Glassy Dynamics in Physics & Beyond
(Landscape & Glassy Dynamics)

- Dynamical Mean-Field Theory of Glassy dynamics
  Dynamical mean-field equation via the dynamic cavity method

- How & Why dynamics slow down in rough high-dimensional landscape?
  Analysis of the Langevin dynamics for the p-spin spherical model

- Rough-landscape with a signal ("crystal hunting")
  Dynamics for the spiked-tensor model and generalization

Rough Landscape and Generalized Spin-Glasses

$$\sum_{\langle i,j\rangle} J_{ij} s_i s_j \xrightarrow[\text{random interactions}]{\text{More complex}} \sum_{\langle i_1,\cdots,i_p\rangle} J_{i_1,\cdots,i_p} s_{i_1} s_{i_2} \cdots s_{i_p}$$

$$\overline{J^2_{i_1,\cdots,i_p}} = \frac{p!}{2N^{p-1}}$$

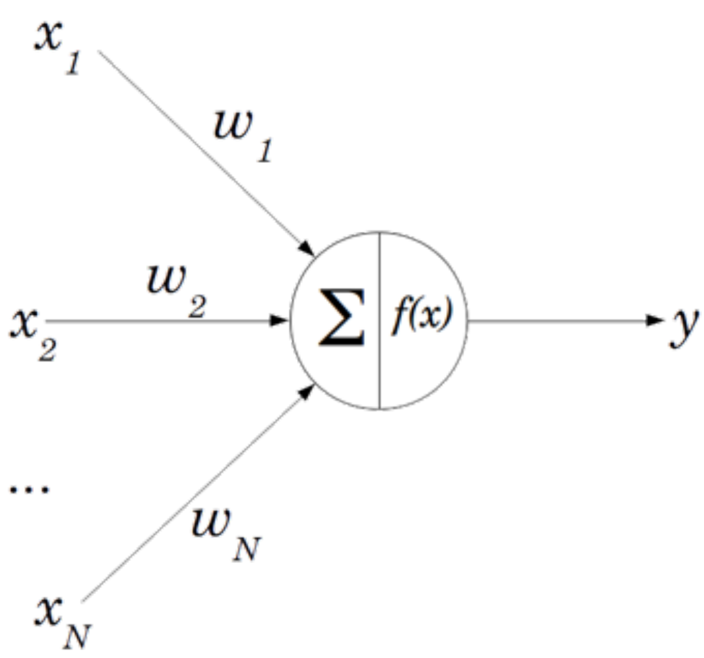
$$\frac{ds_i(t)}{dt} = -\frac{\partial V}{\partial s_i} + \frac{1}{(p-1)!} \sum_{i_2 \neq \cdots i_p} J_{i,i_2,\cdots,i_p} s_{i_2} \cdots s_{i_p} + \xi_i(t)$$

DMFT:

$$\frac{ds(t)}{dt} = -\frac{\partial V}{\partial s} + \frac{p(p-1)}{2} \int_0^t R(t,t') C^{p-2}(t,t') s(t') dt' + \xi(t)$$

$$\langle \xi(t) \xi(t') \rangle = 2T\delta(t-t') + \frac{p}{2} C^{p-1}(t,t')$$

Perceptron



$$E(\{w_i\}) = \sum_{\mu=1}^{M} f(x_\mu \cdot w)$$

$$w^2 = N \qquad M = \alpha N$$

- DMFT for Langevin dynamics
Agoritsas, Biroli, Urbani, Zamponi 2018

- SGD for online learning
Mace, Coolen '97

- DMFT for Glauber dynamics
Horner '92

Non-Conservative Dynamics
(No Landscape)

Mean-field models of recurrent neural networks, large-interacting eco-systems and sheared glasses

$$\frac{dh_i}{dt} = -h_i + \sum_{l \neq i}^{N} J_{il} S_l = -h_i + \sum_{l \neq i}^{N} J_{il} \tanh(g h_l)$$

$$\overline{J_{ij}^2} = \frac{1}{N} \qquad \overline{J_{ji} J_{ji}} = 0$$

DMFT: $$\frac{dh}{dt} = -h + \xi(t) \qquad \langle \xi(t) \xi(t') \rangle = \langle \tanh(g h(t)) \tanh(g h(t')) \rangle$$

Transition to chaos by increasing g
(Crisanti, Sompolinsky '87,...; Kadmon, Sompolinsky PRX 2015)

How & Why Dynamics slow down in rough high -D Landscapes?

