $(|v\rangle, A|v\rangle) > 0$ then A is positive definite.

3 The polar and singular value decompositions

Theorem 3.1 (Polar decompositions)**.** Let A be a linear operator on a vector space V. Then there exists unitary U and positive operators J and K such that

$$A = UJ = KU,$$

where the unique positive operators J and K satisfying these equations are $J = \sqrt{A^\dagger A}$ and $K = \sqrt{AA^\dagger}$. Moreover, if A is invertible then U is unique.

Theorem 3.2 (Singular value decomposition)**.** Let A be a square matrix. Then there exist unitary matrices U and V, and a diagonal matrix D with non-negative entries such that

$$A = UDV.$$

The diagonal elements of D are known as the *singular values of A*.