P-spin spherical model

$$E = - \sum_{\langle i_1, \cdots, i_p \rangle} J_{i_1, \cdots, i_p} s_{i_1} s_{i_2} \cdots s_{i_p} \qquad \sum_{i=1}^{N} s_i^2 = N$$

$$\frac{ds_i(t)}{dt} = - \frac{\partial E}{\partial s_i} + \xi_i(t)$$

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$$\sum_{i=1}^{N} s_i^2 = N$$

$$\frac{ds_i(t)}{dt} = -\frac{\partial E}{\partial s_i} + \xi_i(t) - \lambda(t) s_i$$

$$\lambda(t) = -p\frac{E(t)}{N} + T$$

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$$\lambda(t) = -p\frac{E(t)}{N} + T$$

$$\frac{ds_i(t)}{dt} = -\lambda(t) s_i + \frac{1}{(p-1)!} \sum_{i_2 \neq \cdots i_p} J_{i,i_2,\cdots,i_p} s_{i_2} \cdots s_{i_p} + \xi_i(t)$$

How & Why Dynamics slow down in rough high -D Landscapes?

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DMFT:

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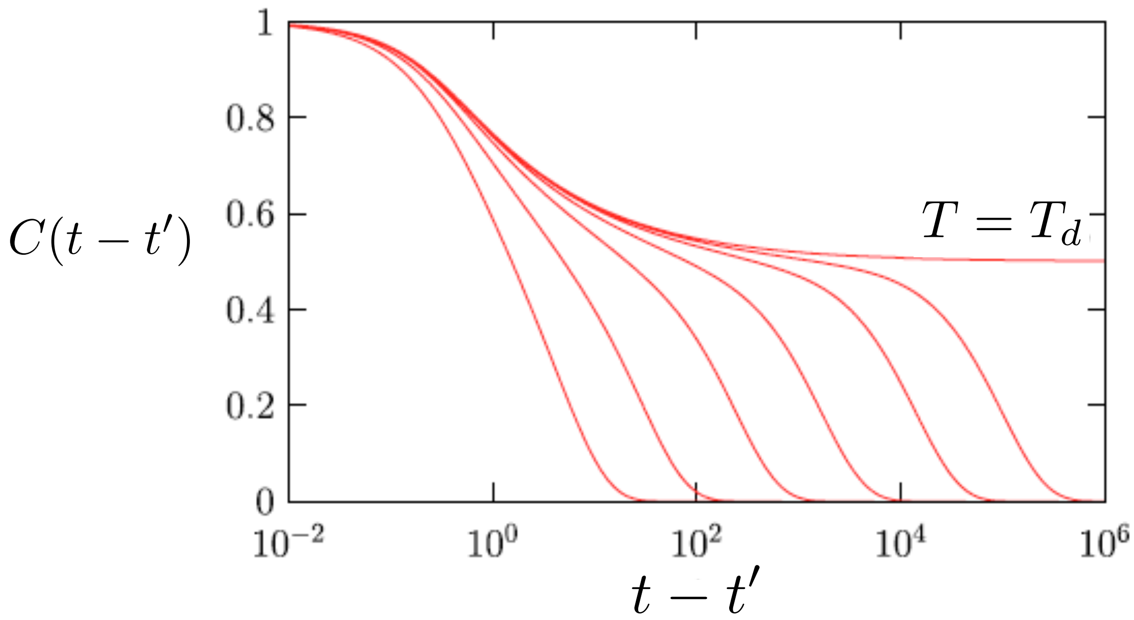
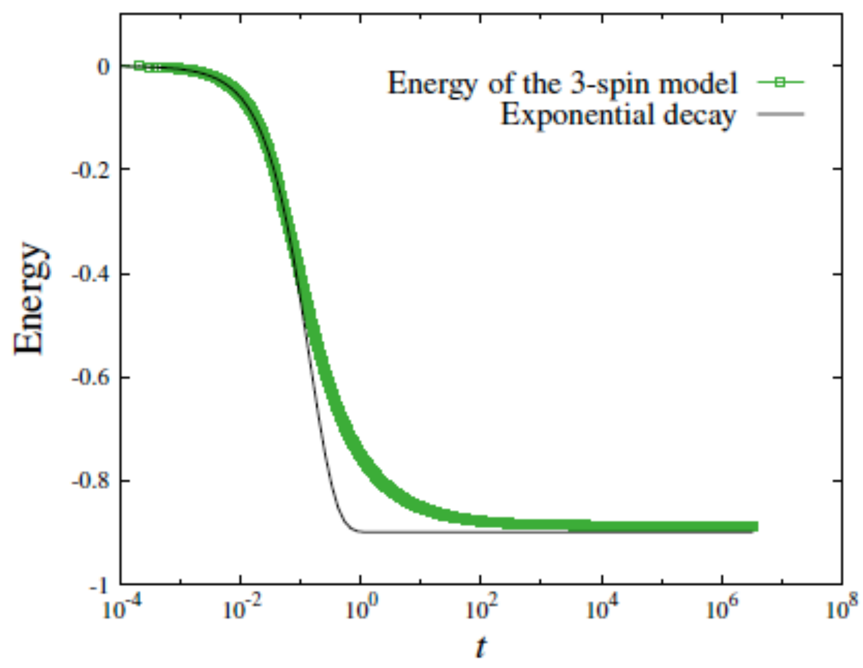
How & Why Dynamics slow down in rough high -D Landscapes?

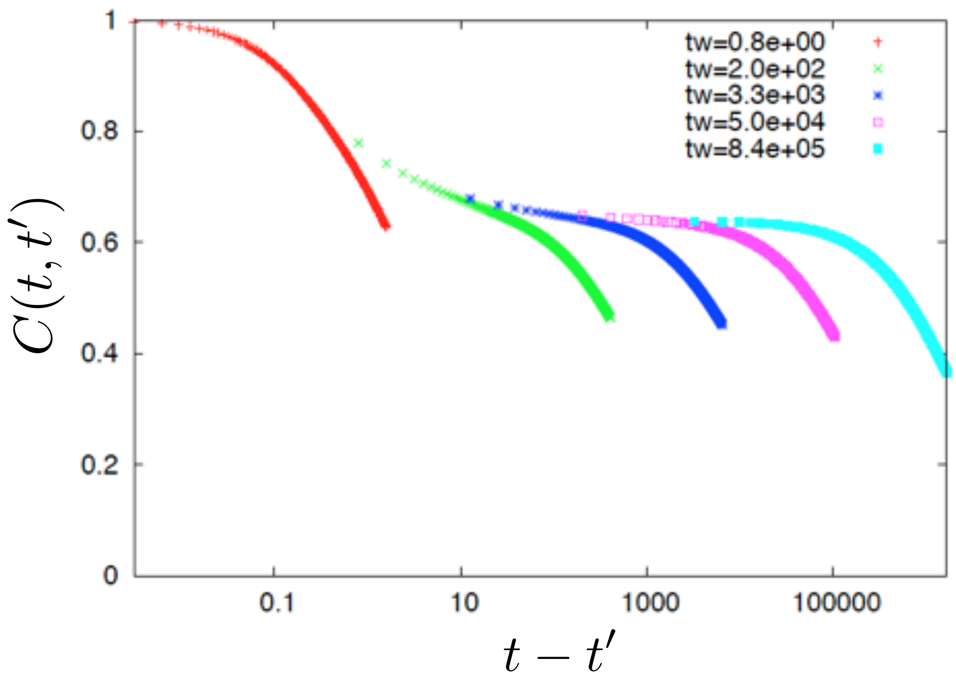
P-spin spherical model: EQUILIBRIUM



How & Why Dynamics slow down in rough high -D Landscapes?

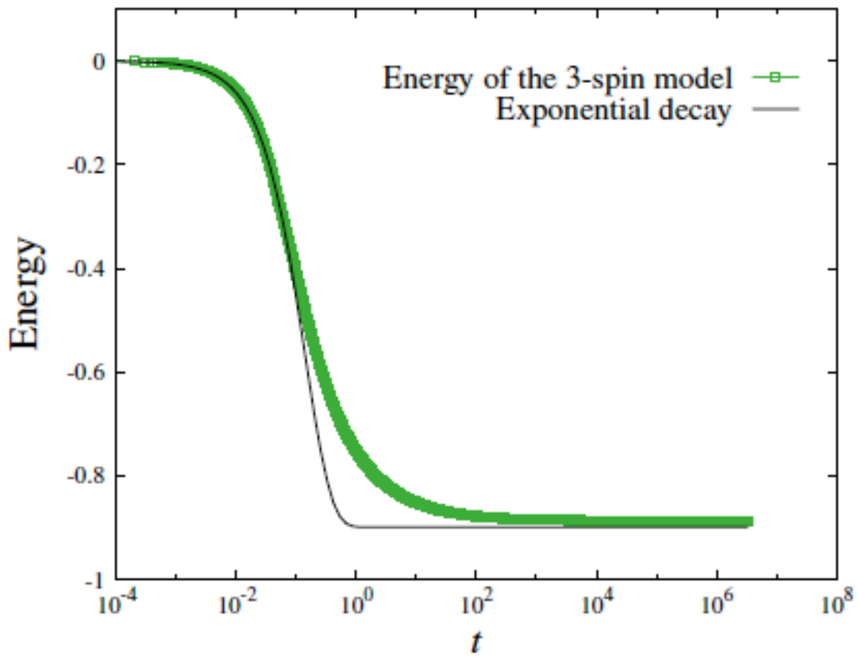
P-spin spherical model: AGING

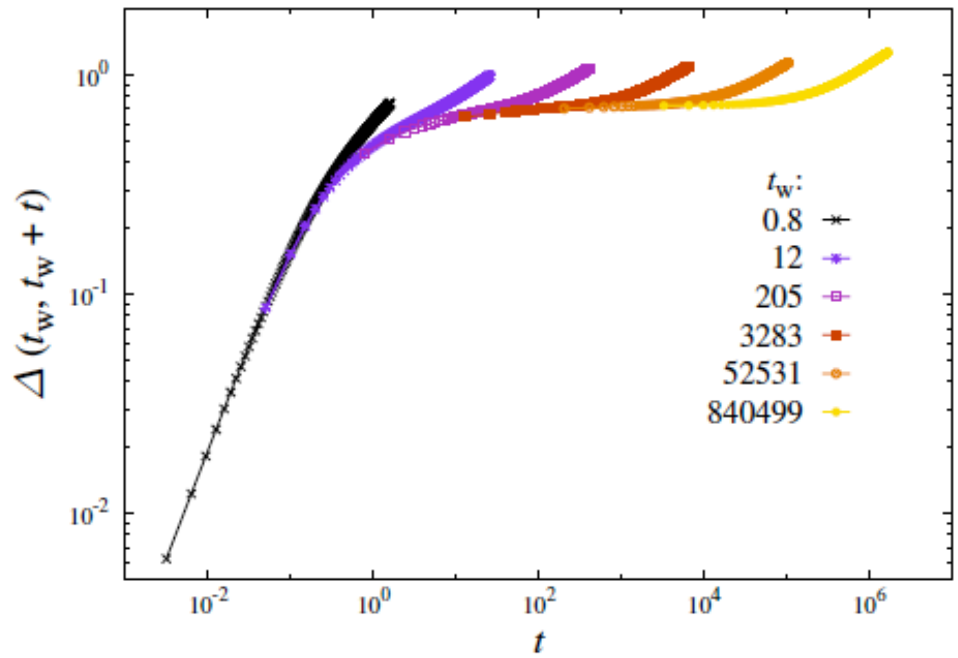




How & Why Dynamics slow down in rough high -D Landscapes?

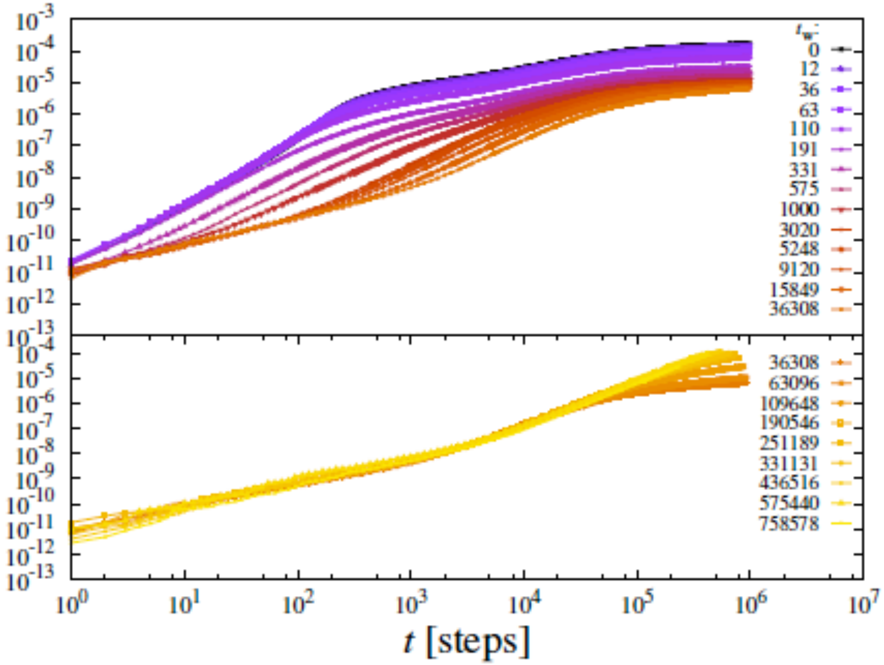
P-spin spherical model: AGING



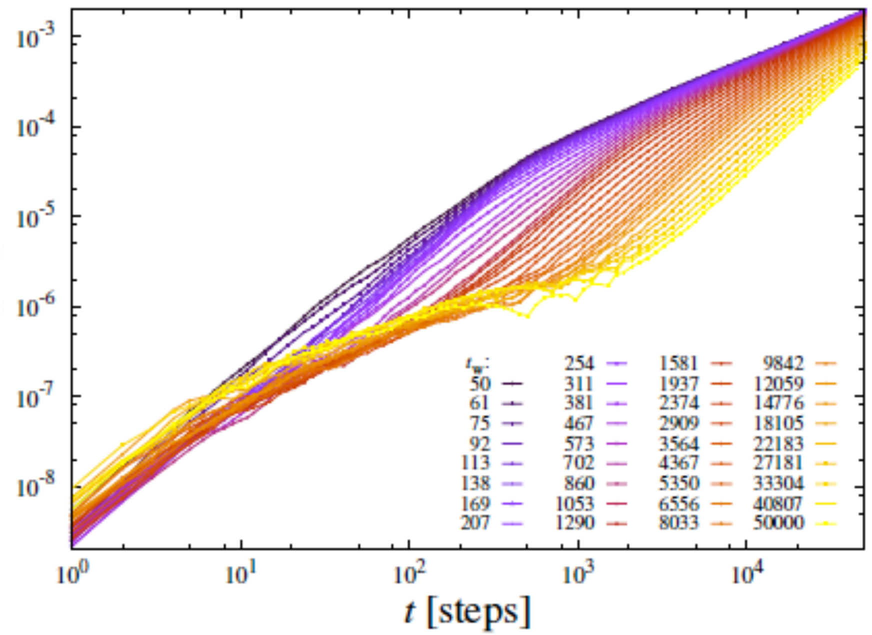


$$\Delta(t,t') = \frac{1}{N}\sum_i \langle (s_i(t) - s_i(t'))^2 \rangle = 2 - 2C(t,t')$$

Correlation-Functions in DNNs



(b) Fully Connected on MNIST, $B = 128$, $\alpha = 0.01$.



(b) Mean square displacement of the under-parametrized model.

Over-parametrized

Under-parametrized

M. Baity-Jesi, L. Sagun, M. Geiger, S. Spigler, G. Ben Arous, C. Cammarota, Y. LeCun, M. Wyart, G. Biroli (2018)
arXiv1803.06969 & ICML 2018

Explanation in terms of changes in the landscape: L. Sagun
More on the transition: M. Wyart